

31 | DEMAND CURVES, AND INCOME AND SUBSTITUTION EFFECTS

Purpose: To show how to find the demand curve for a good from the standard utility maximization problem. To show how to find income and substitution effects of a price change.

Computer file: **utilmax398.xls**

Instructions:

Martha Johnson's utility is a function of the quantities of spaghetti and tacos she consumes. The first task in this problem set is to derive and plot Johnson's demand curve for spaghetti. By definition, a consumer's demand curve is the relationship between a good's own price and the quantity demanded, given income, prices of other goods, and preferences. The point being emphasized here is that the quantity demanded is the consumer's utility maximizing amount of the good — the amount the consumer wants to buy, and is able to buy.

The experiment to find the demand curve is straightforward. Choose a number of prices for spaghetti, and find the corresponding optimal amounts of spaghetti the consumer will choose. Income, prices of other goods, and preferences are held constant. With the price and spaghetti consumption data in hand, plot the demand curve for spaghetti.

Demand curves are assumed to be negatively sloped, a proposition usually called the Law of Demand. Can the Law of Demand be "proved" for the standard case of the theory of consumer behavior? Have enough assumptions been loaded on consumer preferences to assure that every time a good's price falls, the quantity that maximizes utility will rise?

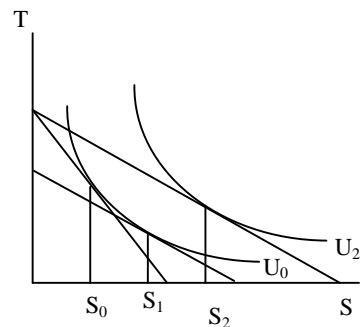
This question is explored using the concepts of income effects and substitution effects of a change in price. The whole issue arises out of the following observations: When the price of a good falls, that good is now relatively cheaper than other goods, and consumers would tend to substitute in favor of buying more of the cheaper good even if income were unchanged. On the other hand, when the price of a good falls, the consumer can buy more of everything, not only the good whose price fell. Purchasing power rises. So there is a sense in which a decrease in price is similar (but not exactly the same) as an increase in income.

Now consider Johnson's choices. When the price of spaghetti falls, we can imagine her moving to a new consumption bundle in two stages. The first is her change in spaghetti consumption due to spaghetti's becoming cheaper, holding her *real income* constant. This change is called the *substitution effect* of the change in price. The second is her change in consumption of spaghetti due to her becoming richer as a result of the price change, richer in terms of *real income*. But what is real income? Economists suggest that a good measure of a consumer's real income is total utility.

What are we to make of this breaking down of a change in consumption into so-called income and substitution effects? The substitution effect of a price change always works in the direction predicted by the Law of Demand. This is due to the assumption that indifference curves are convex when viewed from the origin. But the change in consumption due to an increase in *real* income, holding prices constant, may work to either increase or decrease consumption of the good whose price has changed. If the good is normal, we have nothing to worry about, because the income and substitution effects work in the same direction. But if the good is inferior, a negative income effect works against the substitution effect, and which one dominates depends on which is numerically larger. So theoretically at least, it's possible for a good to violate the Law of Demand. If the income effect is larger than the substitution effect, and works in the opposite direction the good is called a Giffen Good.

You're asked to find the income and substitution effects of a change in the price of spaghetti. Do this in several steps. It must be done by trial and error, as using Goal Seek is not enough. Start from the baseline values of income and prices of the spaghetti and tacos. When you reduce the price of spaghetti, optimal consumption will rise. Call this change in spaghetti consumption the total change. Now find the substitution effect by gradually decreasing money income, maximizing utility at each step of the way, until the *real income* (utility) is the same as where you started. The difference between spaghetti consumption at this point, and the original optimal amount is the substitution effect. The income effect is the difference between the total change and the substitution effect.

The graph at the right may help you compute the income and substitution effects more easily. Original consumption is S_0 , and final consumption is S_2 . The total change is $S_2 - S_0$. S_1 is consumption at the lower price of spaghetti when utility is at its original value. The substitution effect is $S_1 - S_0$, and the income effect is $S_2 - S_1$.



Here are some other things to watch for as you do the problems:

- 1) For the utility function used in these problems, both goods are normal so the Law of Demand is always true
- 2) Finding the substitution effect is tedious, but you should be able to find it in less than ten tries.

MATH MAVEN'S CORNER: The utility function used to generate the graph in [utilmax398.xls](#) is given by

$$U = AS^aT^b$$

where S is the amount of spaghetti consumed and T is the amount of tacos consumed, and A , a , and b are randomly chosen. This is the same utility function as in the last problem set, though

your parameters will be different here. See if you can find the demand function directly. It's also possible to find the exact income and substitution effects analytically. If you want to try this, the exact values for the parameters in your problem are given in the Answer Bin. Good luck!

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Questions

Set income and prices to their baseline values, and find the utility maximizing amounts of the goods.

- 1) How much is taco consumption?
- 2) How much is spaghetti consumption?
- 3) How much is total utility?

With income and the price of tacos at their baseline values:

- 4) When the price of spaghetti is \$2, how much is demanded?
- 5) When the price of spaghetti is \$6, how much is demanded?
- 6) When the price of spaghetti is \$8, how much is demanded?

Continuing on from the last question, with income and the price of tacos at their baseline values:

- 7) When the price of spaghetti is \$10, how much is demanded?
- 8) When the price of spaghetti is \$12, how much is demanded?
- 9) Print out the Graph Paper sheet by selecting the sheet (by clicking on its tab at the bottom of the screen), and clicking on the button to print. Graph the demand curve for spaghetti using the points you found in questions 2 and 4 to 8.

Set all variables to their baseline values, and find the optimal consumption amounts, and the maximum level of utility. Call these baseline consumption and utility. Now reduce the price of spaghetti to \$2 per plate.

- 10) What's the *total change* in spaghetti consumption?
- 11) What would the consumer's money income have to be at the lower price of spaghetti to give her (at best) the baseline level of utility?
- 12) Following on from the previous question, what is the level of spaghetti consumption at the income level you found?
- 13) What is the substitution effect of the price change?
- 14) What is the income effect of the price change?