The relationship between exchange rates, interest rates

- In this lecture we will learn how exchange rates accommodate equilibrium in financial markets. For this purpose we examine the relationship between interest rates and exchange rates. Interest rates are the return to holding interest-bearing financial assets. In the previous lecture we have pointed out that as being a financial asset exchange rates tend to adjust more quickly to new information that goods prices. Like exchange rates, interest rates are also the prices of financial assets and hence adjust quickly to new information.

- The profit-seeking arbitrage activity will bring about an interest parity relationship between interest rates of two countries and exchange rate between these countries.

- A U.S. investor deciding between investing say in New York and in Tokyo must consider several things:
  - the interest rate in the U.S., \( i_S \), (interest rate in a U.S. dollar denominated bond, or rate of return in a U.S. dollar denominated US stock etc), interest rate in Japan \( i_Y \);
  - the spot exchange rate, \( S \); and
  - the future exchange rate for maturity date, forward rate, \( F \).

- If the investor did not lock in a future exchange rate now, the unknown future spot exchange rate would make the investment risky. The investor can eliminate the uncertainty over the future dollar value of the investment by covering the investment with a forward exchange contract.

- If the investor covers the investment with a forward contract the arbitrage between two investment opportunities results in a covered interest parity (CIP) condition:

\[
(1 + i_s) = (1 + i_Y) \frac{F}{S} \tag{1}
\]
which may be rewritten as
\[
\frac{(1 + i_S)}{(1 + i_Y)} = \frac{F}{S}
\]
(2)

- The interest rate parity equation can be approximated for small interest rates by:
\[
i_S - i_Y = \frac{F - S}{S}
\]
(3)

- This later equation says that interest differential between a US denominated investment instrument and a Yen denominated investment instrument is equal to the forward premium or discount on the Yen.

- Example: \(i_S = 5\%, \ i_Y = 3\%\). Suppose \(S = 0.0068\) dollars per Yen. What should be the 90-day forward rate?

\[
0.05 - 0.03 = \frac{F - 0.0068}{0.0068}
\]
\[
F = 0.0068 + 0.02 \times 0.0068 = 0.00694
\]

Thus we expect that a 90-day forward rate of $0.00694 to give a 90-day forward premium equal to the 0.02 interest differential.

- If the forward exchange rates were not consistent with the respective interest rates, then arbitrageurs could profit by immediately changing currency in the spot market, investing it and locking in the profitable forward exchange rate. These actions in the market would increase the spot rate and lower the forward rate, bringing the forward premium into line with the interest differential.

- Suppose the actual 90-day forward rate is not 0.00694 dollars per yen but 0.0071 dollars per yen. Then profit-seeking arbitrageurs could buy Yen spot, then invest and sell the Yen forward for dollars, since the forward price of Yen is higher than that implied by the covered interest parity relation. These actions will tend to increase spot rate and lower the forward rate, thereby bringing the forward premium back in line with the interest differential.
The interest rate parity condition (CIP) can be used to compute effective return on a foreign investment. Re-write (3) as:

\[ i_S = i_Y + \frac{F - S}{S} \]  

(4)

This latter equation says that the return on a US dollar denominated asset (US dollar interest rate) is given by the Japanese interest rate plus the forward premium or discount on Yen. If CIP holds then equation (4) will hold as well.

What happens when an investor does not use the forward market? Then we can not expect effective return on US dollar denominated asset be given by (4) as the investor in question will not be able to get the premium on Yen (or lose the discount). In this case, we say investor has an uncovered investment. The effective return then will be determined by the Japanese interest rate plus the change in the spot exchange rate between today and say 90 days from now. Letting \( i_t \) be the domestic interest rate on a domestic currency denominated asset, say US Dollar, between date \( t \) and \( t+1 \), and similarly \( i_t^* \) stands for foreign interest rate, the effective return on a domestic currency denominated financial asset will be given by:

\[ i_t = i_t^* + \Delta S_{t+1} \]  

(5)

Which in our example will be

\[ i_S = i_Y + \Delta S \]

without time subscript.

