A Translog Cost Function Analysis of U.S. Agriculture, 1939–77

Subhash C. Ray

The translog cost function provides a convenient framework for analyzing U.S. agricultural production in a multoutput context. Treating crops and livestock as two distinct outputs, this study utilizes standard results of neoclassical duality theory to obtain measures of pairwise elasticities of substitution between inputs, price elasticities of factor demands, and the rate of Hicks-neutral technical change. Results obtained from joint GLS estimation of parameters of cost and share equations indicate a declining trend in the degree of substitutability between capital and labor. Price elasticity of demand for all inputs increased over time. The measured rate of technical change was 1.8% per year.

Key words: elasticities of substitution, Hicks-neutral technical change, neoclassical duality, translog.

Over the past four decades major changes have occurred in U.S. agriculture. A new era in agricultural production in the United States began in 1939. World War II gave a boost to agriculture still bogged in the depression. Productivity increases over the period 1939–77 have been quite impressive. Measured crudely by the ratio of the indices of output and input, the productivity index (1967 = 100) increased from 59 in 1939 to 119 in 1977. To study this phenomenon, a multi-input translog cost function incorporating Hicks-neutral technical change (HNTC) was specified. Annual time-series data for the United States was used to estimate the model. Neoclassical duality theory provides an approach for computing the pairwise elasticities of substitution between inputs for each year as well as the annual (own- and cross-) price elasticities of demand for inputs.

The Translog Cost Function for Multiple Outputs

The \(m\)-output-\(n\)-input translog cost function incorporating Hicks-neutral technical change (HNTC) is

\[
\ln C = \ln k + \sum_{i=1}^{m} a_i \ln q_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} \ln q_i \ln q_j + \sum_{r=1}^{n} b_r \ln w_r \\
+ \frac{1}{4} \sum_{r=1}^{n} \sum_{s=1}^{n} f_{rs} \ln w_r \ln w_s + \sum_{r=1}^{n} \sum_{s=1}^{n} g_{rs} \ln q_r \ln w_r + hT.
\]

Here \(C\) is cost, \(q_i\) is output of product \(i\), \(w_r\) is the price of input \(r\), and \(T\) is an annual index of time. This translog (dual) cost function can be regarded as a quadratic approximation to the unspecified "true" cost function. In this context, \(d_{ij} = d_{ji}\) and \(f_{rs} = f_{sr}\) for all \(i, j, r, s\), and \(r\).

Any sensible cost function must be homogenous of degree 1 in input prices. In the translog function (1), this requires that \(\sum_{r=1}^{n} b_r = 1\), \(\sum_{s=1}^{n} f_{sr} = 0\) \((r = 1, 2, \ldots, n)\) and \(\sum_{i=1}^{m} g_{ir} = 0\) \((r = 1, 2, \ldots, n)\). Essentially, the same set of restrictions on parameters follows from the "adding up" requirement of the factor shares.

The translog form is flexible because specific features of technology (like returns to scale or homotheticity) may be tested by examining the estimated model parameters.

Subhash C. Ray is a visiting lecturer, Department of Economics, University of California, Santa Barbara.

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Specifically, if the technology is homothetic, the dual cost function is multiplicatively separable in output quantities and input prices. The cost function \( C = C(q, w) \) is of the form \( h(q) \cdot t(w) \), where \( q \) and \( w \) are the vectors of output quantities and input prices. In the translog case, this requires that \( g_{ir} = 0 \) (for all \( i \) and \( r \)) so that the quadratic interaction terms between output levels and input prices should disappear.

Cobb-Douglas, or some other exact specification, implies constant elasticities of substitution between inputs and homotheticity of technology. Generalized cost functions like the translog allow us to test for specific characteristics of technology. They are not required as prior assumptions. This largely explains the increasing popularity of generalized cost (and production) functions, of which the translog is an example. Berndt and Christensen; Berndt and Wood; Christensen, Jorgenson, Lau; and Binswanger used translog models. Other forms are analyzed in Diewert, Hanoch, and Phillips. Multi-output translog applications are found in Darrough and Heineke, Fuss and Waverman, and Cooley and Bothwell. Hasenampf estimated the multi-output production technology of U.S. railroads using several functional forms.

