Mapping interactions within and between cohesive subgroups *

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Abstract

The structure of interactions and the pattern of influence in an organization can be characterized in terms of a map of interactions within and between cohesive subgroups. I extend the work of Festinger, Schachter and Back (Social Pressures in Informal Groups, 1950, Stanford University Press) who constructed a map based on the pattern of communication within and between apartment courts. In order to generalize Festinger et al.'s approach, I substitute a posteriori subgroups for Festinger et al.'s apartment courts, and I replace the distances of a physical geography with those of a metric multidimensional scaling. I apply the technique to data indicating professional discussions among teachers in a high school. After confirming that discussions are concentrated within a posteriori subgroups at a level that is unlikely to have occurred by chance alone, I construct a map of discussions within and between the cohesive subgroups. The map allows me to characterize the processes of influence at the teacher and school levels through which the school responds to external conditions, and I argue that a map based on blocks of structurally similar actors does not sustain a comparable characterization.

1. Introduction: Mapping relations among actors

Sociologists and others who study organizations and communities have attempted to understand complex patterns of human relations by representing the relations in a map of two or three dimensions (e.g. Moreno, 1934; Festinger et al.,

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1950; Laumann, 1970; Barnett and Rice, 1985; Weller, 1990; Freeman, 1992b; Nakao and Romney, 1993; Krackhardt et al., 1994). Such maps typically consist of nodes or points representing the people, and lines, arcs, or edges, representing the relations among people. Techniques for constructing the maps have become increasingly sophisticated and rigorous, first with the introduction of multidimensional scaling (MDS) to locate actors in a few dimensions (Kruskal, 1964) and then with the extension to maximum likelihood multidimensional scaling (Takane, 1981; Ramsay, 1982) and to simulated annealing procedures to avoid poor local minima (Krackhardt et al., 1994). But even with these improved techniques it is difficult to interpret the results solely in terms of the relations between each pair of actors. Therefore, in order to interpret these maps many researchers have identified clusters of actors who are strongly related and who are plotted close together (e.g. Laumann, 1970; Freeman, 1992b; Nakao and Romney, 1993; Wasserman and Faust, 1994; Krackhardt et al., 1994; Kadushin, 1995, etc.). In fact, MDS often is considered an alternative to cluster analysis (Laumann, 1970; Kruskal, 1977; Davison, 1983; Gaul and Schader, 1986; Borg and Lingoes, 1987; Wasserman and Faust, 1994).

Typically the means for identifying clusters in MDS plots are visual heuristics which may not be generalizable (e.g. Moreno, 1934; Laumann, 1970; Nakao and Romney, 1993; Kadushin, 1995), or numerical criteria which are based more on the plotted locations of actors than on the data from which those locations are derived (e.g. Borg and Lingoes, 1987; Wasserman and Anderson, 1987; Anderson et al., 1992). Doreian et al. (1994) argue strongly that graphically based techniques for identifying blocks of actors are more limited than those techniques which identify blocks according to an explicit criterion defined in the original data. Fortunately, Doreian et al.’s arguments can be addressed without abandoning the plots which facilitate interpretations. For example, Panning (1982) suggested first identifying blocks of actors based on relations according to an explicit criterion and then applying MDS within and between blocks to generate a map of actors and their relations. There are many advantages to Panning’s approach. First, the criterion for determining block membership may be linked with a specific statistical model. Second, by identifying blocks from the original data rather than plotted locations one can be more certain that block membership is salient in terms of the actors’ relations. Third, one can assess the internal validity of the block memberships by obtaining a Monte Carlo sampling distribution for the criterion prior to using the blocks as a basis for interpretation (Jain and Dubes, 1988; Frank, 1995). In this article, I address many of the issues necessary to realize the promise of applying MDS within and between blocks in order to generate a map of a specific type of relation, interaction or exchange, among actors.

In the following section, I establish the importance of representing interaction within specific types of blocks, cohesive subgroups, as the basis for characterizing influence at the individual and organizational levels. In Section 3, I describe an existing criterion for measuring the extent of cohesiveness in subgroups (Frank, 1995). In Section 4, I utilize the criterion to identify cohesive subgroups of teachers in a single school from data indicating the frequency with which each pair of
teachers engages in professional discussions. In Section 5, I construct a map of discussions within and between cohesive subgroups using metric MDS. I interpret the map in Section 6, and compare the map with one based on blocks of structurally similar actors in Section 7. I make some concluding comments in Section 8.

2. Cohesive subgroups as the basis for mapping interactions

There is well established theory regarding how individuals are influenced by, and organizations are composed of, cohesive subgroups of actors who engage in a high rate of interaction. At the level of the individual, social psychologists and sociologists have argued that individuals are most strongly influenced by the members of their primary groups — people with whom they engage in frequent interactions (Cooley, 1909; Roethlisberger and Dickson, 1941; Homans, 1950; Festinger et al., 1950; Roy, 1952; Freud, 1959; Epstein, 1961; Kadushin, 1966; Burawoy, 1979; Durkheim 1984; Jones and Moore, 1988) — and anthropologists have argued that primary groups are integral to understanding people within the contexts of their communities (Bott, 1971; Barnes, 1972). At the level of the organization, Homans (1950) and other theorists have argued that large organizations are composed of essentially non-overlapping subgroups which contain dense interactions (Simmel, 1955; Simon, 1965; Blau, 1977; see Freeman, 1992a for a review). Organizations may consist of integrated subgroups because organizations evolve through the linkages of subgroups (Simon, 1965), because subgroups are imposed on organizations for managerial efficiency (Simmel, 1955; Simon, 1965; Granovetter, 1973), or because subgroups emerge as organizations grow and interactions among actors cannot be sustained at levels high enough to integrate each actor directly into the common organization (Festinger et al., 1950; Newcomb et al., 1965; Robinson, 1981).

Festinger et al. (1950) used cohesive subgroups as the basis for constructing a map of interactions to effectively represent the way in which members of the Westgate apartment complex responded to an external influence. Festinger et al. planted two rumors with the encircled actors in Fig. 1. In the Howe court they planted a rumor that a radio story was to be broadcast about the residents of the complex, and in the Tolman court they planted a story that a magazine article was being written about the residents of the complex. Festinger et al. described the spread of rumor as shown in Fig. 1 in terms of communication between pairs of actors: “number 10, with whom the story was planted, communicated the story to number 9, who in turn repeated it to three other people” (p. 123). Because the people were strangers prior to living in the courts, the physical structure of their environment guided their pattern of interactions; interactions were more frequent within courts than between them. Therefore, Festinger et al. were able seamlessly to reduce their description to one of communication circulating within apartment courts and then between apartment courts. For example, they wrote: “In the
Festinger et al. (1950) characterized the structure of interaction by using apartment courts as a basis for mapping the communication among residents, some of whom were exposed to an external stimulus. In order to study the structure of interaction in other organizations without conducting a similar experiment, I will describe how one can construct a map of the structure of interaction by using a posteriori subgroups obtained from social network data instead of a priori subgroups such as Festinger et al.'s apartment courts, and by defining distances using metric MDS instead of through the geography of physical structures, such as the locations of Festinger et al.'s apartment buildings. These techniques then can be applied to data indicating interaction among actors in any organization, and the map can help researchers to understand the way in which the effects of external forces enter an organization and then are transmitted throughout the organization through the structure of interaction.
3. The criterion for defining cohesion