Suppose in the example we have been considering so far, the US investor did not use the forward market. After 90 days when the investor go to change Yen back to dollars, she finds that the Yen has appreciated against US dollar say by 1 percent. This means that your Yen buys 1 percent more dollars than they did before. This means that effective return on Yen investment then will be given by

\[ i_Y + \Delta S = 0.03 + 0.01 = 0.04. \]
Hence, the return on a foreign investment plus the expected change in the exchange rate (in the value of Yen) is our expected return on a Yen investment.

- If the forward exchange rate is equal to expected future spot rate (Mathematically this means that $E [S_{t+1} \mid \text{given all the available information}] = F_t$) then the forward premium/discount is also equal to the expected change in the exchange rate. In this case we say that **uncovered interest parity, (UIP)** holds.

- More formally UIP condition says that the expected change in spot exchange rate is equal to interest differential.

$$\frac{E(S_{t+1}) - S_t}{S_t} = i_t - i^*$$

where for $E$ denotes the expectation operator. At this level you don’t need to worry about what this operator means, you can simply think $ES_{t+1}$ denoting the expected future value of spot rate.

- As above analysis indicate forward exchange rates incorporate expectations about the future spot exchange rates. If the forward exchange rate is equal to the expected future spot rate, then the forward premium is also the expected change in the exchange rate. In this case, UIP is said to hold.

- Empirical studies indicate that there are small deviations from CIP. These deviations are possible due to presence of transactions cost, differential taxation across countries on the returns from investing in financial markets, government control, and political risk involved in investing in different countries. However, these deviations are small enough to assume that CIP holds true almost exactly in the real world data. Therefore, we can say that profit-seeking arbitrage activities eliminate profit opportunities in the exchange rate markets. Hence, CIP condition can be viewed an equilibrium condition that characterizes the relationship between spot exchange rate, forward rate and interest rates of two countries.
• The problem arise in showing if the UIP holds or not in the data. Extensive studies have shown that UIP does not hold in the data especially for the industrialized countries. This means that percentage change in expected future spot rate is not equal to interest differential. Or, forward rate is not equal to expected future spot rate. Mathematically, this implies that there are deviations from UIP condition stated in (6) above. That is,

\[ i_t - i^*_t - \frac{ES_{t+1} - S_t}{S_t} \neq 0 \]

This means that effective return differential is not equal to zero. There are several explanations given in the literature.

– there should be profit opportunities in the exchange rate market that are being exploited by the investors. That may be possible if the inside trading type of activities are possible and used extensively. In other words, there are informational asymmetries in the market, some investors have more information than others and they make positive profits. Although, this may explain part of the puzzle especially in the very short run, it is hard to believe that these informational asymmetries persist for a long time, especially in financial markets where information flow is very rapid and exchange rates adjust rapidly to new information.

– It is possible to think that investors are systematically making mistakes in predicting the future value of spot exchange rate. That is, \( F_t \neq ES_{t+1} \) for a prolonged period of time. This means that forward rate is a biased predictor of future spot rate. Here biased means that it does not correctly predicts the future value of spot exchange rate on average. In other words, an unbiased predictor means that it predicts on average correctly the future value of a price, say exchange rate, so that over the long run the forward rate is just as likely to overpredict the future spot rate as it is to underpredict. Unbiased predictor does not mean that forward rate is a good predictor. What it
means is that forward rate is just as likely to guess too high as it is too low future spot rates.

There is some evidence that indicates that investors in foreign exchange rate market make systematic mistakes in predicting the future value of spot exchange rate and hence causing systematic deviations from UIP. It may be possible to think scenarios where investors make mistakes in their forecast of future values of asset prices, but the magnitude of these mistakes shouldn’t be that large to account the large deviations we observe in UIP. That is, it is hard to understand why especially over longer time periods investors make big mistakes in a systematic fashion. Over time at least we should expect these errors to shrink a level where deviations from UIP become smaller.

– Another explanation is that there should be a premium to take a risk by not covering the investment. This idea is based on the behavior of investors in taking risk. The effective return differential between two countries should be dependent on the perceived risk on each asset and the risk aversion of the investors. The risk aversion refers to the tendency of investors to prefer less risk. In terms of investments two investors may agree on the degree of risk associated with two assets, but the more risk-averse investor would require a higher interest rate on the more risky asset to induce her to hold it then the less risky-averse investor would. In finance, by risk we mean the variability of return from any given investment. This is because the more variable the return from an investment is, the less certain we can be about its future value.

