Chapter 12
Hypothesis Tests: One Sample Mean

- PSY 295 – Oswald
  • “Where statistics and good times meet.”

Major Points

- An example
- Sampling distribution of the mean
- Testing hypotheses: sigma known
  - An example
- Testing hypotheses: sigma unknown
  - An example

Major Points--cont.

- Factors affecting the test statistic
- Measuring the size of the effect between means
- Confidence limits on the effect between means

An Example

- Returning to the effect of violent videos on subsequent behavior
- Modifying the Bushman study just discussed
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Media Violence

• Does violent content in a video affect subsequent responding?
• 100 subjects saw a video containing considerable violence.
• Then free associated to 26 words that had an aggressive & nonaggressive form.
  – e.g. cuff, mug, plaster, pound, sock

Media Violence

• Results
  – Mean number of aggressive words = 7.10
• Assume we already know that without the aggressive video (H_0):
  – the mean = 5.65
  – and the standard deviation = 4.5
  – These
• Is 7.10 (obtained statistic) larger than 5.65 (H_0), enough to conclude that

Sampling Distribution of the Mean

• We need to know what kinds of sample means to expect if video has no effect.
  – i.e.
    if \( \mu = 5.65 \) and \( \sigma = 4.5 \) under H_0?
  – This is the sampling distribution of the mean.
Sampling Distribution of the Mean

- Depends on
  - $\mu$ of population
    - Why?
  - $\sigma$ of population
    - Why?
    - Why?

Central Limit Theorem

- Given a population with mean $= \mu$ and standard deviation $= \sigma$...
- The sampling distribution of the mean (the distribution of sample means) has
  - mean $= \mu$, and
  - standard deviation $= \sigma$
- The distribution approaches

Demonstration

- Let population be very skewed
- Draw samples of 3 and calculate means
- Draw samples of 10 and calculate means
- Plot means
- Note changes in means, standard deviations, and shapes
**Parent Population**

- Skewed Population

**Sampling Distribution \( n = 3 \)**

- Sample size \( n = 3 \)

**Sampling Distribution \( n = 10 \)**

- Sample size \( n = 10 \)

**Demonstration**

- Means have stayed at 3.00 throughout—except for minor sampling error
- Standard deviations have
- Shapes have become with increasing \( N \)
  - see superimposed normal distribution for reference
Testing Hypotheses: $\sigma$ known

- $H_0$: $\mu = 5.65$
- $H_1$: $\mu \neq 5.65$ (Two-tailed/ test)
- Calculate $p$ (sample mean) = 7.10 if $\mu = 5.65$
- Use $z$ from normal distribution
- Sampling distribution would be normal given $N = 100$

$z$-test for one Sample Mean

$z = \frac{X_i - \bar{X}}{\sigma / \sqrt{n}}$

Score is $X_i$  

Score is $\bar{X}$

Using $z$ To Test $H_0$

- Calculate $z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{7.1 - 5.65}{4.5 / \sqrt{100}} = 3.22$
- If $z > \pm 1.96$, reject $H_0$ at $\alpha = .05$ (2-tailed)
- $3.22 > 1.96$
  - Conclusion: Reject $H_0$ (accept $H_1$)
  - The mean difference is statistically significant.
  - Interpretation...
Testing When $\sigma$ Not Known

- Assume same example, but $\sigma$ not known
- Can’t substitute $s$ for $\sigma$ directly because See next slide.
- …Do it anyway, but call answer $t$ instead of $z$.
- Compare $t$ to tabled values of $t$ – just like you did for the $z$ table.

$\sigma^2 = 138.89$

Population variance = 138.89

$N = 5$

10,000 samples

58.94% < 138.89

$t$ Test for One Mean

- Same as $z$ except for
- For Bushman, $s = 4.40$

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} = \frac{7.1 - 5.65}{\frac{4.40}{\sqrt{100}}} = \frac{1.45}{.44} = 3.30$$

Degrees of Freedom

- Skewness of sampling distribution of variance decreases
- $t$ will differ from $z$ less as
- Therefore need to adjust $t$ accordingly
- $df = N - 1$
- $t$ based on $df$
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### t Distribution

<table>
<thead>
<tr>
<th>df</th>
<th>.10</th>
<th>.05</th>
<th>.02</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.812</td>
<td>2.228</td>
<td>2.764</td>
<td>3.169</td>
</tr>
<tr>
<td>15</td>
<td>1.753</td>
<td>2.131</td>
<td>2.602</td>
<td>2.947</td>
</tr>
<tr>
<td>20</td>
<td>1.725</td>
<td>2.086</td>
<td>2.528</td>
<td>2.845</td>
</tr>
<tr>
<td>25</td>
<td>1.708</td>
<td>2.060</td>
<td>2.485</td>
<td>2.787</td>
</tr>
<tr>
<td>30</td>
<td>1.697</td>
<td>2.042</td>
<td>2.457</td>
<td>2.750</td>
</tr>
<tr>
<td>100</td>
<td>1.660</td>
<td>1.984</td>
<td>2.364</td>
<td>2.626</td>
</tr>
</tbody>
</table>

$d_f = 100 - 1 = 99$, use 100 here

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### Conclusions

- With $N = 100$, $t_{.05}(99) = 1.98$
- Because $t = 3.30 > 1.98$,
- Conclude that viewing violent video tends to lead to word responses than normal.

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### Factors Affecting $t$

- Difference between sample and population means
- Magnitude of sample variance
- Sample size

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### Factors Affecting Decision

- Significance level $\alpha$
- One-tailed versus two-tailed test
- Remember Type I and II errors
Size of the Effect

- We know that the difference is statistically significant.
  - That...
- Population mean = 5.65,
  Sample mean = 7.10
- Difference is nearly 1.5 words, or 25% more violent words than normal.

Effect Size

- Later we will express mean differences in terms of ...
  - 1.45 units is $1.45/4.40 = 1/3$ of a standard deviation.
  - Relative to the standard deviation of the group, the mean difference is $1/3$ st. dev.

Confidence Limits on Mean

- Sample mean is a ...
- We want
  - Probability that interval computed this way includes $\mu$ is 95%

\[
CI_{.95} = \bar{X} \pm t_{.025}\frac{s_x}{\sqrt{n}}
\]

For Our Data

\[
CI_{.95} = \bar{X} \pm t_{.025}\frac{s_x}{\sqrt{n}}
\]
\[
= 7.1 \pm 1.98 \times 0.44
\]
\[
= 7.1 \pm 0.87
\]
\[
= 6.23 \leq \mu \leq 7.97
\]
**Confidence Interval**

- The interval does not include 5.65, the population mean without a violent video (H_0).
- Consistent with ___.
- Confidence interval and ___ tell us about the magnitude of the effect.
- What can we conclude from confidence interval?

**Review Questions**
(Do these on your own. They are NOT the only things to study)

- What is the sampling distribution of the mean?
- Why do we need it?
- Why did I assume that the population mean was 5.65 and its st. dev. was 0.45?
- What statistic is used when the population variance (or st. dev.) is not known?

**Review Questions**

- What did the shape of the sampling distribution of the variance illustrate in this lecture?
- How does t differ from z?
- What are degrees of freedom (df)?
- What factors affect the value of t?

**Review Questions--cont.**

- What factors affect our conclusions?
- What do effect sizes tell us?
- What is the difference between confidence limits and a confidence interval?
- How do we interpret a computed confidence interval?