11 PRODUCTION FUNCTIONS AND ISOQUANTS

Purpose: To illlustrate production functions, isoquants, and total product curves.

Computer file: **cdprod98.xls**

Background information:

The *production function* is the economist's description of the technology of production. It defines for every combination of inputs to the production process the maximum output that can be produced. If you did the exercise on utility functions and indifference curves, it may help you to know that the function representing the production function here is of the same general form as the first utility function in that problem set. The difference in meaning here is that the dependent variable is the output of a good per unit time, and the independent variables are inputs into the production process.

Isoquants, or constant quantity curves, show all values of inputs for which output is constant. Isoquants are formally similar to the indifference curves in the utility function problems. The numerical value of the slope of an isoquant is the marginal rate of technical substitution. It is analogous to the marginal rate of substitution in the model of utility and choice, and shows the extent of substitutability between a pair of inputs.

A *total product curve* shows the relationship between output and a single variable input, holding all other inputs constant. There will, in general, be a different total product curve for each value of the fixed input(s). The marginal product of an input is the change output per unit change in a variable input, holding all other inputs constant. The marginal product is the slope of a total product curve. The Law of Diminishing Returns says that the marginal product of an input will eventually decline as more of a variable input is used.

Most of the questions in this problem set entail manipulating the graph of the production function to show the isoquants and total product curves. To maneuver the graph, click on any of the vertices. Small black dots appear on all of the vertices. You can drag any black dot to change the orientation of the grpah. [To do this, put the mouse pointer on the black dot, and depress the left mouse button. If you hold the button down and move the mouse, the graph will change form and move with the pointer. Releasing the mouse button redraws the graph, showing the new orientation.]

Experiment by moving the graph of the production surface to different orientations, then proceed to answer the questions.

Economists generally impose fewer theoretical restrictions on production processes than on preferences. For example, it is commonly allowed that marginal products of some inputs be negative. And the marginal rate of technical substitution is not always assumed to be increasing as inputs are gradually substituted for each other. In fact, a good deal of economic analysis is devoted to production processes in which the inputs are used in fixed proportions – the isoquants appear as right-angles.

MATH MAVEN'S CORNER: The production function used to generate the graph in this problem set is given by $Q = AL^a K^b$ where L is the amount of labor used and K is the amount of capital used. Exponential functions of this sort are commonly used by economists to describe production functions. They are called Cobb-Douglas functions, though the termis sometimes reserved for the special case in which the exponents sum to one.

An isoquant is the implicit function relating L and K for a given Q. The marginal product of an input is the partial derivative of output with respect to that input. For example, the marginal product of labor in this case is $aAL^{a-1}K^b$, or aQ/L.

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Questions

1) For the part of the function shown in the graph, is the marginal product of labor always positive? (yes or no)

2) For the part of the production function shown in the graph, is the marginal product of capital always positive? (yes or no)

3) Reorient the graph so that it shows the isoquants between labor and capital. Print the graph (or save it as a file) and turn it in with your other answers.

4) For the part of the production function shown in the graph, are the isoquants <convex>? (yes or no)

5) For the part of the production function shown in the graph, do isoquants intersect? (yes or no)

6) Reorient the graph so it shows the total product curves as a function of capital. Print the graph (or save it as a file) and turn it in with your other answers.

   [If you save it as a file, be sure to change the file name to end in a <2> so you don't overwrite the file saved in question 3.]

7) Do the total product curves obey the Law of Diminishing Returns so long as some of each input is employed? (yes or no)