Students' Reactions and Adjustments to Fundamental Curricular Changes:
What Are "Mathematical Transitions"? How Can We Study Them?

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Abstract: The “Navigating Mathematical Transitions” project is a 3-year study that examines students' experiences as they move between "traditional" mathematics curricula and those inspired by the NCTM standards (1989). In this paper, we focus on our method of analysis, particularly the ways that we have conceptualized "mathematical transitions". We describe our analytical framework, which consists of four categories that summarize changes that students' experience during the curricular shift: (1) student achievement, (2) significant differences students notice and report, (3) changes in disposition towards mathematics, and (4) changes in learning approach. We give examples and raise issues with each of these categories, in an attempt to explore what aspects of students' experiences our framework does or does not allow us to capture.

Introduction

The release of NCTM’s *Curriculum and Evaluation Standards* (1989) ushered in lively debate in U. S. mathematics education, and after a decade of curriculum development, assessment studies have begun to examine the effectiveness of Standards-based materials (e. g., Schoen, Hirsch, & Ziebarth, 1998). But very little attention has been given to students' experiences as they move between programs of curricula and pedagogy that differ dramatically in conceptions of thinking, knowing, and doing mathematics. These differences create boundaries that students must cross to achieve their academic and career goals.

This research report will present emerging analyses from the “Navigating Mathematical Transitions” project, a three-year study of how high school and college students cope with changes in mathematics curriculum and teaching. We are studying students as they move between relatively traditional and substantially different mathematics programs inspired by NCTM Standards (1989) at two junctures: junior high to high school and high school to college. We have completed two years of data collection on approximately 80 students, and we now report on our emerging analytical framework and present outlines of specific individual cases of students’ experiences and adjustments. This work follows closely upon our presentation at PME-NA 2000 (Smith et al., 2000) with a stronger focus on our methodology.

We draw upon both cognitive and situated perspectives on learning and development to conceptualize and study students’ mathematical transitions. We assume that students carry a diverse body of conceptions and feelings from their prior mathematical experiences in and out of school. Specifically, we presume that students bring and are oriented by the following sorts of cognitions: (1) emergent goals for their future; (2) mathematical knowledge (e.g., procedural skills and understandings of basic concepts); (3) beliefs and attitudes about mathematics and themselves as learners; and (4) strategies and plans for achieving personal goals. However, students’ experiences are not solely individual, and thinking and learning are not located exclusively inside their heads. What students see as salient, and what they do to learn mathematics (or not) depends on a wide range of social, cultural, and institutional factors.

We distinguish between general developmental changes, such as increased freedom and responsibility for learning that typically accompany the move from junior high to high school and high school to college, and mathematical discontinuities and transitions. We use the term mathematical discontinuities to refer to marked differences between students’ prior notions and their current perceptions of how they are expected to think and act mathematically. Mathematical transitions are students’ responses to those discontinuities: How they consciously experience and understand the difference(s), how they respond (or not), and how they understand the results of these responses.

Methods

This project was designed to study these mathematical transitions in one geographical context (south central Michigan). Two Standards-based curricula, Connected Mathematics Project (CMP) materials for junior high (Lappan, Fey, Friel, Fitzgerald, & Phillips, 1995) and the Core-Plus Project materials (CPMP) for high school (Hirsch, Coxford, Fey, & Schoen, 1996), were developed in Michigan and have been adopted widely throughout the state. The University of Michigan has implemented Harvard Consortium materials (Hughes-Hallett, Gleason, et al.,
in all calculus and pre-calculus classes, and Michigan State University has retained a more traditional calculus and pre-calculus curriculum. So the region provides a promising context for studying students' mathematical transitions. We “follow” high school and college students across 2.5 years of mathematics work in fundamentally different curricula—assessing their performance, learning of key content, daily experience, beliefs, and goals.

We examine mathematical transitions at two high schools and two universities. At one high school and university, students move from traditional curricula to those associated with current reforms. At the other high school and university, the move is opposite, from reform to more traditional curricula. We are following approximately 20 volunteers at each site. This 2 x 2 research design is summarized below.