In view of the widespread use of the translog specification in recent years, a note of warning is in order. As a quadratic approximation to the unspecified "true" cost function, the translog has some limitations. In a recent Monte Carlo study by Guilkey and Lovell, a translog approximation to a CES production function performed rather poorly in estimating the elasticity of substitution between inputs, when the true parameter was as high as 3.0. Therefore, the results reported here and elsewhere should be used with caution. It is difficult to define returns to scale in the multioutput case. However, in the special case of linear homogeneity of the cost function in the output levels, we may assert constant returns to scale (CRS). The conditions for CRS in equation (1) are \( \sum_{i=1}^{n} a_i = 1, \sum_{i=1}^{n} d_{ij} = 0, \) and \( \sum_{i=1}^{n} g_{ir} = 0 \) for all \( j \) and \( r \). (See Darrough and Heineke or Berndt and Christenson for restrictions implying homotheticity, CRS, and other features of the technology.)

One problem with multi-output, multi-input cost functions is that, even for a few outputs and inputs, the number of parameters to be estimated is large. For an \( m \)-output, \( n \)-input model with matrices \( (d_{ij}) \) and \( (f_{sr}) \) symmetrical, one needs to estimate \( \frac{1}{2}(m + n)(3 + m + n) \) parameters. This does not include the intercept and the rate of Hicks-neutral technical progress. For example, in this case with two outputs and five inputs, we must estimate thirty-seven parameters. In general, it is difficult to obtain a sample large enough to estimate the full cost function. Moreover, with time-series observations needed on so many variables, we are likely to encounter severe multicollinearity. Thus, estimating the cost function as a single-equation model even with restrictions imposed for linear homogeneity in the input prices may be either impossible or inappropriate.

Estimating the full dual system (i.e., cost and share equations together) leads to much higher efficiency. Such joint estimation can compensate for the information inadequacy in the equation (1) alone. From the translog cost function and Shephard's lemma, we derive the input share equations:

\[
(2) \quad s_i = b_i + f_{i1} \ln w_1 + \ldots + f_{in} \ln w_n + g_{i1} \ln q_1 + \ldots + g_{im} \ln q_m,
\]

where \( (i = 1, \ldots, n), s_i = w_i/x_i/C, \) and \( x_i \) is the quantity of the \( i \)-th input. These shares must add up to 1. This is true for all prices and outputs. It requires \( \sum_{i=1}^{n} b_i = 1, \sum_{i=1}^{n} f_{ir} = 0, \) and \( \sum_{i=1}^{m} g_{ir} = 0 \) (for \( r = 1, 2, \ldots, n \)). Note that these are the same conditions implied by linear homogeneity of the cost function in input prices. If we additionally assume marginal cost pricing for the outputs, we obtain the relations

\[
(3) \quad y_k = \frac{\partial \ln C}{\partial \ln q_k} = \frac{\partial C/\partial q_k}{q_k/C} = p_k q_k/C
\]

for each output \( k \). This leads to the "revenue share" equations, as follows:

\[
(3) \quad y_k = a_k + d_{k1} \ln q_1 + \ldots + d_{km} \ln q_m + g_{k1} \ln w_1 + \ldots + g_{kn} \ln w_n,
\]

where \( (k = 1, \ldots, m) \).

While few people will disagree with the cost-share equations, the revenue-share equations in (3) may not be readily accepted. Some might argue that agricultural products are inelastically supplied and government support programs tend to repudiate the marginal cost-pricing assumption. This could be a valid objection, but the way out dependent variables
are measured in equation (3) will reduce or eliminate this problem. Instead of using price indices of agricultural products to obtain \( p_k q_k / C \), we directly measured the numerator \( (p_k q_k) \) as market sales revenue plus government payments. It is assumed that government payments are designed to help maintain farm income by equating effective average revenue to marginal cost. This is illustrated by figure 1. Assume that farmers expect a price equal to \( BD \) and produce output equal to \( OD \). At this level price would equal marginal costs. However, from the demand curve, the market-clearing price for this supply is only \( CD \). We assume in our model that the government subsidy, however paid, equals \( BC \). The effective average revenue equals marginal cost at the actual production level.

