

# 13 | MORE COST MINIMIZATION

Purposes: To illustrate cost minimization by emphasizing input marginal products. To show how to derive the firm's total product curves for a single variable input.

Computer file: **costmin298.xls**.

Background information:

Consider again a university that produces student credit hours from two inputs, labor, L, and capital, K. Just as in the previous problem set, the provost faces the standard cost minimization problem – for given technology and input prices, how can total cost be minimized for a given level of production.

The difference here is in the form of the information the provost has about the technology of production, the university's production function. In this problem set she knows directly the marginal products (MP) of the two inputs. She can compute the marginal rate of technical substitution from the marginal productivities, but those data are not displayed on the spreadsheet. Indeed, the MRTS(L for K) is equal to the marginal product of labor ( $MP_L$ ) divided by the marginal product of capital ( $MP_K$ ).

This gives us a different, and, to many people, more intuitive explanation of the decision rule for minimizing the cost of production. The decision rule for minimizing cost is to choose inputs so that  $MP_L/P_L = MP_K/P_K$ .  $MP_L/P_L$ , labor's marginal product divided by its price, is the extra output due to spending another dollar on labor. Costs are minimized when the marginal product per dollar spent on all inputs is the same.

If the marginal product per dollar differs between any two inputs, output can be increased (or cost lowered), by shifting resources in favor of the input with the *higher* MP per dollar.

In the spreadsheet, you are asked to find the minimum cost following this rule. Use Goal Seek to do this. First set output to the desired amount. Then use Goal Seek to set the *difference* between the marginal products per dollar to zero by changing the amount of labor.

One new exercise here is showing how to find the *short-run total product curve* for a single variable input, given some amount of a fixed input. You're asked to set the capital amount to 60, and find the relationship between output and the single variable input, labor. Doing this for several amounts of labor allows you to find – and plot – the total product curve. In a subsequent exercise you will learn about the relationship between these short-run total product curves and a firm's short-run cost curves.

Here are some things to watch for and learn as you do the problems.

- 1) At the cost minimizing input choices, the marginal product per dollar spent on labor equals the marginal product per dollar spent on capital. The isoquant and cost constraint are tangent to each other, just as in the previous problem set.
- 2) You should be able to manage the cost minimization problem both in terms of marginal products, and in terms of the marginal rate of technical substitution.
- 3) The firm's total product curve is the relationship between output and a single variable input. Be sure you can tell by looking at your graph of the total product curve whether or not the technology obeys the Law of Diminishing Returns.

Here is a trick to help you get answers to questions 9 to 15:

Finding the values of labor and output for a given level of capital can be tricky, because you cannot change the capital amount directly on the spreadsheet. Here's how to proceed:

- (i) Set the value for labor to the value called for in the problem.
- (ii) Select the cell containing the current value of capital.
- (iii) Enter Goal Seek from the Tools menu.
- (iii) Click on the To Value box, and enter the numerical value of the capital.
- (iv) Click on the By Changing Cell box, then click on the cell containing the current value of output. The cell address should appear in the By Changing Cell box.
- (v) Click on OK, and, if everything goes well, Excel will find a solution. To update the worksheet with the solution, click on OK.

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**MATH MAVEN'S CORNER:** The production function used to generate the graph is given by  $Q = AL^a K^b$  where L is the amount of labor and K is the amount of capital. The cost constraint is given by  $C = P_L L + P_K K$ , where C is total cost, and  $P_L$  and  $P_K$  are the prices of L and K, respectively. The values of A, a, and  $\beta$  are chosen randomly, but can be seen in the Answer Bin. The math problem is to minimize C subject to the production function with output set to the required output. It can be solved either of two ways: substitute the constraint into the definition of total costs so only one of the inputs is a variable and then minimize, or use the Lagrangian Theorem. Satisfy yourself, if you wish, that the marginal rate of technical substitution of L for K is  $(a/\beta)(K/L)$  in this case. The  $MRS_{L \text{ for } K}$  in general is  $-dK/dL|_{dQ=0} = (\partial Q/\partial L)/(\partial Q/\partial K)$ .

## COST MINIMIZATION

### Questions

Set output and input prices to their baseline values. Set labor use to 40.

- 1) What is the total cost of 27 units of output when labor is 40?
- 2) What is the marginal product of labor when output is 27 and labor is 40?
- 3) Continuing on from the last question, what is the marginal product of capital when labor is 40 and output is 27?
- 4) Continuing on from the last question, if the firm buys another dollar's worth of labor, by how much does output increase?
- 5) Continuing on from the last question, if the firm buys another dollar's worth of capital, by how much does output increase?
- 6) If the firm wants to minimize total cost, should it be using more labor or more capital? (enter labor or capital).
- 7) Continuing on from the last question, what's the best level of labor to use to produce 27 units of output?
- 8) What are total costs at the best amount of labor (and capital)?

Set all variables to their baseline values. Now suppose the firm is operating in the short-run with fixed capital of 60 units.

- 9) For capital equal 60, and labor equal to 1, what is output?

Continuing on from the last question:

- 10) For capital equal to 60, and labor equal to 5, what is output?
- 11) For capital equal to 60, and labor equal to 10, what is output?
- 12) For capital equal to 60, and labor equal to 20, what is output?

Continuing on from the last question:

- 13) For capital equal to 60, and labor equal to 30, what is output?
- 14) For capital equal to 60, and labor equal to 40, what is output?
- 15) For capital equal to 60, and labor equal to 50, what is output?

Print out the Graph Paper sheet by selecting it clicking on the print button.

- 16) Use the data you generated answering questions 9 through 15 to plot the total product curve for the firm as a function of labor.