ABSTRACT
The objective of the current symposium is to bring together diverse perspectives to push the field on the multiple important roles procedural knowledge has for promoting mathematical proficiency. The participants present theoretical and empirical work that illustrates the range of benefits of procedural knowledge for mathematical competence, including conceptual understanding. The 4 empirical presentations focus on middle- and high-school students learning mathematical topics such as algebra in typical classrooms in the US and Asia and in researcher-led classrooms. The results illustrate how instruction on procedures can be beneficial when carefully interweaved with concepts, when grounded in story contexts, or when focused on multiple ways to solve the same problem.

GENERAL OVERVIEW
The objective of the current symposium is to bring together diverse perspectives to push the field on the multiple important roles procedural knowledge has for promoting mathematical proficiency.

The last few years has seen a resurgence of interest in the role of procedural knowledge in mathematical competence (e.g. Baroody, 2003; Rittle-Johnson & Siegler, 1998). Procedural knowledge (knowing how step-by-step to determine solutions) and conceptual knowledge (a well-connected understanding of why) are often pitted against one another concerning which one is developmentally and/or instructionally more important. Although the mathematics education research community is moving away from such a dichotomy, a majority of research and theory focuses on the benefits of conceptual knowledge for procedural knowledge (e.g. Kilpatrick, Swafford & Findell, 2001). The range of benefits of procedural knowledge for mathematical competence, including conceptual understanding, has received much less attention.

According to the National Research Council synthesis of the literature on mathematics learning (Kilpatrick et al., 2001): “Without sufficient procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematical problems. The attention they devote to working out results they should recall or compute easily prevents them from seeing important relationships (p. 122).” At the same time, the report cautions against students practicing procedures they do not understand. In general, the field has recognized the importance of automatizing certain procedures in order to free-up mental resources. However, it has made less progress in considering additional benefits of procedural knowledge, such as abstraction of related concepts or gains in cognitive flexibility.

The current symposium brings together diverse perspectives to illustrate the multiple important roles procedural knowledge has for promoting mathematical proficiency. The presenters represent psychological and educational perspectives, are situated within a variety of instructional approaches, use diverse methodologies (analysis of typical
classroom instruction in different cultures, of alternative classroom instruction, and of alternative small group instruction), and are both new and established researchers in the field, including editors of the 1986 seminal book on the topic (James Hiebert) and the recently published book revisiting the topic (Arthur Baroody).

Format: The symposium will begin with a brief historical and theoretical overview of the relations between conceptual and procedure knowledge in mathematics. This overview will be followed by four presentations of new empirical work addressing the role of procedural knowledge and procedural instruction in mathematical competence. These empirical studies all focus on students learning more advanced mathematics topics, such as algebra, offering an important extension of past research, which has focused primarily on learning arithmetic. To ensure critical reflection on the research with the audience, we have invited a long-standing and prominent researcher of students’ mathematical knowledge to begin the discussion of the ideas presented in the symposium. Throughout the symposium, we will focus on finding balance, theoretically and instructionally, between conceptual and procedural knowledge.

SUMMARY OF INDIVIDUAL PROPOSALS

**Paper 1: The Relation Between Conceptual and Procedural Knowledge: Past, Present, and Future**  
Arthur Baroody, University of Illinois

This historical perspective summarizes and evaluates different views of how conceptual knowledge and procedural knowledge interact. Dating back to the ancient Greeks, traditional educators have essentially proposed a skills-only or a skills-first view, whereas reformers have at least implicitly advocated a concepts-only, concepts-first, or, a concepts-skills-intertwined view (e.g., Brownell, 1935 Kilpatrick et al., 2001). Because promoting procedural fluency effectively depends on integrating procedural and conceptual instruction, Resnick and Ford (1981) and Hiebert (1986) called for research that helped illuminate such integration.

Spurred in part by the on-going crises in mathematics education in this country, there is a renewed interest in how conceptual and procedural knowledge are related. One influential effort in cognitive psychology is Siegler’s (e.g., Shrager & Siegler, 1998) effort to construct a computer simulation of single-digit addition development. Unfortunately, it does not adequately take into account the development of conceptual knowledge and its role in guiding the invention of procedures (cf. Fuson & Burghardt, 2003; Ambrose et al., 2003). Alternative models are needed to account for these findings.

