In the year 2003 alone, the *American Economic Review* published 14 articles reporting regression analyses involving individual or family income variables, and the May *Papers and Proceedings* issue contained almost that many again. In some cases, the income variables were dependent variables; in others, they were regressors used to explain dependent variables ranging from child health in the United States to borrowing and lending behavior in Ghana. Without exception, the measured income variables were short-term values even though, in most cases, it appeared that the relevant economic construct was a longer-term value.

Many influential economic studies have recognized that the use of current income as a proxy for long-run income can generate important errors-in-variables biases. Perhaps the most famous examples are the seminal studies by Franco Modigliani and Richard Brumberg (1954) and Milton Friedman (1957), which analyzed the properties of consumption functions estimated with current rather than permanent income variables as the regressors. Another instance is the literature (e.g., Lee A. Lillard, 1977) suggesting that inequality as measured in cross sections of annual earnings overstates the inequality in lifetime earnings. A recent offshoot of that literature—exemplified by Peter Gottschalk and Robert Moffitt (1994), Haider (2001), and Michael Baker and Solon (2003)—has attempted to partition the upward trend in earnings inequality into persistent and transitory components. Still another recent example is the burgeoning literature on intergenerational income mobility (surveyed in Solon, 1999), which has found that the association between parents’ and children’s long-run income is susceptible to dramatic underestimation when current income variables are used as proxies for long-run income.

Nevertheless, applied researchers often ignore the distinction between current and long-run income. Most researchers who do attend to the issue assume the textbook errors-in-variables model and impute the noise-to-signal ratio by estimating restrictive models of income dynamics on the basis of short panels of income data spanning only a segment of the life cycle. In this paper, we reconsider the appropriateness of the textbook errors-in-variables model and we find that it does not accurately characterize current earnings as a proxy for lifetime earnings. Thanks to a remarkable new dataset, we are able to generate detailed evidence on the association between current and lifetime earnings, including its evolution over the life cycle, without having to resort to an arbitrary specification of the earnings dynamics process.

Our empirical analysis uses the 1951–1991 Social Security Administration (SSA) earnings histories of the members of the Health and Retirement Study (HRS) sample. Despite some limitations discussed in Section II, these data provide nearly career-long earnings histories, which are based on relatively accurate administrative data and pertain to a broadly representative national sample. In Section I, we develop simple models to illustrate some important aspects of the association between current and...
lifetime earnings and to demonstrate the implications for errors-in-variables biases in applied econometric research. In Section II, we describe the dataset and our econometric methods. In Section III, we present our evidence on the connections between annual and lifetime earnings. Section IV summarizes our findings and illustrates their usefulness with a brief application to intergenerational earnings mobility.

I. Models

Following Friedman (1957), most analyses of current income variables as proxies for unobserved lifetime income variables have adopted the textbook errors-in-variables model:

\[ y_{it} = y_i + v_{it}, \]

where \( y_{it} \) is a current income variable such as log annual earnings, observed for individual \( i \) in period \( t \); \( y_i \) is a long-run income variable such as the log of the present discounted value of lifetime earnings; and \( v_{it} \), the measurement error in \( y_{it} \) as a proxy for \( y_i \), is assumed to be uncorrelated with \( y_i \) (and each of its determinants). Often, the current income variable \( y_{it} \) has been adjusted for stage-of-life cycle with a regression on a polynomial in age or experience or by subtracting out the cohort mean. Throughout this section, we will suppress intercepts by expressing all variables as deviations from their population means.

The textbook errors-in-variables model in equation (1) is effectively a regression model that assumes the slope coefficient in the regression of \( y_{it} \) on \( y_i \) equals one. One familiar implication of that restriction is that, if \( y_{it} \) proxies for \( y_i \) as the dependent variable in a linear regression equation, ordinary least squares (OLS) estimation of that regression equation consistently estimates the equation’s slope coefficients. Another well-known implication is that, if \( y_{it} \) proxies for \( y_i \) as the sole explanatory variable in a simple regression equation, the probability limit of the OLS estimator of the equation’s slope coefficient equals the true coefficient times an attenuation factor equal to \( \text{Var}(y_i)/[\text{Var}(y_i) + \text{Var}(v_{it})] \).

These oft-used results no longer apply if the textbook errors-in-variables model incorrectly characterizes the relationship between current and lifetime income. In Section IA, we explain our reasons for suspecting that the slope coefficients in regressions of current income variables on lifetime variables vary systematically over the life cycle and do not generally equal one. In Section IB, we show how such departures from the textbook model alter the standard results on errors-in-variables bias.