Methodologists have developed and employed various techniques for identifying a posteriori cohesive subgroups from data indicating the extent of interaction between pairs of actors (e.g. Roethlisberger and Dickson, 1941; Katz, 1947; Bock and Husain, 1950; Festinger et al., 1950; Cartwright and Harary, 1956; Hubbell, 1965; Johnson, 1968; Matula, 1972; Phillips and Conviser, 1972; Alba, 1973; Davis, 1977; Seidman and Foster, 1978; Mokken, 1979; Everett, 1983; Reitz, 1988; Arabie and Hubert, 1990; Borgatti et al., 1990; Freeman, 1992a; Freeman, 1993). Many of the earliest techniques were based on heuristics or visualization which did not utilize an explicit criterion for determining subgroup boundaries (e.g. Katz, 1947; Hubbell, 1965). More recent approaches have employed graph theoretic criteria for identifying members of cohesive subgroups (e.g. Alba, 1973; Seidman and Foster, 1978; Mokken, 1979; Everett, 1983; Borgatti et al., 1990). But the graph theoretic criteria are deterministic and rigid (Wasserman and Anderson, 1987; Anderson et al., 1992; Frank, 1993), and when employed can result in a failure to identify a few, non-overlapping subgroups (Freeman, 1992a,b; Frank, 1995; Kadushin, 1995).

But Festinger et al.'s map facilitated an analysis of communication in part because the cohesive subgroups were non-overlapping. In social network data, one can identify non-overlapping subgroups by employing stochastic criteria to define permeable subgroup boundaries, instead of applying rigid criteria which result in overlapping subgroup boundaries. For example, Frank (1995) presented a reduced form of the Pl model (Fienberg and Wasserman, 1981; Holland and Leinhardt, 1981; Wasserman and Galaskiewicz, 1984; Fienberg et al. 1985; Frank and Strauss, 1986; Wang and Wong, 1987; Strauss and Ikeda, 1990; Wasserman and Pattison, 1994) and established a stochastic criterion for identifying cohesive subgroups. For n actors and \( n(n - 1) \) pairs of actors \( i \) and \( i' \), define:

\[
X_{ii'} = \begin{cases} 
1 & \text{if actor } i \text{ indicates interacting with actor } i', \\
0 & \text{otherwise,} 
\end{cases}
\]

\[
samegroup_{ii'} = \begin{cases} 
1 & \text{if actors } i \text{ and } i' \text{ are members of the same subgroup,} \\
0 & \text{otherwise.} 
\end{cases}
\]

Frank's logit model then can be written as:

\[
\log \left( \frac{P[X_{ii'} = 1]}{1 - P[X_{ii'} = 1]} \right) = \theta_0 + \theta_1 \text{ samegroup}_{ii'}. \tag{1}
\]

In Model (1), \( \theta_1 \) is associated with the increase in the probability that two actors interact if they are members of the same subgroup and \( \theta_1 \) then becomes the criterion to be maximized to identify cohesive subgroups; subgroups are identified by iteratively reassigning actors so as to maximize \( \hat{\theta}_1 \), the estimate of \( \theta_1 \) based on the emergent subgroups (see Frank, 1995 for details of the algorithm).

Maximizing \( \theta_1 \) has many advantages over the use of other stochastic criteria (e.g. Bock and Hussein, 1950; Borgatti et al., 1992: FACTIONS; Freeman, 1993).
Table 1
Association between common subgroup membership and the occurrence of interactions between actors

<table>
<thead>
<tr>
<th>Subgroup membership</th>
<th>Interaction occurring</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Different 0</td>
<td>No $X_{ij} = 0$</td>
<td>Yes $X_{ij} = 1$</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Same 1</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

$n(n - 1) - \sum g n_g(n_g - 1)$

$(n_g$ represents the number of actors in subgroup $g$).

First, $\hat{\theta}_1$ is defined in an explicit model, which can be modified depending on the researcher’s theoretical definition of cohesiveness. Second, even when $\hat{\theta}_1$ is large, it is possible for interactions to occur across subgroup boundaries, and so deviations from Model (1) do not result in the identification of overlapping subgroups nor in the failure to identify at least one cohesive subgroup of more than one actor. Third, Frank showed how $\hat{\theta}_1$ is related to many other criteria which have been used as the basis for assessing the cohesion of subgroups. But because $\hat{\theta}_1$ can be defined as half the log-odds of Table 1 ($\hat{\theta}_1 = 0.5 \log[AD/CB]$), the value of $\hat{\theta}_1$ reflects deviations from the model associated with cell B (interactions which occur between subgroups) as well as cell C (unrealized potential for within subgroup interaction) and therefore there is no benefit to minimizing one type of deviation at the expense of the other. Fourth, $\hat{\theta}_1$ can be used as a basis for assessing the internal validity of the subgroups, thus allowing one to establish the extent to which the cohesiveness of the subgroups is more than would be found by applying the algorithm to random data. Fifth, Frank established the performance of the algorithm in terms of the extent to which the obtained $\hat{\theta}_1$ exceeded the expected value of $\hat{\theta}_1$ in random data.

In Model (1), $\theta_1$ is defined for logit models for binary data. That is, interaction either occurs or does not occur, as shown in Table 1. While the pattern of binary data may provide enough information to identify cohesive subgroups, it likely will not provide enough information to reveal within subgroup structures which ultimately will be represented in a map of interactions. Freeman (1992b) responded by characterizing the extent to which individuals were ‘core’ or ‘peripheral’ to their subgroups, terms which reflect the centrality of the individuals in their subgroups. I will represent internal subgroup structures by applying MDS to the pattern of weighted interactions (which might indicate the frequency or intensity of interaction) within each subgroup. Therefore, in Table 2 I recast Table 1 to reflect weighted interactions, by defining $X_{ij'} = 0, 1, 2, 3, \ldots$, Maximum Weight ($W_m$). To identify cohesive subgroups from weighted, or valued, interactions 1 then will maximize the log-odds of Table 2, still denoted as $\theta_1$. This approach essentially treats the progression of weights from 1 to $W_m$ as linear, although one can estimate...
Table 2
Association between common subgroup membership and the extent of interaction between actors (weighted data)

<table>
<thead>
<tr>
<th>Subgroup membership</th>
<th>Interaction occurring</th>
<th>Yes $(X_{ii'} = 1 (\text{Weight})_{ii'})$</th>
<th>No $(X_{ii'} = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Different 0</td>
<td>A</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>Same 1</td>
<td>C</td>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

\[ W_m(n(n-1) - \sum_{g} n_g(n_g - 1)] - \sum_{g} n_g(n_g - 1)] \]

(n\textsubscript{g} represents the number of actors in subgroup \textsubscript{g}).

the size of the increment for each step in the weights (Agresti 1984; Wasserman and Galaskiewicz, 1984; Fienberg et al., 1985).