Note that the revenue shares \( (y_k = p_k q_k / C) \), unlike the cost shares \( (s_k = x_k w_k / C) \), need not add up to 1. Even under competitive conditions, total revenue may exceed or fall short of costs in the short run. Moreover, economic profits should be calculated after the opportunity cost of entrepreneurial efforts and imputed wages for family labor are considered. Here we assume that changes in these two inputs (family labor and entrepreneurship) are not related to output changes. This assumption permits us to calculate marginal costs of the outputs from the translog cost function. Hence, we are really approximating a variable cost function rather than a total cost function.

Fuss and Waverman in their study of Canadian telecommunications used the revenue shares only for products whose price elasticity of demand was absolutely greater than 1. For this analysis, we found including the revenue-share equations led to more acceptable parameter estimates, including correct signs of the price elasticities of input demands computed from the estimated model. Since concavity of the cost function at all data points requires that these elasticities be uniformly negative, we retained the revenue shares in our model.

Some additional econometric considerations are the following: (a) Because cost shares must add to 1, one of the share equations from (2) is redundant. (b) The individual equations of the system are seemingly unrelated in the sense of Zellner. (c) Restrictions must be imposed across equations to ensure uniqueness of estimated parameters which occur in more than one equation.

The system was estimated using the joint generalized least squares procedure. Although any one parameter appearing in several equations has the same estimated value, the associated asymptotic \( t \) statistics may differ. We found that the \( t \)-values were higher in the share equations than in the cost equation itself. In all such cases, we accepted the higher value.

A Taylor’s series approximation to any function is defined relative to a specific point of approximation. In this case better results were obtained using 1967 values for scaling (rather than the sample means). By using the 1967 values, we approximate the cost function at that specific point. All time-series data were converted into index numbers with base year 1967 before estimation.

### Variable Definitions and Data Description

In this model, we have two outputs: \( q_1 \) is livestock output and \( q_2 \) is crop output. Both are measured by index numbers of production \((1967 = 100)\). Five inputs are included: hired labor \((w_1)\); farm capital, consisting of farm real estate, motor vehicles, and machinery; fertilizers and lime; (purchased) feed, seed, and livestock; and miscellaneous inputs (pesticides, ginning, electricity, telephone).

In the dual system, input prices are used rather than physical quantities. They are wage of hired labor \((w_1)\); user cost of farm capital \((w_2)\); fertilizer price \((w_3)\); a divisio index of feed, seed, and livestock prices \((w_4)\); miscellaneous input prices measured by the price

![Figure 1. Illustration of price support program](image-url)
index of all commodities purchased for farm production \( (w_b) \). The trend variable, \( T \), included as an explanatory variable to capture HTNC starts with \( T = 39 \) for 1939 and increases by 1 annually.

The dependent variables are as follows:

- \( s_1 \) is (total wages paid to hired labor) \( \div \) (farm production expenses);
- \( s_2 \) is (repairs and operation expenses on capital items + depreciation and other consumption of farm capital + taxes on farm property + interest on farm mortgages + net rent paid to nonfarm landlords) \( \div \) (farm production expenses);
- \( s_3 \) is (expenses on fertilizers and lime) \( \div \) (farm production expenses);
- \( s_4 \) is (expenses on feed, seed, and livestock purchased) \( \div \) (farm production expenses);
- \( s_5 \) is (miscellaneous expenses) \( \div \) (farm production expenses);
- \( y_1 \) is (livestock marketing revenue) \( \div \) (farm production expenses);
- \( y_2 \) is (crop marketing revenue + government payments + inventory adjustments) \( \div \) (farm production expenses); and

\( C \), index of farm production expenses (1967 = 100).

The data have been taken from various U.S. government publications cited in references (1957, 1972, 1976 and 1978).\(^1\)

**Empirical Results**

Alternative versions of the model were estimated to test for constant returns to scale and homotheticity. The final choice of model was based on various \( F \) tests. (Note that the standard \( F \) test proposed by Fisher is to be modified in the context of a joint generalized least squares estimation. See Theil for an exact description of the test). In view of the adding up requirement of the input shares, one equation (for miscellaneous inputs) was excluded from the estimated system.

