Flexibility in the use of mathematical procedures

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**Overview.** Doing mathematics involves the use of mathematical procedures. Throughout the K-16 school mathematics curriculum, particularly after the introduction of symbolic algebra, students learn and are expected to use procedures. However, as reports assessing US mathematics achievement can attest (e.g., Beaton et al., 1996; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999), many US students merely develop rote knowledge of procedures. As an educational outcome, rote knowledge is problematic in that it is resistant to transfer, error-prone, and inflexible (e.g., Anderson & Lebiere, 1998). Rote knowledge has direct links to low academic achievement in mathematics – students with rote knowledge have great difficulty on both near- and far-transfer problems (Hiebert & Carpenter, 1992), particularly in the domain of algebra (Blume & Heckman, 1997; Kieran, 1992). Instead of rote knowledge, our educational goals for the use of mathematical procedures involve *flexibility*, including the ability to select appropriate procedures for particular problems and modify procedures when conditions warrant (National Research Council, 2001). However, such a non-rote outcome for procedural learning has not been adequately considered in prior research, despite its clear potential for improving student achievement in mathematics.

This proposal seeks funds to continue ongoing research on the development of mathematical flexibility. Funds will be used to support graduate student research assistants who will engage in several kinds of activities, including the analysis of existing data and
collaboration on at least one paper to be submitted for publication. Financial assistance will ultimately lead to a resubmission of a recently rejected application for a NSF CAREER grant.

**Rationale.** The acquisition of procedural skill has been extensively studied in cognitive psychology, particularly in the past 40 years. There is convincing evidence that skills are learned through practice (Anderson & Lebiere, 1998; Fitts, 1964). A learner initially executes a skill step by step, through a slow, methodical process of executing factual instructions. With practice, the learner is able to perform the skill more quickly and efficiently. With continued practice, a learner is able to perform the skill even more quickly and efficiently, but at the cost of increasing inflexibility and rigidity in behavior, with the endpoint being a state of automaticity (Anderson, 1992; Schneider & Shiffrin, 1977; Shiffrin & Schneider, 1977).

While the rapid, efficient, and automatic execution of a procedure is certainly one desired outcome, it is not the only important goal from an educational standpoint. Automatic executors are often derailed when they encounter problems that require very slight modifications of well-practiced procedures. While students with automatized knowledge may know a set of procedures very well, they may have great difficulty applying these procedures in problems or situations that are not identical to the settings in which they were learned. If student achievement in mathematics is to be maximized, it is necessary to foster the development of students who not only can execute a procedure automatically but also flexibly. Those with more flexible knowledge, defined here as (a) possessing knowledge of multiple solution procedures, and (b) possessing the ability to select appropriate procedures for completing particular kinds of problems (Blöte, Klein, &
Beishuizen, 2000; Blöte, Van der Burg, & Klein, 2001; Carroll, 2000; Hiebert & Wearne, 1996; Resnick, 1980; Resnick & Ford, 1981), can solve problems that automatic executors cannot, particularly near- and far-transfer problems. Thus, flexibility has great potential to lead to gains in student achievement.

The recently issued “Adding It Up” report from the National Research Council (2001) echoes this sentiment by identifying procedural fluency as one strand of mathematical proficiency, which they define as, “knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently” (p. 121). Automatic execution fails to address all of the goals set forth by the “Adding It Up” report, particularly with respect to appropriate use of procedures and flexible performance. Mathematically proficient students should be able to move in and out of automatic execution as experts do (Ericsson & Charness, 1994; Ericsson, Krampe, & Tesch-Romer, 1993); they should know “what to do and why” (Skemp, 1978, 1987); they should know “why [the procedure] has the steps it has, why it has certain conditions on its applicability, and why it succeeds when those conditions are met and the steps are followed accurately” (VanLehn, 1990, p. 38).

Numerous interventions have been undertaken to foster the development of more flexible learning outcomes for procedures. A common assumption among these interventions is that procedural knowledge must be connected or associated with more principled, “conceptual” knowledge (e.g., Hiebert & Lefevre, 1986). Connections to conceptual knowledge are fostered through a variety of activities, including the use of concrete manipulatives (e.g., base-ten blocks in elementary school), the generation of non-standard solution algorithms by students, and the in-class sharing and comparing of
student-generated solution strategies. Promising results have been obtained using such activities, particularly in the domain of arithmetic (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fennema et al., 1996; Fuson et al., 1997; Hiebert et al., 1996).