4. Application: Identifying cohesive subgroups from professional discussions among teachers

As an example, I analyze weighted data indicating the extent to which pairs of teachers engaged in professional discussions in a high school called ‘Our Hamilton High.’ In January of 1993, each teacher listed the five (or fewer) teachers with whom he or she had most often discussed professional matters during the 1992–1993 school year. The teachers also were asked to weight the frequency of discussions (1 = less than once a month, 2 = two to three times a month, 3 = once or twice a week, and 4 = almost daily). I have chosen to analyze data in this school because the school represents a small bounded organization which is ideal for the study of the social network. But the boundaries of the school are permeable, as the teachers are exposed to external factors which affect the way in which they think and behave in the school (see Section 6). The map of teachers’ interactions within and between cohesive subgroups will help me to infer how these external forces are translated into teachers’ actions and sentiments, serving as a prototype for analyses of interactions of members of other organizations.

The teachers and their discussions are represented in Table 3. The value in each cell in Table 3 indicates the extent of professional discussion listed by the teacher in the corresponding row with the teacher in the corresponding column.

\[ W_m(n(n-1) - \sum_{g} n_g(n_g - 1)] - \sum_{g} n_g(n_g - 1)] \]

\[ W_m(n(n-1)) \]

\[ W_m(n(n-1)) \]

1 The subgroups identified from these weighted data are similar to those identified by Frank (1995) for unweighted data. Subgroups A and B in Table 3 are consistent with subgroups A and D in Frank (1995). Subgroup C in Table 3 represents a consolidation of subgroups B and C in Frank (1995), and subgroup D in Table 3 draws peripheral members from subgroups B and C in Frank (1995).
Table 3
Professional discussions among teachers at 'Our Hamilton High' partitioned by cohesive subgroup boundaries

<table>
<thead>
<tr>
<th>Sub-group ID</th>
<th>Subgroup and teacher ID</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1A</td>
<td>7</td>
</tr>
<tr>
<td>1A</td>
<td>1</td>
</tr>
<tr>
<td>1A</td>
<td>2</td>
</tr>
<tr>
<td>1A</td>
<td>9</td>
</tr>
<tr>
<td>2B</td>
<td>23</td>
</tr>
<tr>
<td>2B</td>
<td>22</td>
</tr>
<tr>
<td>2B</td>
<td>6</td>
</tr>
<tr>
<td>2B</td>
<td>20</td>
</tr>
<tr>
<td>2B</td>
<td>17</td>
</tr>
<tr>
<td>2B</td>
<td>8</td>
</tr>
<tr>
<td>3C</td>
<td>5</td>
</tr>
<tr>
<td>3C</td>
<td>15</td>
</tr>
<tr>
<td>3C</td>
<td>10</td>
</tr>
<tr>
<td>3C</td>
<td>14</td>
</tr>
<tr>
<td>3C</td>
<td>3</td>
</tr>
<tr>
<td>3C</td>
<td>16</td>
</tr>
<tr>
<td>3C</td>
<td>19</td>
</tr>
<tr>
<td>3C</td>
<td>4</td>
</tr>
<tr>
<td>4D</td>
<td>18</td>
</tr>
<tr>
<td>4D</td>
<td>4</td>
</tr>
<tr>
<td>4D</td>
<td>13</td>
</tr>
<tr>
<td>4D</td>
<td>21</td>
</tr>
<tr>
<td>4D</td>
<td>12</td>
</tr>
</tbody>
</table>

Teachers and subgroups have been ordered based on the ratio of in-to out-degree. N = 24.

(letters indicating subgroup membership appear on the diagonal). For example, teacher 7 (in subgroup A) indicated engaging in professional discussions with teacher 2 (in subgroup A) less than once a month, and teacher 1 (in subgroup A) indicated engaging in professional discussions with teacher 15 (in subgroup C) almost daily. The cohesiveness of each subgroup is characterized by the concentration of discussions within each subgroup as indicated by the large values along the block diagonal of the matrix. The value of $\theta_1$ for the partitions in Table 3 is 1.22.

Of course, the iterative partitioning algorithm may not have recovered the true subgroup memberships. Iterative partitioning algorithms are fairly simple and may not perform as well as genetic algorithms (e.g. Freeman, 1993), simulated annealing procedures (e.g. Hajek, 1988), taboo searches (e.g. Borgatti et al., 1992), or other algorithms which seek to avoid local maxima in a criterion. But, given the
extent to which interactions were concentrated within subgroups in ‘Our Hamilton High’, Frank (1995) established through simulated data that the iterative partitioning algorithm was four times as likely to assign two teachers who were in the same true subgroup to the same observed subgroup as it was to assign two teachers in different true subgroups to the same observed subgroup. For the data in Table 3, the performance of the algorithm would be enhanced to the extent that teachers in ‘Our Hamilton High’ indicated engaging in discussions more frequently (as defined by the weights) with subgroup members than with those in other subgroups with whom they engaged in discussions. Therefore it is quite likely that the subgroups represented in Table 3 are identical, or highly similar, to the ‘true’ subgroups in the data.

Before using the subgroups in Table 3 as the basis for constructing a map of interactions I will establish that the discussions are concentrated within subgroup boundaries to an extent that is unlikely to have occurred by chance alone. Following Jain and Dubes (1988) and Frank (1995), I repeatedly applied the iterative partitioning algorithm to data simulated under conditions in which subgroup membership was not related to the extent to which pairs of teachers engaged in discussions. First, taking the distribution of discussions listed by each teacher as given, I randomly reassigned the discussions of each teacher to others throughout the school. For example, teacher 7 listed four others with whom she engaged in discussions. Her nominations and corresponding weights (2, 1, 3, 1) were randomly reassigned to other teachers in the school without regard for subgroup membership, as were the nominations and corresponding weights for each teacher in the school. I then applied the algorithm and obtained the value for $\hat{\theta}_1$. I repeated this process 1000 times to obtain a Monte Carlo generated sampling distribution of $\hat{\theta}_1$. The value of $\hat{\theta}_1$ associated with the subgroups of teachers in ‘Our Hamilton High’ based on the given data was greater than all values of $\hat{\theta}_1$ obtained when the algorithm was applied to the simulated data. From this I determine that the probability that the data in Table 3 could have been obtained if teachers engaged in discussions without regard for subgroup membership is less than 0.001; I reject the hypothesis that teachers engaged in discussions without regard for subgroup membership. The subgroups identified among the teachers in ‘Our Hamilton High’ are more than boundaries imposed to facilitate an analysis of a pattern of interactions — they represent an empirical tendency of teachers to interact within the identified subgroup boundaries.