Ideally, we should first test for linear homogeneity in the cost function with input prices. This, however, cannot be tested in our complete model [equations (1), (2), and (3) all included]. The reason is that the adding up restrictions on the input-share equations (2) imply linear homogeneity of the cost function in the input prices. As an alternative one might try to test for such linear homogeneity in the cost function taking the equation (1) separately as a single-equation model. This, although theoretically possible, proved to be impractical in our case due to severe multicollinearity.

A test for profit-maximizing behavior was illustrated by Appelbaum. He imposed linear homogeneity on the cost-function parameters, adding up restrictions on share-equation parameters separately and then tested for uniqueness of parameters across equations. It was assumed that if firms really maximized profits (and were efficient in the neoclassical sense) parameters appearing in cost and share equations simultaneously would have unique estimated values. This test of internal consistency could not be carried out in our case because the problem of singularity for the cost-function estimation remained even after the parameter restrictions were imposed on equation (1). Equations (2) and (3) provide the additional information necessary to resolve this multicollinearity problem. It should be theoretically possible to obtain estimates of the parameters of the cost function (1) except for the intercept and the time trend factor from the share equations (2) and (3). However, our attempt to so obtain these parameters failed to yield acceptable estimates. In particular when equation (1) is excluded and parameters are estimated from (2) and (3) only, the price elasticities of input demand computed from such parameters are not of the right sign at every data point. We need to include all the equations—(1), (2), and (3)—to estimate the parameters of the cost function.

The model was first estimated to test for constant returns in the special sense of constant average variable costs. Linear homogeneity of the cost function in factor prices and internal consistency were assumed. The computed \( F \)-value (with 8 and 224 degrees of freedom) was 62.025. This leads to a strong rejection of CRS. Next we tested for separability of the cost function in outputs and input prices. Such functional separability indicates homotheticity of the technology. The computed \( F \)-value (with 10 and 244 degrees of freedom) was 4.98 and significant. Consequently, we rejected homotheticity as well. The model finally retained was the profit maximizing, linear homogenous version (in input prices). The estimated parameters of the system and the associated asymptotic \( t \)-values are reported in table 1.

\(^1\) For detailed references to tables from which variables were taken and an explanation of the construction of farm capital price index, see Ray, pp 10-11.
Table 1. Estimated Coefficients of the Translog Cost Function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>(Asymptotic) t Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$ (intercept)</td>
<td>-7.150465</td>
<td>-1.1201</td>
</tr>
<tr>
<td>$r$ (trend)</td>
<td>-0.018004</td>
<td>-9.9244</td>
</tr>
<tr>
<td>$a_1$</td>
<td>1.152584</td>
<td>3.0331</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.170444</td>
<td>2.2545</td>
</tr>
<tr>
<td>$d_{11}$</td>
<td>0.038830</td>
<td>0.5493</td>
</tr>
<tr>
<td>$d_{12}$</td>
<td>-0.139765</td>
<td>-1.9625</td>
</tr>
<tr>
<td>$d_{22}$</td>
<td>0.022331</td>
<td>0.1869</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.595642</td>
<td>4.3981</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.780157</td>
<td>4.0054</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.104822</td>
<td>1.6336</td>
</tr>
<tr>
<td>$b_4$</td>
<td>-0.679955</td>
<td>-4.0591</td>
</tr>
<tr>
<td>$b_5$</td>
<td>0.199374</td>
<td>0.5233</td>
</tr>
<tr>
<td>$f_{11}$</td>
<td>0.004591</td>
<td>0.4106</td>
</tr>
<tr>
<td>$f_{12}$</td>
<td>-0.011978</td>
<td>-1.3456</td>
</tr>
<tr>
<td>$f_{13}$</td>
<td>0.029050</td>
<td>5.6084</td>
</tr>
<tr>
<td>$f_{14}$</td>
<td>0.015430</td>
<td>1.2087</td>
</tr>
<tr>
<td>$f_{15}$</td>
<td>-0.037093</td>
<td>-4.6166</td>
</tr>
<tr>
<td>$f_{21}$</td>
<td>0.021669</td>
<td>1.4739</td>
</tr>
<tr>
<td>$f_{22}$</td>
<td>0.004641</td>
<td>1.4011</td>
</tr>
<tr>
<td>$f_{23}$</td>
<td>-0.054936</td>
<td>-4.9146</td>
</tr>
<tr>
<td>$f_{24}$</td>
<td>0.040604</td>
<td>7.1280</td>
</tr>
<tr>
<td>$f_{25}$</td>
<td>0.029263</td>
<td>6.4161</td>
</tr>
<tr>
<td>$f_{32}$</td>
<td>-0.010268</td>
<td>-1.4728</td>
</tr>
<tr>
<td>$f_{33}$</td>
<td>-0.052687</td>
<td>-5.8779</td>
</tr>
<tr>
<td>$f_{34}$</td>
<td>0.101900</td>
<td>4.7802</td>
</tr>
<tr>
<td>$f_{35}$</td>
<td>-0.052126</td>
<td>-3.4954</td>
</tr>
<tr>
<td>$g_{11}$</td>
<td>0.101302</td>
<td>1.1485</td>
</tr>
<tr>
<td>$g_{12}$</td>
<td>-0.079501</td>
<td>-3.6859</td>
</tr>
<tr>
<td>$g_{13}$</td>
<td>-0.067489</td>
<td>-2.9179</td>
</tr>
<tr>
<td>$g_{14}$</td>
<td>-0.023776</td>
<td>-2.3451</td>
</tr>
<tr>
<td>$g_{15}$</td>
<td>0.204859</td>
<td>7.7450</td>
</tr>
<tr>
<td>$g_{16}$</td>
<td>-0.034142</td>
<td>-2.0256</td>
</tr>
<tr>
<td>$g_{17}$</td>
<td>-0.028521</td>
<td>-1.1805</td>
</tr>
<tr>
<td>$g_{18}$</td>
<td>-0.012118</td>
<td>-0.3740</td>
</tr>
<tr>
<td>$g_{19}$</td>
<td>0.014151</td>
<td>1.1624</td>
</tr>
<tr>
<td>$g_{21}$</td>
<td>0.009516</td>
<td>0.2784</td>
</tr>
<tr>
<td>$g_{22}$</td>
<td>0.016971</td>
<td>0.8961</td>
</tr>
</tbody>
</table>

