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This chapter is organized into four major sections. The first section presents a short review of travel cost methods of non-market valuation. The second section provides the theoretical details of an approach referred to as the random utility model (RUM). The third part of the chapter is devoted to explaining the nature of the welfare measures derived from RUMs. The shape of the resulting benefits function is discussed and related to the value per fish approximations. The final part of this chapter makes some theoretical points related to the repeated RUM.

2.1 The Travel Cost Method

Estimating the demand for fishing involves determining where and how often people go fishing under different price regimes. If these two facets of demand are related to the quality of a fishing site, then standard economic surplus measures can be used to measure the benefits of changes in the quality of sites. The formal mechanism for this type of indirect valuation of environmental quality is known as weak complementarity.

Since most recreational goods do not have market prices, the travel cost method is often used to estimate the demand for these goods. While recreation may not have market prices, to visit a recreation site people will typically incur some costs, e.g., the costs of travel. The basic idea underlying the travel cost method is to exploit the variation across people in visits and travel costs to uncover the demand for a recreational site.

Traditional travel cost models were developed for use on single recreation sites, and these models can do a good job of predicting current demand for a particular site. Two problems with single-site models are relevant here. First, with a single-site model, the quality of the site generally does not vary across the sample of anglers; each respondent faces the same site quality. Thus, one is unable to statistically identify the relationship between site visitation and site
quality. In some special cases, it is possible to identify site quality in a single-site model. For example, a parameter on site quality could be identified if site quality changes over the season or if individual's perceptions of quality vary and these perceptions are measured.

Second, to appropriately predict changes in demand due to changes in quality characteristics, demand models should include relevant substitute sites to account for anglers' abilities to go to or come from alternative fishing sites. For this reason, single-site models are not recommended for predicting changes in demand unless they have included the prices and quality characteristics of substitute sites. Single-site models that include the prices and characteristics of other sites can be estimated, but they become rather unwieldy as the number of substitute sites grows. Moreover, single-site models are only appropriate for measuring changes in quality for the site at which the demand equation is estimated.

Another difficulty with traditional travel cost approaches is that the models can only be used to estimate demand for sites which an individual visits. However, for any given choice occasion, an individual can only visit one site. In the literature, the preponderance of unvisited sites is referred to as an extreme corner solution problem (Bockstael, Hanemann and Strand). Because of estimation difficulties that result in extreme corner solution situations, most demand models are unable to incorporate all relevant substitute sites. Morey et al. (1995) provide a general discussion and illustrate some systems of demand equations incorporating such corners.

To deal with this difficulty, more recent recreational demand models use discrete-choice models to characterize demand. In these models, a single element must be chosen from a set of alternatives. In the recreational fishing context, at any point in a season, anglers must choose where to fish from the many possible fishing sites. This is referred to as the site choice decision. Discrete choice models are particularly useful in the estimation of site choice because such models handle the extreme corner solution problem and can do so for a large number of sites. In doing so, discrete choice models are capable of including the prices and qualities of relevant substitute sites in the estimation of the site choice model.

McFadden pioneered the use of discrete choice models in economics and elaborated the properties of such models in several articles (1974, 1978, 1981) that provide a link between utility maximization and discrete choice. Since the discrete utility maximization presumes that the choices are random from the perspective of the researcher, the models are often referred to as random utility models (RUM). Also, since discrete choice models are frequently estimated using the multinomial logit distribution, these models are often referred to as multinomial logit
(MNL) models. Other important literature on RUM/MNL models has been summarized by Maddala; Ben-Akiva and Lerman; and Greene (1992).

Bockstael, Hanemann and Strand helped bring RUM/MNL models of site choice to the attention of recreation demand analysts. In addition to elaborating the properties of discrete choice models, they review techniques for valuing improvements in water quality. More recently, Bockstael, McConnell, and Strand have reviewed the state of the art in recreational demand modelling. There are several empirical examples of RUM/MNL models that have been specifically developed to estimate recreational fishing site choices. Some applications that have examined various implementation issues include the work of Parsons and Kealy (1992); Parsons and Needleman; Adamowicz et al.; Feather (1994); Peters et al.; Kaoru et al.; and Kling and Thompson. Freeman provides a review in the context of marine fishing (1993).

While discrete-choice models are well suited to explaining site choice, a single application of a discrete choice model cannot provide any information about seasonal participation. In the literature, there are two distinct seasonal participation approaches that utilize discrete-choice travel-cost models of site-choice. The first links a more traditional seasonal participation model with the site choice model. The second repeats the discrete-choice framework over the course of a season. In the repeated model, seasonal participation is the outcome of repeated decisions on where to go, with the option "not to go" included as an alternative in every repetition.

The first approach is more common because the data demands are not as great as those of the repeated models. Detailed information about site choice is only needed for one trip in a season, and this information is utilized to determine site choice over the entire season. Site choice is then combined with demographic information and used to explain the number of trips over the course of a season. Thus, the first model requires detailed information for one trip along with the total number of trips over the course of the season. However, in most of these models, the theoretical link between the seasonal model and the site choice model is not explicitly developed.

Most models which follow the first approach are very close derivatives of the models presented in Bockstael, Hanemann, and Strand, and in a related article by Bockstael, Hanemann, and Kling. These models use the inclusive value index from the site choice model as an explanatory variable in the participation model. The inclusive value index (IV) is the expected value of indirect utility from the site choice model (see section 2.2.1). While this linkage was initially ad hoc, recent work has provided a more explicit theoretical justification for such a
model (Hausman, Leonard and McFadden). Early applications of the IV approach treated the dependent variable (trips) as a continuous variable. Recent variants of these models improve the econometric model by treating the number of trips as a discrete variable, i.e., they use count data models (Feather et al.; Hellerstein and Tomasi; Hausman et al.; Creel and Loomis) or even hurdle count models (Shonkwiler and Shaw, 1996a; Haab and McConnell; Feather and Hellerstein).

The main alternative to the IV approach for linking the site choice model to a participation equation is the expected price approach (Feather, 1992; Feather, Hellerstein, and Tomasi). The expected price approach uses the site choice model to derive site probabilities that are then used to construct expected price indices and expected quality indices which get used in a participation function (Feather et al.; Feather and Hellerstein). Additional theoretical justification for a related version of such a model has been discussed by Shonkwiler and Shaw (1996b) -- see also Parsons and Kealy, (1995).

While the linked site choice and participation models such as the IV or expected price approaches have their strengths, they are not amenable to situations where site quality varies across the season because of changing temperature or catch rates. If site quality does change over the season, then so do site choice probabilities, IVs, expected prices, and expected qualities. To use the IV or expected price methods in this case, one would need to estimate separate equations for each period of uniform site quality. Alternatively, the repeated RUM can easily accommodate time varying site quality.

In repeated RUMs, trips are explained by the outcome of repeated site choice decisions where not going is also an option. To be of use, repeated discrete-choice models typically require detailed information about every trip. Perhaps because of this, applications of the repeated RUM are relatively rare in applied recreational demand modelling. Modelling the participation decision as the outcome of repeated site choice decisions was discussed in Bockstael, Hanemann, and Strand, and was empirically implemented by Carson and Hanemann. Morey, Shaw and Rowe have developed a model that explains trip behavior in a similar manner as repeated discrete choice models but can be estimated if detailed information is only available for some trips. Morey, Rowe and Watson provide an empirical application of the repeated RUM and compare repeated RUM results with the results of other approaches. Other recent repeated RUMs include Needelman and Kealy; and Montgomery and Needelman.

A related model, developed by Jones and Sung, has features of both of the above models. Jones and Sung's model uses detailed information about one trip to estimate a site choice model.
The time between trips is then used in a survival model to estimate the rate of trips. The rate of trips is then estimated as a function of the IV from the site choice model. Using the rate of trips, the number of trips a person would take over the season can be calculated. The structure of the Jones and Sung model is like a continuous version of a repeated discrete choice model. However, the estimation of the model is more similar to the linked models in that a seasonal participation equation is estimated using the results of a site choice model. Unlike the usual linked participation and site choice models, the Jones and Sung approach is amenable to seasonal changes in site quality.

Another related model, developed by Tay, involves a one time discrete choice model in which the elements are all possible combinations of sites, trip lengths, and trip frequencies. In this way, the site choice and seasonal problems are solved simultaneously. However, the number of potential sites in Michigan limits the applicability of this type of portfolio model.

As mentioned above, it is the variation in site visitation and travel costs that permits demand to be estimated. Travel involves inputs of market goods such as fuel, but travel also requires time. Since time spent travelling could be put to other desirable uses, many economists have argued that the opportunity cost of time must be accounted for if travel costs are to be correctly specified. Shaw provides a review of some of these issues and the literature which addresses them. Other relevant articles include Bockstael, Strand and Hanemann; McConnell and Strand; McConnell (1992); and Larson (1993).

Most studies are interested in the demand relationship as a step toward estimation of the benefits of changes in site quality characteristics. McFadden (1978, 1981); Small and Rosen; and Hanemann (1982 and 1983) have laid much of the foundation for the calculation of benefit measures from discrete choice models. While most researchers follow the same approach to derive welfare measures from the discrete choice portions of a model, there is some variation in technique when seasonal benefits are calculated from models with discrete choice components. For example, the approaches of Bockstael, Hanemann, and Kling; Creel and Loomis; Hausman et al.; Jones and Sung; and Morey, Rowe, and Watson all differ somewhat. Welfare measurement will be discussed further in section 2.3 of this Chapter. These alternative approaches will be examined in the context of the repeated RUM model -- an explicitly utility theoretic model of site choice and participation. In addition, final section will examine the relationship between the true repeated RUM seasonal welfare measure and several bounds on seasonal welfare measures that have appeared in the literature.
2.2 The Random Utility Model

The theory of choice in the RUM context has been explained by others (McFadden, 1974 and 1981; Small and Rosen; Hanemann 1982). Reviews in the context of transportation are provided by Train and by Ben-Akiva and Lerman. Useful presentations geared toward recreational demand are given in Bockstael, Hanemann, and Strand; Bockstael, McConnell and Strand; and Freeman. Econometric methods for estimating RUM models have been reviewed by McFadden 1981; and Morey 1994a. Here, the basic theory of random utility models is reviewed with an eye toward recreational site choice.