Caveats and conjectures for future research will be offered and illustrated. For example, a procedural advance does not necessarily indicate a conceptual advance, and, without a conceptual basis, will probably be short-lived or not generalized.
**Paper 2:** The development of flexible procedural knowledge in equation solving  
Jon Star, Michigan State University

The results of a recent study investigating the development of flexible procedural knowledge of linear equation solving will be presented. 65 6th graders received 30 minutes of procedural instruction on the operators that can be used to solve equations (e.g., adding a constant to both sides, combining like terms), and, in 3 hours of problem-solving practice over 3 days, were left to discover how these steps can be used to solve linear equations. Students were randomly assigned to two conditions: a control condition, where students solved a progression of new equations, and a treatment condition, where students were asked to re-solve problems that had been previously completed but using a different ordering of steps. A post-test assessed students’ ability to solve familiar and novel equations, their conceptual knowledge about equation solving, and their procedural flexibility, which was operationalized by investigating whether students knew multiple procedures for solving equations and/or a variety of ways that operators can be used on particular problems for more efficient solutions. Both control and treatment students become competent equation solvers on familiar and transfer problems, discovered an efficient algorithm for solving linear equations, and showed improved conceptual knowledge. But treatment students also showed significantly greater flexibility.

**Paper 3:** Using real-world contexts to strengthen links from procedural knowledge to improved conceptual knowledge

Bethany Rittle-Johnson & Page McMullen, Vanderbilt University

Conceptual and procedural knowledge often develop in a mutually supportive, iterative process. However, conceptual knowledge more reliably impacts procedural knowledge than vice versa. In the current study, we evaluated one potential instructional technique for strengthening the link from improved procedural knowledge to improved conceptual knowledge – using story contexts to build off of informal knowledge. Fifty-six sixth-grade students learned to solve simple algebra equations (e.g. $28 = 5 \times a + 3$) in problems with or without story contexts. Students had no prior instruction on solving equations and were randomly assigned to instructional group. In their classrooms, students received instruction and practiced solving equations that were presented with story contexts or without. Although both groups made similar gains in procedural knowledge, students in the context-instruction group made greater gains in conceptual knowledge than the no-context-instruction group. Thus, eliciting real-world knowledge via story contexts strengthened the links from improved procedural knowledge to improved conceptual knowledge, illustrating how well-designed instruction on procedures can support gains in conceptual understanding.

**Paper 4:** Conceptual and procedural relationships in mathematics teaching re-examined from an international perspective.  
James Hiebert, University of Delaware
The results of the TIMSS 1999 Video Study of (Eighth-Grade) Mathematics Teaching have just been released. They show, among other things, that the higher-achieving countries in the sample differ along many dimensions considered in the U.S. to be critical for high-quality instruction. In particular, the two highest achieving countries in the sample, Hong Kong and Japan, are at the opposite ends of the spectrum with respect to what kinds of problems were presented to students: the large majority of problems in Hong Kong focused on procedures whereas the majority of problems in Japan focused on concepts. The U.S. showed a procedural emphasis, closer to Hong Kong. But, when the problems were worked on with students, the picture changed. Both Hong Kong and Japan showed attention to conceptual development, even when the problems were presented as procedural; the U.S. showed almost no attention to conceptual development. Especially instructive are examples of Hong Kong teachers working through procedures with students by attending to a rather unique mix of procedural and conceptual features. Examples of this kind of teaching might help to create new images in the U.S. for how conceptual and procedural knowledge can interact and strengthen each other.

**Paper 5:** Benefits of prior procedural knowledge for learning from conceptually-oriented instruction on functions

Mindy Kalchman, DePaul University

Results from a teaching experiment will be discussed with respect to how conceptual and procedural knowledge are inter-related as children develop mathematical understandings over-time. In the context of a bigger study, an experimental approach to teaching functions was developed and the same unit taught to a cross-sectional sample of students in grades 6, 8, and 11. The experimental approach focused on developing children’s conceptual understanding of functions with the expectation that procedural knowledge would eventually be learned as a more “efficient” way of working the concept.

Somewhat surprisingly, older students’ prior knowledge of conventional procedures seemed to facilitate problem solving. The younger students (both sixth and eighth graders) were at a loss for how to express and encode their conceptual knowledge and thus offered mainly partially correct solutions to intermediate and advanced problems. The older students, on the other hand, were able to incorporate their previous experiences with a procedure-oriented curriculum with the highly conceptual approach taught in the experiment. These findings suggest that prior procedural-oriented instruction could be construed as an asset rather than as an impediment and that curriculum for even young students learning sophisticated mathematics should have concept and procedure interactively embedded in their learning experiences.

**REFERENCES**


