TESTING RATIONAL EXPECTATIONS AND EFFICIENCY IN THE FOREIGN EXCHANGE MARKET

BY RICHARD T. BAILLIE, ROBERT E. LIPPENS, AND PATRICK C. McMahan

Forward and spot exchange rates are modelled as an unrestricted bivariate autoregression from weekly data on the New York foreign exchange market for June, 1973 to April, 1980. The null hypothesis that the forward exchange rate is an unbiased estimate of the corresponding future spot exchange rate is tested by means of a nonlinear Wald test and is rejected for all six currencies considered. The results cast doubt on a central assumption in many current models of exchange rate behavior.

1. INTRODUCTION

The advent of flexible exchange rates in 1973 has led to a great amount of empirical research concerning the relationship between forward and spot exchange rates. Considerable attention has been focussed on the efficient markets hypothesis for foreign exchange over a wide variety of different currencies. If the foreign exchange market is efficient, in the sense that all available information is used rationally by risk-neutral agents in determining the spot and forward exchange rates, then the expected rate of return to speculation will be zero. Thus, under the assumption of rational expectations and risk neutrality, an hypothesis can be derived, in which the forward rate is an unbiased predictor of the future spot rate. This is clearly a joint hypothesis since it includes the assumption of rational expectations and the assumption that the risk premium for the forward rate is zero. An empirical test which rejects this joint hypothesis can be interpreted either as rejecting rational expectations, or as indicating that the risk premium is nonzero and time varying, or that both assumptions are inappropriate. Moreover, the issue of market bias has attracted attention since Keynes [12] developed a normal backwardation theory which implies that speculators require a premium for undertaking risk.

Much econometric work has considered the implementation and testing of the rational expectations hypothesis; see Wallis [18] and Hoffman and Schmidt [10]. Specifically, in regard to the foreign exchange market, use of the forward rate as a measure for exchange rate expectations is appealing since, as noted by Frenkel [3, 5] among others, it can be regarded as an observable expectations series of agents in the market, thus circumventing the problem of how best to generate an “expectations” series. Indeed, this assumption is a central building block in the monetary approach to the exchange rate. The seminal paper by Dornbusch [2] implicitly assumes that the forward rate is an unbiased predictor of the future spot rate. However, this is an empirical question and much of the previous evidence has been inconclusive and has not paid sufficient attention to the joint

1 We are very grateful to Ralph Bailey for writing appropriate computer programs, and to two anonymous referees for their very helpful comments.
nature of the hypothesis. Many authors have been primarily concerned with the rational expectations aspect of the hypothesis. For example, on one of the few occasions when Frenkel [4] did reject the null hypothesis he concluded that it was not inconsistent with the rational expectations hypothesis and attributed the rejection to individuals' lack of experience with hyperinflation rather than the existence of risk aversion.

In this paper our objective is to subject the proposition of unbiasedness to detailed and rigorous empirical testing. We model the 30 day forward rate and the spot rate as a jointly dependent bivariate autoregressive process. Weekly data are used for both variables and the efficient market hypothesis is seen to imply certain nonlinear restrictions on the coefficients of the bivariate autoregressive process. The hypothesis is then tested by means of a Wald test which is asymptotically equivalent to the likelihood ratio test.

2. METHODOLOGY

Assuming the logarithm of the spot rate $s$, and the logarithm of the forward rate $f$, to be a linear nondeterministic jointly covariance-stationary process, it follows from the multivariate form of Wold's decomposition (see Hannan [8]) that the process can be represented by a unique infinite order bivariate moving average process. For a suitably chosen value of $p$, this can be approximated by the bivariate autoregression

$$
\begin{bmatrix}
1 - \alpha(L) & -\beta(L) \\
-\gamma(L) & 1 - \lambda(L)
\end{bmatrix}
\begin{bmatrix}
s_t \\
f_t
\end{bmatrix}
= 
\begin{bmatrix}
\epsilon_t \\
\omega_t
\end{bmatrix}
$$

where

$$
\alpha(L) = \sum_{j=1}^{p} \alpha_j L^j, \quad \beta(L) = \sum_{j=1}^{p} \beta_j L^j, \quad \gamma(L) = \sum_{j=1}^{p} \gamma_j L^j,
$$

$$
\lambda(L) = \sum_{j=1}^{p} \lambda_j L^j,
$$

$L$ is the lag operator, and both variables $s$ and $f$ are measured as deviations from their means. By letting $u'_t = [\epsilon_t, \omega_t]$, then $E(u_t) = 0$ and

$$
E(u_t u'_{t-j}) = \begin{cases} 
\Omega, & j = 0, \\
0, & j \neq 0.
\end{cases}
$$

