CUBISM, invented so recently as 1908, is an art of the past which no one any longer practices. But everyone who has anything to do with modern art recognizes the importance of the style, an importance that goes beyond the quality of individual pictures and resides in their bearing on the entire tradition of the past. At no other time, with respect to no other style, does one find the same sureness and intensity of the production of entirely new forms. Its advent has been compared to the Renaissance but from a historical viewpoint it is absolutely singular, perhaps the most astounding transformation in the history of art. All previous developments of such a striking character had, like the transition from Medieval to Renaissance art, taken hundreds of years, or at least the lifetime of two or three artists to be accomplished. Cubism became at once the most influential movement on the continent and directly inspired a host of derivatives—such as Italian Futurism and Der Blaue Reiter of Munich. It needed but one decade to become the most vital force in the art world.

What is even more astonishing is the fact that the artists participating in the movement not only created a revolution, they surpassed it, going on to create other styles of an incredible range and dissimilarity, and even devised new forms and techniques in sculpture and the ceramic arts. The reason for this may seem self-evident. The artists most active in the movement—Picasso, Braque, Gris, Villon, for example—were people who had always been highly diverse in the matter of personal styles. The range of Picasso’s styles was noted at his first one-man show when he was only twenty.1 Reviewers were astonished to find Paris street scenes in the man-

1 See Wilhelm Boeck and Jaime Sabartes, Picasso (New York, ’957), PP. 32-36.
net of Lautrec bearing the same date as a harlequin in a haunting romantic style. But the extraordinary inventiveness of the early century does not depend so much upon a fortuitous marshalling of talent as it does upon the primacy of the self.

Style, for these spiritual descendents of Post-Impressionism, was not a costume identifying one with others or merely marking out one's independence. To attain a style was part of a continuous process—a process of eliciting from the self constantly new attitudes towards art and life, and of disclosing novel relationships of the personality in kinship with artistry. Having invented some new mode, they considered it only the manifestation of a temporary immediate role. For men like Picasso, Braque, and Matisse the process of form creation had itself become inviting as a practice, had become aligned with the matter of self-fulfillment. Their ideal was the liberation of that process from the habituated methods and inhibiting traditions of any single form. This is—almost an expected attitude from a man like Picasso, who had already mastered the world of traditional painting at the age of fourteen and who absorbed everything else at a glance.

Pablo Picasso was the most important practitioner of Cubism, but the style was also developed by Georges Braque who had followed Picasso's experiments with great interest. We cannot say for sure that the form had a cultural existence independent of its inventors, that Braque would have invented Cubism had Picasso never been born, but the conditions for it were certainly "in the air." It is odd, after all, that the two men worked in ways so similar that their Cubist works can be told apart only with difficulty. Too, the immediate impact of the style on the painting of the avant-garde, and even on its writing, is suggestive of some sort of inevitability.

If one wishes to find a parallel to Cubism in the sense of an inexorable force revolutionizing a field of intellectual endeavor, he will find the closest thing to it in modern physics. Not only was the impact similar, so was the apparent inevitability: it is possible to suppose that the theory of relativity would have been invented whether the genius Einstein had happened on the scene or not. Furthermore, as Hunter writes, "many analogies have been drawn between Cubism and modern science, between the 'simultaneity' of vision (or shifting points of view) Picasso and Braque applied to nature, and space-time physics."

As it happens those analogies are as specious as they are ubiquitous.

They misrepresent both Cubism and modern physics. Because they emerge with great frequency in the contexts of cultural history and art criticism—pretending to serve as unitary principles of modernity—it is important that some energy be devoted to discrediting them. In order, however, to give a somewhat comprehensive coverage of the typical absurdities of critical thinking about Cubism it might be best to discuss the matter in terms of the notion of a "fourth dimension."

The term "fourth dimension" has a unique history. Supposed by many to be part of the esoteric vocabulary of contemporary science and only that, the term is as well a household word among many nonscientific groups connected with very exotic varieties of speculation. It was invented during the seventeenth century by an Englishman, Henry More, the most mystical of an obscure group of philosophers known as the Cambridge Platonists. In his *Enchiridion Metaphysicur* More proposed a fourth dimension as the realm necessary for the Platonic ideal to occupy. Given this curious background it is not at all surprising that the term and the idea should have been cordially welcomed by an intellectual curiosity of recent times known, variously, as "psychic science," "occultism," "spiritualism," and "the lunatic fringe."