A primary objective was to measure the elasticities of substitution between pairs of inputs during the four decades covered by the data. We used standard results from neoclassical duality theory to compute these elasticities of substitution from the data using estimated parameters. Uzawa showed that in a profit-maximizing, competitive model, the Allen partial elasticities of substitution between inputs $i$ and $j$ can be derived from the (dual) cost function as

$$
\delta_{ij} = (\partial^2 C/\partial w_i \partial w_j) / (\partial C / \partial w_i \cdot \partial C / \partial w_j).
$$

In the translog model, we obtain

$$(4) \quad \hat{\delta}_{ij} = (\hat{f}_{ii} + \hat{s}_i \cdot \hat{s}_j) / (\hat{s}_i \cdot \hat{s}_j).$$

In equation (4) the hats indicate estimated values. When $i = j$, we obtain

$$(5) \quad \hat{s}_{ii} = [\hat{f}_{ii} + \hat{s}_i (\hat{s}_i - 1)] / \hat{s}_i^2.$$

The pairwise elasticities of factor substitution computed from our model and the data set are reported in table 2.

It has been proved (see, for example, Binswanger) that the (own-) price elasticities of demand for individual inputs can be obtained from equation (5) as

$$(6) \quad \hat{e}_{ii} = \hat{s}_i \cdot \hat{s}_{ii}.$$  

Similarly, the cross-partial elasticities of demand for inputs are

$$(7) \quad \hat{e}_{ij} = (\partial x_i / \partial w_j) \cdot w_j / x_i = \hat{s}_{ij} \cdot \hat{s}_j$$

for pairs of inputs $i$ and $j$. These cross elasticities of demand are not symmetrical as are the substitution elasticities. Price elasticities of input demands are reported in tables 3 and 4.

The estimated parameters reported in table 1 may be used to compute partial and overall scale economies at each data point. These scale economies measure the relative changes in outputs when expenses change but input prices are held constant. These are reciprocals.

