Review

The goal of semantics is to supply an account of what sentences (and their sub-pieces) actually mean. This account should be helpful in explicating relationships between sentences such as CONTRADICTION, SYNONYMY (aka paraphrase) and ENTAILMENT. This goal can be achieved using the concept ‘truth’.

Truth-conditional semantics treats the meaning of a sentence $S$ in terms its truth-conditions: the way the world would have to be in order for $S$ to be true. For example, if someone asks you what the sentence “Moldova is landlocked” means, you might well reply:

$$S \text{ is landlocked if and only if the nation named by “Moldova” has no border with any ocean.}$$

Examples like 1 are called ‘T-sentences’ [Davidson, 1967]. It’s important to keep the two parts of the T-sentence straight: the distinction between the natural language sentence whose meaning is to be explicated and the truth conditions that constitute the explication of that meaning. The former are in the object language, the latter in the metalanguage. Our metalanguage includes set-theoretical requirements that models will have to meet in order for the sentence to be true in those models. It is often helpful for the metalanguage to use an obviously different notation, so that it is clear that the translation from object language to metalanguage is not circular.

<table>
<thead>
<tr>
<th>notation</th>
<th>idea</th>
<th>example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\in$</td>
<td>membership</td>
<td>$48 \in {2, 4, 6, 8, 10, 12, \ldots}$</td>
</tr>
<tr>
<td>$</td>
<td>\</td>
<td>$</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>empty set</td>
<td>$</td>
</tr>
<tr>
<td>$\subseteq$</td>
<td>subset</td>
<td>$SENIORS \subseteq MSU_{UGRAD}$</td>
</tr>
<tr>
<td>$\cup$</td>
<td>union</td>
<td>$MSU_{UGRAD} = FRESH \cup SOPH \cup JUNIOR \cup SENIOR$</td>
</tr>
<tr>
<td>$\cap$</td>
<td>intersection</td>
<td>$SUCCESSFUL = TALENTED \cap HARDWORKING$</td>
</tr>
</tbody>
</table>

Figure 1: Set theory quick reference card

We will legislate a solution to the problem of reference by defining a model to include

1. the individuals that exist
2. what words refer to

For instance, let $NATION$ be the set of all countries in the world. This completes the definition of #1. Then, let us say that the object-language word “Moldova” refers to the set theoretical element $m$ which is a member of $NATION$. We might say also that “Italy” denotes a different element $i$ of $NATION$, “France” denotes $f \in NATION$ etc. The word “landlocked” shall denote the subset of $NATION$ that in fact has no oceanfront property. All of this information can be summarized in table such as 2 (overleaf) where the denotations of proper names of countries are specified to be model-theoretic individuals, while the denotation of the adjective “landlocked” is a set.
Writing up such a table indeed says what words refer to, accomplishing definition #2 and completing the specification of a model. Using the notation \([x]\) for “the interpretation of \(x\) on an assumed model” one can state truth conditions precisely.

<table>
<thead>
<tr>
<th>sentence (object language)</th>
<th>truth condition (metalanguage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moldova is landlocked</td>
<td>([\text{Moldova}] \in \text{[landlocked]})</td>
</tr>
</tbody>
</table>

The model in 2 satisfies this condition, because the denotation of “Moldova” is the element \(m\), the denotation of “landlocked” is the set \(\{m, s, a\}\) and indeed \(m \in \{m, s, a\}\). Correspondingly, this model fails to meet the truth condition for “France is landlocked” correctly deriving that sentence’s falsehood in this situation.

A pleasant consequence of all of this is that the pretheoretic meaning-relationships CONTRADICTION, SYNONYMY and ENTAILMENT now receive a systematic explanation. Two sentences \(S_1\) and \(S_2\) are contradictory if and only if their truth conditions \(TC(S_1)\) and \(TC(S_2)\) cannot both be satisfied in the same model. For instance, 3a and 3b are CONTRADICTORY in exactly this sense: given conventional meanings of the individual words, no model can satisfy both of their truth conditions.