The most interesting, not to say bizarre, turn given the geometry of higher dimensions was in the 1870's by the Leipzig astronomer Zollner. He was interested in the "experiments" of an American medium named Slate who claimed direct intercourse with the spirit world and whose exhibitions consisted of causing objects to disappear and reappear. To account for these phenomena the astronomer propounded a pseudomathematical theory that has come to be universally accepted by spiritualists, occultists and, most recently, by the "flying saucerists."

He postulated that for the real physical phenomenon there is really a space of four or more dimensions, of which we, because of our limited endowment, can appreciate only a three-dimensional section $x_4=0$. He argued that an especially gifted medium who, perhaps, is in touch with beings outside this world of ours, can remove objects from it, which would then become invisible to us, or he can bring them back again. He attempts to make these relations clear by picturing beings who are restricted to a two-dimensional surface and whose perceptions have this limitation. We may think of the mode...
of life of certain animals, e.g. mites. If an object is removed from the surface on which these creatures live, it would appear to them to disappear entirely (that is how it is conceived) and it was in an analogous fashion that Zollner explained Slate’s experiments.4

The idea that Cubism might have something to do with the fourth dimension was broached very early by Guillaume Apollinaire, the poet and champion of avant-garde art, who asserted this in a lecture in 1911 and in a little book on Cubism later on.5 One might assume that Apollinaire, whose writing could never claim clarity as a virtue, was using the term “fourth dimension” metaphorically, in the way Susanne K. Langer uses “virtual space” to describe a space that is not the space of the world of conventional magnitude and yet has a sense of form and reality nonetheless.6 From his words it seems far more probable that he was speaking of Cubism as an artistic formulation of Zollner’s theories, that is, as a depiction of a spatial realm not accessible to the ordinary senses.

Whether or not Zollner and Apollinaire were correct about dimensionality and Cubism respectively no one but a genuine medium could guess. For the ordinary person it is an open—if curiously irrelevant—question. The other variation on the dimensionality theme, however, has to do with the Theory of Relativity and thereby stakes a claim to the absolutely monumental prestige of the principal scientific development of the twentieth century.

For people already familiar with the notion of the fourth dimension as an invisible set of relations it was only a short step to the idea of temporality as a fourth dimension. The development of the four-dimensional space-time world of modern physics afforded the opportunity to take that step. As it happened, it was a misstep, but the direction was very, very appealing. That can be seen in the work of one of the most popular novelists of the modern era.

In 1895 H. G. Wells introduces us to a fashionable dinner party where "the fire burned brightly, and the soft radiance of the incandescent lights in the lilies of silver caught the bubbles that flashed and passed in our glasses."8 One of the group is the Time Traveler who sets out to contro-

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vert some ideas that are almost universally accepted. He begins by arguing that a mathematical line, a line of nil thickness, has no "real" existence (in the prosaic as opposed to abstract sense). All present agree. Nor, he says, has a mathematical plane. Again, agreement all around. Neither, then, he proceeds, can a cube which has only length, breadth, and thickness have a real existence. At this, of course, they protest. But, Time Traveler urges, can an instantaneous cube exist? "Clearly," he goes on, "any real body must have extension in four directions: it must have length, breadth, thickness and duration.... There are really four dimensions, three of which we call three planes of space, and the fourth, time."9

Thus, by the end of the nineteenth century, serious fiction as well as pseudo-scientific writing had prepared a certain segment of the reading public to accept the idea of time as a dimension. And the appearance of the space-time world of modern physics confirmed in these people's minds the justice of the notion.