The following notation will be used throughout the chapter:

- \( Y \): Income
- \( k, j \): represent sites
- \( K \): total number of alternative sites
- \( z \): Hicksian composite good
- \( p_k \): price of a trip to site \( k \) (travel cost)
- \( *_k \): indicator function: equals one if site \( k \) is chosen, equals 0 otherwise.
- \( q_k \): quality at site \( k \)
- \( U(\@) \): utility function
- \( V(\@) \): deterministic portion of utility function (from researcher's perspective)
- \( \epsilon_k \): stochastic portion of utility for site \( k \) (from researcher's perspective)

Using the above notation, the decision of which site to visit on any choice occasion can be written as follows:

\[
\begin{align*}
\text{Max}_{\{z, *\}} & \quad U(z, q) \\
\text{s.t.} & \quad Y = \sum_k p_k *_k + z \\
& \quad \sum_k *_k = 1 \\
& \quad \sum_k *_k q_k = q \\
& \quad *_k \in \{0,1\}.
\end{align*}
\]

The first constraint is the budget constraint. The indicator function, \(*\), reflects the condition that only one alternative can be chosen, and this notion is embodied in the second and last constraints. The third constraint indicates that the quality variable entering the utility function is the quality of the site actually chosen. Thus, the model formulation states that the individual receives utility from a composite good and from the characteristics of the single discrete alternative that is chosen. Since utility is only received from alternatives that are chosen, non-use values cannot be measured in this setting.
Budget exhaustion implies that

\[ z = Y - E_k p_k^* \]  \hspace{1cm} (5)

i.e., any income not spent on the chosen alternative is spent on the composite good. From the definition and restrictions on the \(*_k\) functions, it is clear that only one of the sites can be chosen during any choice occasion. Hence, conditional on having chosen site \(k\), the budget constraint can be restated as

\[ z = Y - p_k \]  \hspace{1cm} (6)

Now, substituting for \(z\), the conditional indirect utility for the choice occasion, (maximum utility conditional on choosing \(k\)), is

\[ U_k = U_k(Y - p_k, q_k) \]  \hspace{1cm} (7)

where \(U_k\) is referred to as the conditional indirect utility function.

The problem in (4) can now be stated as choosing the site \(k\) that maximizes the conditional indirect utility \(U_k\), that is,

\[ \text{Max}_{(k)} U_k(Y - p_k, q_k) \]  \hspace{1cm} (8)

The solution to (8) will be a set of demand functions for the discrete alternatives which can be expressed as follows

\[ \delta_j = \begin{cases} 1 & \text{if } U_j = \max\{U_1, \ldots, U_K\} \\ 0 & \text{o.w.} \end{cases} \]  \hspace{1cm} (9)

where \(*_j\) is the discrete demand function for alternative \(j\). With the assumption of utility maximization, when one observes an individual choosing site \(j\) from \(K\) alternatives, then by revealed preference it can be inferred that \(U_k = \max\{U_1, U_2, \ldots, U_K\}\) for that individual. The choice implies a set of utility inequalities. If one observes an individual who chooses \(k'\), then \(U_k > U_k\) for all \(k \neq k'\).\footnote{In RUMs, each \(U_k\) includes an error term. Indifference can then be ignored because the probability of a tie is zero with extreme value or normal errors (McFadden, 1981).}

The model becomes the "random" utility model by recognizing that not all attributes that affect utility can be observed by the researcher. Thus, researchers can only predict the
probability that any alternative is the best in the choice set. The discrete demand indicators $*_{k}$ are each expressed in probabilistic terms as follows:

$$B_k = \text{Prob}[*_{k}=1] = \text{Pr}[U_k = \max\{U_0, U_1, \ldots, U_K\}] = \text{Pr}[U_k > U_0, \ldots, U_k > U_{k-1}, U_k > U_{k+1}, \ldots, U_k > U_K]$$

(10)

The choice probabilities serve as the expected demand functions for each alternative in the choice set. While the decisions are stochastic from the researcher's perspective, it is important to stress that the underlying choice behavior of individuals is made without uncertainty.

In the next section, econometric specifications of the RUM are discussed. Before doing so, note that if the choice set includes a non-fishing alternative $k=0$, $U_0=U(Y, 0)$, i.e., all income is spent on the composite good. As mentioned in section 2.1, including a non-fishing alternative and repeating the above model over choice occasions results in the repeated RUM. The repeated RUM will be discussed further in section 2.4.

**2.2.1 Econometric Methods for Implementing the RUM.**

In the RUM approach, utility is typically written as the sum of a deterministic portion of utility that can be measured (estimated) by the researcher and a stochastic portion that is unobservable for the researcher but remains known to the individual making the choices. The latter terms are the errors which usually enter additively. Thus, conditional utility is typically written as

$$U_k = V_k + r_k$$

(11)

where $V$ is the deterministic portion of utility to be estimated.

After choosing a functional form for $V$, the deterministic portion of utility can be parameterized. For example, researchers commonly adopt a linear form for the deterministic portion of the conditional indirect utility functions, in which case

$$V_k = : (Y-p_k) + $q_k.$$  

(12)

Here the $:$ is the marginal utility of income as well as the negative of the coefficient on price, and $\$q_k$ is a vector of marginal utilities associated with a vector of site characteristics $q_k$. In specifying $V$, it is important to bear in mind that the choice theory outlined above requires that $V$ has a form such as $V(Y-p,q)$ since $(Y-p)$ entered $V$ through the composite good $z$. 


To derive an econometric model, one assumes a distribution for the error terms in (11). If the errors are assumed to be drawn from a generalized extreme value (GEV) distribution (McFadden, 1978) then the resulting site choice probabilities (10) have closed forms. Alternatively, if the errors are specified as normal, then the resulting choice probabilities have the multinomial probit (MNP) form. In this chapter, discussion will focus on GEV models, but a few comments on normal errors and MNP are warranted.

Until recently, applications of the RUM with normal errors were extremely rare for models with more than a few alternatives. The reason for this is that maximum likelihood estimation of multivariate normal choice probabilities required numerical evaluation of J-1 normal distributions for each iteration of the MLE procedure -- a computationally intensive task. Recent improvements in simulation-based estimation methods have led to improved methods of estimating multinomial probit models (McFadden, 1989; Keane; Hajivassiliou). Even so, the simulation methods are still computationally intensive, a fact that restricts the size of the models that can be practically estimated (i.e., the number of alternatives in the choice set).

The principal advantage of the multinomial probit (MNP) structure is that MNPs permit very general patterns of correlations among alternatives in a given choice set (more will be said about the effects of this below when independence of irrelevant alternatives, IIA, is discussed). The underlying errors for the MNP can be specified quite generally. However, the ability to do so is still limited by computational considerations and identification problems (Chen, Lupi and Hoehn, 1997). For example, if there are J alternatives, then there are J(J+1)/2 potentially distinct elements in the covariance matrix, though not all of these elements can be identified. For the large J typifying the recreational site choice problems, estimation of MNP will also require some simplifying parameterization of the covariance matrix.

Formulating the MNP as a random parameters (varying parameters) model results in an extremely flexible error structure (Hausman and Wise). Random parameter models specify a distribution for all or some of the $s in the indirect utility function. The distribution of the $s can be interpreted as representing the distribution of tastes in the population. The random $s can then be decomposed into a mean effect and an error term. The error terms for the $s are estimable, and they result in a pattern of correlations among the errors for the alternatives in the choice set. The end result is a MNP that parsimoniously parameterizes the covariance matrix and can permit fairly general correlations across alternatives in the choice set. The random parameters MNP has recently been applied in the area of environmental economics: see Layton
for an application to stated preference valuation data and Chen and Cosslett for an application to recreational fishing site choice.

The random parameters MNP has been used to capture correlations across repeated choices by an individual, thereby capturing some of the panel effects that might exist in data that contains multiple observations for each decision maker sampled. This area of research is in its infancy, but a recent application in the recreational demand arena is by Train. Train actually formulates a random parameter logit model, but the effect is the same. While currently limited in the number of alternatives that can be included in the choice set, MNP models with flexible covariance structures will become more feasible for large choice sets with continued advances in computing power.

The remainder of this section focusses on econometric models that are amenable to traditional MLE methods. A common approach is to assume that the errors are in the family of GEV distributions (Johnson and Kotz, McFadden, 1978). Extreme value distributions take their name from the property that the maximum of an extreme-value variate will also have an extreme value distribution. That the distribution of the maximum is known is a desirable property for specifying probabilities of the RUM form in (11). A second, and related, property of the extreme value distributions is that they yield closed-form choice probabilities. The closed form for the choice probabilities facilitates maximum likelihood estimation of the parameters of the choice probabilities. The next section discusses some of the properties of a simple extreme value distribution, and the following section presents the same for the nested logit form of the GEV.

### Multinomial Logit (MNL)

The simplest form of a GEV distribution is the Type 1 Extreme Value (EV) distribution. When the errors in (11) are assumed to be i.i.d. across individuals and across alternatives and have a Type 1 EV distribution, choice probabilities will have the multinomial logit (MNL) form

\[
\pi_j = Pr(\delta_j = 1) = \frac{\exp(V_j)}{\sum_{j=1}^{J} \exp(V_j)}
\]

This form of the MNL is also referred to as the conditional logit (Greene, 1993). From the above expression, it is clear that the \( B_j \) will depend not only on the price and quality of site \( j \), but on the
prices and qualities of all sites. Therefore, the RUM model provides a convenient form for relating the demand for a site to the prices and qualities of all relevant substitute sites.