(3) a. Moldova is landlocked
   b. Moldova borders the Indian Ocean.

Similarly, any model satisfying the truth conditions of an example like 4a will necessarily also satisfy the truth conditions for 4b. This provides an account of what ENTAILMENT actually is.

(4) a. Mary was laughing and dancing
    b. Mary was laughing.

The pre-theoretical sentence-relation SYNONYMY can be analyzed as mutually-entailing truth conditions. Any model satisfying the truth conditions of 5b will also be a model that satisfies the truth conditions of 5a and vice versa.

(5) a. John sold a car to Mary
    b. Mary bought a car from John
Semantic composition

The meaning of a sentence depends on its syntactic structure. To work out the truth conditions of 3a rules like the ones in 6 can be used. These rules state how the meaning of a parent node is defined in terms of the meaning of its children; the rules are “compositional.” The rules in 6b all basically say that the meaning of the parent is the same as the meaning of the child.

(6) a. \[
\begin{array}{c}
\text{IP} \\
\text{NP} \\
\text{IP}'
\end{array}
\] = true iff NP is a proper name and [NP] ∈ [I'].

b. \[
\begin{array}{c}
\text{I}' \\
\text{VP} \\
\text{I'}
\end{array}
\] = [VP] \[
\begin{array}{c}
\text{VP} \\
\text{V'}
\end{array}
\] = [V'] \[
\begin{array}{c}
\text{NP} \\
\text{N'} \\
\text{N} \\
\text{Name}
\end{array}
\] = [Name] \[
\begin{array}{c}
\text{V'} \\
\text{V}
\end{array}
\] = [V]

Transitive verbs

The simplest analysis is to view transitive verbs as denoting 2-place relations. These can be represented by sets of pairs. For instance, it could be that Charles loves Camilla and Diana but not Fergie. In model 7, Camilla loves him back reciprocally whereas Diana is fed up.

\[
\begin{align*}
\text{[Charles]} &= c \\
\text{[Fergie]} &= f \\
\text{[Camilla]} &= a \\
\text{[Diana]} &= d \\
\text{[loves]} &= \{(c, a), (c, d), (a, c)\}
\end{align*}
\]

The rule of semantic composition for transitives (8) roughly says “the denotation of a V' that has an NP complement is the set of individuals occupying the first component of any pairs in the denotation of the V whose second component matches the meaning of the NP.”

(8) \[
\begin{array}{c}
\text{V'} \\
\text{V} \\
\text{NP}
\end{array}
\] = \{ X \mid [V](X)([NP]) \} where NP is a proper name.
Quantifiers

The meaning of a determiner is naturally thought of as a relation between the meanings borne by a common noun and those contributed by predicate verb phrases. This is the standard semantics for quantified NPs. Let $A$ and $B$ stand for the meaning of the $N'$ and the $I'$ respectively.

\[\begin{align*}
\text{a. } & \llbracket \text{all} \rrbracket (A)(B) \text{ true iff } A \subset B \\
\text{b. } & \llbracket \text{some} \rrbracket (A)(B) \text{ true iff } A \cap B \neq \emptyset \\
\text{c. } & \llbracket \text{the} \rrbracket (A)(B) \text{ true iff } A \subset B \text{ and } |A| = 1
\end{align*}\]

\[(10)\]

Intersective Adjectives

Some kinds of adjectives “narrow down” the denotation of a common noun.

\[(11)\]

Using the rules given so far, let's derive the truth conditions for “All happy Beatles love Queen Elizabeth II” in the model 13. Assume the syntactic analysis in 12.

\[(12)\]

\[(13)\]
Directional Entailingness

There are lots of determiners in natural language, and their rich variations in meanings constitute a fecund area of study. However, all determiner meanings \( D \) can be viewed as relating two sets: \( A \), the meaning of the subject’s common noun and \( B \) the meaning of the \( \Gamma' \). What happens when, through alternative word choice or the use of intersective adjectives, \( A \) or \( B \) get smaller while \( D \) stays constant? Consider manipulating \( B \) by adding the word “fast.” The set of fast-walkers ought to be a subset of the walkers. Exactly this restriction has been applied to the \( \Gamma' \) meaning in both 14a and 14b. Curiously, 14b is a valid inference whereas 14a is invalid.