A. Life-Cycle Variation

Several fragments of evidence suggest that the association between current and lifetime income variables changes over the life cycle. Anders Björklund (1993), the closest predecessor to our study, uses Swedish income tax data from 1951 to 1989 to conduct a direct comparison of current and lifetime income. He finds a strong life-cycle pattern in the correlation between current and lifetime income. In his words (1993, p. 381), “the correlations are quite low—and in some cases even negative—up to around 25 years of age and are rather high after 35 years of age. In general the correlations are around 0.8 after the age of 35.” Unfortunately, the correlations in income levels reported by Björklund do not map directly into magnitudes of errors-in-variables biases in the sorts of regression estimation that economists commonly do. In the next subsection, we develop measures of association between current and lifetime earnings that do have direct implications for errors-in-variables biases.

Another indication of life-cycle-related departures from the textbook errors-in-variables model, noted by Stephen Jenkins (1987) and Nathan D. Grawe (forthcoming), involves the estimation of intergenerational mobility models such as the regression of sons’ log lifetime earnings on fathers’ log lifetime earnings. If sons’ log annual earnings as a proxy for the dependent variable obeyed the textbook errors-in-variables model, the estimated intergenerational elasticity would have the same probability limit, regardless of the age at which the sons’ earnings were observed. On the other hand, if the slope coefficient in the regression of sons’ log annual earnings on sons’ log lifetime earnings deviates from one in a way that evolves over the life cycle, then analyses observing sons’ earnings at different ages will yield systematically different elasticity estimates. Solon’s (1999) survey of the intergenerational mobility literature reveals precisely
such a pattern—the studies that estimate the smallest elasticities tend to be those that observe sons’ earnings early in their careers. Correspondingly, several studies (e.g., Robert T. Reville, 1995) that have explicitly investigated the effects of varying the ages at which sons’ earnings are observed have found that the estimated intergenerational elasticities increase substantially as the sons’ earnings are observed further into their careers.

Notwithstanding the strong tradition of assuming that current income variables as proxies for lifetime income variables follow the textbook errors-in-variables model, indications that this assumption is false should not be surprising. Any realistic model of income evolution over the life cycle would contradict the traditional assumption. As an extremely simple example, suppose that \( y_{it} \), the log real earnings of worker \( i \) in year \( t \) of his career, follows

\[
y_{it} = \alpha_i + \gamma_i t,
\]

where initial log earnings \( \alpha_i \) varies across the population with variance \( \sigma^2_\alpha \), and the earnings growth rate \( \gamma_i \) varies across the population with variance \( \sigma^2_{\gamma} \). Heterogeneity in earnings growth is a natural consequence of heterogeneity in human capital investment, and its empirical importance has been documented by Jacob Mincer (1974), Baker (1997), Haider (2001), and Baker and Solon (2003), among others. For simplicity, assume zero covariance between \( \alpha_i \) and \( \gamma_i \), infinite lifetimes, and a constant real interest rate \( r > \gamma_i \). Then the present discounted value of lifetime earnings is

\[
V_i = \sum_{s=0}^{\infty} \exp(\alpha_i + \gamma_i s)(1 + r)^{-s} \\
= \exp(\alpha_i)[(1 + r)/(r - \gamma_i)],
\]

and the log of the present value of lifetime earnings is thus

\[
\log V_i \equiv \alpha_i + r - \log r + \gamma_i / r.
\]

It follows that the slope coefficient in the regression of current log earnings on the log of the present value of lifetime earnings is

\[
\lambda_i \equiv \frac{\text{Cov}(\log V_i, y_{it})}{\text{Var}(\log V_i)} \equiv \frac{\sigma^2_\alpha + (t\sigma^2_\gamma / r)}{\sigma^2_\alpha + (\sigma^2_{\gamma} / r^2)}.
\]

The main thing to note about this result is that, contrary to the textbook errors-in-variables model, \( \lambda_i \) generally does not equal one. Instead, it starts at a value less than one at the outset of the career and then increases monotonically over the life cycle. It reaches one when \( t = 1/r \) and then exceeds one afterward. The intuition is that the workers with high lifetime earnings tend to be those with high earnings growth rates. Consequently, when comparing the current earnings of those with high and low lifetime earnings, an early-career comparison tends to understate their gap in lifetime earnings, and a late-career comparison may overstate it. Note that the common practice of adjusting current earnings for the central tendency of earnings growth over the life cycle does not undo this result. The result is due to heterogeneous variation around the central tendency.