If investors differ in their risk taking behavior we may observe that deviations from UIP and hence, changes in risk and risk aversion are associated with changes in effective return differential (that is interest differential). That is,

\[ i_t - i_t^* - \frac{ES_{t+1} - S_t}{S_t} = f(\text{risk, risk aversion}) \]
The left hand side of this equation is the effective return differential (or deviations from UIP). The right hand side can be viewed as the risk premium.

- Since CIP condition $i_t - i_t^* = \frac{F_t - S_t}{S_t}$ holds almost exactly, subtracting expected change in exchange rate from both sides:

$$i_t - i_t^* - \frac{ES_{t+1} - S_t}{S_t} = F_t - S_t - \frac{ES_{t+1} - S_t}{S_t}$$

(7)

or

$$i_t - i_t^* - \frac{ES_{t+1} - S_t}{S_t} = \frac{F_t - ES_{t+1}}{S_t}$$

(8)

Thus, we find that the effective return differential (or deviations from UIP) is equal to the percentage difference between forward and expected future spot exchange rate. The right hand side of (7) is usually considered to be a measure of risk premium in the forward exchange rate market. If effective return differential is zero, then risk premium will be zero. If it is positive, then there is a positive risk premium on the domestic currency, because the expected future spot price of foreign currency is less than the prevailing forward rate. In other words, traders are offering to sell foreign currency for domestic currency in the future will receive a premium, in that foreign currency is expected to depreciate (relative to domestic currency) by an amount greater than the current forward rates. Conversely, traders wishing to buy foreign currency for delivery next period will pay a premium to the future sellers to ensure a set future price.

The relationship between interest rates and inflation

- The real interest rate reflects the nominal interest rate with an adjustment for inflation. In other words, real interest rate is the nominal interest rate adjusted for inflation.

- Generally, the nominal interest rate will tend to incorporate inflation expectations. The relationship between interest rates and inflation is given by the Fisher
equation:

\[ i = r + \pi \]  \hspace{1cm} (9)

where \( i \) is the nominal interest rate, \( r \) is the real interest rate and \( \pi \) is the expected inflation rate. An increase in \( \pi \) will tend to increase the nominal interest rate.

• If the real rate of interest is the same across countries, then the Fisher can be combined with CIP equation:

\[ i_\$ - i_Y = \pi_{US} - \pi_J = \frac{F - S}{S} \]  \hspace{1cm} (10)

This latter equation says that if real interest rates are the same internationally, then nominal interest rate differential differ solely by differences in expected inflation.

• Note that relative exchange rate is given by the inflation differential and assuming that PPP, Fisher equation, and interest rate differential hold then real interest rates are equalized across countries.

The expected exchange rates and the interest rates

• The pattern of interest rates over different time periods for different investment opportunities is known as term structure of interest rates. There are several interest rates. Short run interest rates, long run interest rates, namely 1 month, 3-month, 6-months etc. There are several theories explaining the structure of interest rates on different investment opportunities over time.

  – Expectations: the long term interest rates tend to equal to the average of short-term rates expected over the holding period. The expected return that will be generated from holding a 10 year bond should be on average be the the sum of holding a series of short term bonds, say 30-day bond rates.

  – Liquidity premium: Long term investment instruments must incorporate a risk premium since investors prefer short term investments. As the term of
holding an instrument rises, the interest rate on that instrument should rise as well.

– Preferred Habitat: There exists separate markets for short and long term assets, with interest rates determined by conditions in each market.

• Under conditions of freely flowing capital across countries, the term structures in different currencies infer expected exchange rate changes, even if forward exchange markets for these currencies do not exist.

• If the term structure lines for two currencies are:
  
  – parallel, then exchange rate changes are expected to be constant;
  
  – diverging, then the high interest rate currency is expected to depreciate at an increasing rate over time;
  
  – converging, then the high-interest rate currency is expected to depreciate at a declining rate relative to the low-interest rate currency.