Table 2. Elasticities of Substitution between Pairs of Inputs (Selected Years)

<table>
<thead>
<tr>
<th>Year</th>
<th>$s_{12}$</th>
<th>$s_{13}$</th>
<th>$s_{14}$</th>
<th>$s_{15}$</th>
<th>$s_{23}$</th>
<th>$s_{24}$</th>
<th>$s_{25}$</th>
<th>$s_{34}$</th>
<th>$s_{35}$</th>
<th>$s_{45}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1939</td>
<td>0.8546</td>
<td>4.6947</td>
<td>1.5196</td>
<td>-0.4423</td>
<td>1.2192</td>
<td>0.3137</td>
<td>1.5857</td>
<td>-0.2677</td>
<td>-6.5114</td>
<td>-0.9661</td>
</tr>
<tr>
<td>1949</td>
<td>0.8035</td>
<td>5.2288</td>
<td>1.4198</td>
<td>-0.9056</td>
<td>1.2334</td>
<td>0.4834</td>
<td>1.7205</td>
<td>0.1438</td>
<td>-7.2964</td>
<td>-0.5339</td>
</tr>
<tr>
<td>1959</td>
<td>0.7542</td>
<td>5.5655</td>
<td>1.4974</td>
<td>-1.1354</td>
<td>1.2057</td>
<td>0.5006</td>
<td>1.6592</td>
<td>0.2851</td>
<td>-5.5520</td>
<td>-0.3295</td>
</tr>
<tr>
<td>1969</td>
<td>0.6722</td>
<td>6.2039</td>
<td>1.6595</td>
<td>-1.4686</td>
<td>1.1776</td>
<td>0.4984</td>
<td>1.5773</td>
<td>0.3898</td>
<td>-3.5189</td>
<td>-0.1885</td>
</tr>
<tr>
<td>1977</td>
<td>0.6196</td>
<td>6.9961</td>
<td>1.8846</td>
<td>-1.5140</td>
<td>1.1667</td>
<td>0.4522</td>
<td>1.4787</td>
<td>0.3344</td>
<td>-3.0374</td>
<td>-0.1094</td>
</tr>
<tr>
<td>Sample Mean</td>
<td>0.7482</td>
<td>5.7250</td>
<td>1.5272</td>
<td>-1.1473</td>
<td>1.2117</td>
<td>0.4865</td>
<td>1.6607</td>
<td>0.2249</td>
<td>-5.9461</td>
<td>-1.0278</td>
</tr>
</tbody>
</table>

Note: $s_{14}$ is elasticity of substitution between labor and capital; $s_{15}$ is elasticity of substitution between labor and feed, and livestock; $s_{23}$ is elasticity of substitution between labor and miscellaneous inputs; $s_{24}$ is elasticity of substitution between capital and fertilizer; $s_{25}$ is elasticity of substitution between capital and feed, seed, and livestock; $s_{34}$ is elasticity of substitution between capital and miscellaneous inputs; $s_{35}$ is elasticity of substitution between fertilizers and feed, seed, and livestock; $s_{45}$ is elasticity of substitution between fertilizers and miscellaneous inputs.
of the cost elasticities with respect to output. If only one output \((q_i)\) is varied, while other outputs and input prices are held constant, the partial scale economy may be measured as

\[
SCE_i = \frac{(C/q_i)}{(\partial C/\partial q_i)} = 1/(\partial \ln C/\partial \ln q_i).
\]

This is the inverse of the revenue share of output \(i\) defined in equation (3). The overall scale economy is obtained as

\[
SCE = \left[\sum_i (1/SCE_i)\right]^{-1}.
\]

It may be shown that for any given proportionate increase in each output simultaneously, the computed value of \(SCE\) is greater than 1, when total costs increase by a lower proportion than the outputs. Similarly, \(SCE\) is less than 1 when costs increase by a greater proportion than outputs. The overall scale economy may thus be interpreted as measuring the returns to scale in a multiproduct situation. In this particular study, the scale economies relate to variable costs only. The partial and overall scale economies are reported for selected years in table 5. The computed values of \(SCE_i\) \((i = 1, 2)\) may be used to obtain the marginal costs of each output, using the relation

\[
MC_i = \frac{1}{SCE_i} \cdot \frac{\dot{C}}{q_i} \quad (i = 1, 2).
\]