Of course, the exact result in equation (5) is particular to the very simple assumptions of the model. A more realistic model would incorporate many additional features including transitory earnings fluctuations, nonzero covariance between initial earnings and earnings growth, nonlinear growth, and shocks with permanent effects. While these features would lead to a more complex relationship between \( \lambda_i \) and \( t \), they clearly would not overturn the main qualitative results—that \( \lambda_i \) does not generally equal one and should be expected to vary over the life cycle.

Figure 1 provides a pictorial version of the argument. The figure contains the life-cycle log earnings trajectories of workers 1 and 2, with worker 2 attaining higher lifetime earnings. Both trajectories display the familiar concave shape documented and analyzed by Mincer (1974), and worker 2 experiences more rapid earnings growth through most of the life cycle. The horizontal lines depict the log of the annuitized value of each worker’s present discounted value of lifetime earnings. The difference between the two workers’ log lifetime earnings therefore is simply the vertical distance between the two horizontal lines. But how well is that...
where the error term \( \varepsilon_i \) is uncorrelated with the regressor vector \( X_i \). Starting with the case of left-side measurement error, suppose that \( y_i \) is the log of lifetime earnings, which is not observed and hence is proxied by \( y_{it} \), log annual earnings at age \( t \). In accordance with the discussion in the preceding subsection, we do not assume the textbook errors-in-variables model in equation (1). Instead, we generalize that model to

\[
y_i = \beta'X_i + \varepsilon_i,
\]

(6)

where \( \varepsilon_i \) is uncorrelated with \( \varepsilon_i \) and \( \beta \) is the slope coefficient in the linear projection of \( y_{it} \) on \( y_i \), need not equal one and may vary over the life cycle. By construction, \( v_{it} \) is uncorrelated with \( y_i \), and we will continue to maintain the textbook model’s assumption that it also is uncorrelated with each separate determinant of \( y_i \) (\( X_i \) and \( e_i \)). Then, if OLS is applied to the regression of \( y_{it} \) on \( X_i \),

\[
y_{it} = \lambda_i y_i + v_{it},
\]

(7)

difference estimated if it is proxied by the difference in log earnings at a particular age? If the worker with higher lifetime earnings has a steeper earnings trajectory, the current earnings gap between the two workers early in their careers tends to underestimate their gap in lifetime earnings (and could even have the opposite sign). As the workers mature, this downward bias becomes less severe until age \( t^* \), when the horizontal distance between the current earnings trajectories equals the distance between the horizontal lines. That is the age at which the textbook errors-in-variables model is correct. For at least some of the life cycle beyond that age, the gap in current earnings tends to overstate the gap in lifetime earnings.

B. Implications for Errors-in-Variables Biases

Suppose we wish to estimate the regression model

\[
y_i = \beta'X_i + \varepsilon_i,
\]

(6)

where \( \varepsilon_i \) is uncorrelated with \( \varepsilon_i \) and \( \beta \) is the slope coefficient in the linear projection of \( y_{it} \) on \( y_i \), need not equal one and may vary over the life cycle. By construction, \( v_{it} \) is uncorrelated with \( y_i \), and we will continue to maintain the textbook model’s assumption that it also is uncorrelated with each separate determinant of \( y_i \) (\( X_i \) and \( e_i \)). Then, if OLS is applied to the regression of \( y_{it} \) on \( X_i \),

\[
y_{it} = \lambda_i y_i + (\lambda_i e_i + v_{it}),
\]

(8)

the probability limit of the estimated coefficient vector for \( X_i \) is \( \lambda_i \beta \) instead of \( \beta \). In the textbook case where \( \lambda_i = 1 \), measurement error in the dependent variable does not result in inconsistent estimation of \( \beta \). More generally, however, the OLS estimator is inconsistent, and the inconsistency varies as a function of the age at which annual earnings are observed.