The marginal effects of these probabilities are presented here for reference in later sections

\[
\frac{\partial \pi_j}{\partial V_k} = \begin{cases} 
\pi_j(1-\pi_j) > 0 & \text{if } j = k \\
-\pi_j \pi_k < 0 & \text{if } j \neq k
\end{cases}
\] (14)

Naturally, the sign of \( \frac{M_B}{M_{\delta k}} \) will depend on \( \frac{M_V}{M_{\delta k}} \). Thus, with the typical linear form \( V_k = (Y-p_k) + \$q_k \), own probability effects of changes in \( q_k \) are positive (negative) if \( \$ \) is positive (negative). Cross probability effects are positive (negative) if \( \$ \) is negative (positive). Clearly, all sites are strictly substitutes in a model with MNL choice probabilities. In fact, alternatives in any RUM are necessarily strict substitutes.

Figure 2.1 depicts the shape of site choice probabilities. The probability is on the vertical axis and a generic site quality variable, \( q_j \), is on the horizontal axis. The domain of \( B_j(q_j) \) is the real line. The shape of the curve assumes \( q_j \) is positively valued (for a disamenity or a site price, the graph is simply reversed). The horizontal axis could just as well be \( V_j \). As \( V_j = 64 \), \( B_j = 61 \), and as \( V_j = 6 \), \( B_j = 60 \). The slope of \( B_j \) is largest at \( B_j = 0.5 \) and approximately 0 when \( V_j \) is very high or very low. Thus, the site demand is most responsive to changes in characteristics when the changes "make the site one of the best." When a site has very low utility, changes in characteristics increase demand, but unless the changes are large the demand is not very responsive. As the site begins to be "one of the better sites," any further changes make the site "one of the best." Once the site is the best, changes in site quality do not draw more individuals, that is, once it is best, improvements in quality don't change it's status as best, so the site choice probability is very flat.
when site utility is very high relative to other sites. Finally, note that even though the probability function graphed in Figure 2.1 is based on a MNL form, other forms such as nested logit or normal will have very similar shapes. For example, if the site choice probability is normal, the general shape will be the same as in Figure 2.1, though the tails of the normal density are not as fat as for the logit density.

**Expected Indirect Utility with MNL**

The MNL choice probabilities were derived by assuming that the underlying errors were iid Type 1 EV. A property of this distribution says that if \( u_i \) are Type 1 EV with location \( V_i \) and scale 1, then the max of \( u_i \) is distributed as Type 1 EV with location \( IV = \ln(3, e^{VI}) \) (Johnson and Kotz; McFadden, 1978; Ben-Akiva and Lerman). Therefore, the expected value of the maximum conditional indirect utility is also Type 1 EV and its expected value is given by

\[
E[\max(U_1, \ldots, U_J)] = \ln(\sum_{m=1}^J e^{V_m}) + K
\]

\[
= IV + K
\]

where \( K \) is Euler's constant and equals 0.577... (Johnson and Kotz). The expression for the maximum is often referred to as the "log sum" formula (Williams) or the "Inclusive Value Index" (McFadden, 1978). Here, the "inclusive value" (IV) terminology more commonly found in the econometrics literature is adopted. IV+K gives the expected indirect utility of the choices in the choice set \( J \). The IV notion will be used later in the discussion of welfare measures and in the decomposition of NL choice probabilities.

Note for later use that

\[
\frac{\partial IV}{\partial p_j} = -\mu \pi_j \quad \text{and} \quad \frac{\partial IV}{\partial q_j} = \beta_q \pi_j
\]

where : is the negative of the parameter on price and \( $q \) is the utility parameter on site quality (Hanemann 1983; McFadden 1981). Also note that as the quality of a particular site increases, IV approaches the indirect utility for that site. Specifically, as \( V_j \rightarrow 64 \), IV \( \rightarrow 6V_j \). That is, as the measured utility of a site gets large (relative to the other sites), the expected maximum of the utilities approaches the utility of that site.
Expected Indirect Utility and Roy’s Identity

By utilizing the expected indirect utility function, one can illustrate that it satisfies an analogue of Roy’s identity (McFadden, 1981). First, note that for linear in income conditional indirect utilities, the expected indirect utility is (ignoring Euler’s constant):

\[ E[V] = IV + \mu Y = \ln \left( \sum_j e^{(u_j - \mu p_j)} \right) \]  

(17)

where :  is the negative of the parameter on price, i.e., the estimated marginal utility of income. The \( E[V] \) function is the ”expected” indirect utility function with the underlying income terms being retained. Utilizing \( E[V] \), one can show that Roy’s identity applies to the ”expected” indirect utility function as follows:

\[
\frac{\partial E[V]}{\partial p_j} = -\frac{\partial IV}{\partial p_j} = \frac{\pi_j}{\mu}
\]

(18)

where the last step makes use of the fact that \( MIV/Mp_j = -\pi_j \). The derivation of the choice probabilities from the expected indirect utility function reinforces the interpretation of the choice probabilities as the expected demand functions for the discrete alternatives.

Independence of Irrelevant Alternatives

The choice probabilities that have the MNL form suffer from the well-known problem of independence of irrelevant alternatives (IIA), sometimes referred to as red bus/blue bus problems. What IIA says is that the ratio of choice probabilities for \( j \) and \( k \) are unaffected by the attributes of other alternatives. That is, \( B_j/B_k = \exp(V_j)/\exp(V_k) \). The implications of this mathematical property is that when a particular site improves, the decreases in other site probabilities will be spread out so as to preserve the pre-existing ratios between unaltered sites. Of course, even if the model exhibits IIA, the choice probabilities still depend on the quality of all sites, and these site choice probabilities do change in response to changes in quality at their own site.

IIA does not affect the ability of the model to fit the baseline choice probabilities. Rather, IIA affects the way the choice probabilities respond to changes in quality. MNL
probabilities will respond in a way that keeps the ratios of site choice probabilities of all sites where quality does not change the same. This property is counter-intuitive if some of the alternatives are perfect or close substitutes for one another. The classic example discusses a hypothetical choice of transportation mode where the choices are to take a red bus, a blue bus, or a car. One would expect that any change in the red bus that increased the probability of using the red bus would result in a corresponding decrease in the probability of choosing the blue bus, leaving the probability of choosing car unaffected. However, the IIA property implies that the ratio of the probabilities of choosing blue bus and car must stay the same. Hence, some of the increase in red bus probability will be drawn from the probability of choosing the car as well as from the probability of choosing the blue bus.

Consider an example in the context of lake and river recreation site choices. Suppose there are three sites: A is a lake (red bus) and B is a similar lake (blue bus), while C is a river (car). In this case, when the quality of A increases, one might expect that relatively more trips are drawn from B than from C. This cannot happen if the error terms exhibit IIA. Consider a numerical example. Suppose before a change that Pr(A)=1/6, Pr(B)=1/3, and Pr(C)=1/2. If after the change Pr(A) goes to 1/3, then by IIA the new Pr(B)=8/30 and the new Pr(C)=12/30. Here, the Pr(A or B) goes from 1/2 to 18/30, and Pr(C) goes down more than Pr(B). In the extreme case where A and B are perfect substitutes (red and blue busses), we would expect that P(A or B) stays at 1/2 so that Pr(C) is unchanged and any increase in pr(A) is completely offset by decreases in Pr(B).

The IIA property result is fundamentally due to treating the errors across alternatives as independent. Any model, even a simple multinomial probit, that is based on the assumption that the errors across alternatives are independent, will embody a property similar to IIA (McFadden, 1981; Hajivassiliou; Ben-Akiva and Lerman; Chen et al., 1997). Therefore, relaxing IIA type substitution patterns requires relaxation of the assumption that the errors are independent across alternatives.

Nested Logit, NL

More general forms of the GEV distribution result in models that partially relax IIA. The most widely known version of the GEV results in choice probabilities that have the nested logit (NL) form. The NL models allow correlations across alternatives but retain the assumption that the underlying error vector is i.i.d. across individuals. This section briefly outlines some features of choice models with GEV errors and draws on McFadden (1981) and Morey (1994a).
To specify the NL, subdivide the alternatives into $M$ mutually exclusive groups that are expected to be correlated to one another. Let $J^m$ denote the number of alternatives in group $m$. For example, group the buses and group the cars, or group the lakes and group the rivers. As the NL nomenclature implies, the groups of alternatives are referred to as nests.

In the nested logit, the joint probability of choosing $j$, where $j$ is a member of group $m$, is

$$
Pr(\delta_{jm} = 1) = \frac{\exp\left(\frac{1}{1-\sigma_m} V_{jm}\right) \left(\sum_{j=1}^{J^m} \exp\left(\frac{1}{1-\sigma_m} V_{jm}\right)\right)^{-\sigma_m}}{\sum_{m=1}^{M} \left(\sum_{j=1}^{J^m} \exp\left(\frac{1}{1-\sigma_m} V_{jm}\right)\right)^{1-\sigma_m}}
$$

where $F_m$ is a parameter of the underlying GEV distribution for the errors. If $F_m$ equals 0, the above NL probabilities reduce to the MNL. McFadden has shown that $F_m$ is approximately equal to the correlation between alternatives in group $m$ (as cited in Maddala, p71). Frequently, the NL model is specified by defining $2_m = 1 - F_m$. $2_m$ is then referred to as the dissimilarity coefficient for group $m$. When $2_m = 1$ the MNL results, and as $2_m$ goes to 0 the alternatives in $m$ become more like one another than they are like the alternatives in other groups. Thus, $2_m$ measures the degree of dissimilarity between alternatives in different nests.

In nested models, it is often convenient to decompose the joint probability into the product of the marginal probability and the conditional probability as follows

$$
Pr(\delta_{jm} = 1) = Pr(\delta_m = 1) Pr(\delta_j = 1 | \delta_m = 1)
$$

$$
\pi_{jm} = \pi_m \pi_{j|m}
$$

Using the above, the conditional probability of choosing site $j$ from group $m$ is

$$
\pi_{jm} = \frac{\exp\left(\frac{1}{1-\sigma_m} V_{jm}\right)}{\sum_{j=1}^{J^m} \exp\left(\frac{1}{1-\sigma_m} V_{jm}\right)}
$$
This conditional probability in (21) has the MNL form with scale factor $1/(1-F_m)$. The scale factor $1/(1-F_m)$ is unidentified at this level. If the conditional probability in (21) were estimated in isolation, the estimated parameters would be estimates of $(1-F_m)$ -- for linear in parameters utility functions.