The historical background of the idea of time as a dimension in science begins with the Michelson-Morely experiments which attempted to establish the velocity of light through a hypothetical "ether," involves the theoretical merit of the theories of Einstein and the value of the hypotheses of Lorentz, and, all in all, is extremely technical both in conception and lineage. Suffice it to say that the popular (and quite mistaken) view of space-time is embodied in the casual statement: "Time- is the fourth dimension of space." That view assumes that the prosaic or historical meaning of time as past, present, and future states is destroyed. It holds, in other words, that time in the sense of duration or of a sequence of moments is an illusion. In this respect time is viewed similarly to Zollner's mysterious fourth dimension. In effect, all that has ever happened or ever will happen is presumed to have occurred simultaneously. Thus, everything is, from the classical standpoint, coexistent; it is only because we perceive of it in segments that we say "time passes." Time, metaphorically speaking, is a yardstick, a given space; some people (e.g., Nero) are at fifteen inches, others are at two feet, and so on and so on. Of course, since no one can see the yardstick at all except as he moves on it we quite naturally term our sequence of perceptions "temporal."

The above description may make the popular notion of simultaneity seem slightly more ridiculous than it actually is, but it is set down here in
its most adumbrate and least elaborate form. Actually, the bare skeleton of this conception is very ancient—at least as old as Zeno \((335-265\text{ B.C.})\) who composed the famous paradoxes to prove that time and change are illusory.

The idea that the scientific conception of simultaneity has something to do with both prosaic dimensionality and Cubism seems to have become more and more assimilated into thinking about the art of the period since the time it was first expressed. For example, so justifiably respected a critic as Sigfried Giedion talked about it in his famous Charles Eliot Norton Lectures during 1938 and 1939:

Cubism breaks with Renaissance perspective. It views objects relatively; that is, from several points of view, no-one of which has exclusive authority. And in so dissecting objects it sees them simultaneously from all sides—from above and below, from inside and outside.... Thus, to the three dimensions of the Renaissance which have held good as constituent facts throughout so many centuries there is added a fourth one-time.... The presentation of objects from several points of view introduces a principle which is intimately bound up with modern life-simultaneity. It is a temporal coincidence that Einstein should have begun his famous work, *Elektrodynamik bewegter Korper*, in 1905 with a careful definition of simultaneity."

One finds this also in Kahnweiler's 1920 essay *The Way of Cubism.* And almost the same interpretation recurs all through Moholy-Nagy's influential design text *Vision in Motion* of 1947.\(^1\) It appears to have been something of an article of faith with the Bauhaus.

Giedion's opinion, however, had been expressed in a much more explicit way in an article written by the minor Cubist, Metzinger, very early in the century and hinted at in the book *Du Cubisme* by him and Gleizes later on.\(^2\) His idea was to justify the Cubist method of drawing with the new physics by attempting to show that Cubism, while apparently irrelevant to reality did, in fact, present a truer picture of things because it represented time as the new theories did, as a dimension. According to Metzinger, what the Cubist did was to present as simultaneous, successive

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\(^0\) Sigfried Giedion, *Space, Time and Architecture* (Cambridge, 1940, p 357.

moments of vision." The view has become increasingly fashionable since and is, indeed, a cliche of contemporary criticism.

Curiously enough, there has been from the very first a tendency to apply the ideas of physics to Cubism without any attempt to "check out" either the ideas themselves or their applications. And as a flagrant falsehood is perpetuated it begins to sound more and more reasonable since it is met with more and more often.

Now, one has only to examine Cubist painting by its major practitioners to observe that their forms could not possibly have been arrived at by the procedure outlined by Metzinger. No conceivable superpositioning of given objects would produce a Ma Jolie (Fig. 36) unless the fractioning of those elements were carried out to an extremity altogether uncalled for by the explanation. In fact, a Cubist image is made up not of elements of fractured objects but, instead, is built from fragments of elements. That is to say, one does not discover there a piece of a vessel, a segment of an eyeball, a part of a table; one finds instead no more than the lines and strokes that might represent such things were they brought into other relationships.

Still, what is most peculiar about Metzinger's theory, in view of its prominence, is that at the very time he was propounding it, Einstein was proving the impossibility of establishing the simultaneity of any two events that do not occur approximately, that is, side by side. So far as the Special Theory of Relativity is concerned, the sole difference between it and classical science lies precisely in Relativity's denial of the absoluteness of the simultaneity of spatially separated events. Had the Cubists really been consistent with the new developments in physics they would have demolished simultaneity! After all, the presentation of simultaneous images had been common practice in architectural and machine drawing since the Renaissance, by way of elevations and projections which showed at once the top, sides, front and back of an object.