We combine a systematic program of classroom observation (to assess the enacted curriculum) with a broad program of assessment of students’ experiences, learning, and personal goals. We assess each student on six dimensions: (1) achievement (course grades and grade point averages [GPA]), (2) learning of key ideas, (3) daily course-related experience, (4) career and educational goals, (5) beliefs about self and mathematics, and (6) strategies for coping with changes and challenges. Fundamental tools are self-report (journals and e-mail), survey instruments, and individual interviews (2-3 times each semester). This rich and diverse corpus of data is compiled into individual case reports describing and summarizing each student's experience.

Results

In our analyses of the data thus far, individual differences have predominated over easily recognized general patterns, even within a particular site. If we had to summarize our findings in one sentence, our choice might be, “Different students react to curricular shifts differently in each school context.” In looking at the data collected so far, we are in the process of formulating a decision rule to determine which students have or are currently experiencing a mathematical transition. This decision rule includes four categories that summarize changes students have experienced during the curricular shift: (1) student achievement, (2) significant differences students notice and report, (3) changes in disposition towards mathematics, and (4) changes in learning approach. Our challenge has been to determine what levels of change in these four areas constitute a mathematical transition. At present, our decision rule states that we will characterize a student as having experienced a mathematical transition if we find significant change in two or more of these four categories.

In previous papers, we have written more extensively about the results from our first year of data collection. In this paper, we have chosen to step back from our results and consider our methods a bit more closely. As mathematical transitions have not been previously studied in the educational literature, we have had to define and operationalize constructs necessary for our analyses. In this paper, we focus particularly on the way in which have answered the question, "What is a mathematical transition?" We look closely at the four components of our decision rule, giving examples and raising issues about what our framework captures and perhaps obscures. We then bring the four components together and apply our decision rule to a small number of students' cases, and again ask what our framework allows us to capture and what it perhaps fails to illuminate.

Mathematics achievement

In characterizing students' mathematical transitions, one obvious place to start would be to look at students' grades in their mathematics courses. Since achievement is one measure of students' learning, it is reasonable to consider that this construct should provide some sense of whether a mathematical transition has been experienced. However, our data suggests two trends of student achievement, one emerging from the college site and one from the high school site, which question whether (and how) grades should be used as a measure of mathematical transitions.

The first trend that we noticed is at the college sites: students' grades often drop in the move from high school to college (Star, 2001). For example, at the U-M site, 17 of the 19 students in our first year sample had drops in their overall GPAs in their first semester. At the college sites, it seems likely that a drop in grades may be more related to general developmental changes than those associated with mathematics. Thus, if a drop in grades were considered to be an indicator of a mathematical transition, it is likely we would falsely identify many college students as having experienced a transition (e.g., "false positives"). A second trend that we noticed at the high school sites is that there is typically very little change in students' grades as they move from middle to high school (Jansen & Herbel-Eisenmann, 2001). High performers in middle school tend to remain high performers in high school, and vice versa. Thus, using achievement as an indicator of mathematical transitions might lead us to fail to...
identify many high school students who did in fact experience a mathematical transition (e.g., "false negatives"). These two observations suggest that achievement should be approached with caution as an indicator of mathematical transitions. While we feel that grades do indicate something, achievement data do not portray as clear a picture as one might initially expect. Further support for caution in the use of achievement has emerged at our high school sites, where we have found that teachers' grading practices vary from classroom to classroom and often incorporate features which are only marginally related to student learning of mathematics (e.g., organization and neatness; Jansen & Herbel-Eisenmann, 2001).

