Quantifiers

The meaning of a determiner is naturally thought of as a relation between the set denoted by a common noun and that denoted by a postnominal predicate. Let $A$ and $B$ stand for the meaning of the $N'$ and the $I'$ respectively.

(1) a. $\llbracket \text{all} \rrbracket (A)(B)$ true iff $A \subset B$

   b. $\llbracket \text{every} \rrbracket (A)(B)$ true iff $A \subset B$

   c. $\llbracket \text{some} \rrbracket (A)(B)$ true iff $A \cap B \neq \emptyset$

   d. $\llbracket \text{the} \rrbracket (A)(B)$ true iff $A \subset B$ and $|A| = 1$

(2) \[
\begin{array}{c}
\text{IP} \\
\text{NP} \\
\text{I'} \\
\text{Det} \\
\text{N'}
\end{array}
\]

Where Det is a quantifier. $\llbracket \text{Det} \rrbracket (\llbracket N' \rrbracket)(\llbracket I' \rrbracket)$

Although we have only considered single-word determiners so far, there are also multi-word determiners. The truth condition of $\llbracket \text{Fewer than six N'} \ I' \rrbracket$ is that $|\llbracket N' \rrbracket \cap \llbracket I' \rrbracket| < 6$.

(3) multiword quantifier truth condition

| at least five $A \ B$ | true iff $|A \cap B| \geq 5$ |
| all but three $A \ B$ | true iff $|A| - 3 = |A \cap B|$ |

Directional Entailingness

What happens when, through alternative word choice or the use of intersective adjectives, $A$ or $B$ get smaller while the quantifier $D$ stays constant?

Consider manipulating $B$ by adding the word “fast.” The set of fast-walkers ought to be a subset of the walkers. Exactly this restriction has been applied to the $I'$ meaning in both 4a and 4b. Curiously, 4b is a valid inference whereas 4a is invalid.

(4) a. If every man walks, then every man walks fast.

   b. If no man walks, then no man walks fast.

The pattern in 4 exposes a semantic distinction between “every” and “no.” “No” is downward entailing in its second argument, whereas “every” is not. The pattern in 5 confirms that “every” is upward entailing in its second argument, whereas “no” is not.

(5) a. If every man walks, then every man moves.

   b. If no man walks, then no man moves.

Is “every” downward entailing in its first argument? Yes. The validity of the inference in 6 attests this property.

(6) If every person walks, then every man walks

Consider negative polarity items such as ever, at all, any and give a damn. They seem to only be acceptable under the influence of some negative word (example 7).
(7)  a. I don’t want to go to London at all.
   b. *I want to go to London at all
   c. She never gave a damn about you
   d. *She gave a damn about you
   e. Susan seldom makes any comments in class
   f. *Susan always makes any comments in class

NPIs are acceptable as the A element with “every” but not in the B element.

(8)  a. Every fan who could ever dream of being on TV can be in the audience for TRL.
   b. Every voter who can conceive at all of a better world will support the Kyoto protocol.

(9)  a. *Every fan can ever be in the audience for TRL.
   b. *Every voter will support the Kyoto protocol at all.

(10)  a. No fan can ever be in the audience for TRL.
   b. Few fans can ever be in the audience for TRL.

(11)  a. No voters will support the Kyoto protocol at all.
   b. Few voters will support the Kyoto protocol at all.

A correct empirical claim about NPIs appears to be that they may grammatically appear in downward-entailing semantic environments; even with quantifiers like few that don’t appear to be negative in the way that n’t and no are.

Comparative determiners such as more...than also seem to license negative polarity items as in 12. The examples in 13 suggest that it is only the first argument that licenses the NPIs, though.

(12)  a. Phil reads more books than any other student.
   b. Phil reads more books than I ever thought at all possible.

(13)  a. *Phil reads more books ever than other students.
   b. *Phil reads more books at all than other students.

Is a comparative like more...than downward entailing in its first argument? The validity of the inferences in 14 supports this characterization.