What, then, is meant when a scientist says that time is the fourth dimension of space? It means that he is speaking very loosely; what he should say is: "Time is a fourth dimension of the space-time world," or more exactly: "Time is one of four space-time parameters in a physical theory." And what he then means has to do with mathematical expressions exclusively, for the Theory of Relativity sought to resolve all distinctions

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as to determinations of temporality and position into the unity of purely numerical determinations.

The particularity of each "event" is expressed by the four numbers \(x_1, x_2, x_3, x_4\), whereby those numbers among themselves have reference to no differences so that some of them \(x_1, x_2, x_3\) cannot be brought into a special group of "spatial" coordinates and contrasted with the "time" coordinate \(x_4\) .... The direction into the past and that into the future are distinguished from each other in this form of the concept of the world by nothing more than + and - directions in space, which we can determine by arbitrary definition.

What has happened is that the concept of an inertial system has been displaced by that of a field which depends on the concept of the total field that Einstein said is "the only means of description of the real world.... The space aspect of things is then completely represented by a field, which depends on four coordinate parameters; it is a quality of this field. If we think of the field as being removed, there is no "space" which remains, since space does not have an independent existence." \(^{16}\)

The author is no less aware than his reader that a thorough understanding of these matters presupposes a more than rudimentary grasp of the calculus. The point, though, is that that particular attainment is quite unnecessary to an understanding of Cubism. Surely, it must be obvious to any careful reader that the space of painting cannot accommodate the field concept of modern physics; those two things have nothing in common. What is more, the paintings do not represent such a concept symbolically. The fragmentations of Cubist art did not derive from simultaneous presentations of shifting points of view, but even if they had they would be unconnected with the Theory of Relativity. Thus, it can be argued that the entire notion of a hermetic connection between Einstein's theory and Cubism is false. And, in fact, Einstein himself voided the connection. Responding to an essay sent him in manuscript which argued more-or-less the position I have been attacking in these pages, Einstein—after a very generous explanation of his own sense for some real connection between art and science—commented: "Now, as to the comparison in your paper, the essence of the Theory of Relativity has been incor-


rectly understood in it, granted that this error is suggested by the attempts at popularization of the theory.” He then said pretty much what has been said here regarding the theory and closes with the remark that ”this new artistic ‘language’ has nothing in common with the Theory of Relativity.”

Cubism has nothing to do with the Theory of Relativity and that is the end of the matter. To argue this, however, is not to assert the absence of any significant relationship between the painting style and modern

science—or, more definitively, between Cubism and the total culture to which science has contributed so vast an influence. Besides, the prominence of a belief in some kind of hermetic geometry associated with the paintings done between 1907 and 1913 is inescapable and should somehow be accounted for. It is due, most probably, to the sheer appearance of the analytical Cubist works which, in their typical form, are possessed of a peculiarly uncanny space.

In Cubism, Cezanne's method of rendering solid objects in such a way that they retained both their volume and a frontal stability was brought to a sort of zenith. In Picasso and in Braque (Fig. 37) the objects...
or areas have a certain kind of stability but there is no enclosure of space in the old sense. Their fascination with continuities and discontinuities makes it very difficult for us to separate figures from their background. For anyone it is easy to see that in Figure 38 plane P lies between O and Q; the arrangement is of an ordered set. But with regard to Figure 39, a schematic drawing that takes the Cubist image as a point of departure, the viewer is unable to establish the position of P in space; possibly it lies both ahead and behind the other lines and planes. This ambiguity is not due to a transparency of the -forms-to some curious crystallinity-but is instead the product of the arrangement of continuities and discontinuities. So it is in Cubist paintings. Lines are prolonged through planes, edges vanish before they intersect other edges. All this makes for an increased complexity of the frontal plane and for an uncleanness of relations among the parts of the pictures. Therefore the space of a Cubist work becomes highly elusive. Although it affords a definite illusion of depth the space of Cubism is an uncanny sort of space on which the viewer can never put his finger or rest his mind. We cannot refer to the picture plane as a screen behind which objects, or parts of objects, exist. But neither can we say that the picture is without the illusion of space, merely frontal, for at times the space appears even to protrude out into the world. There is something here far more complex in its meanings than the old counterchange devices of the pattern-making arts in which figures, like the stripes of the zebra or the squares on a chessboard, are constantly competing with their backgrounds. Briefly, Cubism amounts to a maximization of the unordered set or series; it is a glorification of the non-a priori constructions of Paul
Cézanne. That aspect of the style has surely inspired a great deal of far-fetched speculation about its connection with seemingly mysterious inventions in the narrow regions of thought with which art historians are normally unfamiliar.

