

# 14 | MARGINAL PRODUCTS AND MINIMIZING COST

Purpose: To illustrate cost minimization using marginal products directly, rather than the idea of marginal rate of technical substitution.

Computer file: **costmin398.xls**

Background information:

This problem set takes a different tack to understanding cost minimization for a given level of output. Instead of relying on the difficult idea (at least for people who don't know calculus) of the marginal rate of technical substitution, we consider the rule for cost minimization rewritten so that it emphasizes the marginal products of inputs, and the notion of marginal product per dollar.

Regardless of one's technical background, many people find the intuition in the approach taken in this problem set to be more accessible.

Again take a typical firm that wants to minimize the total cost of producing a particular level of output. The firm uses two inputs, labor and capital, that can be bought at fixed prices  $P(L)$  and  $P(K)$ . We can find (for any amounts of the inputs) the *marginal product* of, say,  $L$  as the increase in output achieved from employing an extra unit of labor, holding capital constant. The *marginal product per dollar* spent on labor is therefore  $MP(L)/P(L)$ . This is the increase in output the firm can achieve from spending another dollar on labor. Similar definitions hold for capital.

A firm wanting to minimize cost should always seek a mix of inputs,  $L$  and  $K$ , such that the marginal products per dollar spent on each should be equal. That is, we want  $MP(L)/P(L) = MP(K)/P(K)$ . Why? Suppose the firm is producing the desired output with  $MP(L)/P(L) > MP(K)/P(K)$ . Spending one dollar less on capital will reduce output by capital's marginal product per dollar. But because the marginal product per dollar for labor is greater than that for capital, less than a dollar's worth of labor must be bought to achieve the desired output. Costs are reduced by buying more of the input whose marginal product is higher.

The graph in this problem set displays directly the marginal products per dollar for each input. You can determine by simple inspection of the height of the graphs whether cost is minimized, and if not, which input should be more intensively employed.

To find the cost minimizing amounts of labor and capital, use Goal Seek to make the difference between their marginal products per dollar equal to zero by changing the amount of labor.

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**MATH MAVEN'S CORNER:** The production function used to generate the graph in this problem set is the same exponential form given by  $Q = AL^a K^b$  where  $L$  is the amount of labor used and  $K$  is the amount of capital used. Once again the actual values for the parameters appear in the Answer Bin, and you can use them to answer the questions by more conventional means – pencil, paper, and a hand calculator.

## MARGINAL PRODUCTS AND MINIMIZING COST

### Questions

Set output and input prices to their baseline values. Set labor use to 12.

- 1) What is the total cost of 23 units of output when labor is 12?
- 2) What is the marginal product of labor when output is 23 and labor is 12?
- 3) Continuing on from the last question, what is the marginal product of capital when labor is 12 and output is 23?
- 4) Continuing on from the last question, if the firm buys another dollar's worth of labor, by how much would output increase?
- 5) Continuing on from the last question, if the firm buys another dollar's worth of capital, by how much does output increase?
- 6) If the firm wants to minimize total cost, should it be using more labor or more capital? (enter labor or capital).
- 7) Continuing on from the last question, what's the best level of labor to use to produce 23 units of output?
- 8) What are total costs at the best amount of labor (and capital)?

Set all variables to their baseline values, and find the best amounts of labor and capital to produce 20 units of output.

- 9) What is the cost minimizing amount of labor?

Continuing on from the last question, now lower the price of labor to \$5.

- 10) Which input now has the higher marginal product per dollar? (labor or capital)
- 11) What's the new cost minimizing amount of labor?