Next, consider nonjointness in the outputs. In the absence of jointness in production, the marginal cost of one output is independent of the production of the other. Stated rigorously in the translog model, the condition for nonjointness is \(d_{12} = -a_1a_2\). This is a nonlinear restriction and not directly tested in this linear model. However, from the estimated parameters, \(d_{12} = -0.139765\) whereas \(\hat{a}_1 \cdot \hat{a}_2 = 1.349035\). This is not a conclusive test, but it does suggest that nonjointness of the two outputs does not hold.\(^2\)

Finally, consider Hicks-neutral technical change. The rate of technical change in U.S. agriculture captured by the HNTC coefficient on the time variable measured up to 1.8% per year. It is statistically very significant, and

2 Nonjointness of the outputs would require that the marginal cost of one should be independent of the level of production of the other, i.e.,

\[
\frac{\partial (MC_i)}{\partial q_j} = \frac{\partial C}{\partial q_i} \frac{\partial q_i}{\partial q_j} = 0
\]

(for all \(i\) and \(j\)). It may be shown that when the above condition holds, we obtain the relation,

\[
\frac{\partial \ln C}{\partial \ln q_i} \cdot \frac{\partial q_i}{\partial q_j} = -\left(\frac{\partial \ln C}{\partial \ln q_i}\right) \cdot \left(\frac{\partial \ln C}{\partial \ln q_j}\right).
\]

This is a necessary condition for nonjointness and must hold for all levels of \(q\) and \(w\). If we set \(q_i = w_i = 1\) for all \(i\) and \(j\), the condition reduces to \(d_{ij} = -a_1a_2\). Note that this is a necessary but not a sufficient condition for nonjointness.
may be compared with other estimates of productivity increase. Schultz found that during the period 1910–50 (at 1946–48 prices) the index of productivity measured by the ratio of output to input indexes in agriculture increased at a 1.35% annual rate. For the subperiod 1924–50, the annual growth rate was 2%. Such simple measures incorporate both technological changes and factor substitution due to price changes. We also computed a similar exponential trend using data from U.S. Department of Agriculture (USDA) Statistical Bulletin No. 612, obtaining an annual HNTC rate of 1.77%. The measure of technological change obtained from our translog model is net of factor substitution and, therefore, should be more precise. Still, all these measures of productivity change are of comparable size. Recall that technological change neutrality was assumed specifically in order to obtain a single measure of productivity growth. Hence, the possibility that technical change in U.S. agriculture was biased in favor of specific inputs was ruled out rather than tested in this analysis.

From the results of this work the following broad conclusions may be drawn about U.S. agriculture:

(a) Farm capital; fertilizers; and feed, seed, and livestock all substitute for hired labor in varying measures. However, the degree of substitution between labor and capital is much smaller than between labor and fertilizers. The high degree of substitutability between labor and fertilizers is consistent with the steady decline in labor use and the steep increase in fertilizers use.

(b) Farm capital is a substitute for all other inputs. The possibility of substituting fertilizers for farm capital may be of interest for developing economies. Intensive use of biochemical inputs (principally inorganic fertilizers) may be made in those countries instead of massive agricultural mechanization in order to increase production.

(c) Substitutability between labor and capital has declined, while that between labor and fertilizers or labor and feed, seed, and livestock has increased.

(d) The (own-) price elasticities of demand for all the inputs are less than 1 in absolute value. Farm labor has the highest price elasticity of demand. Each input experienced increase in its price elasticity of demand over time. This probably reflects greater use of purchased rather than farm-supplied inputs.

(e) The overall scale economy is less than 1, implying that the average variable costs rise when both outputs increase.

(f) Nonhomotheticity of technology (evidenced by the F-tests) indicate that the cost function is not multiplicatively separable in outputs and input prices. This violates Solow's condition for the existence of an aggregate production function for U.S. agriculture.

We may compare these findings with those of Binswanger, who also used a translog approximation for the cost function for U.S. agriculture. There are some major differences between his model and ours. He estimated a single output cost function using pooled cross-section and time-series data for forty-eight states for the years 1949, 1954, 1959, and 1964. Moreover, he estimated only the cost-share equations and inferred the cost function parameters from them. Further, he assumed biased (rather than neutral) technical change and specified his share equations accordingly. Finally, his definition of inputs is somewhat different than here. For instance, he treated land and machinery as separate inputs, while in our model they are pooled into farm capital.

