12 | COST MINIMIZATION

Purposes: To show how a cost minimizing firm chooses inputs in the long-run. To show how to derive a firm's long-run total cost curve.

Computer file: **costmin198.xls**.

Background information:

Consider a firm, say a university, that produces an output, for example, the number of student credit hours, from two inputs, labor, L, and capital, K. The university's provost (chief academic officer) can buy any amounts of the two inputs used in production at fixed, known prices. The problem she faces is to choose the amounts of labor and capital that will minimize the total cost of producing a specified output. In other words, this is the standard cost minimization problem in the theory of the firm.

The production function for the university gives the current technology of production. Given the technology of production, the cost minimizing amounts of the inputs are those for which (i) the firm can produce the required output, and (ii) the marginal rate of technical substitution of labor for capital equals the ratio of the price of labor to the price of capital. Another way to say this is that the university should be on a cost constraint (isocost line) where the constraint is just tangent to the given isoquant.

Once you find the cost minimizing amounts of inputs, you will examine the effects of changing input prices on the university's minimum cost of production, and you will derive the school's long-run total cost curve.

The spreadsheet shows a few of the university's isoquants, and a cost constraint for the current level of output and input use. The baseline value for output is 30, and the baseline values of labor and capital are $9 and $4, respectively. You can return to these values by clicking on the appropriate button on the spreadsheet. Excel computes the amount of capital needed to produce the current output of 30 units of instruction.

Get a feel for how the model works by setting output to 30, and choosing some different values for labor. Notice how the cost of production changes every time you choose a different amount of labor to produce the given output. Notice also that the isocost line (cost constraint) changes every time you change labor to show the effects on total cost.

The central question here is how to produce the required output at minimum total cost. The answer is to choose input amounts that make the Marginal Rate of Technical Substitution of L for K equal to P(L)/P(K). Here's how to use Goal Seek to find the answer.

Set output to its required amount, and make sure input prices are at the amounts you want.

(i) Select the cell containing the current value for the MRTS.
(ii) Enter Goal Seek from the Formula menu.
(iii) Click on the To Value box, and enter the numerical value of the price ratio of the inputs using the keyboard.
(iv) Click on the By Changing Cell box, then click on the cell containing the current value of labor. The cell address should appear in the By Changing Cell box.
(v) Click on OK, and, if everything goes well, Excel will find a solution. To update the worksheet with the solution, click on OK.

The firm's long-run total cost curve shows the minimum cost of producing any output, given input prices and the technology of production, when all inputs are variable. You can find points on the LRTC by solving the firm's cost minimization problem for a succession of output levels, holding input prices constant. That's just what you're asked to do in the problems, and you will plot the LRTC from the points you find.

Here are some things to watch for and learn as you do the problems.

1) At the cost minimizing input choices, the ratio of the input prices (PL/PK) is equal to the MRTS (L for K). The isoquant and cost constraint are tangent to each other.

2) Lowering an input price will lead the firm to use more of the input whose price has fallen.

3) Note the similarity between the cost minimization problem firm's face and the utility maximization problem consumers face.

4) Note that the MRTS of labor for capital is equal to the ratio of marginal product of labor to the marginal product of capital (MP_L/MP_K)

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MATH MAVEN'S CORNER: The production function used to generate the graph is given by $Q = AL^a K^b$ where L is the amount of labor and K is the amount of capital. The cost constraint is given by $C = P_L L + P_K K$, where C is total cost, and P_L and P_K are the prices of L and K, respectively. The values of *A, a,* and *ß* are chosen randomly, but can be seen in the Answer Bin. The math problem is to minimize C subject to the production function with output set to the required output. It can be solved either of two ways: substitute the constraint into the definition of total costs so only one of the inputs is a variable and then minimize, or use the Lagrangian

Theorem. Satisfy yourself, if you wish, that the marginal rate of technical substitution of L for K is $(a/\beta)(K/L)$ in this case. The $MRS_{L\ for\ K}$ in general is $-dK/dL\rfloor_{dQ=0} = (\partial Q/\partial L)/(\partial Q/\partial K)$.

COST MINIMIZATION

Questions

Set output and input prices to their baseline values. Set labor use to 50.

    1) What is the total cost of 30 units of output when labor is 50?
    2) How much capital is used in this case?

Continuing on from the last question:

    3) What is the marginal product of labor?
    4) What is the marginal product of capital?
    5) What is the MRTS of labor for capital?

Be sure output and input prices are at their baseline values.

    6) What is the minimum cost of producing 30 units of output?
    7) How much labor is used at the cost minimizing solution?
    8) How much capital is used at the cost minimizing solution?

Continuing on from the last question:
    9) At the cost minimizing solution, what is the ratio of the price of labor to the price of capital?
    10) At the cost minimizing solution, what is the MRTS of L for K?

Set all variables to their baseline values. Questions 11-16 generate the data to find the firm's long-run total cost curve.

    11) What's the minimum cost of producing zero units of output?
    12) What's the minimum cost of producing 10 units of output?

    13) What's the minimum cost of producing 20 units of output?
    14) What's the minimum cost of producing 35 units of output?
    15) What's the minimum cost of producing 40 units of output?
    16) What's the minimum cost of producing 45 units of output

17) Print out the Graph Paper sheet by selecting it (by clicking on its tab at the bottom of the screen), and clicking on the button to print. Graph the firm's total cost curve using the points you found in questions 6 and 11 to 16.

Set all variables to their baseline values. Now raise the price of labor to $12/unit.

18) What's the new minimum cost of producing 30 units of output? (Compare to question 6.)

19) Following on from the last question, what's the new best level of labor use? (Compare to question 7.)