5. Constructing the map

5.1. MDS within each subgroup and between subgroups

Using a metric version of MDS (as is described by Gower (1984) and as available in MDSC(X) and Borgatti et al.’s (1992) UCINET IV), I generated a map of interactions within and between the cohesive subgroups indicated in Table 3.
Within each subgroup, define:

$$\delta_{ii'} = \text{the true distance between teachers } i \text{ and } i' \text{ based on the data. Here, } \delta_{ii'} \text{ is defined in terms of } W_m/(\text{average rate of interaction as given in } X_{ii'} \text{ and } X_{i'i}). \text{ For example, } \delta_{ii'} = 4/3 \text{ if teacher } i \text{ indicated engaging in professional discussions almost daily with } i'(X_{ii'} = 4) \text{ and teacher } i' \text{ indicated engaging in professional discussions a couple of times a month with } i(X_{i'i} = 2). \text{ The maximum distance was defined as } 4/(0.5).$$

$$d_{ii'} = \text{the plotted distance between teachers } i \text{ and } i'.$$

Note that the true distances between teachers are based on the average of $$X_{ii'}$$ and $$X_{i'i}$$ and, as such, are insensitive to the direction of the interaction. I used MDS to minimize the stress function for the $$n_g$$ teachers within each subgroup:

$$\text{Stress} = \sum_{i=2}^{n} \sum_{i'=1}^{i-1} (\delta_{ii'} - d_{ii'})^2 / \sum_{i=2}^{n} \sum_{i'=1}^{i-1} (d_{ii'} - \bar{d})^2.$$  \hspace{1cm} (2)

Of course, MDS does not operate on the $$\delta$$'s, but on some function of the $$\delta$$'s. By restricting that function to a simple multiple of $$\delta$$, the metric of the original data can be preserved in the map.\footnote{In many applications, MDS is implemented to reduce the effect of outliers of the original distribution of $$\delta$$'s, resulting in the loss of the original metric of the data. The metric can be preserved by restricting the transformation to:}

$$f(\delta_{ii'}) = \beta_1 \delta_{ii'} + \epsilon_{ii'}.$$  \hspace{1cm} (3)

Then, using the estimate of $$\beta_1$$, one can convert the coordinates produced by MDS into the metric of the original $$\delta$$'s.

The same MDS procedure used within subgroups can be applied to the between subgroup discussions, where the distances between the centers of subgroups $$g$$ and $$h$$ can be defined as $$4/(\text{average density of discussions between teachers in subgroups } g \text{ and } h). \text{ The definition of between subgroup distances is different from other applications of MDS within and between subgroups which define proximities between subgroups in terms of the similarity of subgroup members' patterns of interactions instead of the direct interactions between members of the subgroups (e.g. ALSCAL). Because the distances between subgroups are defined in the same metric as the distances within subgroups, the plotted distances between subgroup centers will be in the same metric as the plotted distances between teachers within subgroups and thus the maps of interactions within each subgroup can be combined with the map locating subgroups into a single map of interactions within and between subgroups.
5.2. Combining within and between subgroup scalings

One could construct a map of interactions within and between subgroups by applying MDS with constraints (Borg and Lingoes, 1981), fixing the mapped distances between teachers within their subgroups and between the centers of each subgroup. But the only remaining flexibility would remain in rotating each subgroup about its center and in reflecting each subgroup about its vertical axis. These last modifications would not greatly affect the stress, and might be chosen as much to improve the esthetics of the plot as to minimize stress. Therefore, I rotated the teachers within each subgroup about the subgroup center so as to reduce the number of between subgroup interactions which cut across the space of a subgroup. This was achieved by minimizing a rotation function \( RF \) based on a polar coordinate transformation of the plotted locations of teachers within each of the \( G \) subgroups and of subgroups relative to one another:

\[
RF = \sum_{g=1}^{G} \sum_{i=1}^{n_g} \sum_{i'=1}^{n_{i'}} \left( \alpha_i - \omega_{\text{subgroup}(i')} \right)^2.
\]

Where:

- \( i \) represents the \( i^{th} \) teacher in subgroup \( g \),
- \( r_i \) represents the number of interactions which teacher \( i \) either listed, or was listed by, teachers who are not in teacher \( i \)’s subgroup,
- \( \text{subgroup}(i') \) represents the subgroup in which teacher \( i' \) is a member,
- \( \alpha_i \) represents the angle at which teacher \( i \) is plotted in its subgroup (with the zero angle at 3 o’clock by convention), and
- \( \omega_{\text{subgroup}(i')} \) is the angle at which \( \text{subgroup}(i') \) is located relative to where \( \text{subgroup}(i) \) is located (with the zero angle at 3 o’clock by convention).

If \( \alpha_i - \omega_{\text{subgroup}(i')} \) is greater than 180 degrees then

\[
\alpha_i - \omega_{\text{subgroup}(i')} = 360 - (\alpha_i - \omega_{\text{subgroup}(i')}).
\]

I used the radius of each teacher’s location relative to the center of its subgroup as a weight in the function, assigning more weight to those teachers who were furthest from the center (and whose misplacement would cause more lines to cut across the subgroup space). Specifically, I minimized:

\[
RF = \sum_{g=1}^{G} \sum_{i=1}^{n_g} \sum_{i'=1}^{n_{i'}} \frac{\text{radius}_i}{\max(\text{radius}_i)} \left( \alpha_i - \omega_{\text{subgroup}(i')} \right)^2.
\]

Minimization was achieved by evaluating the function for rotations of each subgroup. For example, in considering subgroup \( g \), each teacher’s \( \alpha \) was increased
by 10° and then the function was evaluated. The angles were again increased by
10°, and the function was evaluated. This continued until the total increment
equaled 360°, returning all teachers to their initial positions. The above procedure
was repeated beginning with a reflection of the initial positions about the vertical
axis. The reflection represents a comparable within subgroup scaling in terms of
the value of the stress but the reflected positions may fit better with regard to
cross-subgroup interactions (note that MDS with constraints would likely not
evaluate the reflection of the teachers within their subgroups). Since $a_{\text{subgroup}}(c')$
is defined relative to the angles between subgroups, the rotations of subgroups $g$ and
$h$ are independent. The order of evaluation did not matter, and the function was
evaluated for one subgroup at a time.

I applied MDS within and between subgroups and rotated teachers within
subgroups to construct the map in Fig. 2. Beyond partitioning teachers into
subgroups, the map in Fig. 2 reveals the structure of interaction within each
subgroup. The structure is not based on ideal types (e.g. Moreno, 1934; Guetzkow,
1951) but on a two dimensional representation of the interactions between sub-
group members (Bavelas, 1950; Leavitt, 1951; Smith, 1973; Freeman, 1992a,b;
Dulad-Vincent et al., 1994). For example, subgroup B can be characterized in
terms of a central dyad (teachers 20 and 23), two teachers closely associated with
the dyad but who do not engage in direct discussions with one another (teachers 17
and 22), and two peripheral members (teachers 6 and 8). Each of the subgroups
can be similarly characterized, as can the structure of subgroups, which might be
characterized in terms of a central subgroup (C), and three competing factions (A,
B, and D). Most importantly, because the metric in the original data is preserved
within and between subgroups, sets of distances in the map can be compared. For
example, the members of subgroup A are mapped, on average, about 1.68 units
apart, reflecting the fact that the density of discussions within subgroup A is 2.42,
or almost once a week ($1.68 = \frac{W_m}{\text{density within subgroup A}} = \frac{4}{2.42}$). The
average distance of 1.68 between members of subgroup A can be sensibly com-
pared with the average distance of 2.48 between members of subgroup B who
engage in discussions on average about once a month ($2.48 = \frac{W_m}{\text{density within
subgroup B}} = \frac{4}{1.6}$). The distances within the boundary of subgroup A can also
be compared with the distances between members of different subgroups, such as
the 16 units that separate the members of subgroups A and C.