With these two observations in mind, we devised a strategy that enables us to use achievement as an indicator of potential mathematical transitions but at the same time minimizes the number of false positives and negatives. This strategy involves comparing students' math grades with their overall GPAs, across multiple time periods, in order to identify students who experienced "significant" change in their math grade as compared to their overall grade, where "significant" will be defined below. The importance of this type of comparison between math grade and semester GPA is illustrated by the example of two U-M students, Jack and Teresa. Both Jack and Teresa did very well in high school, both in math classes (grades of 4.0 for both) and in their overall GPA (4.0 and 3.9, respectively). And both struggled in their first semester of U-M math, with each earning a 2.3, or a drop of 1.7 and 1.6 points, respectively. However, Jack's overall GPA fell to 2.6, while Teresa's GPA only dropped to 3.5. From our perspective, Jack represents a case of someone who experienced a general developmental change: Upon coming to college, all of his grades (including math) suffered a major drop. In contrast, in Teresa's case, something unusual seemed to be happening in math class; she did relatively well in all of her classes except for math. Cases such as Teresa, where grades indicate that something noteworthy may have been happening around her math class, are worth further investigation. In cases such as Teresa, there is a mismatch between the pattern of achievement in overall GPA and in mathematics; this mismatch is the criterion that we will use to indicate which students should be flagged. More specifically, a "significant" achievement change occurred when the change in students' math grade differed from the change in overall GPA by more than 0.5 grade points (on a 4-point scale). In the example above, Jack's GPA dropped 1.7 points (from 4.0 to 2.3), and his math grade dropped 1.3 points (4.0 to 2.6), so the difference between these two grade changes is 0.4 points. Teresa's GPA dropped 0.4 points (3.9 to 3.5), while her math grade dropped 1.7 points (4.0 to 2.3), for a difference of 1.3 points. By our definition, Teresa has experienced a significant change in achievement, while Jack has not.

In addition, our extensive conversations about grades have lead to a search for different ways to measure students' learning of mathematics content. One alternative that we are currently considering makes use of the problem-solving interviews that were conducted at each site. These interviews give us a window into the ways that students come to understand the mathematics that they are studying. At present, our thinking into the ways in which problem-solving interviews can be analyzed is preliminary.

**Significant differences**

In order to understand students' experiences as they moved between traditional and standards-based curricula, we asked them in interviews to describe what they noted as different between their previous and their current mathematics classes. Often the interviewer began with an open prompt like, “What, if anything, do you feel is different in math class this year?” Some students had a lot to say in response to this question, while others simply reported, “It’s just different.” So we often probed with questions that prompted students to consider possible differences in specific categories, such as the textbooks or the homework problems.

As we began our initial analysis of the differences reported by students during the first year of the project, we found that the differences could be classified based on their origin. Three categories emerged from this initial analysis: teachers/teaching, curriculum, and site policy. We define Curricular differences as those differences that had their origin in the written or intended curriculum. For example, differences noted by students in the types of problems presented in the textbooks were coded as Curricular differences. These differences also included observations about the difficulty of the content. For example, Kevin, a student at PHS, noted that the mathematics expected of him in moving from a reform to a traditional classroom was more complex, but focused on computation over connections between representations in mathematics: “This year is more about multiplying the numbers and last year was more about inserting them into the equation and getting them to where they like bond between two ways of doing it.” Teachers/Teaching differences represent those differences that had their origin in the teaching or the teacher’s personal choices. These differences included those reported by students like Pablo at MSU, who remarked that one of the instructors "just didn’t care" about students or his teaching. As another example, a PHS student, Bethany, did not like that her 9th grade teacher presented a single method for solving problems; she liked choosing the way to solve the problem that she preferred. Site Policy differences represent those differences that had
their origin in the decision of the sites' mathematics departments rather than individual teachers. For example, at U-M, the Mathematics Department, not the curriculum (the Harvard Calculus materials) or the instructors, developed the course component of group homework. Thus, when students noted this as a difference from high school, it was categorized as a difference in Site Policy.

Some differences by their nature consisted of complex interactions between categories, and, as such, were classified in the intersection of two or more of the categories. For example, at PHS, the use of graphing calculators depended on both the curriculum and the teacher’s classroom decisions. As a result, differences in the use of the graphing calculators at PHS were classified as an interaction between Curriculum and Teachers/Teaching. However, at MSU, the Mathematics Department determined where and how graphing calculators were used; in some courses, graphing calculators were not allowed on the exams. Thus, noted differences in the use of graphing calculators at MSU were coded as differences in Site Policy.