As a style, Cubism began and ended with the assumption that nature and art are two utterly dissimilar phenomena, the one absolutely accidental and informal, the other rigorously formal and self-sufficient. In this it was the summing up of all that had been present in the hygienic, detached art of Georges Seurat and represented the furthest extension of Cézanne’s preoccupation with completeness and order. It was the first non-ornamental painting to represent nothing but itself, to assume that a painting is only a painting just as a building is a building and that a picture ought look no more like a segment of real space than a house ought to resemble a baker’s roll.

It is interesting that in such formal painting, which represents the ultimate removal of art from the literary content of Classicism and Romanticism, actual words so often appear. The fact is instructive. For there is a world of difference between writing as a literary process and writing as script or printed type, that is, as lettering. A letter is just one sort of mark or combination of marks. And marks need not be judged by any relation to signification unless they are in a conventional context. The Cubists eliminated conventional relations from words—from, say, "Ma jolie," the title of a popular song—and treated them as visual forms, as shapes. Since a Cubist painting is subject to all the demands related to our perception of the work our sense of artistic form is not violated simply
because the day’s dance hall tune has been introduced into the painting. None of this is like the syntax of Cezanne who had to realize the visible in order to realize the form. For the Cubists Cezanne’s mimetic requirements did not exist. They divorced, for the first time in history, the means of representation from the objects represented.

The Cubists substituted for the objects rendered by the artist the objects created by him, that is, the pictures themselves. This is not to say that Cubism was without a class of themes. Its most frequent subjects are still-life objects drawn, like Cezanne’s, from casual bohemian life. But in principle, at least, it did not matter what was painted. Picasso spoke of the mode of representation being distinct from anything represented: "Art has always been art and not nature. And from the point of view of art there are no concrete or abstract forms, but only forms which are more or less convincing lies.... Cubism is no different from any other school of painting." 18

Looking back, one can see that the thing was bound to emerge from its past. Whistler and Pater had talked rather like that during the eighties. It was only a question of time before someone—if not Picasso or Braque, then Matisse—attempted to turn painting into an art of pure construction like musical composition. It was one of those ideas whose time had come. Moreover, the style ensignied the emergence of an utterly new critical attitude towards all art.

When Henri Matisse said that upon seeing the Giotto frescoes at Padua he did not trouble to recognize which scene of the life of Christ he had before him but perceived "instantly the sentiment which radiates from it and is instinct in every line and color," 19 he was putting into a painter’s language what "Formalist" critics Roger Fry and Clive Bell were calling "Significant Form." In 1914 Bell wrote: "The representative element in a work of art may or may not be harmful; always it is irrelevant. For, to appreciate a work of art we need bring with us nothing from life, no knowledge of its ideas and affairs, no familiarity with its emotions. Art transports us from the world of man’s activity to a world of aesthetic exaltation.... It is a world with emotions of its own." 20 By 1928, in a new edition of his book, Bell was arguing that realistic forms in an otherwise good composition tended, because of their extra-aesthetic references, to

19 Matisse, p. 32.
the canvas. The goal of the collage artist was the attainment of maximum concreteness and the notion of painting as handicraft was, perhaps, realized most completely in the *papier colles* of the Cubists. Prefabricated pattern matter was used, but if it was something like a newspaper it could not be read except as a formal element of the composition because it was inverted, turned on its side, or smeared with pigment. And there was still the ambiguous space of earlier work. In *Guitar and Clarinet* by Braque (Fig. 40 we can't quite settle upon the true relationship of the paper to the cardboard. Are the forms cut from the paper or are they the result of the cardboard overlaying it? Or is one created in one way, and another in the other? The work is clearly Cubist, but as Picasso and Braque began to give to collage elements a new concreteness they also began to lend them the function of reconstituting the picture plane that early Cubism had destroyed.

