EXAMPLE 2
SIMPLE FEED MILL PROBLEM

Regional Feed Mill with Five Products

The manager desires to maximize net returns to his fixed resources by finding the right combination of the five products

---- per unit figures -----

Net Income ($)

Economic situation:

- Hog Finisher Feed (10 ton) 62
- Dairy Supplement (10 ton) 64
- Turkey Finisher Feed (10 ton) 56
- Custom Blending (truck load) 61
- “Old Red” Dog Food (100 bags) 36
(Simple Feed Mill Example Continued)

Resources available:
- Mill Capacity (ton) 120,000
- Regular Labor (hr) 32,000
- Nutritionist / Sales Rep (hr) 1,600

Also have a contract to custom blend 500 loads of feed

Resources required:  

<table>
<thead>
<tr>
<th>Product</th>
<th>Mill Cap</th>
<th>Reg Labor</th>
<th>Nutritionist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hog Feed (10 tons)</td>
<td>10</td>
<td>2.3</td>
<td>0.12</td>
</tr>
<tr>
<td>Dairy Feed (10 tons)</td>
<td>10</td>
<td>2.1</td>
<td>0.19</td>
</tr>
<tr>
<td>Turkey Feed (10 tons)</td>
<td>10</td>
<td>2.2</td>
<td>0.10</td>
</tr>
<tr>
<td>Custom Blend (1 load)</td>
<td>14</td>
<td>1.8</td>
<td>0.00</td>
</tr>
<tr>
<td>Dog Food (100 bags)</td>
<td>0.6</td>
<td>5.6</td>
<td>0.08</td>
</tr>
</tbody>
</table>

(Simple Feed Mill Example Continued)

How can this information be expressed in tableau form?

<table>
<thead>
<tr>
<th>Constraint</th>
<th>SIGN</th>
<th>B_i</th>
<th>幻 1(10 t)</th>
<th>(10 t)</th>
<th>3(10 t)</th>
<th>4(Load)</th>
<th>5(100 bgs)</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mill Cap</td>
<td>≤</td>
<td>120000</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>36</td>
<td>62</td>
<td>64</td>
<td>56</td>
<td>61</td>
<td>36</td>
</tr>
<tr>
<td>RgLab (hr)</td>
<td>≤</td>
<td>32000</td>
<td>2.3</td>
<td>2.1</td>
<td>2.2</td>
<td>1.8</td>
<td>5.6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Nutrst (hr)</td>
<td>≤</td>
<td>1600</td>
<td>0.12</td>
<td>0.19</td>
<td>0.10</td>
<td>0</td>
<td>0.08</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cntrct (Ld)</td>
<td>=</td>
<td>500</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
(Simple Feed Mill Example Continued)

- Analysing the feed mill problem with a Simplex algorithm (AELP)
  - A DOS based program that handles the tableau information directly
- Provides similar post optimality information as SOLVER

- Going to illustrate this program

(Simple Feed Mill Example Continued)

- Results
  - Profit
    - $769002
    - Used to cover fixed resources
  - Activities in solution
  - Shadow prices and slack information
  - Cost of forcing in nonoptimal activities
  - Price Ranges
  - RHS Range (or resource range)
(Simple Feed Mill Example Continued)

- **To solve this problem the tableau is modified**
  - Desirable to get rid of the inequalities and replace with equalities
    - For less than equal to constraints add a positive slack activity with a zero objective function value
    - For equal to constraints add an artificial activity with a very large negative objective function value
    - For greater than equal to constraints, add an artificial activity with a very large negative objective function value and a negative slack activity with a zero objective function value
  - Need to find an initial feasible solution
    - Use the positive slacks or the artificial activity