Moving on to the case of right-side measurement error, suppose that the log of lifetime earnings is one element \( x_i \) in the regressor vector \( X_i \). Because \( x_i \) is not observed, it is proxied by \( x_{it} \), log annual earnings at age \( t \). Analogously to equation (7) for \( y_{it} \), we express the linear projection of \( x_{it} \) on \( x_i \) as

\[
x_{it} = \lambda_i x_i + v_{it},
\]

(9)

where \( v_{it} \) again is assumed to be uncorrelated with \( X_i \) and \( e_i \). If \( x_i \) is the only element in \( X_i \) and OLS is applied to the linear regression of \( y_{it} \) on \( x_{it} \), the probability limit of the estimated slope coefficient is

\[
\text{plim } \hat{\beta} = \frac{\text{Cov}(x_{it}, y_i)}{\text{Var}(x_i)} = \theta_i \beta,
\]

(10)

where

\[
\theta_i = \frac{\text{Cov}(x_{it}, x_i)}{\text{Var}(x_i)} = \frac{\lambda_i \text{Var}(x_i)}{\lambda_i^2 \text{Var}(x_i) + \text{Var}(v_{it})}.
\]

(11)

\(^2\text{When this assumption fails, as it sometimes does, neither the textbook analysis nor our extension is applicable. When Var}(v_{it}) = 0, \text{equation (7) specializes to the rescaling of variables often discussed in introductory econometrics textbooks (e.g., section 2.4 of Jeffrey M. Wooldridge, 2006). See section 4 of Joshua D. Angrist and Alan B. Krueger (1999) for an excellent overview of errors in variables, including nonclassical measurement error.}\)
The inconsistency factor $\theta_i$, sometimes referred to as the “reliability ratio,” is most simply interpreted as the slope coefficient in the “reverse regression” of $x_i$ on $x_{it}$. In the textbook case where $\lambda_i = 1$, this factor simplifies to the familiar attenuation factor $\text{Var}(x_i)/[\text{Var}(x_i) + \text{Var}(v_{it})]$. More generally, the factor $\theta_i$ also depends on the value of $\lambda_i$. Indeed, with $\lambda_i < 1$ and sufficiently small $\text{Var}(v_{it})/\text{Var}(x_i)$, $\theta_i$ can exceed one so that the errors-in-variables bias is an amplification bias rather than an attenuation bias.

Two further results about right-side measurement error are worth noting. First, if $x_i$ is just one element in the regressor vector $X$, the attenuation factor for its estimated coefficient is the same as the last expression in equation (11), except with $\text{Var}(x_i)$ replaced by the residual variance from the auxiliary regression of $x_i$ on the other regressors in $X$. Second, if the measurement error in $x_{it}$ as a proxy for $x_i$ is treated with an instrumental variable (IV) correlated with $x_i$ but uncorrelated with $e_i$ or $v_{it}$, the probability limit of the conventional IV estimator of the coefficient of $x_i$ is the coefficient divided by $\lambda_i$.3

The results presented in this subsection deliver two key messages. First, with plausible departures from the textbook errors-in-variables assumptions, the familiar textbook results about OLS and IV estimation are overturned. Measurement error in the dependent variable is not innocuous for consistency, and measurement error in the explanatory variable can induce either amplification or attenuation inconsistency in OLS estimation as well as in IV estimation. Second, some of the estimation inconsistencies from using log annual earnings as a proxy for log lifetime earnings can be summarized with just two simple parameters: the slope coefficient in the “forward regression” of log annual earnings on log lifetime earnings, and the slope coefficient in the “reverse regression” of log lifetime earnings on log annual earnings. In Section III, we will estimate those two parameters and examine how they vary over the life cycle.

3 The inconsistency of conventional IV estimation in the presence of nonclassical measurement error has been discussed previously by Thomas J. Kane et al. (1999), John Bound and Solon (1999), and Bonggeun Kim and Solon (2005).

II. Data and Methods

A. Data

Most U.S. studies of the relationship between current and lifetime income variables have been based on longitudinal survey data from only a limited portion of the respondents’ careers. In contrast, like Björklund’s (1993) study of Swedish income tax data, our study is based on nearly career-long earnings histories. This information is now available for a U.S. sample because, in accordance with an agreement with the SSA, the University of Michigan’s Survey Research Center asked the participants in its HRS to permit access to their Social Security earnings histories for 1951–1991.4 The HRS sample is a national probability sample of Americans born between 1931 and 1941, and about three-fourths of the respondents agreed to permit access to their Social Security earnings histories. As shown in Haider and Solon (2000), in terms of observable characteristics, the respondents who granted access appear to be surprisingly representative of the complete sample. The earnings data supplied by the SSA round the earnings observations to the nearest hundred dollars, with a distinction made between zero and positive amounts less than $50.