Students at every site were able to report at least a few differences that they observed. In fact, participants may have simply reported differences because they were asked to do so, rather than because those differences were particularly salient. Thus, our next step was to determine for which students the differences between curricula had been significant. Differences were deemed to be significant to a particular student if he/she (a) reported them spontaneously, or (b) repeatedly mentioned them, or (c) gave them particular emphasis or attributed particular impact to them. Thus, the principal objective here was to require some indication of importance for and/or impact on the student.

Determining whether or not a student had reported a difference with sufficient emotion or emphasis to be deemed “significant” proved to be challenging. The emotion with which students communicated differences could not be easily determined from the transcripts. Often we had to rely on the interviewer’s memory of students’ responses during the interview, which made reliability analyses, both within and across sites, difficult to conduct. In addition, since interviewers both had a relationship with the participants and regularly requested information on the differences they noted, there is a potential for bias in judging the emotion and intensity of students’ statements. Due to these problems, the frequency of reported differences, rather than the emotion accompanying statements, often determined the significance of the differences in our analysis.

Disposition towards mathematics

Another factor in our model of mathematical transitions is whether students have experienced a change in their disposition towards mathematics. The word “disposition” is used colloquially to refer to an attitude; thus we are interested in capturing changes in students’ attitudes toward mathematics. More specifically, we use this factor to refer to students’ interest in, attitudes about, beliefs toward, motivation to succeed in, and enjoyment of mathematics. Data on attitudes has come primarily from interviews; data on beliefs has come from survey data. Changes in attitude or belief may be accompanied by changes in actions, such as a decision to take more mathematics, but such changes are neither necessary nor sufficient. At present, our criteria for “significant” change in attitudes and beliefs are not clearly and objectively defined but rather are decided through discussion of individual cases by the project team. In the absence of convincing data, we score the disposition factor as “no change.” Thus, our current scheme attempts to provide a label (yes or no) as to whether a student experienced a change in mathematical disposition. In addition, for those students labeled “yes”, we try to characterize the disposition change as either becoming more positive or more negative.

For example, consider two students’ mathematical dispositions from PHS: Kevin and Stacy. Both of these students exhibited a change in their attitudes toward their mathematics courses, but in different ways. Kevin was in the advanced track of mathematics courses. His disposition became more negative in high school: he found his high school mathematics classes “more boring” than middle school classes, and he wanted to be invited by the high school teachers to become more involved in class. In contrast, Stacy, who was in the lower track of mathematics courses, had a positive change in her disposition. She expressed a preference for high school mathematics due to fewer story problems, more equation solving, and a more “direct” approach to the mathematics.

At present, we are grappling with two issues related to how we operationalize disposition toward mathematics: (1) differences between our conceptualization and that of the literature on “mathematical disposition;” and (2) the challenges of rigorously assessing students’ beliefs about the discipline of mathematics. With respect to this first issue, other characterizations of mathematical disposition make more explicit connections to the discipline of mathematics and also articulate ways in which a mathematical disposition has features that are unique to mathematics, as opposed to dispositions in other content areas (e.g., Yackel & Cobb, 1996). What would these alternate conceptualizations of mathematical disposition afford our analysis? A second challenge to our notion of mathematical disposition is methodological: how do we assess students’ dispositions? We are currently working on
the development of a more explicit method for qualitatively assessing students’ beliefs about mathematics as expressed in interview data, and we hope that it can supplement what we have already learned about disposition from our existing, less formal methods and from survey data. Clearly, the construct of “mathematical disposition” is a valuable one in understanding students’ transitions; however, we have come to realize that assessing disposition will require more careful work.

### Learning approach

The fourth and final category that plays a role in our decision rule for determining mathematical transitions is the student’s approach to learning mathematics. By learning approach we mean the autonomous actions that a student undertakes to learn mathematics. Thus a change in learning approach means that the student changes the kinds of actions they undertake to learn mathematics. The qualifier “autonomous” is included to distinguish actions that a student freely undertakes from those that are more or less mandated by teachers’ decisions in the classroom. Students’ approach to learning mathematics is how they organize themselves within the zone of their own autonomous activity; this zone may include actions undertaken in the mathematics classroom, but it should also include actions undertaken on the students’ own time (e.g., individual study strategies) and activities that involve others as resources (e.g., going to see the teacher or professor away from class time, seeking tutorial help, and studying with peers). Some commonly reported changes in students’ learning approach include changes in how the textbook was used as a course resource; the use of discussions outside class with the teacher, classmates, and friends as resources; and the sustained use of help-rooms and math help centers, particularly at the college level.

