Linguistics 401, section 2
Hale

Semantics: set theory and truth conditions

(1) Truth-conditional semantics
   real-world knowledge, truth as such – not part of semantics
   which pieces of real world knowledge bear on sentences’ truth – the very stuff of semantics

Meaning relationships

Entailment

(2) a. Mary was laughing and dancing.
   b. Mary was dancing.

(3) a. Mary is a great dancer.
   b. Mary is a dancer.

If S1 entails S2, it’s impossible for S1 to be true and S2 to be false in any situation.

Synonymy

(4) a. John sold a car to Mary.
   b. Mary bought a car from John.

(5) a. John is in front of Mary.
   b. Mary is behind John.

Synonymous sentences are true in exactly the same situations: mutually entailing.

Contradiction

(6) a. John is asleap.
   b. John is awake.

(7) a. John claimed that semantics is the funnest part of linguistics.
   b. John denied that semantics is the funnest part of linguistics.

Contradictory sentences cannot both be true in the same situation.

Compositionality

(8) a. The horse behind Pegasus is bald. true if TC₁
   b. The horse behind the horse behind Pegasus is bald. true if TC₂
   c. The horse behind the horse behind the horse behind Pegasus is bald true if TC₃

The truth conditions of (8)(c) are systematically related to the truth conditions of (8)(b).

compositionality The meaning of an expression is determined by the meaning of its parts (and the way in which they are combined).

G. Frege (1892)
1 Model-theoretic semantics in two pages

- standardize on completely unambiguous ‘picture’ of situations: a **model**
- ingredients: **individuals, sets** of individuals, ...

Formalize real (or imaginary) entities in a situation as individuals, formalize properties as sets of individuals. An **assignment function** links particular words to particular entities in a model. Compositional rules, specifying how the meanings of children combine to yield the meanings of their parents permit the derivation of truth conditions for arbitrarily-long sentences.

1.1 Set theory

The things collected together in sets are the **members** of the set. Members can be anything:

- the set of Beatles
- the set of presidents of the USA in 2003
- colors in crayola 4-pack
- the positive integers less than 7
- the positive integers greater than 7
- the set of English sentences
- empty set

the number of members of a set is its **cardinality**. \[ |E| = 4 \quad |\emptyset| = 0 \]

The thing about a set is, you’re either in it or not: \[ 4 \in S_7 \quad \text{“Kissed from Mary John”} \not\in L_{\text{English}} \]

Mick \not\in E \quad 1001 \in \mathbb{G}_7

If one set has all the members of another unequal set, they are in the (proper) **subset** relation.

\[
(9) \quad \begin{array}{ll}
\text{a. } & \{a, b, c\} \subset \{s, b, a, e, g, i, c\} \\
\text{b. } & \{a, b, j\} \not\subset \{s, b, a, e, g, i, c\}
\end{array}
\]

Operations are usually defined on pairs of sets \( A, B \):

- **union** anything in either set \( A \cup B = \{x | x \in A \text{ or } x \in B\} \)
- **intersection** members common to both sets \( A \cap B = \{x | x \in A \text{ and } x \in B\} \)
- **difference** members in one but not the other \( A - B = \{x | x \in A \text{ and } x \notin B\} \)

Compositional rules gives the truth conditions of the S in terms of the meaning of the DP and VP:

\[
(10) \quad \begin{array}{ll}
\text{a. } & \text{When DP is a proper name, a sentence of the form [DP VP] is true if and only if [DP] is a member of [VP].} \\
\text{b. } & \text{[DP Name | ]} = \text{[Name]} \\
\text{c. } & \text{[VP V | ]} = \text{[V]} 
\end{array}
\]
The way the world is and the denotations of individual words are given the model and the assignment function, respectively.

\[
\begin{array}{c}
S \\
| \\
DP \quad VP \\
| \\
Name \quad V \\
| \\
Ringo \quad sings \\
\end{array}
\]

<table>
<thead>
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<th>model-theoretic object</th>
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<tbody>
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<td>Ringo</td>
<td>r</td>
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<td>Michael</td>
<td>m</td>
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<tr>
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(11) A situation pretty close to the real world:

(12) A hypothetical world where Yellow Submarine, Act Naturally, etc were never written:

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What is the meaning of \([_{VP}VP\text{ and }VP]\) ?

Rules needed to accommodate determiners:

(13) a. \([\text{No N VP}]\) is true if and only if \([N] \cap \{VP\} = \emptyset\)
     b. \([\text{All N VP}]\) is true if and only if \([N] \subseteq \{VP\}\)

A rule for intersective adjectives:

(14) \([N\text{Adj} N] = [\text{Adj}] \cap [N]\)

(15) All happy Beatles sing,

Derive the falsehood of this sentence in a model where Paul is the only happy one of the four.