Under the further assumption of $u_t$ being normally distributed the application of ordinary least squares to each equation will result in asymptotically efficient estimates of the parameters

$$
\theta' = [\alpha_1 \ldots \alpha_p \beta_1 \ldots \beta_p \gamma_1 \ldots \gamma_p \lambda_1 \ldots \lambda_p].
$$
The resulting estimates, based on $T$ observations, will have the asymptotic distribution

$$\sqrt{T}(\hat{\theta} - \theta) \sim N\{0, \Omega \otimes M^{-1}\},$$

where $M = \text{plim}(X'X T^{-1})$ and the matrix $X$ contains observations on the variables $(s_{t-1}, \ldots, s_{t-p}, f_{t-1}, \ldots, f_{t-p})$.

It is also convenient to express equation (1) as a first order system in companion form as

$$
\begin{bmatrix}
    s_t \\
    s_{t-1} \\
    \vdots \\
    s_{t-p+1} \\
    f_t \\
    f_{t-1} \\
    \vdots \\
    f_{t-p+1}
\end{bmatrix} =
\begin{bmatrix}
    \alpha_1 \alpha_2 & \cdots & \alpha_p \beta_1 \beta_2 & \cdots & \beta_p \\
    1 & 0 & \cdots & 0 \\
    1 & \cdots & 1 & 0 \\
    \gamma_1 \gamma_2 & \cdots & \gamma_p \lambda_1 \lambda_2 & \cdots & \lambda_p \\
    1 & 0 & \cdots & 0 \\
    0 & \cdots & 0 & \cdots & 0 \\
    0 & \cdots & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
    s_{t-1} \\
    s_{t-2} \\
    \vdots \\
    s_{t-p} \\
    f_{t-1} \\
    f_{t-2} \\
    \vdots \\
    f_{t-p}
\end{bmatrix} +
\begin{bmatrix}
    \epsilon_t \\
    0 \\
    \vdots \\
    0 \\
    \omega_t \\
    0 \\
    \vdots \\
    0
\end{bmatrix}
$$

which can be written as

$$Y_t = A Y_{t-1} + \eta_t.$$  

We can now turn to the relationship between the spot rate and the forward rate. With weekly data the 30 day forward rate is approximately a 4 period forecast of spot rates, and under the assumption of rational expectations and market efficiency it follows that

$$s_{t+4} = f_t + a_t,$$

where the disturbance $a_t$ is such that $E(f,a_t) = 0$. One of the key properties of the theory is that the forward rate contains all relevant and necessary information required to predict future spot rates. In order to obtain testable restrictions in terms of the parameters of model (1), it is convenient to express (4) in terms of information known at time $t - 1$.

From equation (3)

$$f_t = g'AY_{t-1} + \omega_t,$$

where $g'$ is a $1 \times 2p$ row vector, composed entirely of zeros, except for unity in the $(p + 1)$ element. From (3) it can also be shown that

$$Y_{t+4} = A^5Y_{t-1} + \sum_{j=0}^{4} A^j\eta_{t+4-j}.$$
or

\[ s_{t+4} = e' A^5 Y_{t-1} + \xi_t \]

where \( e' = [1 \ 0 \ldots \ 0] \), so that \( e' A^5 \) is a row vector whose elements are optimal 5 step ahead prediction weights (see Baillie [1]).