Our analysis is for male HRS respondents born between 1931 and 1933, who were about 19 years old at the beginning of the 1951–1991 earnings period and about 59 at the end. Thus, for the 821 men in our analysis, we have annual earnings information for every year over the major portion of their careers.5 The other main strength of our dataset is that the Social Security

4 Because of the highly confidential nature of the data, the earnings histories are not part of the HRS public release datasets, but are provided only through special permission from the Survey Research Center. For information on accessing “HRS restricted data,” see the HRS Web site at http://hrsonline.isr.umich.edu. For more general information on the HRS, see the Web site or F. Thomas Juster and Richard Suzman (1995).

5 This sample is restricted to workers with positive earnings in at least ten years during the 1951–1991 period. This criterion, which excludes only 33 individuals, is less restrictive than the usual practice in survey-based earnings dynamics studies of requiring positive earnings in every year (e.g., John M. Abowd and David Card, 1989; Baker, 1997). Within this sample, our main analysis includes years of zero earnings, but we also will report results from an analysis based only on the positive earnings observations.
earnings histories tend to be more accurate than the survey reports of earnings used in most previous research. Indeed, Bound and Krueger’s (1991) influential study of errors in earnings reports in the Current Population Survey (CPS) used Social Security earnings data as the “true” values against which the CPS measures were compared.

The strengths of the Social Security earnings data are accompanied by two serious limitations. First, the earnings data pertain only to jobs covered by Social Security. According to SSA (1999, Table 3, B2), the percentage of earnings covered by Social Security has exceeded 80 percent ever since the coverage extensions effected by 1957, and exceeded 85 percent over most of our sample period. Between 1951 and 1956, however, this percentage ranged between 66 percent and 79 percent. Accordingly, in addition to our analyses for 1951–1991, we also will report results for 1957–1991.

Second, the Social Security earnings in our data are measured only up to the maximum amount subject to Social Security tax. In some years, the proportion of observations that are “right-censored” is quite large. For the 821 men in our sample, Table 1 displays the median observed earnings, the percentage in the sample with zero earnings, the taxable limit, and the percentage with earnings at the taxable limit for each year from 1951 to 1991. The table shows that, in the early years, very few sample members are earning enough to approach the taxable limit. As their earnings grow over their careers, however, the taxable limit becomes more constraining, especially in the years when the taxable limit is low relative to the general earnings distribution. The worst year is 1965, when 62 percent of the sample is right-censored. Afterward, the degree of censorship lessens as the taxable limit is progressively increased. By 1991, only 9 percent of the sample is right-censored. Although some previous studies of current and lifetime earnings have used annual earnings data with less severe right-censorship, their observation of earnings usually has been limited to relatively short segments of the life cycle. In effect, they have used restrictive models of earnings dynamics to impute missing earnings data over most years of their sample members’ careers.

If not for the right-censorship, we would follow Björklund’s (1993) approach of directly summarizing the observed joint distribution of annual and lifetime earnings. Because of the right-censorship, however, we are forced, instead, to estimate the joint distribution in a way that imputes the censored right tails of the annual earnings distributions. We describe our methods in the next subsection.

### B. Econometric Methods

As explained above in Section IB, our ultimate goal is to summarize the association between

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annual and lifetime earnings in terms of two types of parameters: one is $\lambda_t$, the slope coefficient in the regression of log earnings in year $t$ on the log of the present value of lifetime earnings; the other is $\theta_t$, the slope coefficient in the reverse regression of log lifetime earnings on log earnings in year $t$. If we had complete data, we would estimate these parameters simply by applying least squares to the forward and reverse regressions of the relevant variables.

Because of the censorship of the Social Security earnings data at the taxable limit, however, we cannot observe the exact value of annual earnings in the cases where earnings are right-censored; furthermore, in those cases, we also cannot compute the present value of lifetime earnings. We therefore apply a three-step procedure for estimating the $\lambda$ and $\theta$ coefficients. First, we use a limited-dependent-variable model to estimate the joint distribution of uncensored annual earnings histories for the 4,000 simulated observations left-censored at $50. Observations of zero earnings and of positive earnings less than $50 are both included as observations left-censored at $50.$

Having estimated each year’s mean and variance in the cross-sectional Tobits, we still need to estimate the autocorrelations between years. To obtain those estimates, we apply the conventional bivariate Tobit maximum likelihood estimator separately for each of the $41 \times 40/2 = 820$ distinct pairs of years in our 1951–1991 period. With those autocorrelations estimated along with the mean and variance for every year, we have an estimated version of the entire joint distribution of uncensored annual earnings over all 41 years.

