Investigating Student Thinking about Estimation: What Makes a Good Estimate?

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This work is supported by a grant from the US Department of Education, Institution for Education Sciences. Correspondence can be addressed to Jon R. Star, 513C Erickson Hall, College of Education, Michigan State University, East Lansing, MI, 48824, 517-432-0454 (voice), 517-353-6393 (fax), jonstar@msu.edu, www.msu.edu/~jonstar.
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A large majority of students have difficulty doing simple calculations in their heads or estimating the answers to problems (e.g., Case & Sowder, 1990; Reys, Bestgen, Rybolt, & Wyatt, 1980). For example, on the third National Assessment of Educational Progress in Mathematics (NAEP), less than half of the 13-year-olds tested could mentally calculate $60 \times 70$, and only 38% thought that $945 \times 1000$ should be solved mentally (Lindquist, 1989; Lindquist, Carpenter, Silver, & Matthews, 1983).

This disuse or inability to use mental math or estimation is a significant barrier to using mathematics in everyday life. We often must make quick computations or judgments of numerical magnitude without the aid of calculator or paper and pencil. In addition to being a fundamental, real-world skill, the ability to quickly and accurately perform mental computations and estimations has two additional benefits: 1) It allows students to check the reasonableness of their answers found through other means, and 2) it can help students develop a better understanding of place value, mathematical operations, and general number sense (National Research Council, 2001). These benefits are encapsulated in the recent “Adding It Up” report from the National Research Council: “The curriculum should provide opportunities for students to develop and use techniques for mental arithmetic and estimation as a means of promoting deeper number sense” (2001, p. 415).

Why do students have difficulty estimating? One explanation might be that students may not know what makes an estimate a good estimate, perhaps in part because this issue is not as straightforward as it might appear.
What Makes an Estimate Good?

The existing literature on estimation identifies two criteria that might be useful in evaluating the goodness of an estimate.

The first, simplicity, refers to how easy it is to compute an estimate (Reys & Bestgen, 1981; LeFevre, Greenham & Waheed, 1993). For example, to compute an estimate for $31 \times 46$, one could round both numbers to the nearest ten and multiply $30 \times 50$. Alternatively, one could round only one number to the nearest ten and multiply $30 \times 45$. For many elementary school students, it seems plausible that the first method is easier.

The second, proximity, refers to how close the estimate is to the exact answer (Reys & Bestgen, 1981; LeFevre, Greenham & Waheed, 1993). For example, to compute an estimate for $31 \times 46$, one could round the 31 and multiply $30 \times 46$, or one could round the 46 and multiply $31 \times 50$. The former differs from the exact value by $1 \times 46$, while the latter differs by $31 \times 4$, so that former is more proximate and thus potentially a better estimate.

However, these two criteria interact in quite subtle ways. The demands of an estimation task may suggest that proximity is paramount, even if the resulting calculation is far from simple to perform. For example, one has to generate a fairly accurate estimate of expenses when resources are tight. Alternatively, some situations call for a very ‘rough and dirty’ estimate, meaning that simplicity is critical, even if the estimate is far from the exact value. Furthermore, decisions about emphases on simplicity and proximity are made at the expense of each other (LeFevre, Greenham & Waheed, 1993). These decisions are determined by individuals, as what is easy to compute varies across a wide
range of factors related to number facts and knowledge, development, and confidence level (Reys et al., 1982).

In addition, some strategies may present a false illusion of consistently producing more proximal estimates than others. For example, it might appear that rounding only one number to the nearest ten always produces a more proximal estimate than rounding both numbers to the nearest ten. But it turns out that the more proximal of these two strategies actually depends on the problem. (Consider which strategy is more proximal for 39 * 41 versus 37 * 39.)

Thus, the question of what makes an estimate good is quite complex and nuanced. In the present study, our goal was to learn more about how students think about estimation, particularly what makes an estimate good, in order to understand students’ difficulties with estimation. This work was part of a larger project that investigated the effects of a particular instructional intervention on students’ ability to estimate, but the present paper only reports our analysis of student pairs talking about and working on multiplication estimation problems.

Educational Significance

Why is it important to investigate how students think about what makes an estimate good? First, students’ perceptions of good estimates might be very different from experts and potentially idiosyncratic or even flawed. Star and Madnani (2004) found that students’ perceptions of what makes a strategy (for solving linear equations) good were frequently idiosyncratic and often diverged from expert views. Second, knowing what students label as good estimates may help educators and curriculum developers incorporate this information into more effective teaching strategies and
materials. And finally, such an investigation could also inform practitioners of the language and reasoning used by students to discuss good estimates and provide details regarding how students make sense of various strategies – information which might ultimately assist student learning of computational estimation strategies that they could use in everyday situations.

Method

Sixty-six fifth-grade students learned about computation estimation strategies by viewing a series of worked examples and by answering questions about the hypothetical students’ strategies in the worked examples. For example, one worked example showed two hypothetical students Allie and Claire estimating the solution to $27 \times 43$. Allie estimated by truncating, where she covered the ones digits, multiplied the tens digits ($2 \times 4$), and ultimately added two zeros to this product to arrive at an estimate of 800. Claire estimated by rounding the two operands to their nearest ten (30 and 40, respectively) and then multiplying these two values together to get an estimate of 1200. Below the worked example, the student pair were asked “How is Allie’s way similar to Claire’s way?” Students were randomly paired, and they studied worked examples such as this one during a two-day intervention in their intact math class.