---

### Modified tableau with added slacks and artificial activities and a initial feasible solution defined

<table>
<thead>
<tr>
<th>Constraint</th>
<th>SIGN</th>
<th>BI</th>
<th>Op Cost</th>
<th>(10 t) Hog Fd</th>
<th>(10 t) Dairy Fd</th>
<th>(10 t) Trky Fd</th>
<th>Custom</th>
<th>(100 bags)</th>
<th>(t)</th>
<th>(hr)</th>
<th>(hr)</th>
<th>(Ld)</th>
<th>Artfcl 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cj</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Slack 1</td>
<td></td>
<td></td>
<td></td>
<td>120000</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-999</td>
</tr>
<tr>
<td>2 Slack 2</td>
<td></td>
<td></td>
<td></td>
<td>32000</td>
<td>0</td>
<td>2.3</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3 Slack 3</td>
<td></td>
<td></td>
<td></td>
<td>1600</td>
<td>0</td>
<td>0.12</td>
<td>2.1</td>
<td>2.2</td>
<td>1.8</td>
<td>5.6</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4 Artfcl 1</td>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td>-999</td>
<td>0</td>
<td>0.19</td>
<td>0.10</td>
<td>0</td>
<td>0.08</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
<td>-499500</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-999</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-999</td>
</tr>
<tr>
<td>(Z-C)</td>
<td></td>
<td></td>
<td></td>
<td>-499500</td>
<td>-62</td>
<td>-64</td>
<td>-56</td>
<td>-938</td>
<td>-36</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

---

### Tableau

- **Cj**
- **SIGN**
- **BI**
- **Op Cost**
- **(10 t) Hog Fd**
- **(10 t) Dairy Fd**
- **(10 t) Trky Fd**
- **Custom**
- **(100 bags)**
- **(t)**
- **(hr)**
- **(Ld)**
- **Artfcl 1**
(Simple Feed Mill Example Continued)

- **SIMPLEX method** is used to solve the linear programming tableau

- **Steps involved**
  1. First find the most profitable activity to bring into solution; moving to a better “corner point”
     - The most negative “Z-C”
  2. Determine what activity currently in solution most limits the selected most profitable activity
     - Divide the positive \( a_{ij} \) into the \( b_i \)
  3. Pivot selected activity into solution at activity determined in step 2 and adjust tableau to reflect the new activity in solution
     - The substitution effect
     - The feedback effect
  4. Calculate new “Z-C” values
  5. If all “Z-C” values are zero or positive have an optimum solution, if not repeats steps 1 to 5

---

(Simple Feed Mill Example Continued)

**Final tableau after five iterations**
(numbers have been rounded for illustration purposes)

<table>
<thead>
<tr>
<th>Constraint (Mill Cap)</th>
<th>S</th>
<th>B_i</th>
<th>Opp Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_j )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Hog Fd</td>
<td></td>
<td>8818.0</td>
<td>62 64 56 61 0</td>
</tr>
<tr>
<td>2 Dog Cw</td>
<td></td>
<td>1024.2</td>
<td>36 0 0 0</td>
</tr>
<tr>
<td>3 Dairy Fd</td>
<td></td>
<td>2420.6</td>
<td>64 0 1.0 0</td>
</tr>
<tr>
<td>4 Custom</td>
<td></td>
<td>500 61 0 0</td>
<td>0 1.0 0 0 0 0 1.0</td>
</tr>
<tr>
<td>( Z )</td>
<td>769001.9</td>
<td>62 64 60.59 61</td>
<td>36 4.45 5.32 43.8 -10.8</td>
</tr>
<tr>
<td>( (Z-C) )</td>
<td>769001.9</td>
<td>0 0 4.59</td>
<td>0 0 4.45 5.32 43.8 -989</td>
</tr>
<tr>
<td>( R_{upper} )</td>
<td></td>
<td></td>
<td>39061</td>
</tr>
<tr>
<td>( R_{lower} )</td>
<td></td>
<td></td>
<td>19686</td>
</tr>
</tbody>
</table>