On subtracting (5) from (6) we obtain

\[ s_{t+4} - f_t = (e' A^5 - g' A) Y_{t-1} + \nu_t, \]

where \( \nu_t = \xi_t - \omega_t \), and is at most an MA(4) process. We now define the operator \( E_{t-1} \) to denote the expectation of a random variable at time \( t - 1 \) based on all relevant information available at time \( t - 1 \). Since \( E_{t-1}(a_t) = 0 \), it follows from equations (4) and (7) respectively that

\[ E_{t-1}(s_{t+4}) - E_{t-1}(f_t) = 0 \]

and that

\[ E_{t-1}(s_{t+4}) - E_{t-1}(f_t) = (e' A^5 - g' A) Y_{t-1}. \]

On combining (8) and (9) we see that the hypothesis of rational expectations and efficiency in the foreign exchange market implies that

\[ e' A^5 - g' A = 0, \]

which generates highly nonlinear restrictions on the parameters \( \theta \). One way of testing the theory would be to estimate the unrestricted model (1) and then to re-estimate subject to the restrictions (10) and to compute a likelihood ratio test. However, full maximum likelihood estimation of the restricted model is difficult and a computationally simpler procedure is to estimate just the unrestricted model by ordinary least squares and to compute a Wald [17] type test. Such a test statistic will be asymptotically equivalent to the likelihood ratio test (see Silvey [15]). Consequently, we wish to test

\[ H_0 : \quad e' A^5 - g' A = 0, \quad \text{versus} \]

\[ H_1 : \quad e' A^5 - g' A \neq 0. \]

It should be noted that in part of their Appendix, Hansen and Hodrick [9] mention the possibility of testing for unbiasedness by examining the cross-equation restrictions on a vector time series model, but reject this approach as being too complex. Since our paper was written, a paper by Hakkio [7] has appeared in which he estimates a vector autoregressive model and conducts a likelihood ratio test on the cross equation restrictions. However, when estimating under the null hypothesis Hakkio [7, p. 669] finds that “generally 600 iterations were required to obtain three significant digits,” suggesting the possibility of a poorly identified model. Furthermore because of problems of estimating under the null hypothesis Hakkio [7, p. 671] was forced to take \( p = 4 \) while a likelihood ratio test rejected \( p = 4 \) in favor of \( p = 12 \). This problem is also apparent in his Appendix 2 which presents single-equation portmanteau statistics which exhibit marked autocorrelation in two of his models. Clearly, a vector autoregression with too few lagged terms may result in biased test statistics.
By writing the $2p$ vector of parameter constraints as

$$\begin{equation}
(11) \quad r(\theta)' = e'A^2 - g'A,
\end{equation}$$

the appropriate test statistic is

$$\begin{equation}
(12) \quad W = r(\hat{\theta})'[D'\{\hat{\Omega} \otimes (X'X)^{-1}\}D]^{-1}r(\hat{\theta}),
\end{equation}$$

where $D = \partial r(\theta)/\partial \theta$. An expression for $D$ can be found by using the matrix differentiation result due to Schmidt [14], that

$$\frac{\partial R(A^l)}{\partial R(A)} = \sum_{j=0}^{l-1} A'^j \otimes A'^{l-1-j}$$

where $R$ denotes the row stacking operator. This result has been extensively used in the derivation of various properties of predictions from dynamic models; for example, see Baillie [1]. In this particular case

$$\begin{equation}
(13) \quad D = \left[ \begin{array}{c}
\sum_{j=0}^{4} (e'A'^je)A^{4-j} \\
\sum_{j=0}^{4} (g'A'^je)A^{4-j} - I
\end{array} \right]
\end{equation}$$

so that $e'A'^je$ and $g'A'^je$ select the $(1, 1)$ and $(p + 1, 1)$ elements of $A'^j$ respectively and hence the Wald statistic (12) can be straightforwardly computed. Under the null hypothesis, $W$ has an asymptotic $\chi^2$ distribution with $2p$ degrees of freedom.

