1. (pts) Write down the basic growth D.E. using $P(t)$ for the population at time $t$. Then solve for $P$.
\[
\frac{dP}{dt} = kP \\
\frac{dP}{P} = kd t \\
\ln P = kt + c \\
P = ce^{kt}
\]

b. Using your expression from 1a, find the population 3 years from now if the current population is now 3000 and was only 1000 the year before. (Hint: let $t = 0$ for the earliest year for which we have data.)

Initial conditions: $P(0) = 1000$

\[
P(0) = 1000 \\
1000 = c e^{0} = c \\
2. P = 1000 e^{kt} \\
\ln P = \ln 1000 + (\ln 3) t \\
k = \ln 3 \quad (\ln 3) t = 1000 (e^{kt}) = 1000 \cdot 3^{t}
\]

\[
P(1) = 3000 \\
3000 = 1000 e^{k} \\
P(4) = 1000 \cdot 3^{4} = 81,000
\]

2. (pts) Set up, but do not solve, the D.E. describing the following situation. Let $S(t)$ be the lbs of salt in truck at time $t$.

\[
\frac{dS}{dt} = \text{(flow of salt in)} - \text{(flow of salt out)} \\
= (2.5) - \frac{S(t)}{100+2t}
\]

\[
\frac{dS}{dt} = 10 - \frac{S}{100+2t}
\]
3. (pts) a. Show the D.E. below is homogeneous in the sense that it can be made separable with a change of variables.

\[ 2xyy' = (x^2 + y^2)dx \]

\[ \frac{M(tx, ty)}{N(tx, ty)} = \frac{(tx)^2 + (ty)^2}{t^2[(x^2 + y^2)]} = t^2 \frac{M(x, y)}{N(x, y)} \]

which is hom.

\[ N(tx, ty) = 2t(tx)(ty) = t^2(2xy) = t^2 N(x, y) \]

\[ \Rightarrow \text{D.E. is hom.} \]

b. Solve the D.E. and find the general solution.

Let \( y = vx \) so \( dy = vdx + xdv \)

\[ 2x(vx)(vdx + xdv) = (x^2 + v^2x^2)dx \]

\[ 2x^2v^2dx + 2x^3v - x^2dx + v^2x^2dx \]

\[ (2x^3)(v - dv) = (x^2 + v^2x^2 - 2x^2v^2)dx \]

\[ = (x^2 - 2x^2v^2)dx \]

\[ = \frac{x^2}{1 - v^2}dx = \ln \frac{1}{1 - v^2} = \ln cx \]

\[ \frac{v - dv}{1 - v^2} = \frac{x^2}{2x^3}dx \]

\[ \int \frac{-1}{1 - v^2} (-2v)dv = \frac{x^2}{2x^3}dx \]

\[ v^2 = \frac{1}{cx} \]

\[ y = x\sqrt{\frac{1}{cx}} \]

4. (pts) Determine if \( f(x) = \cos x \) and \( g(x) = \sin x \) are linearly independent.

\( W(\cos x, \sin x) = \det \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix} = \cos^2 x + \sin^2 x = 1 \) for all \( x \)

\( W \neq 0 \Rightarrow \cos x, \sin x \) are l. indep.

5. (pts) Using the formal definition of \( L[f(t)] \), determine \( L[e^t] \)

\[ L[e^t] = \int_0^\infty e^{-st}e^t \, dt = \int_0^\infty e^{t(1-s)} \, dt = \lim_{b \to \infty} \int_b^{b(1-s)} e^{t(1-s)} \, dt \]

\[ = \lim_{b \to \infty} \frac{1}{1-s} \left[ e^{b(1-s)} - e^0 \right] = \frac{1}{1-s} \lim_{b \to \infty} \left[ e^{b(1-s)} - 1 \right] = \frac{1}{1-s} \lim_{b \to \infty} \left[ \frac{1}{e^{b(1-s)}} - 1 \right] \]

\[ \text{for } s > 1 \]

\[ = \frac{1}{1-s} \begin{pmatrix} 0 \end{pmatrix} = -\frac{1}{1-s} = \begin{pmatrix} 1 \\ s-1 \end{pmatrix} \]
b. What is the appropriate form for the \( y_p \) of the above D.E.

\[ y_p = Ae^{2x} \]

c. Use your \( y_p \) above to find the general soln to the D.E.

\[ y_p = Ae^{2x} + 2Ae^{2x} \]

\[ y'' = 4Ae^{2x} + 4Axe^{2x} = 4Ae^{2x} + 4Axe^{2x} \]