The marginal probability of choosing an alternative from group $m$ is given by

$$
\pi_m = \frac{\left( \sum_{j=1}^{j^m} \exp\left( \frac{1}{1-\sigma_m} V_{jm} \right) \right)^{1-\sigma_m}}{\sum_{m=1}^{M} \left( \sum_{j=1}^{j^m} \exp\left( \frac{1}{1-\sigma_m} V_{jm} \right) \right)^{1-\sigma_m}}
$$

(22)

It is sometimes convenient to rewrite the marginal probabilities at this level by exploiting the fact that $\exp(\ln(x))=x$. In particular, the terms in the parentheses in (22) can be rewritten as follows

$$
\left( \sum_{j=1}^{j^m} \exp\left( \frac{1}{1-\sigma_m} V_{jm} \right) \right)^{1-\sigma_m} = \exp\left( \ln \left[ \sum_{j \in J_m} \exp\left( \frac{1}{1-\sigma_m} V_{jm} \right) \right] (1-\sigma_m) \right)
$$

(23)

$$
= \exp \left( (1-\sigma_m) IV_m \right)
$$

where $IV_m = \ln \left( \sum_{j \in J_m} \exp\left( \frac{1}{1-\sigma_m} V_{jm} \right) \right)$

In (23), $IV_m$ is the inclusive value index for group $m$ and represents the expected value of the maximum indirect utility for all the sites within the group $m$ (ignoring Euler's constant). Again, letting $Z_m = 1-F_m$ and substituting the $IV_m$ back into (22) yields
As a result, the marginal probabilities all have the MNL form where the IV\(_m\) are treated like variables that are parameterized by \(2\).

The above decomposition and rearranging of the joint probability of selecting alternative jm is useful for illustrating a common estimation method for NL models. Since MNL models are relatively easy to estimate, a sequential estimation strategy for the NL model involves estimating the conditional site choice parameters in (21) for each of the m groups. Since the estimation of the conditional probabilities recovers estimates of \(S/(1-F_m)\), one can use the estimates to calculate the IV\(_m\) terms. Next, the IV\(_m\) terms are used to estimate the marginal probabilities in (24). At this stage, any variable that does not vary within group m, but does vary across the m can be included among the variables in the second stage. The second stage yields estimates of the \(2_m\).

The above sequential estimation strategy is consistent but it is not efficient (Amemiya; McFadden, 1981). It is not efficient because the IV terms are functions of the lower level parameters, and this information is ignored in the sequential estimation. However, since each step only requires estimation of a MNL model, the sequential strategy is easy to implement. Besides being inefficient, the covariance matrix for the sequential estimates is not consistently estimated by the conventional MNL covariance matrix. The sequential standard errors overstate the precision of the upper level estimates because the IV terms are treated as non-stochastic even though they are random variables (Amemiya, McFadden, 1981).

McFadden (1978 and 1981) has discussed sufficient conditions for consistency between a probabilistic choice system and RUM. In the MNL, these conditions are satisfied for all values of the estimated parameters. In the NL, the conditions are only satisfied for estimated values of \(2\) that lie in the unit interval. The conditions are not guaranteed by MLE estimates of the model parameters (McFadden, 1981). While values of \(2\) greater than 1 can be consistent with RUM, Herriges and Kling (1996b) have shown that in practice, there is little leeway for values of \(2>1\) to be consistent with RUM. If the estimated values of \(2\) exceed 1 or are less than zero, then this can be taken as evidence that the nests are poorly specified (Ben-Akiva and Lerman; McFadden, 1981; Maddala).
Marginal effects of the NL choice probabilities are presented here for comparison with those of the MNL (14) -- see also Herriges and Kling (1996c).

As \( F \) goes to 0, these results reduce to the partials for the MNL model (14). As with the MNL, the own effects are positive and the cross effects are negative. In both the MNL and NL formulations of the RUM, all the alternatives are strict substitutes. Again, as with MNL, the sign of \( M_{B_k} / M_{q_k} \) will depend on \( M_{V_k} / M_{q_k} \). So with the typical linear form \( V_k = (Y - p_k) + \$q_k \), own probability effects are positive (negative) if \( \$ \) is positive (negative). Cross probability effects are positive (negative) if \( \$ \) is negative (positive).

In later sections, these marginal effects are used to illustrate properties of RUMs. Note that while the expressions differ slightly whether the RUM has MNL or NL choice probabilities, the marginal effects will have the same signs. Hence, many of the properties of RUM models that are established for MNL will apply to NL models.

**Expected Indirect Utility with NL**

McFadden (1978) presents a formula for the expected indirect utility for general RUM's based on GEV errors. In the NL case, the expected maximum indirect utility can be expressed in a manner that mirrors the expression for MNL. Let \( IV_{NL} \) denote the inclusive value index from the two level nested logit. \( IV_{NL} \) can be expressed as

\[
IV_{NL} = \ln \left( \sum_m \exp \left( \theta_m IV_m \right) \right)
\]

where \( IV_m = \ln \left( \sum_{j \in J} \exp \left( \frac{1}{\theta_m} V_{jm} \right) \right) \)

\[(26)\]
where the similarity to the IV measure for MNL models is apparent (see equation 15). The expected value of the maximum conditional indirect utilities is equal to $\text{IV}^{\text{NL}} + K$. Also note that the marginal effect on IV of a change in utility still recovers the site probability,

$$\frac{\partial \text{IV}^{\text{NL}}}{\partial \nu_k} = \pi_k^{\text{NL}} \tag{27}$$

McFadden (p. 228, 1981) provides the proof of the above for general GEV forms. Recognizing these two results, many of the properties of RUM that are derived for the MNL in later sections directly apply to NL.


2.3 Welfare Measures in RUMs

In the random utility maximization model, anglers choose the best site. The separable nature of the problem implies that anglers only get utility from changes in quality when they actually use the site where quality changes. In other words, only use values matter. For any individual, the total benefits of a change in site quality will depend on the difference between the indirect utility of the old best site and the indirect utility of the new best site. Thus, changes in site quality only have value when they occur at the best site or when they reach a level that the site where quality changes becomes the best site. This insight is key to understanding the welfare measure for RUMs and will be key to understanding the shape of the aggregate benefits function derived from the RUMs.

Deriving the welfare measures for the RUM is complicated by the fact that the individual-specific terms, $\epsilon$, are unknown to the researcher. For explication, it is helpful to begin by ignoring this uncertainty and focusing on the underlying logic of the welfare measure. Under certainty and full knowledge of the error terms for each individual, compensating variation for a change in prices and quality from $(p^0,q^0)$ to $(p^1,q^1)$ would be calculated as follows:

$$
\begin{align*}
\text{Max}_{j \in J} \rho U(Y - p_j^0, q_j^0, \epsilon_j) &= \text{Max}_{j \in J} \rho U(Y - p_j^1 - CV, q_j^1, \epsilon_j) \\
\text{Max}_{j \in J} \left[ V(Y - p_j^0, q_j^0) + \epsilon_j \right] &= \text{Max}_{j \in J} \left[ V(Y - p_j^1 - CV, q_j^1) + \epsilon_j \right]
\end{align*}
$$

(34)

where the second line makes use of the fact that the error terms are assumed to be additive.

Further simplifications of (34) can be made if the deterministic portion of the conditional indirect utility function, $V(Y - p_0, q_0)$, is assumed to be linear-in-parameters; that is, suppose that $V_k = (Y - p_0) + \beta q_k + \epsilon_k$. In this case, compensating variation (CV) is defined as follows:

$$
\begin{align*}
\text{Max}_{j \in J} \rho \left[ \mu(Y - p_j^0) + \beta q_j^0 + \epsilon_j \right] &= \text{Max}_{j \in J} \rho \left[ \mu(Y - p_j^1 - CV) + \beta q_j^1 + \epsilon_j \right]
\end{align*}
$$

(35)

Additional simplifications of expression (35) can be made by further exploiting the fact that the model is linear-in-income and that the marginal utility of income term does not vary over the alternatives. In this case we can write CV as follows:
From these expressions for CV, it is clear that if a policy change does not affect the best site before or the best site after a change, then the CV for that change will be zero. This is a direct consequence of the fact that use value is being measured. If a policy change does not affect sites that are being used, then the policy does not generate use value.

To illustrate the use-value nature of the CV measure, suppose there is an increase in quality for a single alternative, j. In this case, the compensating variation for some individual can be written as

\[
CV = \frac{1}{\mu} \left( \left( \max_{j \in A} \left[ -\mu p_j^1 + \beta q_j^1 + \epsilon_j \right] \right) - \left( \max_{j \in A} \left[ -\mu p_j^0 + \beta q_j^0 + \epsilon_j \right] \right) \right) \tag{36}
\]

where the notation \( m^0 \) means that under the initial quality conditions alternative m was chosen.

For purposes of describing their CV, individuals can be classified into three types. First, there are individuals who initially chose site j and continue to do so after the quality change (\( m^0 = j \)). For these individuals, the welfare measure is given by \( CV = q_j \). Second, there are individuals who did not initially choose site j, but are induced to choose j following the quality change (\( m^0 \neq j, m^1 = j \)). For the second group, there is a welfare gain, but its magnitude depends on the level of utility of the site they initially visited. The change in value for these individuals is less than \( q_j \) because \( U_{m^0} > U_j \) for these individuals. Third, there are individuals who do not choose site j before or after the change in quality at site j (\( m^0 \neq j, m^1 \neq j \)). Since the utility of the best site does not change for these individuals, there is no change in well being, and \( CV = 0 \). Thus, the aggregate CV will only be greater than zero only if there is a change in quality at what was initially the best site for some individuals or if the change in quality induces some individuals to switch to the site where quality changes.