An alternative approach is to consider the model

$$s_{t+1} - f_t = \Psi'x_t + \eta_t$$

where the forward premium is regressed on a vector of explanatory variables $x_t$, and $\Psi$ is the vector of parameters associated with $x_t$. Then under the efficient market hypothesis $\Psi = 0$ and, if the sampling interval is less than the period of maturity,

$$E(\xi_t\xi_{t+j}) = 0 \quad \text{for} \quad j \geq l.$$ 

This approach is used by Hansen and Hodrick [9] who take 90 day contracts, so that $l = 13$, and arbitrarily include the last two forecast errors in $x_t$. For the 1970's they cannot reject the hypothesis for six out of seven currencies. When $x_t$ includes the last forecast error on all other currencies, Hansen and Hodrick can reject the null hypothesis in three out of five cases. Using the same methodology for the 1920's they reject for five out of six currencies.

In our study $x_t$ corresponds to $Y_{t-1}$ and under the null hypothesis all information should be contained in $f_t$. Frenkel [3, 4, 5] considers a model of the
form

$$s_{t+4} - f_t = \Psi f_t + \xi_t$$

and tests the hypothesis that $\Psi = 0$ and $\xi_t$ is white noise. However, Frenkel uses monthly data and is generally unable to reject the null hypothesis. It may be that a test based on monthly rather than weekly data is less powerful due to loss of information. To gain some insight into this we also estimated, by ordinary least squares, the model

$$S_{t+1} = \alpha + \beta F_t + \xi_t,$$

where $S_t$ is the logarithm of the four weekly spot rate and $F_t$ is the logarithm of the four weekly forward rate. The hypothesis of interest is given by the constraint that $\alpha = 0$ and $\beta = 1$ and that $\xi_t$ is serially uncorrelated.

3. DATA

Daily observations on spot and 30 day forward rates on various currencies were taken from the New York foreign exchange market. For each week an observation for the spot rate was recorded on the Thursday and the forward rate was recorded on the Tuesday. This method of recording ensured that exactly 30 days separated each spot and its corresponding forward rate. When an observation was unavailable due to the foreign exchange market being closed, an observation on an adjacent day was chosen and the observation point of the corresponding series was also moved to ensure a 30 day gap between the observations. Observations were recorded on six different currencies in terms of their value against the U.S. dollar. For U.K., West Germany, Italy, and France observations covered the period June 1, 1973 to April 8, 1980, realizing 362 data points. For Canada and Switzerland the same quality data were only available from December 1, 1977 to May 15, 1980, which provided 128 observations.

When estimating the testing model (14), every fourth observation on the spot and forward rates was taken. This produced 90 observations for the U.K., West Germany, Italy, and France and 32 observations for the other two currencies.

4. RESULTS

The logarithms of the spot and 30 day forward exchange rate series were analyzed for each currency. Analysis of the autocorrelation functions of the transformed variables failed to detect any evidence of seasonal behavior. Since model (1) is essentially the most general model under the alternative hypothesis it seems likely that it is important to choose carefully the value for $p$, the maximum lag. Three different criteria and forms of test were examined in this regard: firstly the value for $p$ was found that minimized the Akaike Information Criterion (AIC) of:

$$-2 \ln \text{likelihood} + 2 \text{ (number of parameters)}.$$
Secondly, for each estimated model, general diagnostic statistics were computed. In particular, for each separate equation of the model the modified Box-Pierce portmanteau statistic

\[ Q_i(m) = T(T + 2) \sum_{j=1}^{m} \frac{1}{(T-j)} r_{ij}^2, \quad i = 1, 2, \]

was calculated, where \( i \) is the number of the equation and \( r_{ij} \) is the residual autocorrelation of lag \( j \) from the \( i \)th equation. Under the null hypothesis that the \( i \)th equation’s residuals are uncorrelated, \( Q_i(m) \) has an asymptotic \( \chi^2 \) distribution with \( m - p \) degrees of freedom. Although this statistic is of some use in detecting misspecification of the model for the \( i \)th equation, it is more useful to apply an appropriate multivariate test to test the null hypothesis that the residuals are a multivariate white noise process. Under such a null, Hosking \[11\] has recently shown that

\[ H(m) = T^2 \sum_{j=1}^{m} \frac{1}{(T-j)} \text{trace } C_j^t C_0^{-1} C_j C_0^{-1} \]

has an asymptotic \( \chi^2 \) distribution, with \( 4(m - p) \) degrees of freedom, where \( C_j \) is the residual autocovariance matrix of lag \( j \) and is defined

\[ C_j = \frac{1}{T} \sum_{t=j+1}^{T} \hat{u}_t \hat{u}'_{-j}. \]