The subgroup boundaries imbedded in Fig. 2 also reveal structural holes within
and between subgroups (Burt, 1992). For example, within subgroup C, the limited
amount of direct discussion between teachers 14 and 15 in the upper right and
teachers 4 and 16 in the lower left constitutes a structural hole within the subgroup
(only one of the four possible interactions occurs). This hole is filled primarily by
teachers 5, 10, and 19, each of whom engage in direct discussion with three of the
four disconnected teachers. In Burt’s language, the action of teachers 5, 10, and 19
would be less constrained than that of teachers 4, 14, 15, and 16. There are also

\footnote{Figs. 2 and 3 were produced using a SAS program culminating in proc gplot.}
Fig. 2. Professional discussions within and between cohesive subgroups among teachers at 'Our Hamilton High'. Solid lines within subgroups, broken lines between, thickness is proportional to frequency of discussion. Scale = $W_n/(\text{density of discussion})$.

structural holes between teachers in different subgroups. For example, teachers 1 and 2 are the only members of subgroup A who engage in discussions with the teachers in subgroup B. Without these bridging teachers there would be a hole between subgroup A and subgroup B. Most importantly, the hole is filled primarily through discussions involving teacher 1 in Subgroup A. Without this teacher, the
Table 4
Characteristics of teachers by cohesive subgroups

<table>
<thead>
<tr>
<th>ID</th>
<th>Race</th>
<th>Gender</th>
<th>Subject field</th>
<th>Room</th>
<th>Years teaching</th>
<th>Moral Pal</th>
<th>Moral Pal</th>
<th>Moral Pal</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Black</td>
<td>Female</td>
<td>Business</td>
<td>204</td>
<td>25</td>
<td>0.30719</td>
<td>-1.10587</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Black</td>
<td>Female</td>
<td>Social Studies</td>
<td>202</td>
<td>20</td>
<td>0.81606</td>
<td>-0.78682</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Black</td>
<td>Female</td>
<td>English</td>
<td>215</td>
<td>21</td>
<td>0.61764</td>
<td>-0.15872</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Black</td>
<td>Female</td>
<td>Business</td>
<td>203</td>
<td>14</td>
<td>0.28530</td>
<td>0.40910</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Other</td>
<td>Female</td>
<td>Special Education</td>
<td>205</td>
<td>24</td>
<td>-0.51569</td>
<td>-0.37477</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>White</td>
<td>Female</td>
<td>Foreign</td>
<td>177</td>
<td>12</td>
<td>-0.08439</td>
<td>-0.08439</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>White</td>
<td>Female</td>
<td>English</td>
<td>210</td>
<td>5</td>
<td>-0.42454</td>
<td>0.98521</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Other</td>
<td>Female</td>
<td>English</td>
<td>212</td>
<td>12</td>
<td>0.06467</td>
<td>-0.22482</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>White</td>
<td>Female</td>
<td>Special Education</td>
<td>204</td>
<td>11</td>
<td>0.31899</td>
<td>-0.51393</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>White</td>
<td>Male</td>
<td>Science</td>
<td>183</td>
<td>30</td>
<td>0.34734</td>
<td>-0.31899</td>
<td></td>
</tr>
<tr>
<td>10</td>
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<td>-0.06197</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>White</td>
<td>Male</td>
<td>Math</td>
<td>171</td>
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<td>-0.02972</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>White</td>
<td>Male</td>
<td>Music</td>
<td>158</td>
<td>28</td>
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<td></td>
</tr>
<tr>
<td>13</td>
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<td>Male</td>
<td>Physical Education</td>
<td>105</td>
<td>1</td>
<td>0.20916</td>
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<td></td>
</tr>
<tr>
<td>18</td>
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<td>Male</td>
<td>Physical Education</td>
<td>213</td>
<td>17</td>
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<td>0.59406</td>
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</tr>
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<td>27</td>
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<td>-0.12905</td>
<td></td>
</tr>
</tbody>
</table>

discussions between teacher 2 and 17 would be the only remaining direct link between members of subgroups A and B. All other communication would be indirect, mediated primarily by members of subgroup C.

6. Interpretation of the subgroups and the map

6.1. Characteristics of teachers by subgroup

I have indicated each teacher's race, gender, room assignment, subject field and seniority by subgroup in Table 4. As was the case in Frank (1995) one can observe that teachers are more likely to be members of a subgroup with others of the same race (a $\chi^2$ test of independence and Fisher's exact tests produced $p < 0.02$). All of
the teachers in subgroup A are black, and all but one of the teachers in subgroups C and D are white. Subgroup membership also aligns with gender (a $\chi^2$ test of independence and Fisher's exact tests produced $p < 0.01$). All of the teachers in subgroup A, and all but one in subgroup B, are female, while all of the members of subgroup C and all but one of subgroup D are male. After race and gender, teachers align by subject field. Subgroup B consists mostly of Special Education and English teachers, subgroup C mostly of Math and Science teachers, and subgroup D contains all of the Physical Education teachers. Subgroup membership is also associated with room location, with teachers in subgroup A mostly occupying rooms in one section of the second floor, teachers in subgroup D mostly occupying rooms in a different section of the second floor, and members of subgroup C teaching in rooms mostly on the first floor. These findings are supported by field work. For example, the black female teachers in subgroup A could frequently be observed talking in the halls during the breaks between classes, as could some of the teachers in subgroup C.

I also have indicated each teacher's emphasis on 'moral agency' in Table 4 (the teacher ID's were assigned according to their rank order on moral agency 4). Moral agency was measured by teachers' responses to survey items described in Frank (1995) (reliability = 0.74). In brief, the moral agent keeps firm control of the classroom and what is taught, emphasizing the inculcation of moral values over the teaching of a subject matter. As is shown in Table 4, the extent to which a teacher emphasizes the moral agency orientation depends on the teacher's subgroup membership. A Kruskal–Wallis test indicated that the probability of achieving the differences in moral agency among the subgroups by chance alone was 0.06. Using Tukey's Honestly Significant Differences, the members of subgroup A were significantly higher on moral agency than were the members of subgroup B.