By 1918 the process of de-spatializing things by breaking them up
into lines and marks had reversed itself. Out of the strokes the solid zone was reinstated. This period in the work of Picasso (Fig. 42), Braque, Gris, and Leger is often called "synthetic" Cubism (after Apollinaire, who also invented the term "analytic" Cubism for the earlier style). The paintings were still characterized by great artifice and ambiguity and were still conceived of as the composite results of the operations of the artist. Gris, in his letters, described the process as a matter of beginning with an abstraction and seeing something into it, the objects coming into being only by virtue of the abstract forms that existed beforehand in the mind of the painter.

Certainly, it is hard to imagine Picasso painting his *The Red Tablecloth* in so strictly deductive a manner. The work is extremely rich in its orchestration of color, that is, in the transpositions and groupings of hues.

and tones. And it is never possible to find identical pairings or triplings. Still, Picasso always makes it obvious that the artist is at work imposing order on reality. He breaks up a border for no reason having anything to do with natural appearances or sunlight. He has painted the cast of a classical head black. And his melon is a pale violet. Only an exceptional painter would feel the need of a lavender melon there. Why not a red one to match the tablecloth? It lends itself to the hue, after all. Partly, I suppose, because this is the home of the successful Picasso, a wealthy aesthete, and in France watermelon are fed to the swine. But a violet melon? Is this curiously colored fruit a pure whim on the artist's part? No, the melon is part of a passage from the blue sky through the window to the tablecloth. One feels that it could not be else but lavender and yet be so absolutely right. Again, the ability to divorce the means from the subject-matter manifests itself in an original and exciting form. But it seems all too imaginative to be contained by Gris's explanation.

Probably, Gris's was not an accurate description of the method of other synthetic Cubists. But it is of great interest because it so clearly affirms the confidence of the artists in the priority of formal constructions independent of nature. Their assuredness parallels the faith of the Formalist critics. And these moderns had relatives in every field. Gertrude Stein's poetry, Schoenberg's music, the philosophical thought of Wittgenstein and the Vienna Circle, Cubism, and Formalist criticism all typify a general tendency in twentieth-century life towards the "construct" that is sufficient unto itself, ordered according to its own laws, dissociated from any moral cause, and associated exclusively with the hermetic experience of man confronted by man's creations. Nowhere is the tendency better exemplified than in physical science where the discipline of mathematics was being explored with a new passion for the purely structural, just as the Cubists explored painting in terms of its purely artistic side.

In the world of theoretical clarities the closest parallel to Cubism will be found in the modern logics, for they too separate the mode of representation from the thing represented. Usually the three main schools of

22 It should be clearly understood that the pursuit of mathematical knowledge has been forever undertaken without much regard for its extramathematical functions. To this extent parallels between the new mathematics and painting are strained compared to ones that might be drawn with modern music. For music has always been nonrepresentational and the new emphasis on formal structure inaugurated by Schoenberg, Stravinsky, and Hindemith merely reinforced awareness of the fact. A similar statement could be made about the relation of modern mathematical thought to its tradition. Thus, so far as their relation to the past was concerned, the Cubists and Gertrude Stein were far more radical
example from number theory to make this clear. In 1742 one C. Goldbach had conjectured that every even number, excepting 2, could be represented by the sum of two prime numbers. Thus 4=2+2, 8=5+3, 10=7+3, 18=13+5, and so forth. The fact is that there is no known even number for which the partitioning into primes is not possible, but that does not prove Goldbach's conjecture true. To prove it false, of course, all one has to do is to discover an even number for which the rule does not hold. It would appear, then, that either there is such a number or there is not, that is:

(I) Either all even numbers can be written as the sum of two prime numbers,
Or there are even numbers for which this does not hold, tertium non datur.