In the second step of our procedure, we use our estimated joint distribution of uncensored earnings for 1951–1991 to perform the following simulation. First, we take 4,000 random draws from the estimated joint distribution of the 41 years of annual earnings.$^7$ Then, for each of the 4,000 simulated earnings histories, we calculate the present discounted value of lifetime earnings. In the main version of the simulation, we perform the discounting by (a) using the personal consumption expenditures deflator to convert each year’s nominal earnings to a real value and (b) assuming a constant real interest rate of 0.02. In the end, we have a simulated sample of 4,000 observations for which we observe the present discounted value of lifetime earnings as well as each year’s earnings.

Finally, for this sample of 4,000 individuals, we apply OLS to the regression of each year’s log annual earnings on the log of the present value of lifetime earnings, and thereby produce a $\hat{\lambda}_t$ for each year from 1951 to 1991. Similarly, we obtain a $\hat{\theta}_t$ for each year by applying OLS to the reverse regression of the log of the present

6 In the simulation described below, our treatment of zero-earnings observations as left-censored observations from a lognormal distribution causes our simulated observations to include no zeros, but instead small annual earnings values less than $50. The purpose of the simulation is to generate observations for the present discounted value of lifetime earnings. For that purpose, the difference between annual earnings of zero or a few dollars is of practically no consequence.

7 To implement the simulation, we need the estimated autocovariance matrix to be positive semidefinite (as the true one must be). Our procedure for imposing the restriction of positive semidefiniteness is described on the AER Web site (www.e-aer.org/data/sept06/20040284_data.zip).
value of lifetime earnings on each year’s log annual earnings. Plotting each of these coefficient estimates over time depicts the life-cycle trajectory of the association between current and lifetime earnings in a way that translates directly into implications for errors-in-variables biases.

III. Empirical Results

In the first step of our estimation procedure, the Tobit analysis described above results in a $41 \times 41$ estimated autocovariance matrix for log annual earnings from 1951 to 1991. The full matrix is available on the AER Web site (www.e-aer.org/data/sept06/20040284_data.zip). Table 2 shows the estimated autocovariances for 1975–1984, a period when our cohort born 1931–1933 is between the ages of about 43 and 52. As shown in the second column of Table 3, the first-order autocorrelations over this period average to 0.89, the second-order autocorrelations average to 0.82, the third-order autocorrelations average to 0.73, and so forth. Table 3 also displays estimated autocorrelations from two other studies of administrative data. The most comparable results reported in Baker and Solon’s (2003) study of Canadian income tax data are the autocorrelations over the 1985–1992 period for the cohort born 1942–1943. Their average autocorrelations, shown in the third column, are fairly similar to ours, but somewhat lower. As shown in the fourth column, the estimates

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Note: Numbers in parentheses are estimated standard errors.

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from Anders Bohlmark and Matthew J. Lindquist (2005), a replication of our study based on Swedish income tax data, are closer to ours than to Baker and Solon’s. Note that this resemblance between other studies’ estimates and ours occurs even though the other studies use uncensored data and therefore can estimate the autocorrelations directly without imposing distributional assumptions. We find it reassuring that, despite the omission of earnings not covered by Social Security and the imputation of right-censored values, our autocorrelation estimates are similar to those from other datasets. Most of these estimated autocorrelations are somewhat higher than those reported by Baker (1997) and Haider (2001) in their analyses of the Panel Study of Income Dynamics, but the survey-based estimates may be biased downward by reporting error.

Another relevant comparison is to an alternative earnings variable available for our sample for 1980–1991. For those years, in addition to the Social Security earnings data, the Survey Research Center also has obtained earnings data from employers’ W-2 reports to the Internal Revenue Service. Unlike the Social Security data, the W-2 variable includes earnings not covered by Social Security, and it is right-censored (for confidentiality reasons) at $125,000, which is far less constraining than the Social Security taxable limits listed in Table 1. On the other hand, the W-2 variable leaves out self-employment earnings and earnings allocated to 401(k) pensions. As shown in the fifth column of Table 3 when we use the W-2 data to reestimate our Tobits for 1980–1991, the first-order autocorrelations average to 0.89, the second-order autocorrelations average to 0.83, and the third-order autocorrelations average to 0.79. As shown in the last column, the corresponding average autocorrelations for the Social Security earnings variable over the same period are 0.91, 0.85, and 0.81. The idiosyncrasies of the alternative earnings measures do not appear to generate major discrepancies in the estimated persistence of earnings.