However, such diagnostic tests of misspecification are based on the alternative hypothesis that the errors follow an \( m \)th order linear process. Such test statistics have high degrees of freedom and may have low power. Accordingly a third category of test statistic was calculated, with likelihood ratio tests for the process being vector AR(\( p + 1 \)), or vector AR(\( p - 1 \)), against vector AR(\( p \)) being computed. All the various test statistics were of some use in detecting model misspecification. Although Sawa \[13\] has shown that the AIC is in general biased toward accepting the larger model it was discovered that the minimum AIC choice of \( p \) was generally supported by the other test statistics for the six bivariate processes considered in this study. The general criteria used was to choose the smallest value for \( p \) which gave satisfactory diagnostic portmanteau statistics and for which the likelihood ratio tests did not indicate the need for further increasing \( p \).

The results presented in Table I give the details for the most appropriate model for each currency. For some currencies the choice of the optimum value for \( p \) was not clear cut, but the Wald statistic (12) appeared robust for overfitted models which used a higher than necessary value of \( p \).

It can be seen from the final column of Table I that the Wald test statistic was significant at the 1 per cent level for all six currencies. It should be noted that first differences were taken of all the series. From a time series perspective this transformation was necessary in order to invoke covariance stationary series.
### TABLE I

**Wald Tests on Weekly Observations in Differences**  
**June 1973 to April 1980**

<table>
<thead>
<tr>
<th>Country</th>
<th>Value of ( p ) that was finally chosen</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
<th>( H(20) )</th>
<th>( H(54) )</th>
<th>Value of ( p ) that minimized the AIC</th>
<th>Likelihood Ratio Tests</th>
<th>Wald Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>9</td>
<td>66.67&lt;sup&gt;b&lt;/sup&gt;</td>
<td>52.15</td>
<td>46.36</td>
<td>178.07</td>
<td>9</td>
<td>21.72&lt;sup&gt;a&lt;/sup&gt;</td>
<td>9.64&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>West Germany</td>
<td>6</td>
<td>51.86</td>
<td>44.69</td>
<td>77.59&lt;sup&gt;b&lt;/sup&gt;</td>
<td>207.63</td>
<td>6</td>
<td>25.63&lt;sup&gt;a&lt;/sup&gt;</td>
<td>19.39&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Italy</td>
<td>10</td>
<td>29.79</td>
<td>52.50</td>
<td>50.20</td>
<td>147.26</td>
<td>7</td>
<td>14.69&lt;sup&gt;c&lt;/sup&gt;</td>
<td>9.57&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>France</td>
<td>6</td>
<td>40.68</td>
<td>47.63</td>
<td>78.01&lt;sup&gt;b&lt;/sup&gt;</td>
<td>186.00</td>
<td>11</td>
<td>33.30&lt;sup&gt;a&lt;/sup&gt;</td>
<td>16.90&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Canada</td>
<td>3</td>
<td>15.74</td>
<td>17.17</td>
<td>68.86</td>
<td>–</td>
<td>3</td>
<td>34.86&lt;sup&gt;a&lt;/sup&gt;</td>
<td>13.05&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Switzerland</td>
<td>3</td>
<td>7.28</td>
<td>18.43</td>
<td>58.70</td>
<td>–</td>
<td>3</td>
<td>25.64&lt;sup&gt;a&lt;/sup&gt;</td>
<td>11.98&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Key: \( Q_1 \) and \( Q_2 \) are computed from \( m = 54 \) lags for the U.K., West Germany, Italy, and France; and for \( m = 20 \) lags for Canada and Switzerland.

The symbols \( 1 \_2 \), \( 1 \_1 \), and \( 1 \) denote \(-2 \ln \) of the likelihood ratio for the choice of lag \( p \); \( 1 \_2 \) is a test that the model is AR(\( p - 2 \)) against AR(\( p \)), \( 1 \_1 \) tests AR(\( p \) - 1) against AR(\( p \)), \( 1 \) tests AR(\( p \)) against AR(\( p + 1 \)), and \( 2 \) tests AR(\( p \)) against AR(\( p + 2 \)). Thus, under the null hypothesis that the lower value of \( p \) is appropriate, \( 1 \_2 \) and \( 1 \) have asymptotic \( \chi^2 \) distributions and \( 1 \_1 \) and \( 1 \) have asymptotic \( \chi^2 \) distributions.