However, if the number theoretician wished to show that Goldbach was right, then he obviously cannot examine all the natural numbers. He would have to prove that it can be inferred from the characteristic of an even number that the number is equal to the sum of two primes. The either-or (I) would have to be formulated precisely:

(2) Either it can be inferred from the characteristic of an even number that the number can be written as the sum of two prime numbers,
Or there exists a procedure for calculating a counterexample.

It is clear that there is no genuine alternative. Yet the two opposing statements are not contradictory. The validity of the choice in form (I) is indisputable if we restrict ourselves to a statement about a finite set of numbers, e.g., the Goldbach conjecture is either true or false for all of the even numbers in the Berlin telephone book.... However, it is an inadmissible generalization to carry over to infinite sets this inference which is justified for finite sets. The consideration of Goldbach's conjecture developed here is equally valid for other statements about natural numbers and also about elements of arbitrary infinite sets.

One can see that, in dealing with infinite sets, it might well be that one could not find, on running through the numbers, one that is neither even but not reducible to two primes, nor a proof of the Goldbach conjecture derivable by means of mathematics.

26 Meschkowski, p. 56.
David Hilbert, who worked out the axiomatics of the non-Euclidean geometries, was also among the first to object to Brouwer's thesis. One of his motives in this seems to have been to defend the nature of what is called set theory, an invention of the German George Cantor whose particular contribution was his theory of infinite sets and transfinite numbers. This development had given all of mathematical thought a purity and an autonomy denied to other forms of effective thinking and Hilbert swore: "No one shall drive us from the paradise which Cantor created for us." In order not to be driven out by Intuitionism he and the Formalist school led by him presented a new foundation for mathematics which justified residence in Cantor's "paradise."

Hilbert acknowledged that the objections of the Intuitionists to unlimited application of the law of the excluded middle were just. But, not wishing to diminish mathematics because of that, he proceeded to "formalize" the method of proof in mathematics. His procedures could not possibly be understood by nonmathematicians unfamiliar with set theory and even a reasonable caricature using sentential logic would presuppose some sophistication about rules of implication, *modus ponens* and *modus tollens*. Suffice it to say that for the Formalists mathematics in the narrow sense is replaceable by an utterly mechanical method for deriving formulae and has nothing to do with the meaning or interpretation of the symbols used. Someone has described this kind of mathematics as "a game played according to simple rules with meaningless marks on paper." Of course, this is said facetiously; there was nothing arbitrary about Hilbert's procedures. Mises wrote that "Certain aggregates of symbols are assumed as premises; these are the axioms, and from them further groups of signs are derived according to the fixed rules in a purely mechanical manner, i.e., without the use of conclusions drawn from their interpretation; the new groups are then the provable theorems. Thus, the entire content of mathematics is, according to Hilbert, transformable, in principle, into a system of symbolic formulas." Hence, the Formalist doctrine is almost exactly similar to the Cubist ideal except in that the one deals in axioms and rules for the combination of signs whereas the other deals in lines and marks and the "rules" for their combination.

As for the law of the excluded middle, which Hilbert was eager to

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28 Mises, p. 132.
admit into his formal system, he wrote: "Operating with the infinite can only be made certain by the finite. The role which is left to the infinite is purely that of an idea—if in Kant’s words an idea is understood to be a concept of reason, which exceeds all experience and by which the concrete in the sense of totality is completed—an idea, moreover, which we may trust without hesitation in the framework which has been fixed by theory outlined and upheld by me here." In this connection it should be made clear that besides the system of formulas Hilbert proposed something else which serves as their justification and is called "metamathematics." This seems to comprise all of the arguments that lead to the proof of consistency in the formal system itself. 'By a consistency proof is meant the proof that a certain 'false' formula, e.g., the formula '1 = 2' cannot be derived.'" Hilbert hoped that he could conduct his consistency proof with "finite" methods that would be recognized by the Intuitionists, thus showing that even the unlimited use of the law of the excluded middle would not lead to an internal contradiction.