Determining what constitutes a “significant” change in a student’s learning approach has been a difficult decision to make. We have decided that a significant change in this category is indicated by the presence of either (1) experimentation with new learning strategies, (2) laying aside old learning strategies, or (3) the use of old strategies in new ways. Because our insight into students’ approaches to learning is dependent on their self-report, we look primarily to interviews and the journals for evidence of significant change.

One issue that potentially complicates our analysis is the relationship between learning approach and changes in achievement. For most students (e.g., those who are concerned about earning high grades) autonomous changes in learning approach tended to be initiated in response to a drop (or a perceived potentially imminent drop) in achievement. When a student is dissatisfied with her math grade or is worried that her grade might drop in the near future, she may attempt new learning strategies in order to remedy the situation. Thus, a change in a student’s learning approach may only be indirectly linked to a change in curriculum, in that the curricular changes resulted in the grade drop. Other factors share this indirect link, such as a more demanding teacher or additional extracurricular activities, and these alternative explanations could be just as responsible for the student’s grade drop as the curricular shift. As a result, in some cases, we have struggled to find evidence that students’ changes in learning approach came as a direct result of the curricular discontinuities.

A second issue with the learning approach category is in our requirement that changes in actions are “autonomous” -- that is, freely undertaken by the student rather than mandated by the teacher or by other outside influences. Particularly at the high school sites, we have found that changes in students’ learning approaches are almost always affected (and in some cases, initiated) by factors other than the student. For example, Stacy, a ninth grade student at PHS, used a strategy in her high school math class that was new to her: reading the textbook before attempting homework problems. She began using this strategy because her high school textbook provided worked examples within each section, and she found reading through the worked examples before starting her homework to be helpful. Stacy did not use this strategy in middle school, most likely because her middle school math textbook did not provide worked examples. Should this change in learning approach be considered autonomous when she could not have used this strategy in the past, even if she wanted to, due to the different structures of the texts?

As was the case with mathematical disposition, we feel strongly about the importance of the category of learning approach in understanding students' experiences during a mathematical discontinuity, but we continue to search for ways to carefully and rigorously determine the "significance" of the changes that students in our sample report.

### Illustrative cases

With the preceding description of the four categories in our mathematical transition decision rule, we now briefly describe 4 cases of mathematical transitions to illustrate some of the diversity in our participants’ experiences. We have selected them from the nearly 80 in our corpus, not because they were typical or representative of experience at any site, but to illustrate different senses or “types” of transition. We include them to
flesh out our basic claim in greater detail: Mathematical transitions in the context of current curricular reforms take different forms. Recall that our decision rule states that a student who had "significant" change in at least two of the four categories is considered to have had a mathematical transition. Each of the four students described below meet this criterion; in fact these four students could be considered to have had a "positive" transition, as each is happier in his or her current curriculum as compared to the previous one. Yet, despite these surface similarities, these four students represent very different flavors of mathematical transitions.

**Stacy**, a student at PHS, experienced a mathematical transition by virtue of showing significant change in her disposition toward mathematics and also by noting significant differences between her previous and current math class. She showed no change in her achievement in math class, as she was a consistent “A” student in both her reform (CPMP) junior high and more traditional high school mathematics classes. But despite her success, she lacked confidence in her ability. The elements she valued in her high school math classes reflected this: She preferred clearly stated procedures for solving problems, tests with content clearly specified in advance, “notes” to guide work on those tests, and teachers who kept “order”. The “story problems” in junior high were “harder” because the solution method was often unclear. Stacy's 9th grade experience in Algebra I increased her interest in mathematics; she was drawn to the explicit structure of equation solving. Stacy recognized key differences between CPMP and Algebra I (e.g., non-routine, contextual problems and greater student ownership for mathematical thinking in the former). Thus, movement into a more traditional content improved her disposition toward the subject -- perhaps because she felt more secure and confident in her new setting.