8 Also like these other studies, we find that earnings autocorrelations are somewhat smaller early in the life cycle.

9 Perhaps the similarity of the autocorrelation estimates should not be a surprise. If one thinks of the log of covered earnings as the sum of log total earnings and the log of the proportion covered, one would expect the autocorrelation of log covered earnings to be approximately a weighted average of the autocorrelations for log total earnings and log coverage. Presumably, both of these autocorrelations are highly positive. If they are not very different from each other, then “averaging in” the coverage autocorrelation will not produce a large bias in estimating the earnings autocorrelation.
of the present value of lifetime earnings. To focus on the life-cycle variation in \( \lambda_t \), we express \( t \) on the horizontal axis as year minus 1932, which gives the approximate age of our 1931–1933 cohort in each year.10 In contrast to the textbook assumption that \( \lambda_t = 1 \) throughout the life cycle, \( \hat{\lambda}_t \) begins at 0.24 at age 19, increases steadily until it rises to about 1 at age 32, and then declines some in the late forties. The main implication is that, contrary to the textbook errors-in-variables model, using log current earnings to proxy for log lifetime earnings as the dependent variable can induce an errors-in-variables bias. Most importantly, using current earnings in the twenties causes a large attenuation bias. A constructive implication is that the bias is small if one uses current earnings between the early thirties and the mid-forties, when the textbook assumption that \( \lambda_t = 1 \) is reasonably accurate.

The lower portion of Figure 2 shows the estimated life-cycle trajectory of the reliability ratio \( \theta_t \), the relevant parameter for assessing errors-in-variables bias from using log annual earnings to proxy for log lifetime earnings as the explanatory variable in a simple regression. \( \theta_t \) begins at only about 0.2, increases to a fairly flat peak averaging about 0.65 between the late twenties and mid-forties, and then decreases. Our discussion in Section IB showed that theoretically the errors-in-variables bias could be either an attenuation bias or an amplification bias. Our empirical results, however, confirm the conventional presumption that using current earnings to proxy for lifetime earnings as a regressor induces an attenuation bias. The bias is especially large if current earnings are measured early in the life cycle. There is a wide age range in mid-career when the errors-in-variables bias stays about the same, but it remains quite substantial even in that range.

To check the robustness of our main results, we have carried out a series of sensitivity analyses, the results of which are displayed in Figure 3. The first is motivated by the question of how to treat years of zero earnings. Our main results are based on two-limit Tobit estimates that retain observations of zero earnings in the analysis. Because most previous analyses of earnings dynamics, however, have excluded observations of zero earnings, we supplement our main analysis with another that excludes the zeros, codes positive earnings less than $50 as $25, and estimates one-limit Tobits with only right-censorship. As shown in Table 1, zero earnings are especially prevalent in the early years of our sample, both because many of our sample members are not yet working for pay and because the Social Security system’s coverage is less extensive before 1957. We therefore conduct this analysis only for 1957–1991. Excluding the zeros changes the estimates

10 The point estimates plotted in Figure 2 and the associated standard error estimates are tabulated in our electronic Appendix, which also describes our bootstrap procedure for estimating the standard errors.
of the variances and autocovariances in log annual earnings, but because those changes are roughly proportional, the estimated autocorrelations are similar to those in the main analysis. Accordingly, the new estimates of $\lambda_t$ and $\theta_t$ shown in Figure 3 are similar to the estimates from our main analysis repeated from Figure 2.

Our second and third robustness checks explore the sensitivity of our results to our choice of interest rate series. In our main simulation, we calculated the present discounted value of lifetime earnings by (a) using the personal consumption expenditures deflator to convert each year’s nominal earnings to a real value and (b) assuming a constant real interest rate of 0.02. Our third supplementary analysis uses a real interest rate of 0.04, and our fourth discounts nominal earnings by a nominal interest rate series, the annual one-year T-note interest rates. The results shown in Figure 3 are quite similar to those based on our original interest rate series.