2.3.1 Incorporating Uncertainty

The preceding section addressed how welfare would be measured if the \( \epsilon \)'s were known. The essence of the RUM is that the \( \epsilon \)'s are not known to the researcher and are treated as error...
terms. Since the conditional indirect utilities in (34) are random variables, the maximum of these will also be a random variable, so the CV itself will be a random variable.

One way to deal with this is to calculate an expected CV, typically a complex matter. However, McFadden (1981 and 1997) has shown that in certain settings, the expected CV is equivalent to solving the following

\[ E \left( \text{Max}_{j \in J} \mu(Y - p_j^0) + \beta q_j^0 + \epsilon_j \right) = E \left( \text{Max}_{j \in J} \mu(Y - p_j^1 - CV) + \beta q_j^1 + \epsilon_j \right) \]  

(38)

This approach is only valid in the case of linear income effects and a \( \beta \) that does not vary across alternatives (McFadden, 1997). If these assumptions do not hold, then the relevant expectations would be taken at equation (34), and one would have to determine the probabilities for each of the events \((j,k) = (m_0,m_1)\) \(\forall j,k\), i.e., the probabilities for all the combinations of best sites before and after the policy change. For these situations, McFadden (1997) has developed bounds for the CV and has also proposed a Monte Carlo simulation approach for calculating CV for models with non-linear income effects.

As when the errors were treated as deterministic, additional simplifications of expression (38) can be made by further exploiting the fact that the model is linear-in-income and that the marginal utility of income term does not vary over the alternatives. In this case, expected CV can be written as follows:

\[ E[CV] = \frac{1}{\mu} E \left[ \left( \text{Max}_{j \in J} (-\mu p_j^1 + \beta q_j^1 + \epsilon_j) \right) - \left( \text{Max}_{j \in J} (-\mu p_j^0 + \beta q_j^0 + \epsilon_j) \right) \right] \]  

(39)

Actual closed-form solutions for expression (39) generally will not exist. For example, if the error terms are normal, then (39) will require evaluation of J-1 integrals of a multivariate normal distribution. Because there is no closed form solution for these J-1 normals, numerical methods can be used, but they will be very computationally intensive. On the other hand, Chen
and Cosslett have developed a straightforward simulation method for estimating (39) when the errors are normal. Alternatively, if the errors in (39) are in the class of GEV distributions, then closed form expressions for (39) will exist because one of the properties of the GEV is that the maximum of GEV variates is also distributed GEV (McFadden, 1978).

For ease of exposition, the focus will be on the case of Type 1 EV errors, although with minor revisions the results apply to GEV errors. Using the inclusive value notion developed in section 2.2, if the error terms in (39) are Type 1 EV then the expected CV is given by

\[ B = E[CV] = \frac{IV^1 - IV^0}{\mu} = \frac{\ln\left(\sum_{n=1}^{j^1} e^{\gamma_n^1}\right) - \ln\left(\sum_{n=1}^{j^0} e^{\gamma_n^0}\right)}{\mu} \]  

(40)

Note that the Euler's constants have canceled out. The notation using B links the welfare measure to the discussion of the benefits function in Chapter 1. The above equation has long been utilized as a benefits measure for recreational demand models based on the RUM (Bockstael et al.). The expression provides a means of evaluating complex arrays of changes in site quality and prices. Moreover, the expression is well suited to evaluating the impact of the closure of a site or the addition of new sites (note the index on the summation is specified over the sites available in the before and after policy settings).

Another way to derive the CV measure is to utilize the formulation of the expected indirect utility given by \(E[V]\) in (17). Using \(E[V]\) to define CV, it is more apparent that the CV measures are simply differences in IV:

\[ E[V(Y, p, q^1)] = E[V(Y-CV, p, q^2)] \rightarrow IV(q^1) + \mu Y = IV(q^2) + \mu (Y-CV) \]

\[ CV = \frac{1}{\mu} [IV(q^2) - IV(q^1)] \]  

(41)

Clearly, deriving the CV in this manner is only appropriate when the model is linear in income. In the next section, this measure is related to the weak complementarity approach typically used to derive welfare measures for site quality changes in the context of continuous recreational demand systems.
2.3.2 Weak Complementarity in the RUM

Weak complementarity allows the value of changes in the quality of a non-market good to be recovered from the demand curve for a market good (Maler; Bradford and Hildebrandt; Freeman). A prerequisite for using weak complementarity requires that the two goods be Hicksian complements; loosely, this means that the demand curve for the non-market good will depend on the level of quality for the non-market good. If the conditions for weak complementarity are satisfied, then the value of the changes in the non-market good can be measured by changes in the consumer surplus (CS) for the market good. Weak complementarity is the theoretical underpinning for the welfare measurement for indirect non-market valuation methods such as the travel cost method.

Given that the two goods are Hicksian complements, the two conditions for weak complementarity are

1. there exists a choke price for the market good; i.e., \( p^* > P^* \) such that \( \frac{d}{dp} x(p) = 0 \).
2. when the demand equals zero, changes in the non-market good have no value; i.e., \( p > p^* \), \( M/V_M = 0 \) and \( M/e_M = 0 \) where \( e \) is the expenditure function.

The first condition ensures that the area under the market demand curve is bounded so that total consumer surplus exists. The condition implies that the market good is not essential. Since the existence of a choke price is not necessary for the consumer surplus integral to be bounded, the first condition is not necessary for the weak complementarity approach to be valid. However, in order to use the weak complementarity approach, CS does need to exist. The second condition implies that there are not any non-use values associated with the non-market good. That is, consumers will derive utility from the good only if they consume the good.

In what follows, weak complementarity is first discussed in the discrete-choice case where the uncertainty about demand and utility is ignored. Next, weak complementarity in RUMs is considered when the stochastic errors are accounted for.

It is easy see that the conditions for weak complementarity are satisfied for the discrete choices. Here, there is clearly a choke price for the discrete goods (condition 1). Recall from section 2.2, that the demand functions for the discrete alternatives can be expressed as follows

\[
\delta_j = \begin{cases} 
1 & \text{if } U_j = \max\{U_1,...,U_k\} \\
0 & \text{o.w.} 
\end{cases}
\]  

(42)
where \( *_{j} \) is the demand function for alternative \( j \). Clearly there would be \( p_{j} \) that would drive \( *_{j} \) to zero. If there were not a choke price for each of the discrete alternatives it would imply that some of the alternatives were essential. The discrete choice framework would be an inappropriate framework for modelling the choice among essential goods, since in any choice occasion \( K-1 \) of the goods are not consumed by the mutually exclusive nature of choices considered in the RUM.

As discussed above, the individual's utility will only depend on the quality of the good that is chosen. If a good is not chosen, then changes in quality do not affect utility unless the changes induce the consumer to select that alternative. Thus, when alternative \( j \) is evaluated at prices such that \( j \) would not be chosen for any value of \( q_{i} \), any changes in \( q_{i} \) will not yield any change in utility. Therefore, when the stochastic nature of the error terms is ignored, the discrete demand system satisfies the conditions for weak complementarity.

Even when the error terms are treated stochastically, it turns out that the RUM framework satisfies the conditions for weak complementarity. Recall that site choice probabilities can be interpreted as expected demand functions. For choice probabilities that have logit or normal forms, there is no choke price at which this probability is exactly zero. However, the probability does approach zero as price goes to infinity. Therefore, the area under the demand curve yields a closed form integral for choice probabilities that satisfy McFadden's (1981) conditions for probabilistic choice systems consistent with random utility maximization. Further, by examining the nature of the welfare measure from the RUM, one can also see that RUM's satisfy the second condition for weak complementarity: changes in \( q_{i} \) do not affect expected \( V_{j} \) if expected demand for site \( j \) goes to zero.

The weak complementarity measure is illustrated below when the choice probabilities have logit forms. First, the consumer surplus measure is defined as

\[
CS(q_{j}^{'}) = - \int_{\pi_{j}(q_{j}^{'})dp} = \frac{1}{\mu} \ln \left( \sum \exp \left( v_{i} \right) \right)_{\pi_{j}^{*}} - \frac{1}{\mu} \ln \left( \sum \exp \left( v_{i} \right) \right)_{\pi_{j}^{*} \rightarrow \infty} \\
= \frac{IV(q_{j}^{'})}{\mu} - \frac{1}{\mu} \ln \left( \sum \exp \left( v_{i} \right) \right)
\]

(43)
The second term makes use of the fact that $e^{x_60}$ as $x_6-4$. Both terms make use of the fact that $\frac{Mv}{M_{pj}} := B_j$ for all $j$.

The second term in (43) is the inclusive value for the choice set that includes all the sites except $j$. Therefore, the measure given in (43) is equivalent to the negative of the "change in inclusive value" measure for eliminating the site from the choice set. As such, it is no surprise that it equals the consumer surplus for the site.

Also, regarding the second term in (43), while $e^{x_60}$ as $x_6-4$, it is also the case that $e^{x+a_60}$ as $x_6-4$. Adding a constant does not affect this term, so consumer surplus does not depend on $q_i$. $V_j$ goes to minus infinity regardless of the value of $q_i$. Therefore, as price goes to infinity, changes in $q_i$ do not affect welfare. Thus, the conditions for weak complementarity are satisfied.

Making use of (43), the weak complementarity measure is defined as

$$CS(q_j^2) - CS(q_j^1) = \int_p \pi_j(q_j^2) dp - \int_p \pi_j(q_j^1) dp$$

$$= \left( IV(q_j^2) - IV(q_j^1) \right)$$

The second term is identical to the change in inclusive value measure derived in (40) and (41). Therefore, the traditional RUM based welfare measures are equivalent to weak complementarity measures when there are no income effects.

