Beliefs about Mathematical Understanding

Cigdem Hasen, Michigan State University
Jon R. Star, Michigan State University

Hasen is Doctoral Candidate in Teacher Education, and Star, Ph.D., is Assistant Professor of Educational Psychology and Teacher Education, in the College of Education.

Abstract
Thirteen preservice middle school mathematics teachers from a four-year teacher education program in Turkey were interviewed about their beliefs related to mathematical understanding. The analysis yielded four components of mathematical understanding with various subcomponents: Content, reasoning, applications, and procedures. The competitiveness of high school background seemed to have an effect on participants’ beliefs about mathematical understanding. Participants from the most competitive high school background seemed to have richer conceptions of mathematical understanding.

Introduction
Teachers’ knowledge and beliefs have become a recent focus of educational research. Of particular interest is the relationship between the teachers’ beliefs about the nature of and the teaching and learning of subject matter and their classroom practices (Calderhead, 1995; Thompson, 1992). Preservice teachers are likely to have different beliefs from inservice teachers about the nature of and teaching of subject matter since they have not met real classroom conditions yet (Haggarty, 1995). They carry their existing beliefs from pre-college education to their teacher education programs and they learn to teach through the lenses of what they know and believe about teaching the subject matter from those years (Lampert, 1990). Moreover, their beliefs can affect the ways that they conduct lessons in the first few years of teaching (Feiman-Nemser, 2001). Hence, preservice teachers’ beliefs become an important concern in teacher education programs.

Theoretical Background
Beliefs can be characterized as subjective in nature, likely to differ among people, free from social norms of validation, and reflective of differences attributable to various background and environmental factors. They can have varying degrees of conviction and they are not consensual as others might have different beliefs. Beliefs seem to play an important role in how teachers view and/or perceive or develop a conception of knowing, understanding and teaching the content (Calderhead, 1995). In teaching mathematics, teachers’ beliefs about mathematical knowledge and understanding are found to be associated with their teaching practices (Stipek, Givvin, Salmon and MacGyvers, 2001), and with their students’ beliefs about mathematical knowledge and understanding (Hiebert & Carpenter, 1992). Teachers conceptualize teaching and learning with an eclectic collection of different beliefs that they have as a result of their experiences in classroom settings (Thompson, 1992). These experiences include their pre-college experiences and university level courses as students, as well as classroom experiences as teachers. Research on preservice teachers’ beliefs has concluded that the ways preservice elementary teachers have been taught mathematics, which is typically as discrete bits of procedural knowledge, affect the ways that they understand mathematics and relate the ideas to each other in their own teaching (Lampert, 1990).

The nature of mathematical knowledge is generally analyzed within two main domains. The first one, conceptual knowledge, is defined to be rich in relationships and is developed through building relationships between new and existing pieces of knowledge and among the existing pieces of knowledge (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1988). Conceptual understanding emphasizes an awareness of
one’s building those relationships and how one knows what one knows as well as finding out the answers of “why” and “what” questions (Skemp, 1987). The second one, procedural knowledge, includes symbolic system, algorithms and rules such as “step-by-step instructions that prescribe how to complete tasks” (Hiebert & Lefevre, 1986, p.6) or sequences of actions for solving problems (Rittle-Johnson & Siegler, 1988). This type of knowledge and related understanding addresses the answer of “what” (Skemp, 1987).

Methodology

Context

The Turkish pre-college educational system that participants come from is a centralized system with a national curriculum. The high schools in this system can be grouped into three levels of competitiveness depending on how the student population is formed. The most competitive high schools, Exam-Track High Schools (E-HS), take their students from a competitive national exam and the teachers of these schools are also considered to be superior in terms of knowledge and skills in teaching. The medium competitive high schools, Foreign-Language Based High Schools (F-HS), take their students according to their middle school cumulative grades. The least competitive high schools, Regular High Schools (R-HS), take all students regardless of cumulative grade or exam score. Although the curriculum is the same in all high schools, more competitive high schools generally have more in-depth instruction especially in mathematics and sciences compared to the less competitive high schools.

Participants

The study was conducted with seniors in a four-year teacher education program that aims to educate mathematics teachers for the middle school grades (sixth, seventh and eighth grades). The subjects were seven female and six male fourth year students chosen from the students who volunteered for the study based on the type of the high school that they had attended. It was assumed that the competitiveness of the high school would affect participants’ beliefs about the nature of mathematics and the teaching and learning of mathematics. Six of the participants had E-HS, four of them had F-HS and three of them had R-HS background. Participants’ ages ranged from 21 to 23. These 13 students formed 54% of the fourth year cohort. The program offered five courses on mathematics teaching, four courses on educational psychology, nine courses on mathematics, and three semesters of student teaching. The participants had completed all requirements of the program and graduated from the program soon after the study was finished.