But where do the intuitionists stand on this "synthesis"? Brouwer rejected Hilbert's program. In his view an incorrect theory that does not lead to a contradiction is on this account no less incorrect just as a criminal not condemned surely is still a criminal. Hilbert replied that to forbid a mathematician the use of the tertium non datur "would be like denying the astronomer the telescope or the boxer the use of his fists." It was always considered the special glory of mathematics that it recognizes no conflicts of opinion. It lost this renown by discussions of fundamental questions which reach into the realm of philosophy.

In view of this extension into philosophy it is appropriate that the leader of the third party in the controversy over the foundations of mathematics should be best known, not as a mathematician, but as a philosopher. Too, he was an Englishman and the only philosopher of his nation since John Stuart Mill to combine so much intellectual power, wit, cultivation, and passion for liberty." Moreover, he expressed himself on everything from education to war—not always very carefully—and he was one of the most distinguished masters of English prose of the last two

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30 Quoted in Meschkowski, p. 96.
31 Mises, p. 133
32 Meschkowski, p. 97.
says "I am lying" it is clear that if he lies then what he says is false and he must be speaking truthfully. On the other hand, if he speaks the truth he must, therefore, be lying. All that Russell's principle says is that such a problem is inadmissible. His theory of logical types which distinguishes between individuals, sets, families of sets, and so on, representing these, respectively, by variables $x_0, y_0, z_0, \ldots, x_1, y_1, \ldots, x_2, \ldots$, does not permit the admission of a formula such as $x_0(x_0)$, which is how the liar paradox would be symbolized. The arguments become very quickly hard to follow. This is not because Russell's language is opaque; if anything, his language is baffling in its clarity. But his insights are very close to the equivalents of the laws of purest thought and few of us can cope with this kind of strictness. That pristine ideas should be esoteric while impure and complicated ones are accessible to every dunce may seem curious, but that this is so has been recognized by thinkers from very ancient times.

Russell's formulation of the foundations of mathematics entails an analysis of mathematics as a formal system derived from purely deductive principles. There is, here, no concern with nor reference to the empirical side of mathematics, that is, to applied mathematics. However, it should be pointed out that which elements of mathematics are really the most purely deductive or simplest and most plausible remains to be decided. Is the concept of number sequence, of the cardinality of numbers, less simple than the successor, the one-to-one coordination of elements? The answer may depend entirely upon the aesthetic feelings of individual mathematicians about these relationships.

Whether or not Russell's system can ever command universal acceptance is of no consequence in this context. The important thing is that the Logicists (as followers of the Russell school are called) no less than the Intuitionists and the Formalists explore the discipline of mathematics as an exclusively formal activity, just as Cubists explored painting in terms of its purely artistic side. Intuitionism, though it stands somewhat apart from the other schools in being more concerned with the relation of mathematics to experience, is occupied with structural problems, particularly those bearing upon the nature of mathematical proof. On the other hand, the Intuitionists (particularly Brouwer) accept absurdity as a basic notion in mathematics; in a way they may have some closer ties with the artists of the succeeding chapter than to the Cubists. I cannot
forbear, in this connection, quoting from a marvelous little book by Mlle Lucienne Mix:

Even a sketchy outline of any intellectual specialty in our century involves comparisons with bordering activities. It is as if, with the disappearance of the universal man (such as Leonardo da Vinci), more or less conscious influences have worked to establish in society as a whole the connection between specialties which formerly could exist within the mind of one man. These influences, so far as thought is concerned, almost always originate in the material contributed by mathematics or physics, and the terms of comparison follow in this order: mathematics, music, painting, poetry. 38

38 Lucienne Felix, *The Modern Aspect of Mathematics*, trans. Julius and Fancille Hlavaty (New York, 1960), p. 15. This is a fascinating work which deals mostly with the group of French mathematicians who publish under the pseudonym Nicolas Bourbaki. But even this rational Frenchwoman has been led to believe in the simultaneity myth. She writes on page 23: "The Cubist studied forms; any cross section could be useful. He even worked in time-space!"

I should also like to mention here an extremely useful work which I have had no occasion to cite but which has been of considerable assistance: Robert M. Exner and Myron F. Rosskopf, *Logic in Elementary Mathematics* (New York, '959)