### 2.3.4 Valuing Multidimensional Policies: Sequencing and Independent Valuations

The direct evaluation of multiple quality changes that is the norm in the economics literature is not the norm in practice. Hoehn has discussed the valuation of multidimensional policies where environmental quality changes at several "sites" which may or may not be substitutes. In theory, if individual utility for the environmental goods being valued depends on the level of environmental quality for substitutes or complements, then appropriate welfare measures should take these relationships into account. However, Hoehn points out that in practice most benefit-cost studies examine a single isolated policy or project. These studies tend to measure the value of environmental goods such as air quality as if all other levels of environmental quality are at their status quo level. Hoehn has used the term Independent
Valuation and Summation (IVS) to refer to the practice of adding together separate and independently derived environmental valuations. He shows that if several environmental goods are substitutes or complements, then the IVS measure will misstate the appropriate welfare measure for a joint change in the quality of these goods (see also Hoehn and Randall). When IVS is not appropriate, the sequence in which a quality change at a particular site is valued will affect the value attributed to that site. This is the essence of the embedding concept in contingent valuation.

The issues that arise in evaluating the welfare effects of a multidimensional policy are similar to the issues and practice of deriving value of fish estimates and treating them as independent of the level of quality at substitute sites (the separability question). While sequencing effects and the degree of bias inherent in IVS have both been explored empirically in the context of contingent valuation, there has not been any comparable empirical assessment involving indirect valuation methods such as the RUM travel-cost model.

Here the problem with IVS is framed in the context of the RUM travel-cost model. The discussion bears a close relationship to the appropriate method of deriving weak complementarity welfare measures for joint changes in the quality of multiple sites (Bockstael and Kling). Bockstael and Kling stress that, as with evaluation of multiple price changes in welfare economics, for joint changes in the quality of several sites, a path of integration must be followed in order to derive an appropriate welfare measure. Similarly, the technical problem with IVS is that it ignores the path of the integration (or takes an invalid path) when calculating the welfare measure.

To illustrate, consider the correct welfare measure for a multidimensional change in a vector of site quality at three sites. Denote this as a change from $q^0 = (q^0_1, q^0_2, q^0_3)$ to $q^1 = (q^1_1, q^1_2, q^1_3)$. The welfare measure is given by

$$B = \frac{IV^1 - IV^0}{\mu}$$

$$= \int \sum_k \frac{\pi_k}{\mu} dV$$

$$= \frac{1}{\mu} \int \sum_k \pi_k \frac{dv}{dq} dq$$

(51)
Here, the welfare measure is written as areas under marginal valuation functions. The result is equivalent to the weak complementarity measure in (44). In fact, for linear-in-income specifications, the above is one way of deriving weak complementarity and analogous to the approach developed by Bradford and Hildebrandt for continuous-choice situations.

The second step in (51) makes the change of variables from \( V \) to \( q \) and, for convenience, the third step assumes the linear form for \( V \) so that \( dV/dq = \mu \). The welfare measure is then the sum of the areas under the marginal valuations of site quality. The marginal valuations of site quality are simply the site probabilities times the marginal implicit price of quality. The measure \( B \) will equal the IVS measure only if the marginal valuations are independent of one another; i.e., if the site choice probabilities are independent of the level of quality at substitute sites. This condition is violated in any RUM. The violation is made clear by inspection of the RUM inequalities in (10) -- see also the MNL site choice probabilities in (13) or the marginal valuations in (14). In addition, an assumption in (51) is that the IVS is measured from the status quo \( q^0 \). Other assumptions regarding the alternative reference levels of \( q \) could be made. The key point is that IVS ignores the path (the sequence) of the site quality changes.

For policies that are composed of changes in quality that all move in the same direction (all improvements or all decrements), the direction of bias of the IVS measure can be signed. For joint improvements in quality, \( \text{IVS} > B \) and for joint decrements in quality, \( \text{IVS} < B \). The reason for this is that in RUMs all the sites are strictly substitutes -- recall the marginal effects in (14) or (25). As a result, the cross partial of \( B \) is negative (see equation 51) so the marginal valuations are negatively affected by improvements in site attributes at other sites. Ignoring the path will result in integrating marginal valuations that are too large for joint improvements or too small for joint decrements.
One expects that the extent of the bias imparted by using IVS will depend on the number of sites, relative changes in probability at the sites where quality changes, and the range of the changes in site characteristics that is spanned by the policies being considered. Thus, the degree that IVS differs from B will depend on the sensitivity of the welfare measures to the level of quality at other sites.
2.4 The Repeated RUM and Seasonal Welfare Measures

In this section, the repeated RUM is briefly described and several novel properties are established. First, the relationship between the repeated RUM welfare measure and the number of choice occasions is discussed. Next, the repeated RUM welfare measure is used to examine several bounds on welfare measures that have been presented in the literature. Finally, the welfare measure for the repeated RUM is compared to some alternative seasonal welfare measures that have appeared in the literature.

To explain seasonal demand and participation using the repeated RUM, one need only make a few simple additions to the basic RUM. In the repeated RUM, the season is divided into a set number of choice occasions, \( N_{oc} \). In each choice occasion, there is a *don't go* alternative in the choice set; say \( k=0 \). The per choice occasion site probabilities are given by \( B_k \). \( B_0 \) represents the probability of choosing the *don't go* alternative so that the probability of going is \( B_{go} = 1 - B_0 = \sum_{k=0}^{B_k} \). To predict annual trips, one needs to sum the probability over the choice occasions. If the variables in the model do not vary over time, seasonal trips is \( N_{go} = N_{oc}B_{go} \) and seasonal trips to site \( k \) is given by \( N_k = N_{oc}B_k \). If the model includes variables which do vary over time, then the choice probabilities will vary over time. In this case, seasonal trips are given by

\[
N_{go} = \sum_{n=1}^{N_{oc}} \pi_{go}^n
\]

where \( n \) indexes the choice occasions.

The welfare measure for the repeated RUM is a straightforward extension of the RUM welfare measures. For any choice occasion, the welfare measure is given by (41) or (44), where it is understood that "don't go" is one of the alternatives in the set \( J \). To derive a seasonal welfare measure for an individual, one takes the sum of the measures across choice occasions. If the quality variables do not vary over the season, this results in \( B \times N_{oc} \) where \( N_{oc} \) is the number of choice occasions. Alternatively, if the model variables do vary over the season, then the seasonal welfare measure is given by \( \sum_{n=1}^{N_{oc}} B_n \) where \( n \) indexes the choice occasions.

2.4.1 Choice Occasions

On the surface, the seasonal welfare measure from a repeated RUM would seem to be directly proportional to the assumed number of choice occasions \( N_{oc} \). This idea is common in the literature. For example, Montgomery and Needelman state that by treating each day of the
Provided the number of occasions is sufficient to prevent the trimming of observations, i.e., provided $N_{oc} > N_{go}$. The trade-offs between the definition of choice occasions and the valid trip observations are discussed in detail in section 3.1 of Chapter 3.

First, consider the case where the model variables are constant over time. The marginal welfare measure is given by $N_{oc} \times B_j \times M_j / M_j$. From (46), this can be written as $N_{oc} \times B_j \times M_j / M_j$, where the site choice probability, $B_j$, is the unconditional probability of taking a trip to site $j$. In the repeated RUM, the site-choice decision will typically be nested separately from the go/don't go decision. In this case, one can rewrite the unconditional probability of going to site $j$ as the product of the probability of taking a trip times the probability of visiting site $j$ conditional on taking a trip, $B_j = B_{go} B_{j|go}$. Therefore, the marginal seasonal welfare measure can be written as $N_{oc} \times B_{go} \times B_{j|go} \times M_j / M_j$.

A "good" probability model will fit the underlying data. The data reveals the number of trips, $N_{go}$, that occur within the specified number of choice occasions, $N_{oc}$. Thus, "good" estimates of the participation probability will on average equal $N_{go} / N_{oc}$. In a repeated RUM, any change in $N_{oc}$ would require re-estimation of the model and would result in new estimates of $B_{go}$ which "fit" the new data. In addition, the term $M_j / M_j$ can be consistently estimated by the sequential procedure which separately estimates $B_{j|go}$ and the estimation of $B_{j|go}$ is completely independent of $N_{oc}$. Therefore, looking at the marginal welfare measure, any increase in $N_{oc}$ will be offset by a corresponding decrease in $B_j$.

A second way to illustrate the impact of choice occasions makes use of a result developed by Morey (1994b). Morey showed that a seasonal measure of welfare can be bounded below and above by a conditional welfare measure times trips before and after an improvement in quality. Specifically, for improvements, the seasonal welfare measure is bounded by

$$N_{go} \# W_{go} \# N_{oc} \times B \# N_{go} \# W_{go}$$

(53)

where $N_{go}^0$ and $N_{go}^1$ are trips before and after a change in quality and $W_{go}$ is the welfare measure for the change in quality when individuals are constrained to take a trip (the "per-trip" or
"conditional-on-a-trip" welfare measure). The seasonal measure $N_{oc} \times B$ comes from a model that allows trips to change in response to changes in quality. In a NL where "don't go" is one nest and the sites comprise the other nest, $W_{go} = (IV_j^1 - IV_j^0)/IV_j$ where $IV_j$ is defined in equation (15) and $j=1,...,J$; that is, $j$ indexes sites and does not include the "don't go" alternative ($j=0$).

Since $B_{go}N_{oc}$ is an estimate of trips, the bounds can be rewritten as

$$B_{go}^0N_{oc}W_{go} \# N_{oc} \times B \# B_{go}^1N_{oc}W_{go}$$

Again, since a "good" model fits the underlying data, any change in $N_{oc}$ will result in changes in $B_{go}$ so that the product $B_{go}N_{oc}$ remains unchanged.

Both of the above approaches illustrate that the welfare measures from repeated RUM models do not depend on the researcher's choice of the number of occasions. Of course, if the change in the definition of the choice occasions were to change the data on $N_{go}$, the model results might differ. For example, if the $N_{oc}$ is small enough so that it exceeds $N_{go}$ for some individuals, then data will need to be trimmed to estimate the model. Naturally trimming the data can affect model estimates.

### 2.4.2 Bounds on Seasonal Welfare Measures

Above, the Morey bounds were presented in the context of the repeated RUM. While Morey (1994b) developed the bounds using a deterministic seasonal demand model, it is demonstrated here that the bounds hold for the repeated RUM. In addition, the relationship between the true welfare measure, the Morey bounds, and the bounds given by McFadden (1997) and by Hanemann (1983) is established. These bounds have all appeared in the literature in one form or another. However, the relationships between these bounds has not been established.