Surprisingly, despite the promise of these approaches, little or no effort has been devoted to applying them to the learning of algebra procedures in high school. Arguably, flexibility in the use of mathematical procedures is paramount in algebra, given that procedures play a particularly prominent role in mathematics beginning with algebra (Kieran, 1992; Lewis, 1981; Thorndike et al., 1923). Algebra historically has represented students’ first sustained exposure to the abstraction and symbolism that makes mathematics powerful (Kieran, 1992); the symbolic procedures in algebra enable students to consider relationships, variable quantities, and situations in which change occurs (Fey, 1990). In addition to its central role in the discipline of mathematics, algebra also serves as a critical “gatekeeper” course, in that earning a passing grade has become a de facto requirement for many educational and workplace opportunities. Some have gone so far as to refer to algebra as the new civil right (Moses, 1993). As such, algebra is a particularly important venue in which procedural flexibility should be developed. Regrettably, students’ failure to do so has been well documented in national and international assessments (Beaton et al., 1996; Blume & Heckman, 1997; Lindquist, 1989; Schmidt et al., 1999). For example, data from the National Assessment of Educational Progress indicates that many 12th graders can solve only the most simple and routine algebra tasks (Blume & Heckman, 1997).
Clearly much work remains to be done in pursuit of procedural flexibility in algebra for all students. This proposal seeks financial support to continue ongoing work in this area, leading to both publications and applications for external funding.

**Description of prior work.** Three studies investigating the development of procedural flexibility have been conducted to date by the principal investigator.

In Star (2001), an instructional intervention hypothesized to improve students’ flexibility was experimentally evaluated. 36 6th grade students participated in five one-hour problem-solving sessions in which they learned how to solve linear algebra equations. Treatment students participated in the “alternative ordering” task, in which participants were asked to re-solve previously completed problems but using a different ordering of steps. Participating in this treatment led to reliable gains in students’ flexibility. A paper describing these results is currently under review at the *Journal of Educational Psychology*.

Building on this initial work, and with funding from the Dean’s Office as part of a faculty start-up package, two follow-up studies were conducted in Summer 2003. In Study 1, 134 6th grade students participated in five one-hour problem solving sessions on linear equation solving. In order to replicate the results from Star (2001), the alternative ordering treatment was again evaluated. And as additional treatment (as part of a 2 x 2 design), some students were provided explicit, direct instruction on equation solving strategies. Results confirmed the effectiveness of the alternative ordering task on improving students’ flexibility. In addition, explicit strategy instruction was also found to significantly improve flexibility. A paper describing the results of Study 1 was recently presented at AERA (Star, 2004); a publishable manuscript on this data is nearing completion.
An additional study, Study 2, was conducted in Summer 2003 to qualitatively examine the development of mathematical flexibility. 23 6th grade students participated in five one-hour problem solving sessions on linear equation solving, similar to Study 1 and Star (2001). These 23 students were individually interviewed on videotape as they solved problems. Students were asked several kinds of questions, including prompts relating to planning (“Before you begin working on this problem, tell me how you think you will solve it?”), post-solving reflection (“Looking back on this problem, can you go over the steps that you used to solve this equation?”), and the process of equation solving more broadly (“Is there a way to solve equations that you use a lot?”, “Is there a way that always works?”, “Are there some problems that are easier to solve than other problems?”, “How do you know when you’ve solved an equation in the “best” possible way?”). Analysis of students’ responses to these prompts is underway; it is anticipated that several papers can be written from the Study 2 data. Small subsets of this data were used in a paper and a poster that were submitted for the October 2004 annual meeting of North American chapter of International Group for the Psychology of Mathematics Education (PME-NA). (The paper, written in collaboration with Jagdish Madnani, a mathematics education doctoral student in Teacher Education, was recently accepted. The poster, written in collaboration with Marcy Wood, a mathematics education doctoral student in Teacher Education, is still pending.) There is still a great deal of analysis to be done on the remainder of the Study 2 data; completing this analysis is the primary focus of this proposal.

**Research activities under the current proposal.** All of the studies described above significantly advance current understanding of the development of mathematical
flexibility; nevertheless, much work remains to be done in this area. Unfortunately, I have had difficulty getting external funding to continue my research. In 2002-2003, I submitted grants to the National Science Foundation (CAREER program), the Department of Education (Institute for Education Sciences RFA on Cognition and Student Learning), National Institute of Child Health and Human Development (RFA on Student Cognition), and MSU’s Intramural Research Group Program. Despite favorable reviews from each agency and encouragement to reapply, all of my proposals were denied. Reviewers of my proposals have emphasized the importance of establishing a “track record” of published papers on procedural flexibility in order to improve my chances of receiving funding.

Following this advice, it is clear that completing the analysis and write-up of Studies 1 and 2 from Summer 2003 is a top priority. I anticipate that the Study 1 write-up will be completed in Summer 2004. However, given the extensive data analysis needed for Study 2, I seek the assistance of research assistants with whom I can collaborate. If this proposal were funded, undergraduate and graduate students would participate in the continued transcription, coding, analysis, and write-up of the 23 problem solving interviews in Study 2, leading to the completion of at least one paper from this dataset.

In addition, this proposal would lead to a resubmission of a CAREER grant to NSF, due in late July 2005. The CAREER grant will propose a series of studies further investigating procedural flexibility, building on the results from Star (2001) and Studies 1 and 2 from Summer 2003. (Other than financial support, I also seek additional assistance in proposal writing for the CAREER resubmission, including research design and budgeting help.)
References