The superscripts \( a \), \( b \), and \( c \) denote significance at the 1 per cent, 5 per cent, and 10 per cent levels respectively.

### TABLE II

**Wald Tests on Weekly Observations in Differences**  
**February 1975 to April 1980**

<table>
<thead>
<tr>
<th>Country</th>
<th>Value of ( p ) that was finally chosen</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
<th>( H(20) )</th>
<th>( H(54) )</th>
<th>Value of ( p ) that minimized the AIC</th>
<th>Likelihood Ratio Tests</th>
<th>Wald Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>8</td>
<td>68.59&lt;sup&gt;b&lt;/sup&gt;</td>
<td>52.01</td>
<td>44.61</td>
<td>181.83</td>
<td>8</td>
<td>15.55&lt;sup&gt;b&lt;/sup&gt;</td>
<td>12.20&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>West Germany</td>
<td>6</td>
<td>47.93</td>
<td>49.37</td>
<td>55.02</td>
<td>179.99</td>
<td>6</td>
<td>35.36&lt;sup&gt;a&lt;/sup&gt;</td>
<td>31.43&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>Italy</td>
<td>8</td>
<td>33.19</td>
<td>65.16&lt;sup&gt;b&lt;/sup&gt;</td>
<td>52.98</td>
<td>167.23</td>
<td>8</td>
<td>23.39&lt;sup&gt;a&lt;/sup&gt;</td>
<td>13.20&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>France</td>
<td>6</td>
<td>59.20</td>
<td>72.81&lt;sup&gt;b&lt;/sup&gt;</td>
<td>67.04</td>
<td>203.90</td>
<td>6</td>
<td>37.34&lt;sup&gt;a&lt;/sup&gt;</td>
<td>27.21&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

Results based on \( T = 274 \) observations.

Key: See Table I.
The models were also estimated in levels with data that were consequently highly nonstationary. It could be argued that the vector autoregressive model in this situation is merely an assumption and all results are conditional on its appropriateness, although Fuller [6] and Sims [16] have shown some asymptotic tests to be robust to the inclusion of certain types of unstable roots in scalar autoregressions. Analysis of the model in levels, with and without trend terms, generally still gave rise to rejection of the null hypothesis, but the Wald test statistics appeared to be affected by the nonstationarity and do not appear to merit reporting. One possibility is that Iran-type crises might give rise to deterministic influences which invalidate the vector autoregressive assumption. Accordingly the models were re-estimated in an attempt to avoid turbulent periods. In particular the models for the U.K., West Germany, Italy, and France were re-estimated in levels for the period February, 1975 to April, 1980 which avoided the time of the Arab-Israeli war and the period when West Germany had capital controls. The results are given in Table II and again show the Wald statistic to be highly significant. A detailed analysis of the outliers in the residuals of the models was also carried out and dummy variables were included to account for particularly large outliers, such as those due to IMF interventions. Once again, relatively slight changes emerged in the results, with the Wald statistic significant at the 1 per cent level in every case. (Full details of all results are available from the authors on request.)

Table III gives the results for the estimation of equation (14) from the reindexed four weekly data. The joint hypothesis that $a = 0$ and $\beta = 1$ is tested by means of a conventional $F$ test statistic. For West Germany and Switzerland we are unable to reject the null hypothesis, while the hypothesis can be rejected at the 1 per cent, 5 per cent, and 10 per cent levels for Canada, France, and Italy.