Instrument

Semi-structured interviews including 23 main questions were conducted to investigate preservice teachers’ beliefs related to mathematical understanding, teaching and learning mathematics, and high school and college settings that might have affected these beliefs. This paper focuses on participants’ responses to questions about the nature of mathematics and different types of mathematical understanding. Sample questions from the interview include the following:

- What is mathematics for?
- What does it mean to know mathematics?
- Can you tell me what a “mathematical concept” is depending on your own ideas?
- How do you know that a student has understood a mathematical concept?
- Probing or additional questions were asked to explore the emerging issues during the interview.

Procedure

The interviews were conducted in one-on-one settings by the first author and took 45 minutes to one hour. The interviews were transcribed and similarities in the responses among all of the participants were investigated through a predetermined set of response types. These response types were based on both pilot written interviews that were conducted by three preservice teachers and a literature review. The results are combined under categories that include similar type of answers. All names are pseudonyms.
Results and Analysis

The answers given to questions relating to mathematical understanding resulted in four components: Content, Reasoning, Application and Procedures.

Content: Beyond Surface Level Knowledge

Six of the seven participants who mentioned content knowledge as being a part of what it means to know or understand mathematics emphasized that there are more things to be considered besides numbers, operations, and formulas when knowing and understanding mathematics are considered. Moreover, such knowledge can appear in rich ways, like knowing where things come from, as two of the six E-HS, three of the four F-HS, and one of the three R-HS participants mentioned. For example, Yvonne (E-HS) indicated this focus on going beyond surface level knowledge: "...knowing the meaning of the concepts, and where and what they are used for is necessary [knowledge, too]." The participants generally believed that mathematics content should not only include the surface level knowledge of rules and formulas, but also the deeper knowledge of how these formulas came out and what the concepts mean through theorems and proofs. The emphasis on how concepts are formed seems to be an evidence of relational or conceptual understanding (Skemp, 1987).

Reasoning: Logical Thinking and Multiple Views

Twelve of 13 participants expressed that the understanding of mathematics helps in improving habits of thinking in general. One of these habits is thinking logically in studying mathematics, as five of the six E-HS and one of the three R-HS participants mentioned. Another habit of thinking is developing multiple approaches in thinking about not only mathematical tasks but also other situations. Two of the six E-HS, one of the four F-HS, and two of the three R-HS participants mentioned this view, as exemplified by Arthur (F-HS): "We probably don't realize it, but mathematics ... help us in developing multiple views when we come across a situation." The evidence that the participants sought for habits of thinking, or whether one has logical thinking and multiple approaches in studying mathematics, was one's verbal explanation of these habits of thinking and multiple approaches. Four of the six E-HS, three of the four F-HS, and one of the three R-HS participants claimed that the verbal explanation should involve one's own words and the expressions gathered in a meaningful way, as indicated by Mark (F-HS): "I would not want a student to solve for me a problem, I would want him to explain to me verbally [what he has done]."

Application: Mathematical Dimension and Building Connections

All of the participants thought that being able to apply mathematical knowledge to other mathematical tasks or problems and to other contexts such as science and social sciences is an important task of knowing and understanding mathematics. This relates to finding the mathematics within other contexts, or building context-free relations among different types and areas of knowledge (Hiebert & Lefevre, 1986), as indicated by Gail (E-HS): "It [mathematical understanding] can be considered as adding a mathematical dimension to the concepts in other fields."

Application of mathematical ideas to other contexts also means building connections to other mathematical knowledge. This includes the connection between abstract and concrete knowledge and cause-effect relationships, as two of the six E-HS and two of the four F-HS participants mentioned. Emily (E-HS) noted: "Given a concept, if one can build connections between that concept and some other concepts and can comment on the cause-effect relationships between them, then I can say that one has mathematical understanding." Real-life contexts were commonly mentioned both as a special area in the application of mathematical knowledge and also in relation to the nature of mathematics as indicated by Stacy (F-HS): "The mathematical things that we don't see make life go on. It [mathematics] is for life." The participants believed that knowing and understanding mathematics could be seen through the skills such as applying mathematical knowledge to other tasks and fields, building connections through various pieces of knowledge and thinking mathematics within the real-life context. According to the participants, application is not simply a skill of being able to implement rules and procedures on similar
situations but being able to search for the existing mathematical ideas, rules or procedures in different contexts.