6.2. Organizational context of 'Our Hamilton High'

In 1992, 340 students were enrolled in 'Our Hamilton High' which is situated in a small town of about 2000 roughly 90 miles from a major city in the Midwest. Although the town does not function as a suburb of the city, it increasingly serves students who are refugees of the violence and drugs of the city schools. The result is that 'Our Hamilton High' serves two student populations. A middle and lower-middle income population with separate houses on their own lots on the West side of town and a poor, predominantly black, and growing community to the East which consists of small homes, trailer parks and shacks. Many of the students from the poorer community have neither running water nor phones (see Frank, 1993 for more detailed context).

The racial tensions, economic condition, and ethos experienced by the students in 'Our Hamilton High' are very similar to those experienced by students in

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4 One teacher is labeled ‘* * *’ because information regarding his emphasis on moral agency was not obtained.
Hamilton High (Grant, 1988) and observed by Cusick (1983). For example, the racial tensions have led to a decline in enrollment and funding for the school. Many of the more established white families with children who were grown and out of school or in parochial schools were reluctant to support taxes for ‘Our Hamilton High’. As of 1993, the community had not passed a bond initiative in 10 years, and the school’s funding, when adjusted for inflation, had dropped over the past 15 years. While none of the teachers would directly comment on racial tensions among faculty at ‘Our Hamilton High’, the racial segregation of teachers by subgroups represents the legacy of the racial tensions that the faculty experienced in the early 1970s and reflect the divided population that the school serves.

The statistical analysis of moral agency by subgroup can also be interpreted relative to the school’s context. Observations indicated the teachers in subgroup A responded to the concerns of the increasingly large percentage of black students in the school, and when interviewed, these teachers perceived their role to be that of socializing the students so that the students could contribute to the immediate community upon graduation. For example, when asked about the way in which she teaches, teacher 1 stated: “They [the kids] have not had the opportunities to do things and to learn that this is the proper behavior when you do this. And if you are going to advance yourself, you need to do this, and this, and this”. The teacher further explained that she organizes a field trip to the state capitol every year so that the children can broaden their experiences. As a second example, when asked about how she teaches, teacher 2 stated:

Especially with poor ability students...I believe in trying to again make things relevant to the student, I think I try to do that more so than other teachers who did do just facts. ... I try to get them to see things as being significant...I want students...later on seeing themselves as important, seeing themselves as significant.

These quotes suggest that the teachers in subgroup A are highly aware of the needs of the more disadvantaged students in ‘Our Hamilton High’. Further, the teachers respond, in part, as moral agents by initiating experiences which build the values of the students. The teachers’ responses are also consistent with the fact that the teachers of subgroup A, one of whom was married to the black principal, took the principal’s sense of mission more seriously than did the other teachers in the school.

6.3. Interpreting the map of social interactions with respect to the context of ‘Our Hamilton High’

In general, the internal structures of an organization, including the structure of interaction, will affect the way in which the entire organization responds to

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5 One result of the decline in enrollment and financial support is that the school has laid off many of its junior teachers in order to reduce its teaching force. Therefore many of the teachers who remain have been in ‘Our Hamilton High’ for 15 to 25 years. Field work in this school and ethnographic studies of schools (e.g. Grant, 1988) suggest that the pattern of professional discussions among senior teachers is quite stable, reflecting trends that were established in the teachers’ first years in the school.
external influences (Katz and Kahn, 1966; Pfeffer and Salancik, 1978), and the same holds true for the teachers' responses to the students in 'Our Hamilton High'. Although not all teachers responded as directly to the needs of the disadvantaged students as did the teachers in subgroup A, all teachers were influenced by the changing student population through discussions among faculty in the school; even if only one teacher responded directly to the changing student population, that teacher would affect all others in the school through direct or indirect communication. Unfortunately I did not obtain longitudinal data to adequately observe processes of influence among all faculty in 'Our Hamilton High'. But because this is often the case for those who study the social networks of organizations, statistical techniques have been developed for estimating the parameters in models which are based on hypothesized processes of influence, even though the data are only cross-sectional (e.g. Doreian, 1981; Friedkin and Marsden, 1994). Similarly, a map of interactions can be interpreted with respect to a hypothetical cycle of influence that occurs repeatedly and continuously.

In Fig. 2, we may begin describing the process with the members of subgroup A, who emphasize moral agency in response to the large percentage of disadvantaged students who attend 'Our Hamilton High'. The cycle of influence then can be inferred using the weights which represent the frequency of discussions among the teachers (the thickness of the lines in Fig. 2 is based on the maximum of $X_{ij}$ and $X_{ij'}$). As a general example, emphasis on moral agency is cultivated within subgroup A as the teachers in subgroup A engage in discussions with one another on a near daily basis. Then one of the teachers in subgroup A will engage in discussions with a teacher outside the subgroup, thus possibly influencing the member of the other subgroup. As a specific example, teacher 1 establishes her emphasis on moral agency as she engages in professional discussions with teachers 2, 7 and 9 in her subgroup. Then teacher 1 engages in professional discussions with teachers 14, 15 and 16 in subgroup C through which it is likely that she influences teachers 14, 15 and 16 towards the moral agency orientation. In turn, teachers 14, 15 and 16 contribute to the moderate emphasis on moral agency in subgroup C established by teachers 3, 4 and 5.6

The effect of cross-subgroup discussions is not limited to those in subgroup C with whom teachers in subgroup A engage in direct discussions. For example, teacher 15 in subgroup C, who engages in discussions with teacher 1 in subgroup A, also engages in discussions with teacher 18 in subgroup D, and likely has some

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6Note that many of the cross-subgroup discussions which constitute weak ties (Granovetter, 1973) may have their basis in the cross-cutting characteristics of teachers. For example, teacher 1, a member of the all black female subgroup A, engages in frequent discussions with teacher 15 who is the only black member of subgroup C. Also, teacher 1 of subgroup A and teacher 14 of subgroup C are both smokers and engage in discussions in the smoker's lounge. From an individual level, the cross subgroup discussions distinguish a teacher's pattern of discussions and influences from others in the teacher's subgroup (Simmel, 1955). From the organizational, or school, level, the cross-subgroup interactions help integrate the school, linking subgroups so that the members of a given subgroup do not diverge too greatly from the orientations of the others in the school (Blau, 1977).
influence on teacher 18 to emphasize moral agency more than he otherwise would. Again, the effect is not limited to teacher 18 who then engages in professional discussions with the other members of subgroup D, and may influence them to emphasize the moral agency orientation more than they otherwise would. Therefore even teachers who do not respond directly to external forces may be affected by those forces through their direct and indirect interactions with the teachers in subgroup A, although not surprisingly the effect is attenuated with each step in the process.