The intuition behind the bounds is very straightforward. And the basic notion of extending the per-trip welfare measure to a seasonal measure by simply multiplying by trips is well established in the literature and goes back at least to Small and Rosen. The bounds are germane because most RUMs in the literature do not include a participation component. Moreover, the bounds have been used in the literature, so the proof that they do indeed apply with discrete choices validates an intuitively sensible empirical procedure. Another strength of the bounds is pedagogic: the bounds make it easier to convey properties of the repeated RUM and other RUM-based recreation demand models. For example, in the previous section the bounds proved useful in clarifying the role of choice occasions in the repeated RUM.
More generally, Jones and Lupi discuss how conditional-on-a-trip RUMs that exclude relevant substitute activities will tend to underestimate gains and overestimate loses.

To further illustrate how the bounds can help transmit the logic of repeated RUMs, consider the recent model of Montgomery and Needelman (MN). MN estimate a repeated RUM for lake fishing in New York. The authors state that since their model excludes river fishing, which is a potential substitute for lake fishing, their results likely overstate the benefits of toxic cleanups in lakes. Actually, their conditional-on-a-trip welfare measure ($W_{go}$) would underestimate the value of the toxic cleanup since the lake model does not permit trips to be drawn from rivers into lakes. However, the MN model is more than a conditional-on-a-trip RUM; it is a repeated RUM. Thus, the "omitted" river trips are presumably part of the "don't go" alternative. To the extent that total lake trips in their model appropriately adjust to changes in lake quality, their seasonal welfare measures should be unbiased. The point of this discussion is that the Morey bounds help convey the intuition of the repeated RUM. Recall from (53) that the true seasonal measure is bounded by $N_{go,i} \times W_{go}$ for $i=0,1$. In the MN model, the $N_{go}$ only refers to lake trips (not all fishing trips). Thus, so long as their model gets the "right" change in lake trips ($N_{go}$), it gets the right welfare measure -- in theory, any substitution in and out of river trips is implicit in the go/don't go dimension of their repeated RUM. Whether the repeated nested logit can actually get the slope of the lake participation (total lake trips) function "right" is a distinct empirical issue.

**Morey bounds:** For improvements (decreases in prices and/or increases in quality), the Morey bounds are found by multiplying the change in willingness to pay per trip by the trips in the initial state (the lower bound) and by trips in the post policy state (the upper bound).

$$\text{Morey L.B.} = B_{go}^0 N_{oc} W_{go} < B \times N_{oc} < B_{go}^1 N_{oc} W_{go} = \text{Morey U.B.} \quad (55)$$

where $W_{go} = (IV_{j}^1 - IV_{j}^0) / (IV_{j}^1 + IV_{j}^0)$: and $IV_{j}^i$ is the inclusive value from the conditional-on-a-trip site choice model for policy states $i=0,1$ -- see equation (40).

**Theorem 1:** (Morey's bounds for repeated RUM). In the NL repeated RUM where "don't go" is nested separately from sites, if the characteristics of the don't go alternative as well as the error terms are unaffected by policy changes, then for improvements in site characteristics (i.e., $W_{go} > 0$)

$$E[N_{go}^0] W_{go} \# N_{oc} B \# E[N_{go}^0] W_{go}, \quad (56)$$

More generally, Jones and Lupi discuss how conditional-on-a-trip RUMs that exclude relevant substitute activities will tend to underestimate gains and overestimate loses.
and for changes in site characteristics that are losses (i.e., $W_{go} < 0$) the bounds are reversed.

**Proof**: In the repeated RUM, expected trips, $E[N_{go}]$, is given by the number of choice occasions times the probability of going, $N_{oc}B_{go}$. In the bounds, $W_{go}=(IV_j^1-IV_j^0)/$: is the welfare measure conditional on taking a trip. Recall from (43) that the integral of the conditional site choice probability, $B_{k|go}$, will yield $W_{go}$. Similarly, the integral of the unconditional site choice probability, $B_k$, will recover $B$. Consider the case of a decrease in price at site $k$, $p_k^1 < p_k^0$. Combining the above elements establishes the following

$$E[N_{go}^0]W_{go} = N_{oc} \frac{p_k^1}{p_k^0} \int_{p_k^0}^{p_k^1} \pi_{k|go} dp_k$$

$$\leq N_{oc} \int_{p_k^0}^{p_k^1} \pi_{go} \pi_{k|go} dp_k = N_{oc} \int_{p_k^0}^{p_k^1} \pi_k dp_k = N_{oc}B$$

$$\geq N_{oc} \frac{p_k^1}{p_k^0} \int_{p_k^0}^{p_k^1} \pi_{k|go} dp_k = E[N_{go}^1]W_{go}$$

where the inequalities make use of the fact that the unconditional demand for total trips, $N_{oc}B_{go}$, slopes downward. That is,

$$\frac{\partial \pi_{go}}{\partial x_k} = \frac{\partial \pi_{go}}{\partial IV_j} \frac{\partial IV_j}{\partial x_k} = \theta \pi_{go} (1-\pi_{go}) \frac{\beta}{\theta} \pi_{k|go} > 0 \text{ if } \beta > 0 \text{ and } < 0 \text{ if } \beta < 0$$

To use (58) to derive the slope of the demand for total trips, treat price as $x$, and the coefficient on price is $-\theta$. Since $-\theta < 0$, the total demand for trips slopes downward. The bounds are easily generalized for changes in quality by making the appropriate change of variables. Likewise, changes in characteristics at multiple sites are accommodated by taking areas under the site demand functions where characteristics change. The above inequalities will still hold.
Also note that if site characteristics vary over time, then the Morey bounds apply only to periods over which the site characteristics are constant.

The above proof and the inequalities used highlight the interpretation of the Morey bounds as areas under site demand curves that are conditioned on a fixed number of total trips; i.e., they are "trips-constant" site demands. In contrast, the true welfare measure is the area under the unconditional site demand curve. This curve allows total trips to vary appropriately along the path of integration. In this sense, the true measure can be referred to as the area under the "trips-compensated" site demand curve.

An additional point on the Morey bounds addresses complex policies. For many multidimensional policies one may not know a priori if some combination of changes in site characteristics is on net an improvement or a loss. The bounds hold regardless of whether one knows that the policy results in a net improvement or a net decrease, but in order for the direction of the bounds (which is an upper and which is a lower) to be established, one needs to know the sign of $W_{go}$. Therefore, over a sample of $n$ individuals, if $W_{go}$ changes sign for the some individuals, then the lower and upper bounds on the aggregate welfare measure would need to be examined individually. That is, to calculate the aggregate lower bound, loop though the $n$ individuals and keep $W_{n_{go}}^n E[N_{go}^{n_0}]$ if $W_{go}>0$ and keep $W_{n_{go}}^n E[N_{go}^{n_1}]$ if $W_{go}<0$; vice-versa for the aggregate upper bound. The sum of these terms yields the bound on the aggregate measure.$^5$

The Morey bounds will be discussed in more detail following the presentation of some alternative bounds presented in different contexts by Hanemann (1983) and by McFadden (1997).

**Hanemann/McFadden bounds:** McFadden (1997) has proposed bounds on welfare measures that can be used when the underlying model is nonlinear in choice occasion income. Though developed for the nonlinear-in-income case, the bounds can be applied to linear-in-parameters models. Suppose that there is a change in quality at a single site, $k$. Then McFadden's bounds for linear-in-parameters models are

$$N_{oc} B_k^0 (q_k^1 - q_k^0) / N_{oc} B_k^1 (q_k^1 - q_k^0)$$

What does this mean? First, the $N_{oc}$ simply translates per-choice-occasion results to seasonal results. From section 2.3, the terms $(q_k^1 - q_k^0)$ are the true valuations of the change in quality

$^5$ Also note that if site characteristics vary over time, then the Morey bounds apply only to periods over which the site characteristics are constant.
for those individuals who choose site \( k \) before and after the change in quality. For those individuals who initially choose some site other than \( k \) prior to the change, and then switch to site \( k \), \((q_k^1-q_k^0)/e\) overstates their true valuation (recall the discussion following equation (37) in section 2.3). The \( B_k^i \) terms are the unconditional site choice probabilities before and after the change \((i=0,1)\). The use of the probabilities indicates that we are taking expectations of the valuations. Note that by using \( B_k^0 \) the left hand side of (59) is only calculating a welfare gain for those who initially choose the site where quality is changing -- the benefit to those who switch to site \( k \) is ignored. Similarly, by using \( B_k^1 \), the right hand side of (59) attributes the full valuation \((q_k^1-q_k^0)/e\) to all those who choose site \( k \) in the final state -- overstating the benefits for those who have switched to site \( k \).

More generally, when quality (prices) change at multiple sites, the McFadden bounds can be expressed as

\[
\text{H/M L.B.} = N_{go} \sum_k B_k^0 (x_k^1 - x_k^0)/e; \quad \#N_{go} B \quad \#N_{go} \sum_k B_k^1 (x_k^1 - x_k^0)/e; = \text{H/M U.B.}
\]

(60)

where \( x_k \) includes all the variables in the model (prices and quality). Since the lower bound is equivalent to Hanemann's (1983) approximate welfare measure for discrete choice models, these will be referred to as the Hanemann/McFadden (H/M) bounds. The following theorem relates the H/M bounds to the Morey bounds.

**Theorem 2**: For the linear-in-parameters, repeated RUM, the Morey bounds are tighter than the H/M bounds for all policies that do not affect the error terms for any of the alternatives or the characteristics of the don't go option.