Procedures: Steps to Follow
The participants generally viewed mathematical understanding as a multi-dimensional construct. Although most of the participants could not express what conceptual understanding might be, a total of nine participants, including four of the six E-HS, three of the four F-HS, and two of the three R-HS ones, were able to mention a step-by-step pattern for what they believe procedural understanding might be in the context of solving a question or a problem. Mark (F-HS) was one such student; he said that understanding a procedure involving knowing, "The steps that one should follow... If we ask a question, and if the question has several steps, then one should know where to start." If the task is a question, then having procedural knowledge allows one to decide on which steps to take to solve the problem. If it is a problem, then the participants seemed to associate the step-by-step pattern with the problem solving steps, and consider the usage of such a pattern as an indication of procedural understanding. There appears to be a consistency between the participants' beliefs about procedural understanding and the way it is defined in the literature (e.g., Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1988).

Discussion
The analysis showed that participants' beliefs about the nature of mathematical knowledge can be categorized under four components as well as various subcomponents. In general, preservice teachers mentioned the reasoning, application, and procedural components of mathematical understanding more often than the conceptual component. Although most of the participants could not express what conceptual understanding might be, the way they described it through student actions and task examples showed that their beliefs about conceptual understanding aligns towards how it is defined in the literature. Procedural understanding, on the other hand, was expressed through definitions and examples more fluently. High school background appeared to have some effect on the participants' views of the nature of mathematics and teaching and learning mathematics. In particular, preservice teachers from E-HS, which are the most competitive high schools, did not mention content knowledge as frequently as they mentioned the other three components (reasoning, application, and procedures). In contrast, most of F-HS participants mentioned all four components. Content knowledge was also mentioned less frequently than the other three components by the R-HS preservice teachers, who had the least competitive high school background. These differences in students' views likely emerge because of the combined effects of the school type and mathematics teacher, that, more competitive schools have more skillful teachers and thus the probability of having a good mathematics teacher increases for the students of more competitive high schools.

Mathematical knowledge is claimed to include essential relationships between conceptual and procedural knowledge (Hiebert & Lefevre, 1986). Hence, it can be speculated that a rich mathematical understanding includes the relationships between conceptual and procedural understandings that can be used when they are required in any context. Participants generally emphasized reasoning, building connections through other mathematical ideas, and deepening the meaning of the concepts and presented richer conceptions of mathematical understanding. Preservice teachers with richer conceptions, who were mostly coming from E-HS background, did not hesitate in responding to the questions and provided deeper insights when asked to explain more. These participants seemed to exhibit properties of conceptual knowledge and understanding since they mentioned about rich relationships between the concepts (Hiebert & Lefevre, 1986; Ma, 1999; Rittle-Johnson & Siegler, 1988) and emphasized how the concepts are formed (Skemp, 1987).

The way preservice teachers mention real-life and other contextual applications of mathematical ideas seemed to be an indication of reflective conceptions of mathematical knowledge (Hiebert & Lefevre, 1986) since they referred to the relationships among the mathematical ideas independently from the context the ideas presented. In particular, some participants' claims about

150

Academic Exchange – Winter 2004
multiple approaches in dealing with mathematical tasks provide evidence of their richer conceptions of mathematical understanding, as Ma (1999) argues. Moreover, participants generally discussed procedural knowledge and understanding in ways that are consistent with other scholarship in this area (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1988). Preservice teachers with poorer conceptions and with the least competitive high school background, on the other hand, could not provide complete and clear expressions, especially for conceptual understanding. Hence, they were not able to articulate relationships between conceptual and procedural understanding in terms of students’ actions or hypothetical tasks. Moreover, they generally tended to give examples of measurement processes, such as pre and post testing of the students, for analyzing the students’ mathematical understanding, rather than the actual student work. In general, the participants from the most competitive high schools seemed to have richer conceptions of mathematical understanding compared to the participants from the least competitive high schools.

Conclusion
The study reported here attempts to describe preservice teachers’ beliefs about subject matter, to provide preservice education and further inservice development with an insight into the strength and quality of ideas that the teachers have. The results seem to confirm previous findings about the effect of pre-college education on preservice teachers’ beliefs about mathematics (Lampert, 1990) in a different context. However, more investigation with more participants from different year levels is needed to understand differences related to the high school background and how the teacher education program might have affected their beliefs. Further research might also consider mathematical ability as another variable in shaping preservice teachers’ mathematics related beliefs besides high school background. The results of this study might be considered in designing content and pedagogical content courses for preservice mathematics teachers.

References