**TABLE III**

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{a}$</th>
<th>$\hat{\beta}$</th>
<th>$R^2$</th>
<th>$F$ Statistic</th>
<th>Durbin Watson Statistic</th>
<th>$Q(4)$</th>
<th>$Q(13)$</th>
<th>$Q(26)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>.0327</td>
<td>.9560</td>
<td>.99</td>
<td>2.12</td>
<td>1.33$^a$</td>
<td>14.58$^a$</td>
<td>34.85$^a$</td>
<td>83.30$^a$</td>
</tr>
<tr>
<td>(0.0159)</td>
<td>(0.0219)</td>
<td>(.0219)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>West Germany</td>
<td>-.0243</td>
<td>.9684</td>
<td>.99</td>
<td>0.97</td>
<td>1.98</td>
<td>7.56</td>
<td>15.05</td>
<td>24.68</td>
</tr>
<tr>
<td>(0.0197)</td>
<td>(0.0239)</td>
<td>(.0239)</td>
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<td></td>
</tr>
<tr>
<td>Italy</td>
<td>-2.345</td>
<td>.9641</td>
<td>.99</td>
<td>2.94$^c$</td>
<td>1.67</td>
<td>10.59$^b$</td>
<td>17.65</td>
<td>29.42</td>
</tr>
<tr>
<td>(1.1195)</td>
<td>(0.1180)</td>
<td>(.1180)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>-.1736</td>
<td>.8840</td>
<td>.99</td>
<td>4.57$^b$</td>
<td>1.85</td>
<td>2.95</td>
<td>11.27</td>
<td>34.49</td>
</tr>
<tr>
<td>(0.0600)</td>
<td>(0.0397)</td>
<td>(.0397)</td>
<td></td>
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<tr>
<td>Canada</td>
<td>.0565</td>
<td>.6418</td>
<td>.99</td>
<td>5.72$^a$</td>
<td>1.96</td>
<td>0.96</td>
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<td>—</td>
</tr>
<tr>
<td>(0.0172)</td>
<td>(0.1143)</td>
<td>(.1143)</td>
<td></td>
<td></td>
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<tr>
<td>Switzerland</td>
<td>.1141</td>
<td>.7974</td>
<td>.99</td>
<td>1.57</td>
<td>1.55</td>
<td>2.18</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(0.0656)</td>
<td>(1.204)</td>
<td>(.1204)</td>
<td></td>
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Key: Standard errors are given in parenthesis under corresponding parameter estimates; $Q(k)$ is the modified Box-Pierce portmanteau statistic calculated from $k$ lags. The superscripts $a$, $b$, and $c$ denote significance at the 1 per cent, 5 per cent, and 10 per cent levels respectively.
respectively. For the U.K. the $F$ statistic is not significant while there is substantial residual autocorrelation, implying rejection of the unbiasedness hypothesis.

5. CONCLUSION

In this paper the hypothesis that the forward exchange rate is an unbiased predictor of the corresponding future spot rate has been tested using weekly observations on several currencies.

For all the currencies considered the null hypothesis of zero bias is rejected. Due to our testing a joint hypothesis it is possible that the assumption of rational expectations is inappropriate or that the assumption of risk neutrality is invalid. The results are important as the relationship between the forward rate and the expected spot rate is essential to the domestic interest rate, the price level, and the current spot rate. If the forward rate is indeed a biased predictor of the spot rate, this casts doubt on the assumptions made in many macroeconomic models. Our results show that the simple, widespread view that the forward rates contain all relevant information necessary to forecast future spot rates is inappropriate.

Our results using data reindexed to a four weekly basis do not give rise to as many rejections of the null hypothesis. However such an arbitrary procedure for statistical convenience involves a considerable loss of information and appears inappropriate since market participants have the opportunity to overlap their positions in the market.

Our results based on weekly data are not incompatible with portfolio models which suggest that uncertainty about the future will induce risk premium in equilibrium, implying that the forward rate is a biased predictor of the expected spot rate. More work on the sources of uncertainty and the assimilation of new information in foreign exchanges is required.

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3 For data covering the period April, 1973 to May, 1977 Hakkio [7] also generally rejects the null hypothesis of zero bias, but instead of equating this with a biased forward rate he seeks to attribute his results to an inadequate alternative model specification. In our view we cannot accept this interpretation and feel that the only interpretation of our results is to reject the unbiasedness hypothesis.

4 Reindexing on the basis of the other three weeks produced three further replications of equation (14) and the overall results were extremely similar.
REFERENCES