Binswanger found significant substitutability between labor and land as well as between labor and machinery. His average elasticities of substitution were 0.204 between land and labor and 0.851 between labor and machinery. Our average elasticity of substitution between labor and capital was 0.7482. However, Binswanger found complementarity between labor and fertilizers as opposed to substitution in this case. Even recognizing the fact that his data are for an earlier subperiod, this is somewhat surprising. The strong substitutability relation between labor and fertilizers estimated here is more consistent with observed historical trends.

Considering the price elasticities of factor demands, our measures for labor (−0.8389 at the mean) is not too different from his average value of −0.9109. But our elasticity of demand for fertilizers was much less than his. For machinery, his average elasticity was −1.0886 and for land it was −0.3356. Our average elasticity of demand for capital was −0.5297, which is similar to an appropriately weighted average of Binswanger's estimates.

This model was used to simulate the effects of some hypothetical (though not unrealistic) changes in the exogenous variables. The index of the user's cost of farm capital was actually 522.4 in 1979 (1967 = 100). A 10% increase in
real estate and machinery prices along with an interest rate of 16% in 1982 would cause this price index of capital to increase to 999.8. We further assume that farm wages would rise by 12% and fertilizers price by 10% during 1979-82. In that case it would cost in 1982 about 72% more to produce the same amounts of crops and livestock as in 1977.

Using these projected prices we find that the marginal cost of livestock and crops [computed using equation (8)] would be $830.1 million and $690 million, respectively, in 1982. If prices are to correspond to marginal costs, the livestock price index should rise by 66% and crop price index by 60% during 1977-82. Actual levels of production and prices would, of course, be determined by demand and costs together.

The impact of this steep increase in the user's cost of farm capital on the demand for labor may also be calculated from the estimated elasticities. Using the 1977 value of the cross-partial elasticity the 148% increase in the price of farm capital relative to wages causes the demand for farm labor to be 38% higher in 1982 than in 1977. This would probably moderate the out-migration of labor from farming.

Conclusion

This study attempted to analyse the structure of agricultural production in the United States using the translog approximation to the cost function. Neoclassical duality results were extensively used for this purpose. In view of the relatively competitive structure of the farming sector in the United States, this is quite appropriate. Crops and livestock were treated as two distinct outputs instead of being lumped together into an aggregate product. Of primary interest for this study was estimation of the elasticities of substitution between inputs and the price elasticities of factor demands. Labor was found to be a substitute for all other inputs. So was farm capital. The results showed a decline in the substitutability between labor and capital along with an increase in the substitutability between labor and fertilizers. This information is potentially useful for policy. If this trend is to continue, we may expect slowing down in the rate of mechanization and intensified use of chemicals in U.S. agriculture. Another important finding is the rate of Hicks-neutral technical progress in U.S. agriculture. A 1.8% annual growth rate of productivity sustained over decades is quite impressive.

The overall scale economies computed from the cost function indicate that while U.S. agriculture operated under diminishing returns, the returns to scale factor increased over time.

The cost function was tested for homotheticity and rejection of this property indicates that aggregation of different outputs into a single index of agricultural production is invalid. Further, it was found that there is jointness in the production of crops and livestock in the sense that the marginal cost of one is influenced by the output of the other. In this respect, we need to realize that our model with two outputs (although a step in the right direction) is not disaggregative enough. This is an unfortunate limitation imposed by considerations of computational manageability within the scope of this study.

An important limitation is noninclusion of family labor as an input. This is primarily due to the facts that no data were available on the family labor component of the production expenses and no figures were found for the wage rate of family labor on farm. Exclusion of family labor assumes functional separability of the cost function in this and the other inputs. This assumption could not be tested in our model. However, the usual practice of evaluating family labor at the market wage rate is unsatisfactory on two counts. First, this approach ignores the market demand conditions for farm labor. Second, to the extent that family labor consists principally of managerial decision making, it should be treated as a distinct input and not to be added to hired labor.

Ideally, this study should have used farm level behavioral data. Use of aggregate data here (as in all similar models using economy wide observations) introduces a measure of aggregation bias. Besides, as already noted the translog approximation used in this paper has some limitations. Our estimates therefore should be regarded as broad indicators.

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References