Of course, teachers may experience the context of the school differently depending on their formal position in the school. For example, the teachers in subgroup D, many of whom teach Physical Education, interact with a subset of students in a non-academic context, that of coach and athlete. In these non-academic contexts the coaches develop a personal relationship with their students, which Quiroz et al. (1991) described as indicative of the 'pal' orientation (the pal establishes a close personal relationship with the student, characterized by sharing information about personal lives outside of the classroom). In fact, teacher * * of subgroup D was overheard to offer one of his basketball players hot meals and use of a shower and washing machine. Further, the pal orientation carries over to interactions with all students. For example, teacher * * was observed offering to refer a non-varsity athlete to a physician (for an unknown medical problem), indicating that the student had already confided her medical problem to him. While the difference in orientations between teacher * * and the teachers in subgroup A be partly related to the race of the teachers, note that teachers 11 and 18 of subgroup D, both of whom teach physical education and coach but who are white and black, respectively, share teacher * *’s pal orientation. This suggests that the initial impetus towards the pal orientation is as much a function of the role of coach as of teacher’s race.

The fact that teachers respond to the school 'context' differently may result in competing processes of influence within the school. Like the teachers in subgroup A with regard to moral agency, the teachers in subgroup D may reinforce each others’ emphasis on the pal orientation through their frequent discussions. Then when the teachers of subgroup D engage in discussions outside their subgroup they

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Note that I did not interpret the directionality of within subgroup discussions, as subgroup members are likely to have reciprocal influences on one another when taking into account the direct and indirect interactions within subgroups. Further, for these data I make little of the directionality of the influence between subgroups, because the nature of the question “With whom do you engage in professional discussions?” does not suggest a large difference in status between the teacher who listed the discussion and the teacher who was listed. The differences between $X_{ij}$ and $X_{ji}$ may as likely reflect an unreliability in reporting as a distribution of power. Directionality could be inferred in a map based on data that capture the directionality of the relationship, such as data indicating ‘liking’ or the transference of information regarding a rumor, but the more directional the data, the less persuasive is the argument for using the data to identify cohesive subgroups which are based on mutual influences throughout the subgroup. For these data the indication of directionality is secondary to the salience of the cohesive subgroups. I included the arrow heads in Fig. 2 to indicate the potential of this type of map.
are likely to try to influence the members of other subgroups towards the pal orientation. For example, teacher’s emphasis on the pal orientation is established as he engages in discussions with other members of his subgroup on a weekly basis. Then, as he engages in discussions with teachers 5 and 10 (two of the highest ‘pals’ in subgroup C), he possibly influences them to emphasize the pal orientation more than they otherwise would. Teachers 5 and 10 then influence the other teachers in their subgroup as well as some of the teachers in subgroup A, etc.

Until now I have described the pattern of influence essentially from the perspective of the individual, but, just like the map constructed by Festinger et al. (1950), the subgroups imbedded in the map in Fig. 2 facilitate a description of influence from the level of the organization. For example, we may describe moral agency as being cultivated within subgroup A and then spreading to other subgroups where it encounters the competing pal orientation which was cultivated in subgroup D. Indeed, at the organizational level, we may observe in the map a sort of equilibrium of the system, with those in subgroup C who are mixed and moderate in their orientations mediating between the competing orientations of those at the top and bottom of the map. Most importantly, although one may disagree with the interpretation of the pattern of influence given here, the essential point remains that the map facilitates the analysis of influence from the individual and organizational levels.

7. Structural equivalence: An alternative criterion for identifying subsets of actors from sociometric data

The descriptive power of Festinger et al.’s (1950) map and the map in Fig. 2 depends on the cohesiveness of the subgroups. At the individual level, actors are influenced mostly by members of their primary groups. Further, emphasis on an orientation at the school level can be characterized as circulating primarily within the cohesive subgroups and then occasionally crossing subgroup boundaries. But many have argued that blocks of structurally similar actors can also be used to simplify the pattern of interaction among actors (White et al. 1976; Burt, 1982; Panning, 1982; Wasserman and Anderson, 1987; Anderson et al., 1992; Borgatti and Everett, 1994; Doreian et al., 1994; Snijders and Nowicki, 1994). Substantial theory supports the idea that actors’ actions are affected by their structural position, as defined by their pattern of interactions (Radcliffe-Brown, 1940; Merton, 1957; Nadel, 1957; White et al. 1976; Burt, 1982; etc.), and evidence shows that actors who engage in similar patterns of interaction act similarly (e.g. Nadel, 1957; White et al., 1976; Burt 1982, 1987; etc). Of course, it is rare when two actors are structurally equivalent, so blocks identified using a stochastic criterion might be more accurately referred to as blocks of structurally similar actors.

Friedman and Rubin (1967) and Panning (1982) outlined a procedure for identifying blocks of structurally similar actors using stochastic criteria analogous to the one utilized in Section 3 of this article to identify cohesive subgroups. These
criteria treat the interactions which each actor lists as attributes, and then the algorithms iteratively reassign actors to blocks so as to maximize the sums of squares between blocks and minimize the sums of squares within blocks. Thus actors within blocks have similar values on the attributes — that is, they engage in common patterns of interaction throughout the network. Of course, with social network data it is sensible to consider the pattern of interaction in terms of interactions received as well as those initiated (Burt, 1982; Panning, 1982).

I applied an algorithm based on Panning’s (1982) technique (which Arabie and Hubert, 1990 described as elegant) to the data indicating professional discussions among teachers at ‘Our Hamilton High’. I placed a value of 4 on the diagonal of the matrix of professional discussions allowing for direct discussions between teachers to contribute to their structural similarity (technically this blurs the distinction between structural similarity and cohesion, but I wanted to be consistent with the structural similarity approach as it is typically described in the literature). Following the methodology in Section 5 of this article, I constructed the map in Fig. 3 of the professional discussions within and between the structurally similar blocks of teachers. Before constructing the map in Fig. 3 I took the extra step of reducing the distance between each teacher and his or her subgroup center by a factor of 3.8 (this was the least distortion I could introduce which would yield an interpretable plot). This step eliminated the comparability of distances within and between subgroups, but without it subgroup boundaries would have overlapped to the point of uninterpretability.

The difference between the map based on cohesive subgroups in Fig. 2 and the map based on the blocks of structurally similar teachers in Fig. 3 is clear. Fewer professional discussions occur within the blocks in Fig. 3 than within the subgroups in Fig. 2. For example, in Fig. 3 teacher 13 joins with teachers 6, 8, 12, 17, 20, 22 and 23 in block C* in part because teachers 13, 17 and 22 engage in discussions with teacher 21 of block E*. Many more lines cross the block boundaries in Fig. 3 than the subgroup boundaries in Fig. 2 because the discussions which make teachers structurally similar need not occur within the block. These cross-block discussions pull blocks closer to one another, and the sparseness of within block discussions forces members of a common block relatively far apart (the situation would be more extreme had I not artificially reduced the within-block radii, and had I not placed a value of 4 on the diagonals of the matrix of professional discussions). Note that the total area in Fig. 3 is less than 4% of the area in Fig. 2 because the blocks in Fig. 3 are pulled closely together by the between block discussions. More importantly, the map based on cohesive subgroups is more interpretable for the same reason that a description of influence from the school level is more sensible in terms of cohesive subgroups; an orientation such as moral agency can be characterized as circulating within cohesive subgroups, and then occasionally crossing subgroup boundaries. No comparable interpretation can be made for actors located in structurally similar blocks. In fact, to the extent that one might try to characterize patterns of influence relative to structurally similar blocks, one is likely to rely on what cohesiveness exists within the blocks. For example, even White et al. (1976) who identified blocks of structurally similar
Fig. 3. Professional discussions within and between structurally similar blocks among teachers at 'Our Hamilton High'. Solid lines within blocks, broken lines between, thickness is proportional to frequency of discussion. Scale between blocks = \( W_m / \text{(density of discussion)} \), scale within blocks = \( W_m / (3.8 \text{ (density of discussion)}) \).

actors from directed data of who likes whom, confirmed their results and interpretations relative to Sampson's (1969) cohesive subgroups.