**Price Range (hog feed)**

\( P_T = -3.65 \)
\( P_U = 3.17 \)
**LINEAR PROGRAMMING TRICKS**

- Transfer row
  - Keeps track of a shared resource or constraint

**EXAMPLE**: Corn produced and fed in an integrated crops/swine operation

<table>
<thead>
<tr>
<th>Constraint</th>
<th>SIGN</th>
<th>( B_i )</th>
<th>( 1^{(Ac)} )</th>
<th>( 2^{(Bu)} )</th>
<th>( 3^{(Hd)} )</th>
<th>( 4^{(Hd)} )</th>
<th>( 5^{(Bu)} )</th>
<th>( 6 )</th>
<th>( 7 )</th>
<th>( 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_j )</td>
<td></td>
<td></td>
<td>-130</td>
<td>-2.10</td>
<td>5.10</td>
<td>35</td>
<td>2.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 1^{st} ) Corn (Bu)</td>
<td>( \leq )</td>
<td>0</td>
<td>-120</td>
<td>-1</td>
<td>0.65</td>
<td>9.2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 2^{nd} ) Land (Ac)</td>
<td>( \leq )</td>
<td>600</td>
<td>1</td>
<td>4.5</td>
<td>0.15</td>
<td>0.5</td>
<td>2.5</td>
<td>10.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 3^{rd} ) Labor (Hr)</td>
<td>( \leq )</td>
<td>7200</td>
<td>4.5</td>
<td>0.65</td>
<td>9.2</td>
<td>1</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4^{th} ) Barn (Ft²)</td>
<td>( \leq )</td>
<td>54000</td>
<td>2.5</td>
<td>10.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**(Linear Programming Tricks Continued)**

- Transfer column
  - Converts one resource to another resource

**EXAMPLE**: Two types of labor, skilled and unskilled. Skilled labor can perform tasks of either labor type in a greenhouse operation.

| Constraint       | SIGN | \( B_i \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) | \( 5 \) | \( 6 \) | \( 7 \) | \( 8 \) |
|------------------|------|-----------|-------|-------|-------|-------|-------|-------|-------|
| \( C_j \)        |      |           |       |       |       |       |       |       |       |       |
| \( 1^{st} \) Sk-Lb(hr) | \( \leq \) | 20800    | 1.3   | 0.4   | 1     |       |       |       |       |       |
| \( 2^{nd} \) Usk-Lb(hr) | \( \leq \) | 26800    | 1.9   | 0.9   | -1.1  |       |       |       |       |       |
| \( 3^{rd} \) Bench(Ft²) | \( \leq \) | 12000    | 2.2   | 2.2   |       |       |       |       |       |       |
| \( 4 \)           |      |           |       |       |       |       |       |       |       |       |
| \( 5 \)           |      |           |       |       |       |       |       |       |       |       |
(Linear Programming Tricks Continued)

- Other uses of transfer rows and columns
  - Transfer row
    - To control the relative levels of activities in solution
      - Crop rotations
      - Product mix
    - To keep track of flow in and out of a resource
      - Shipping problems
  - Transfer column
    - To transform one product into one or more products
      - Soybeans into oil and meal
      - Can have a cost associated with this conversion
    - To sequence activities
      - Can not plant until tillage has been completed
  - Both transfer columns and rows
    - Transshipment model (covered later in course)

MINIMIZATION WITH LINEAR PROGRAMMING

- Reasons for addressing problems from a minimum cost viewpoint
  - Decision has been made to produce a product
    - Cutting cost does not influence the value of the product
  - Examples:
    - Making hot dogs
    - Formulating paint
    - Blending feed
    - Human diets

- There is no market for the product or service produced
  - Cutting costs is the main goal
  - Examples:
    - Providing recreational service
    - Economic development projects
FROM A PROFIT MAXIMIZATION TO COST MINIMIZATION

EXAMPLE: Manufacturing hot dogs
Purchases ingredients to produce hot dog that meet the government standards and sells packages of hot dogs up to the capacity of the plant to maximize profits

<table>
<thead>
<tr>
<th>Constraint</th>
<th>S</th>
<th>B_i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Meat 1 (cwt)</td>
<td>Meat 2 (cwt)</td>
<td>Meat 3 (cwt)</td>
<td>Water (cwt)</td>
<td>HotDog (Pk)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_j</td>
<td>X</td>
<td></td>
<td>-36</td>
<td>-55</td>
<td>-41</td>
<td>-.01</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Fat (%)</td>
<td>≤</td>
<td>0</td>
<td>23</td>
<td>9</td>
<td>10</td>
<td></td>
<td>-11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Water (%)</td>
<td>≤</td>
<td>0</td>
<td>70</td>
<td>72</td>
<td>82</td>
<td>100</td>
<td>-80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Meat (%)</td>
<td>≥</td>
<td>0</td>
<td>4</td>
<td>12</td>
<td>7</td>
<td></td>
<td>-8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Proc Cap</td>
<td>≤</td>
<td>95000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(From Maximization to Minimization Continued)