(14)  a. If more students danced than teachers sang, then more students danced than teachers sang a ballad.
   b. If more students danced a tango than teachers, then more students danced a tango than nutty teachers.

downward entailing in the first argument \( D(A)(B) \) and \( A' \subset A \rightarrow D(A')(B) \)
downward entailing in the second argument \( D(A)(B) \) and \( B' \subset B \rightarrow D(A)(B') \)
upward entailing in the first argument \( D(A)(B) \) and \( A \subset A' \rightarrow D(A')(B) \)
upward entailing in the second argument \( D(A)(B) \) and \( B \subset B' \rightarrow D(A)(B') \)

Figure 1: Concise definitions of directional entailingness
Conservativity

A candidate semantic universal: natural language determiners are conservative relations

A relation \( Q \) named by a determiner is conservative if and only if, for any properties \( A \) and \( B \), relation \( Q \) holds between \( A \) and \( B \) if and only if relation \( Q \) holds between \( A \) and the things in \( (A \cap B) \).

A conservative relation \( Q \) is one where \( Q(A)(B) \leftrightarrow Q(A)(A \cap B) \). With conservative relations, we only have to look at the part of predicate \( B \) that overlaps with \( A \) to check if the relation holds. Such relations “live on” \( A \). Nonmembers of \( A \) are irrelevant.

If human language determiner meanings are conservative, then sentences whose truth conditions are \( Q(A)(B) \) should be synonymous with sentences whose truth conditions are \( Q(A)(A \cap B) \).

<table>
<thead>
<tr>
<th>constituent</th>
<th>example</th>
<th>kind of set-theoretic object</th>
</tr>
</thead>
<tbody>
<tr>
<td>names</td>
<td>‘W.’, Paul Wolfowitz</td>
<td>individuals</td>
</tr>
<tr>
<td>VPs</td>
<td>snores, occupies Iraq</td>
<td>sets of individuals that do that activity</td>
</tr>
<tr>
<td>Ns</td>
<td>girl, Beatle</td>
<td>set of individuals that have that property</td>
</tr>
<tr>
<td>As</td>
<td>sleepy, trigger-happy</td>
<td>set of individuals that have that property</td>
</tr>
<tr>
<td>([N'N'[CP that VP ]])</td>
<td>girl that sleeps, Beatle that is happy</td>
<td>set of individuals that have both properties</td>
</tr>
</tbody>
</table>

A compositional semantic rule for a class of relative clauses:

\[
(16) \quad \begin{array}{c}
N'_{\text{outer}} \\
N'_{\text{inner}} \\
\text{CP} \\
C \\
\text{that} \\
\text{VP}
\end{array} = [N'_{\text{inner}}] \cap [VP]
\]

Are determiners really conservative? Doing the experiment

The sentence 17 is true in models where the set of Beatles, \( \text{BEATLE} \) is a subset of the set of smokers, call it \( \text{SMOKE} \).

\[(17) \quad \text{Every Beatle smokes.}\]

Said another way, the truth condition for example 17 is that \( [\text{every}] \) holds between the two sets \( \text{BEATLE} \) and \( \text{SMOKE} \). A quick look back at 1b confirms that the meaning of the word “every” amounts to the subset relation on the two sets \( A = \text{BEATLE} \) and \( B = \text{SMOKE} \).

If the meaning of “every” is a conservative relation, then according to the definition (15) example 17 should be synonymous with a sentence where “every” relates \( A = \text{BEATLE} \) and \( B = (\text{BEATLE} \cap \text{SMOKE}) \).

How can we make such a sentence?

\[(18) \quad \text{Every [N' Beatle] [r is a [N' [N' Beatle] that [VP smokes]]]}\]

Indeed, 18 has truth conditions \( \text{BEATLE} \subset (\text{BEATLE} \cap \text{SMOKE}) \). If “every” is really conservative, then 18 should be synonymous with 17. Is “fewer than 6” conservative?
Non-conservative determiner meanings

(19) owt $A B$ is true if and only iff (at least) two things are in $B$ that are not in $A$

<table>
<thead>
<tr>
<th>sentence</th>
<th>truth value</th>
</tr>
</thead>
<tbody>
<tr>
<td>owt cars roll on the highway</td>
<td></td>
</tr>
<tr>
<td>owt animals are muskrats</td>
<td></td>
</tr>
<tr>
<td>owt bachelors are unmarried men</td>
<td></td>
</tr>
<tr>
<td>owt teachers know model-theoretic semantics</td>
<td></td>
</tr>
<tr>
<td>owt teachers are teachers that know model-theoretic semantics</td>
<td></td>
</tr>
</tbody>
</table>

No human language has a determiner whose meaning is non-conservative like owt.