The interpretation of the map in Fig. 3 based on blocks of structurally similar teachers might be viewed as inappropriate because distances between blocks of structurally similar teachers were based on a cohesion criterion. This could be
remedied by defining distances (within or between subgroups) in terms of struc-
tural similarity instead of in terms of direct discussions (e.g. Barnett and Rice,
1985; Nakao and Romney, 1993). But such an approach is an indirect, and
therefore likely inferior, analysis of the original data representing interactions
among teachers (Doreian et al., 1994). To use the map to track the spread of
information or sentiment, linking individual and organizational level analyses, the
lines connecting teachers should represent direct interactions, and, correspond-
ingly, the distances between teachers should be based on direct interactions.
Moreover, the idea of tracing the spread of information or sentiments through
cohesive subgroups is consistent with the characterizations of organizational struc-
ture given not only by proponents of the importance of cohesive subgroups but by
those who have argued that actors' structural roles are critical to understanding
actors' behavior. For example, Nadel (1957) offers a description of the organization
in terms of integrated subgroups:

Sub-groups, like that widest group ‘society at large’, are made up of people in
determinate relationships. And any group is characterized by the kinds of
relationships that occur between the people in question, holding them
together. Now, inasmuch as subgroups are discrete entities, bounded units, at
least certain of these characteristic relationships must be equally bounded,
that is, they must come to an end somewhere, their cessation demarcating the
boundaries of the group. From this point of view, then, we might describe
sub-groups as areas of bounded relationships. But inasmuch as they are also
subdivisions of a wider collectivity and not isolated, self-sufficient units, the
bounded units themselves must be interrelated. (pp. 13–14)

The boundaries of the subgroups are permeable, as captured by the stochastic
criterion, thereby interrelating the subgroups into the organization. But the sub-
groups are bounded and the boundary of each subgroup marks the cessation of
relations within that subgroup, implying that interactions are concentrated primar-
ily within subgroups. Nadel's description of cohesive subgroups is echoed by
Merton (1957) who noted that “sub-groups are structurally constituted by those
who develop distinctive social relations among themselves which are not shared
with other members of the larger group” (p. 287, emphasis added) and by Burt
(1982), who acknowledged “relational models [based on direct interactions among
actors] have a high face validity stemming from their consistency with classic
concepts of communication in face-to-face primary groups as the socializing unit in
society” (p. 221). Although individuals in structurally similar blocks may behave
similarly (e.g. White et al., 1976), we typically conceive of organizations and
collectives as composed of cohesive subgroups which mediate between individual
and higher levels of analysis.

Of course, the identification of cohesive subgroups and structurally similar
blocks need not be mutually exclusive, nor even independent. One can use
cohesive subgroups as a basis for identifying the positions of actors (e.g. Newcomb
et al., 1965; Mizruchi, 1993), and, in particular, one can use graphical representa-
tions of the members of cohesive subgroups to locate the positions of the actors
(Freeman, 1992b). For example, in Fig. 2, teachers 20 and 23 occupy similarly central positions in subgroup B, and teachers 6 and 8 occupy similarly peripheral positions in subgroup B. Further, comparisons of positions across subgroups operationalize recent redefinitions of structural similarity in which two actors occupy a similar position if the actors to whom they are related occupy structurally similar positions. Faust (1988) refers to this as general equivalence, Burt (1991, 1992) refers to this as role equivalence or in terms of secondary holes, and Borgatti and Everett (1994) characterize such relationships in terms of structural isomorphisms. Using Faust’s term, actors who are ‘generally equivalent’ can be identified from a map of interactions such as in Fig. 2. For example, teachers 20 and 23 who are central to subgroup B may be characterized as generally equivalent with teachers 21 and 22 who are central to subgroup D. General equivalence may also be defined relative to between subgroup discussions; teacher 13 in subgroup D, who bridges between teachers in subgroups A and B, might be characterized as generally equivalent with teacher 3 in subgroup C who does the same. Therefore teachers 13 and 3 may occupy similar positions, even though there are no direct and few one-step indirect interactions between them. The similarity is through the common subgroup memberships of those with whom they engage in discussions outside of their own subgroup. The identification of generally equivalent actors can be formalized through techniques such as those described by Anderson et al. (1992) and Wasserman and Anderson (1987) which can be used to identify stochastically equivalent blocks of actors based on parameters representing interactions between each actor and the members of each cohesive subgroup.

8. Conclusion

Festinger et al. (1950) traced the spread of rumor in terms of communications within and between cohesive subgroups as defined by apartment courts. Because the geography of the apartments was known, Festinger et al. were able to construct a map which facilitated their analyses. I have generalized Festinger et al.’s approach by identifying cohesive subgroups from social network data using a stochastic criterion which allowed for interactions within and between subgroups. The criterion also was easily extended to weighted data and facilitated straightforward testing of the internal validity of the subgroups. I then constructed a map of professional discussions within and between subgroups using metric multidimensional scaling.

The techniques that I have presented can be used to analyze the structure of interaction within any organization, although the maps will be most helpful in analyzing data that represents exchange or interaction among actors, as opposed to directed ties such as liking or being attracted to, etc. Given data indicating the extent of interaction between each pair of actors, one can analyze processes from an individual or organizational level through maps of interaction within and between cohesive subgroups. Further, the maps provide a unifying basis for exploring many theoretical characterizations of social networks beyond those
which refer explicitly to cohesive subgroups, including structural holes, weak ties, and structural positions. Finally, because the metric in the maps applies within and between subgroups, maps across organizations can be compared, facilitating the development and testing of theories regarding organizational processes.

Of course, the boundaries of most organizations are permeable (Pfeffer and Salancik, 1978). Therefore as one characterizes the pattern of influence in the organization one characterizes the way in which the members of an organization who respond directly to external forces influence the behavior and attitudes of others in the organization. In this article, I used the map in Fig. 2 of professional discussions among teachers along with ethnographic data to suggest a set of processes through which the teachers at 'Our Hamilton High' influenced one another as all members of the school ultimately responded to the changing student population in the school. One can construct similar hypotheses from maps of interaction in other organizations. Optimally these hypotheses can be explored through longitudinal data, including the possibility that external factors affect the structure of interaction, but I leave that for further research.

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