Chapter 6
The Normal Distribution

PSY 395 – Oswald

Outline

• Distributions and area
• The normal distribution
• The standard normal distribution
• Setting probable limits on a score/observation
• Measures related to $z$

Distributions and Area

• The idea of a pie chart as representing
  – See next slide
• Bar charts say
  – See subsequent slide

Where are the Bad Guys?

Percentage

- Prison: 61%
- Jail: 19%
- Parole: 9%
- Probation: 11%
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Bar Chart of Bad Guys

A more continuous distribution….

Normal Distribution

• We study the normal distribution because many naturally occurring events yield
Normal Distribution

- The normal distribution is a (what are the endpoints?)
- “Normal” here does not mean ; it is a for this specific mathematical function for the curve

Percent Area Under the Normal Distribution

- The mean/median/mode are
- If we split the distribution in half, 50% of the scores of the sample lie (or median, or mode), and 50% of the scores lie (or median, or mode)

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Proportions of Area Under the Normal Distribution

- The entire area under the normal curve can be considered to be equal to a proportion of
- Speaking in proportions, .500 of the scores lie in the bottom half (below the mean, and
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Proportions of Area Under the Normal Distribution

Formula for the Normal Distribution

\[ f(X) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}} \]

- \( X \) is the
- \( Y \) is the
- \( \pi \) and \( e \) are

Histogram w/ Normal data, \( \bar{X} = 0, s = 1 \)

z-scores

\[ z = \frac{X - \mu}{\sigma} \]

- \textbf{z-scores} (or standard scores) are a way of expressing a raw score’s
- Subtract from \( X \)
- Divide by
- This is a (every data point gets changed)
- \( z = \)
z-scores

- A z-score – rather than a raw score – is a better way to let you know than a raw score

- A student gets a 75/100 on a test

- Is this a high score or a low score?

z-scores

- If the average of the class grades is 71 and the (population) standard deviation is 5.2, then the z-score for a grade of 85 would be:

  \[
  z = \frac{X - \mu}{\sigma}
  \]

  \[
  z = \frac{85 - 71}{5.2} = \frac{14}{5.2} = \frac{2}{1}
  \]

- This means that a mark of 85% is actually

- The student would have done as compared to the rest of the class

z-scores

- represents the (which would be 71 in this example)

- represents a score (which would be less than 71)

- represents a score (which would be greater than 71)
**z-scores**

- By definition, for any set of scores:
  - the sum of z-scores will equal
    \[
    (\sum z = 0.00)
    \]
  - \(\mu_z = 0.00\)
  - and a standard deviation
    \(\sigma_z = 1.00\)

- A z-score expresses the position of the raw score above or below the mean in

- For instance
  - \(z = +1\) means that the raw score is 1 standard deviation
  - \(z = -2.00\) means that the raw score is 2 standard deviations

**Reflections on z**

- The mean \(\mu\) and standard deviation \(\sigma\) are always given; they are

- z-scores only reflect the data points’ position to the mean (and nothing more)

- You’re now considering the data as a
  as you’re not looking to infer to

- This means for standard deviation rather than the sample formula whenever you calculate z (i.e.,

**z-score: Another example**

- You take two exams in Math and English, getting the following scores:
  - Math: 70% (class = 55%, \(\sigma = 10\%\))
  - English: 60% (class = 50%, \(\sigma = 5\%\))

- Which test score represents the better performance?

-
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z-score (Example cont.)

• Math mark:
  \[ z = (70-55)/10 \]
  \[ z = +1.50 \]

• English mark:
  \[ z = (60-50)/5 \]
  \[ z = +2.00 \]

\[ z = \frac{X - \mu}{\sigma} \]

Z-score Example Illustration

Math: \[ z = 1.50 \]
English: \[ z = 2.00 \]

The Answer

• Because: \( z = +2.00 \) is greater than \( z = +1.50 \), the English grade of 60% reflects a higher score than does the Math grade of 70%.

• ...even though

z-score: Solving for \( X \)

• The z-score formula can be rearranged to solve for the raw score \( X \):

\[ X = (z)(\sigma) + \mu \]
Z-scores: Solving for $X$

- This formula is used when you know the z-score of a data point, and

Example

- E.g., if a class midterm exam has $\mu = 65$ and $\sigma = 5$, what exam mark has a z-score value of 1.25?

\[
X = (z)(\sigma) + \mu
\]

\[
X = (1.25)(5) + 65 = 6.25 + 65 = 71.25
\]

So, a person whose test is obtained a score of 71.25%

z-scores

- z-score problems ask you to solve for $X$ or solve for $z$

The Standard Normal Distribution

- Transform all $X$ values into z scores, which (always) have mean = 0 and standard deviation = 1

- Define the area under the curve to be 1.0
Tables of $z$

- We use tables to find
- A sample table is on the next slide
- The following slide illustrates areas under the distribution

<table>
<thead>
<tr>
<th>$z$</th>
<th>Mean to Larger Portion</th>
<th>Smaller Portion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>.0000</td>
<td>.5000</td>
</tr>
<tr>
<td>0.10</td>
<td>.0398</td>
<td>.5398</td>
</tr>
<tr>
<td>0.20</td>
<td>.0793</td>
<td>.5793</td>
</tr>
<tr>
<td>1.00</td>
<td>.3413</td>
<td>.8413</td>
</tr>
<tr>
<td>1.50</td>
<td>.4332</td>
<td>.9332</td>
</tr>
<tr>
<td>1.645</td>
<td>.4500</td>
<td>.9500</td>
</tr>
<tr>
<td>1.960</td>
<td>.4750</td>
<td>.9750</td>
</tr>
</tbody>
</table>

Normal Distribution
Cutoff at $z = 1.645$

Area $= .05$
Area $= .95$
Using the Tables

- Based on the $z$ you have,
- Define
- Distribution is __________, so the table negative values of $z$
- Example: Area/proportion of people between $z = +1.5$ and $z = -1.0$
  - See next slide

Calculating areas

- Area between mean and $+1.5 = 0.4332$
- Area between mean and $-1.0 = 0.3413$
- Sum equals $0.7745$
- Therefore about $77\%$ of the observations would be

Converting Back to $X$

- Assume $\mu = 30$ and $\sigma = 5$
- $77\%$ of the distribution

\[
\begin{align*}
  z &= \frac{X - \mu}{\sigma} \\
  \text{Therefore} & \quad X = \mu + z \times \sigma \\
  X &= 30 - 1.0 \times 5 = 25 \\
  X &= 30 + 1.5 \times 5 = 37.5
\end{align*}
\]

Probable Limits

- $X = \mu + z \times \sigma$
- Our last example has $\mu = 30$ and $\sigma = 5$
- We want
- This means $(5\%$ outside the middle, half on both sides)
- Well, the table gives the $z$ for
  - of the normal distribution

\[
\begin{align*}
  X &= \mu + z \times \sigma \\
  X &= 30 + 1.96 \times 5 = 39.8 \\
  X &= 30 - 1.96 \times 5 = 20.2
\end{align*}
\]
Probable Limits--cont.

• We have just shown that in this example that
  lies between 20.2 and 39.8
• Therefore the probability is
  will lie between 20.2 and 39.8

 Measures Related to z

• Standard score
  –
• Percentile score
  – The point below which a fall
• $T$ scores
  – Scores with standard deviation of

Review Questions

• What do we gain ?
• How is a “standard” normal distribution different from any other normal distribution?

Review Questions--cont.

• How do we convert $X$ to $z$?
• How do we use the tables of $z$? ?
• Of what use are ?
• If we know your test score, how do we ?
• What is a $T$ score and why do we care?