Fourth, we have checked whether our results are affected by the HRS oversampling of blacks, Hispanics, and residents of Florida. To do so, we have reestimated the joint distribution of earnings with a Tobit quasi-maximum likelihood procedure that weights each observation’s contribution to the likelihood function by its inverse probability of selection into the sample. The resulting Tobit estimates are very similar to those from our original unweighted analysis, and consequently the new estimates of $\lambda_t$ and $\theta_t$ are again very similar to the main estimates.

Finally, Figure 3 includes the results of Bohlmark and Lindquist’s (2005) replication of our main analysis based on Swedish income tax data. This comparison is particularly interesting because Bohlmark and Lindquist’s data are largely free of the censorship and coverage issues that afflict our U.S. Social Security earnings data. As a result, Bohlmark and Lindquist estimate $\lambda_t$ and $\theta_t$ directly with the forward and reverse regressions involving log current and lifetime earnings, without having to resort to our more complex estimation procedure based on the multivariate normality assumption. Their estimates of $\lambda_t$ in the twenties are somewhat higher than ours, but still much less than one. In general, the patterns of the Swedish and U.S. results are strikingly similar.

IV. Summary and Discussion

All of our analyses tell the same story: contrary to the textbook errors-in-variables model usually assumed in applied research, the slope coefficient in the regression of log current earnings on log annual earnings varies systematically over the life cycle and is not generally equal to one. We can illustrate the usefulness of our results by applying them to the intergenerational mobility regression in which sons’ log of lifetime earnings is the dependent variable and fathers’ log of lifetime earnings is the explanatory variable. As summarized in Solon (1999), most recent research in that literature has devoted considerable attention to the right-side measurement error from using short-run proxies for fathers’ lifetime earnings. Our estimates of $\theta_t$ shown in Figures 2 and 3 confirm the literature’s presumption that right-side measurement error causes an attenuation inconsistency in OLS estimation of the intergenerational elasticity.

The literature, however, has given much less attention to the left-side measurement error from using short-run proxies for sons’ lifetime earnings. Presumably, this neglect reflects an assumption by researchers that, in accordance with the textbook errors-in-variables model, left-side measurement error is innocuous for consistency. All our estimates of $\lambda_t$ suggest that assumption would be fairly well founded if sons’ earnings were measured between the early thirties and mid-forties. Many intergenerational mobility studies, however, have measured sons’ earnings at earlier ages, and this has substan-

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11 This series is available only back to 1954. For the 1951–1953 period, we added 0.003 to the interest rates for three-month T-bills. This adjustment was based on the relationship between the one-year and three-month rates observed for 1954 to 1959.

12 Many researchers have attempted to reduce the attenuation inconsistency by averaging fathers’ log earnings over multiple years. In an analysis summarized in our electronic Appendix, we repeat our estimation of $\theta_t$ except that the new estimates are for five-year averages of log annual earnings, rather than for single years. Our results strongly support the conclusion of Mazumder (2001, 2005) that even estimates based on five-year averages of the earnings variable for fathers are subject to a substantial errors-in-variables bias.
tially affected the findings. Reville (1995), for example, estimates intergenerational elasticities of about 0.25 when he measures the sons’ earnings in their twenties, but his estimates start approaching 0.5 when he observes the sons well into their thirties. This is just the pattern one should expect from the trajectories of \( \lambda_t \) in Figures 2 and 3. An important implication is that many estimates of the intergenerational earnings elasticity have been subject to substantial attenuation inconsistency from left-side measurement error, in addition to the well-known inconsistency from right-side measurement error.

Of course, interpreting evidence on intergenerational earnings mobility is just one example of how our results might be applied. We advise readers, however, to exercise due caution in importing our estimates of \( \lambda_t \) and \( \theta_t \) to other earnings data. We already have mentioned issues of comparability between administrative and survey data. Furthermore, the life-cycle trajectories for our U.S. cohort born 1931–1933 may differ from those for other cohorts and other countries. In addition, as emphasized in Solon (1992), sample selection criteria that affect the sample’s dispersion in earnings also affect the measurement error properties of current earnings as proxies for lifetime earnings. Nevertheless, taking account of our evidence on departures from the textbook errors-in-variables model should enable better-informed analyses of estimation biases in a wide variety of research that uses current earnings variables as proxies for long-run earnings.

REFERENCES


Kane, Thomas J.; Rouse, Cecelia Elena and Staiger, Douglas. “Estimating Returns to Schooling when Schooling Is Misreported.”