(14) a. If every man walks, then every man walks fast.
    b. If no man walks, then no man walks fast.

The pattern in 14 exposes a semantic distinction between “every” and “no.” “No” is downward entailing in its second argument, whereas “every” is not. The pattern in 15 confirms that “every” is upward entailing in its second argument, whereas “no” is not.

(15) a. If every man walks, then every man moves.
    b. If no man walks, then no man moves.

Is “every” downward entailing in its first argument? Yes. The validity of the inference in 16 attests this property.

(16) If every person walks, then every man walks

Examining other cases of (in-)valid inference reveals considerable variation in the directional entailingness of English quantifiers [Ladusaw, 1996].

\[
\begin{array}{c|c|c}
\text{quantifier} & \text{entailment directions} \\
\hline
\text{EVERY}(A)(B) & \downarrow & \uparrow \\
\text{SOME}(A)(B) & \uparrow & \uparrow \\
\text{FEW}(A)(B) & \downarrow & \downarrow \\
\text{NO}(A)(B) & \downarrow & \downarrow \\
\end{array}
\]

downward entailing in the first argument \( D(A)(B) \) and \( A' \subset A \rightarrow D(A')(B) \)
downward entailing in the second argument \( D(A)(B) \) and \( B' \subset B \rightarrow D(A)(B') \)
upward entailing in the first argument \( D(A)(B) \) and \( A \subset A' \rightarrow D(A')(B) \)
upward entailing in the second argument \( D(A)(B) \) and \( B \subset B' \rightarrow D(A)(B') \)

Figure 2: Concise definitions of directional entailingness
Consider **negative polarity items** such as *ever, at all, any* and *give a damn*. They seem to only be acceptable under the influence of some negative word (example 17).

(17)  

a. I don’t want to go to London at all.  
b. *I want to go to London at all  
c. She never gave a damn about you  
d. *She gave a damn about you  
e. Susan seldom makes any comments in class  
f. *Susan always makes any comments in class  

NPIs are acceptable as the $A$ element with “every” but not in the $B$ element.

(18)  

a. Every fan who could *ever* dream of being on TV can be in the audience for TRL.  
b. Every voter who can conceive *at all* of a better world will support the Kyoto protocol.  

(19)  

a. *Every fan can ever be in the audience for TRL.  
b. *Every voter will support the Kyoto protocol at all.  

(20)  

a. No fan can ever be in the audience for TRL.  
b. Few fans can ever be in the audience for TRL.  

(21)  

a. No voters will support the Kyoto protocol at all.  
b. Few voters will support the Kyoto protocol at all.  

A correct empirical claim about NPIs appears to be that they may grammatically appear in downward-entailing semantic environments; even with quantifiers like *few* that don’t appear to be negative in the way that *not* and *no* are.

Comparative determiners such as *more...than* also seem to license negative polarity items as in 22. The examples in 23 suggest that it is only the first argument that licenses the NPIs, though.

(22)  

a. Phil reads more books than any other student.  
b. Phil reads more books than I ever thought at all possible.  

(23)  

a. *Phil reads more books ever than other students.  
b. *Phil reads more books at all than other students.  

Is a comparative like *more...than* downward entailing in its first argument? The validity of the inferences in 24 supports this characterization.

(24)  

a. If more students danced than teachers sang, then more students danced than teachers sang a ballad.  
b. If more students danced a tango than teachers, then more students danced a tango than nutty teachers.  

**References**