**Mimi**, a student at LHS, experienced a mathematical transition since she noted significant differences and also changed her approach toward learning math. As with Stacy, she showed no change in her achievement, as she was an above average student in junior high and high school, in math and in other subjects. She found that the CPMP curriculum required her to think about what problems were asking for, rather than to remember a formula or general solution. This change was challenging at first, but, in her first year of high school, she came to feel that she could understand math—a state that did not generally follow from prior learning via memorization and practice, as was the case in junior high. What she felt she needed to do in order to succeed in math changed; before, memorization and practice allowed her to earn As, but in high school, it became much more important to read and try to understand what problems (particularly word problems) asked her to do. So her important differences were closely tied to changes in how she tried to learn and succeed in mathematics. She preferred understanding what she learned, but it remains unclear whether her disposition toward mathematics has changed significantly in high school.

Although **Matt** looks identical to Mimi from the standpoint of our decision rule (he noticed significant differences and also had a change in learning approach), his experience is quite different from hers. Matt came to MSU from a CPMP high school program and placed into the first semester calculus. He described himself as an undisciplined high school student, doing only what was required to get good grades. He liked elements of his 3-year experience with CPMP but found the pace too slow. When his high school teacher slowed down for other students, he used class time to do his homework. When Matt landed in Calculus I at MSU, he felt lost in the “foreign language” of the traditional curriculum and failed the first test. Shaken, he went to see his faculty instructor who assured Matt that he had been placed in the right class and diagnosed Matt's problems in terms of some weak content and poor learning practices. Under his instructor's guidance, Matt dramatically changed the way he approached the work of Calculus and was eventually successful, earning a grade of 3.5 (on a 4-point scale). Matt saw differences between CPMP and MSU Calculus, both mathematically (e.g., contextual vs. purely symbolic problems) and more generally (e.g., faster pace in college). His approach to learning mathematics changed radically in Calculus and, in fact, led him to expect mastery of the content of his other classes in a way that he never had in high school. Matt's case illustrates that "difficult" transitions can have very positive effects.

**Lissie**, a student at U-M, experienced a mathematical transition by virtue of a significant change in achievement and in her approach to learning, as well as noticing significant differences. Lissie was an "A" student in all of her classes in high school. Although math was not her favorite subject in high school, she enjoyed considerable success in her math classes and particularly valued the individual attention from her teachers that came with attending a small school. Upon her arrival at U-M, she immediately disliked the reform calculus program. She had strong, negative feelings about her instructor; she felt the course moved too quickly; and she disliked the writing and the group work that were integral to the approach. Lissie's grades suffered, yet she was determined to succeed. She hired a tutor and, with his assistance, began devoting a tremendous amount of time and energy to her math class. The changes Lissie implemented in her learning approach ultimately allowed her grade to stabilize. Lissie eventually came to feel that she understood Calculus much more thoroughly and deeply than she did in high school, and she attributed her greater understanding in part to those features of the course that she had initially hated: having to work in groups and do a lot of writing.
Discussion and conclusion

The goal of the work presented in this paper is to understand and describe students' perspectives and experiences as they move between traditional and reform mathematics curricula. We have hypothesized that such fundamental shifts in mathematics curricula serve as potential sites for students to experience mathematical transitions. In this paper, we provide our current answer to the question, what is a mathematical transition? Our conceptualization depends on four components: achievement in mathematics, significant differences that students notice and report, disposition towards mathematics, and approach to learning mathematics. While we feel each component lends strength to our framework, we have also identified challenging issues involved in determining each component's significance for our students. It is our hope that being explicit about the challenges we have faced in our analysis will inspire some feedback about the framework itself. Are these the “right” four factors to use? Are there other factors that we should consider? What do these four factors lend us in conceptualizing a mathematical transition? And what, if anything, does our choice of factors obscure? These are some of the issues we are grappling with as we continue our data collection and analyses.

References


Table 1

Research Design of the Mathematical Transitions Project

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<td>Logan High School (LHS) Various texts -&gt; CPMP</td>
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