Transcripts of the partner interactions were analyzed, with particular attention to the ways that students answered questions relating to students’ perceptions of the goodness of estimates in terms of simplicity and proximity.

Results

Simplicity. Our analysis of the student pair interactions indicated that students talked about simplicity in four ways.
First, some students said simplicity was related to whether the work could be completed “in your head” and did not require the use of paper. For example, when comparing a worked example showing Catherine’s and Marquan’s ways of estimating, one participant said, “you can't really do [Catherine’s way] in your head, you'll get confused what number you're on. So Marquan's way is easier.” On another problem, a student questioned whether a given estimation method was easy and ultimately decided, “I don’t think it is. Because you need a piece of paper.”

Second, some students said simplicity was related to the time needed to use the estimation strategy. For example, when one pair of students discussed the merits of two estimation methods, one student explained that the other student’s way was harder because, “it’s going to take longer.”

However, other students explicitly stated that easier methods do not need to be ones that get estimates quickly. For example, one student explained to her partner why she chose one method instead of another approach by saying, “I think Jenny's way is easiest on this one. I know it's not as quick.” This student went on to discuss that Jenny’s method was easier because it led her to multiply 2 * 7, which is “a fact you know in our head.” These comments imply that this student believed easier methods may not be the quickest, but would involve using mathematical facts that was well known to her.

Third, some students said simplicity was related to the particular strategy used, with some strategies being simpler than others, regardless of the numbers involved. For example, many students felt that estimation methods that involved rounding both operands were easier than methods that involved rounding only one operand. As one
student said, “it is easier just to round both numbers;” similarly, another student said, “it would be less confusing to round both numbers.”

Fourth, some students said simplicity was related to whether a method led to an estimate that was close to the exact value (e.g., proximity). For example, when comparing different methods, one participant stated that a certain method was easier than others because it led to estimates that were “closer to the exact answer.”

*Proximity.* Our analysis indicated that students talked about proximity in two ways.

First, some students said that an estimate’s proximity to the exact value was affected by how close the rounded operands were to the initial operands. For example, when judging whether 10 * 78 or 10 * 80 produced an estimate that was closer to the exact value for 11 * 78, a student explained that 10 * 78 would result in a closer answer because these, “numbers are close[r] to the [original] numbers used in the problem.” Although this student was incorrect (10 * 80 leads to an estimate that is closer to the exact value), this student and others reasoned that the closer the rounded operands were to the initial operands, the closer the estimate would be to the exact value. This thinking was succinctly stated by another student who, when evaluating whether 10 * 32 or 10 * 30 would produce an estimate closer to the exact value for 11 * 32, said, “you can know which estimate is closer by seeing which rounding was closest to the [original] numbers.”

Second, some students looked at how the altered operands impacted the estimate to determine exactly how far away estimates were from the exact value. Such comments arose when students discussed the hypothetical students Yolanda and Anne, and their methods for estimating 11 * 68 and 11 * 18, in order to answer whose estimate was closer
to the exact value for her number problem. To estimate $11 \times 68$, Yolanda multiplied $10 \times 68$, and to estimate $11 \times 18$ Anne multiplied $10 \times 18$. One student reasoned that Anne’s estimate would be closer to the exact value for her problem, “because 10 times 18 is 180, and then 11 is 18 more, [whereas] if Yolanda goes up [one] it is gonna be 68 more.”

Discussion

Analysis of the transcripts revealed that students discussed four characteristics of simplicity of estimation strategies and discussed proximity in two ways. Simplicity comments focused on students’ ability to apply the strategy mentally, the time it took to use the strategy, the estimation method employed by the strategy, and/or whether the strategy led to an estimate that was close to the exact value in a given problem. For proximity, some students noted that the closer rounded operands were to the initial operands in a problem, the closer the estimate would be to the exact value. These statements looked at the problems abstractly whereas other comments regarding proximity discussed explicitly how far estimates were from the exact value by noting how changing the value of the operands impacted the estimates.

What does this information tell us about students’ perceptions of good estimates? It implies that students’ thinking about simplicity and proximity is diverse and one student’s evaluation for whether a strategy is simple may not coincide with another student’s (or expert’s) perceptions. The diversity of comments implies that two students might choose different strategies to solve the same problem, based on their own means of evaluating whether a strategy is simple and/or leads to estimates that are close to the exact answer. As a result, educators should not assume that all students evaluate simplicity and proximity similarly, and that a “one size fits all” approach to these
concepts will work well in any given classroom. Students should be introduced to multiple estimation strategies and gain experience with each of them. Teachers can introduce various statements other students have provided regarding estimation strategies, simplicity, and proximity, and encourage their students to determine and make known which methods they find to be good and why. Such information can enable educators to incorporate students’ thinking and reasoning into the content and instruction, and by hearing the diversity of student views, teachers can better address specific approaches used by each individual.
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