Since the decision was made to produce hot dogs to capacity, the only real decision is to minimize cost

Adjust tableau to cost minimization
LEAST-COST FEED MIX EXAMPLE

PROBLEM: Our regional feed mill wants to minimize the cost of formulating the swine feed
  – This would give him a competitive advantage in this market
  – Not being competitive, he will lose market share and business profitability

A ton of swine feed should have the following nutritional specifications:
  – Not more than 8% fiber
  – At least 35% crude protein
  – At least 1.5% fat

(Swine Feed Mix Example Continued)

Feeds Available:

<table>
<thead>
<tr>
<th>Feed</th>
<th>-- Percentage Level --</th>
<th>Cost ($/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fiber</td>
<td>Protein</td>
</tr>
<tr>
<td>Alfalfa Meal</td>
<td>25.0</td>
<td>17</td>
</tr>
<tr>
<td>Distiller Grain</td>
<td>3.0</td>
<td>25</td>
</tr>
<tr>
<td>Fish Meal</td>
<td>1.0</td>
<td>60</td>
</tr>
<tr>
<td>SBOM</td>
<td>6.5</td>
<td>45</td>
</tr>
</tbody>
</table>
### Tableau for the feed mix

<table>
<thead>
<tr>
<th>Constraint</th>
<th>S</th>
<th>I</th>
<th>G</th>
<th>N</th>
<th>B_i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_j</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-66</td>
<td>-92</td>
<td>-156</td>
<td>-96</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Fiber (%)</td>
<td>≤</td>
<td>8</td>
<td>25</td>
<td>3</td>
<td>1</td>
<td>6.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Protein (%)</td>
<td>≥</td>
<td>35</td>
<td>17</td>
<td>25</td>
<td>60</td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fat (%)</td>
<td>≥</td>
<td>1.5</td>
<td>2</td>
<td>5</td>
<td>7</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bulk (ton)</td>
<td>=</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### RESULTS:
- **Value of the objective function = -90.61**
- **Activities in solution:**
  - Alafalfa Meal = 0.139
  - Distillers Grains = 0.305
  - SBOM = 0.556
- **Shadow Prices and Slack Information:**
  - Fiber (RHS 8.0, Shadow Price 1.04, Slack Level 0.0)
  - Protein (RHS 35.0, Shadow Price -0.38, Slack Level 0.0)
  - Fat (RHS 1.5, Shadow Price 0.00, Slack Level 0.58)
  - Bulk (RHS 1.0, Shadow Price -85.57, Slack Level 0.0)
(Swine Feed Mix Example Continued)

RESULTS CONT.:  

- **Cost of forcing in nonoptimal activities:**  
  - Fish Meal = 48.53

- **Price range over which the optimal solution holds:**
  - Alfalfa Meal: Lower -90.40, Obj. Value -66, Upper 73.76
  - SBOM: Lower -126.12, Obj. Value -96, Upper -87.86

- **RHS range over which optimal solution holds:**
  - Fiber: Lower 4.75, RHS Value 8.0, Upper 10.84
  - Protein: Lower 23.18, RHS Value 35.0, Upper 38.08
  - Bulk: Lower 0.94, RHS Value 1.0, Upper 1.45

---

**EXAMPLES OF BUSINESS USE**

- **Furniture Cutting Program**  
  - Example of making 1000 solid cherry desks with leather tops

![Diagram of desk dimensions]
Furniture Cutting Example Continued

- Many operational and tactical decisions to make
  - Which grade of lumber to use

High Grade Lumber (FAS)
(Higher cost, less defects, better yield, bigger pieces, lower labor cost)

Low Grade Lumber (3 Common)
(Lower cost, more defects, lower yield, smaller pieces, higher labor cost)

Furniture Cutting Example Continued

- Additional operational and tactical decisions to make
  - To glue or not to glue in making pieces
  - Which piece to cut from which grade
  - How to price each piece

❖ The Furniture Cutting Program (FCP) was developed to address these questions
Other models used by businesses

- Spartan Ration models
  - Dairy
  - Swine
  - Beef
  - Other being developed
- Others
  - grain blending
  - fertilizer formulation
  - and many, many more