**Proof**: Consider the case of a decrease in price or increase in quality so that \( N_{go}^1 > N_{go}^0 \) or \( W_{go} > 0 \). The Hanemann/McFadden (H/M) bounds can be applied to the welfare measure conditional on taking a trip, \( W_{go} = (IV_j^1 - IV_j^0)/e \); to establish

\[
3_{k \in A} B_{k|go}^0 (x_k^1 - x_k^0)/e; \quad \#W_{go} \quad \#3_{k \in B} B_{k|go}^1 (x_k^1 - x_k^0)/e;
\]

(61)

where \( A \) is the set of sites available in the pre-policy state and \( B \) is the set of sites available in the pre-policy state. Note that \( A \) and \( B \) do not include the don't go option, \( k=0 \). Multiply the left (right) inequality by expected trips in the pre (post) policy state to derive the Morey upper and lower bounds,

\[
E[N_{go}^0] \left\{ \sum_{k \in A} B_{k|go}^0 (x_k^1 - x_k^0)/e; \right\} \quad \# E[N_{go}^0] W_{go} = \text{Morey's L.B.}
\]

and

\[
E[N_{go}^0] \left\{ \sum_{k \in A} B_{k|go}^0 (x_k^1 - x_k^0)/e; \right\} \quad \# E[N_{go}^0] W_{go} = \text{Morey's L.B.}
\]

(62)

---

6 McFadden (1997) refers to this as \( w_{kk} \). In the linear-in-parameters model \( w_{kk} = (q_k^1 - q_k^0)/e \).
E[N_{go}^{-1}] \{ 3_{k0i} B_{k_{go}}^{-1} (x_k^1 - x_k^0)/\}$: E[N_{go}^{-1}] W_{go} = Morey's U.B.

Note that in the linear-in-utility repeated RUM, \((x_k^1 - x_k^0)/\) = w_{kk} in McFadden's (1997) notation. Also, note that for the policies being considered, the characteristics of the don't-go option are not affected. Hence, the valuation \(w_{00} = 0\). Since there is no change in the utility of the don't-go option, the above inequalities are unchanged by taking the summations over the sites choices and the don't-go option, i.e., over \(A' = A \cup \{0\} \) and \(B' = B \cup \{0\}\). Now, replace \(E[N_{go}^{-1}] \) with \(N_{oc} \ B_{go}^{-1} \) for \(i = 0,1\) and move it inside the summation to obtain

\[ N_{oc} \{ 3_{k0i} B_{go}^{-1} B_{k_{go}}^{-1} (x_k^1 - x_k^0)/\} \# N_{oc} B_{go}^{-1} W_{go} = Morey's L.B. \]

and

\[ N_{oc} \{ 3_{k0i} B_{go}^{-1} B_{k_{go}}^{-1} (x_k^1 - x_k^0)/\} \# N_{oc} B_{go}^{-1} W_{go} = Morey's U.B. \]

since the outer bounds are simply the number of choice occasions times McFadden's per-choice occasion bounds \(3_k B_{i} w_{kk}\), the result holds for changes in site characteristics.

For elimination of a site from the choice set, the theorem holds trivially. As pointed out by Herriges and Kling (1996a), the H/M lower bound is minus infinity; i.e., the initial \(B_k^0\) times an infinite change in prices which have a marginal implicit price of -1 (\$/ = -$/ ). And the H/M upper bound is zero since site elimination is equivalent to an infinite price change and \(B_k^1 = 0\) when \(p_k = 0\).

In Figure 2.3, the results of theorems 1 and 2 are illustrated for the case of a decrease in price at a single site. The graph depicts an unconditional demand curve for trips to site \(k\), which is labeled the "trips - compensated" demand curve. The unconditional demand curve permits total trips to change as site characteristics change. Also depicted are two "trips - constant" demand curves which are the site choice demand curves conditioned on a level of trips, \(N_{go}^0\) for the Morey lower bound, and \(N_{go}^1\) for the Morey upper bound. In the graph, \(N\) represents choice occasions (\(N_{oc}\) above). Consistent with the derivation of the Morey bounds the trips constant demand curves intersect the unconditional demand curve -- the lower bound curve does so at the post-policy price level, \(P_{k}^1\), and the upper bound curve does so at the pre-policy price level, \(P_{k}^0\).

There are several areas labeled in the diagram. The true welfare measure is areas A+B+C. The H/M lower bound is area A. The Morey lower bound is areas A+B. The Morey upper bound is areas A+B+C+D, while the H/M upper bound is areas A+B+C+D+E. Clearly, the Morey bounds are tighter than the H/M bounds because they permit site substitution, though total trips are held constant. Thus, the less elastic is the demand for total trips with respect to site
characteristics, the closer the two trip-constant demand curves, and hence, the tighter are the Morey bounds. Likewise, the less elastic the demand for total trips, the greater the divergence for between the Morey and H/M bounds.

**Figure 2.3:** Bounds on Discrete Choice Welfare Measures.

The fit of the bounds will also depend on two other factors: the magnitude of the change in site characteristics and the relative change in the site choice probability. Notice from the diagram, that if the initial site probability is very small and stays small even after the change, then the demand curves are very steep. In this case the H/M bounds will be relatively tight. On the other hand, if the initial site choice probability is higher, the change in trips for any given change in characteristics is greater, i.e., the demand curve is flatter. The site demand curve is
most elastic around initial probabilities equal to 0.5. Thus, for changes in quality that result in large changes in trips, the H/M bounds will be wide. Note that the Morey bounds can be quite tight for any change in characteristics and for large changes in conditional site probabilities since the performance of the Morey bounds depends on the divergence between the two trip-constant demand curves.

From a purely theoretical standpoint, one can safely recommend that if bounds are needed, one should use the Morey bounds. They are tighter, they are better suited to site elimination, and they require the same amount of information as the H/M bounds.

Before proceeding, it is worth clarifying how the bounds can and cannot be used. The Morey bounds apply for individual decision members (sample members). Therefore, care should be exercised when extrapolating the results from a sample to the population, when transferring the results, or when comparing results across studies. To use the individual bounds to get a bound on the aggregate welfare measures for a sample, one would sum the individual bounds. The sample total trips times the sample average per-trip welfare measure need not bound the aggregate measure. Specifically, the bounds are consistent with the use of \( \sum_s N^0 \times W^0 \) as an aggregate bound and not with the use of \( \sum_s N^0 \times W^0 \times \frac{1}{S} \) where \( s = 1, \ldots, S \) denotes individuals. The former calculation will be referred to as the "trip-weighted CV." The later measure is common in benefits transfer where average per-trip welfare measures from one study are multiplied by total trips in another area. In terms of benefits transfer, one should attempt to transfer the average seasonal benefits "per-angler" rather than the per-trip measures. This highlights the shortcoming of the almost universal practice in the literature of reporting mean per-trip CVs derived from site-choice RUMs, i.e., \( \sum_s W^0 \times \frac{1}{S} \). Instead, the above argument supports reporting the average seasonal welfare measures for models that have participation components, or for models without participation components, the above supports reporting the average trip-weighted CVs. As illustrated, the trip-weighted CVs are at least an appropriate bound on welfare measures. Clearly, the implication for travel cost surveys is that collection of accurate trip data is crucial.

### 2.4.3 Comparison with Other Measures

As mentioned in section 2.1, there are several alternative seasonal welfare measures that have appeared in the literature. In this section, these alternative welfare measures are applied to the repeated RUM to examine what these welfare concepts are measuring in the context of the repeated RUM. One would hope that, were the underlying true model to be repeated RUM, the
alternative concepts would in fact be measuring the same entity. As is shown below, this is not always the case.

**Creel and Loomis:** As demonstrated in section 2.2, the repeated RUM is by definition utility theoretic. However, there are a number of studies that use an alternative approach which links a separate total participation model with a RUM site choice model (see the discussion in section 2.1). Creel and Loomis (CL) use the inclusive value index from a MNL/NL site choice model as a variable to explain trips. In demonstrating how to use the model, CL propose a welfare measure for models that link a seasonal participation model with a RUM site-choice model (as opposed to a repeated RUM). The CL welfare measure and variants of it have appeared in Jones and Sung; Parsons and Kealy (1995); and Creel and Loomis.

The CL approach considers total seasonal utility to be composed of total trips \( T \) times the expected indirect utility of a trip \( (IV+K) \). Their welfare measure is \( T(IV+K) \) in the initial state minus \( T(IV+K) \) in the post-policy state, all divided by the marginal utility of income estimated in the site choice model. The Jones and Sung and the Parsons and Kealy (1995) variants drop the terms involving Euler's constant \( K \).

The Creel and Loomis welfare measure \( W_{CL} \) can be written as follows

\[
W_{CL} = T^1 \left( \frac{IV^1}{\mu} + K \right) - T^0 \left( \frac{IV^0}{\mu} + K \right)
\]

\[
= T^1 \left( \frac{IV^1}{\mu} + K \right) - T^0 \left( \frac{IV^0}{\mu} + K \right) + T^0 \left( \frac{IV^0}{\mu} \right) - T^0 \left( \frac{IV^1}{\mu} \right) \tag{64}
\]

\[
= T^0 \left( \frac{IV^1}{\mu} - \frac{IV^0}{\mu} \right) + \left( T^1 - T^0 \right) \frac{IV^1}{\mu}
\]

where the second step simply adds and subtracts the initial trips times the post-policy surplus. The last step highlights the seemingly appealing interpretation of this measure as the change in value for all the initial trips plus the new total surplus for all the new trips.

However, it can be shown that this measure need not bound the true measure. In particular, the CL measure exceeds the Morey upper bound if \( IV^1 > K \) (it trivially exceeds the Morey lower bound if \( IV \) are positive). The following illustrates the point:
since $T^0 < T^1$ for improvements. In (65), $W_{UB}$ refers to the Morey upper bound. However, there is nothing in the theory of the RUM or the statistical implementation of said model that restricts the IV to be positive -- though changes in IV are positive for improvements.

Now, consider the Creel and Loomis welfare measure applied to a repeated RUM model. It is shown that the CL measure does not equal, nor does it necessarily bound, the true measure.

**Theorem 3:** When applied to the repeated RUM, the CL welfare measure does not equal the true measure, and it can be greater or less than the true measure.

**Proof:** Consider a price change. Applying the CL measure to the repeated RUM yields the following:

$$W_{CL} = T^1 \frac{IV_1}{\mu} - T^0 \frac{IV_0}{\mu} > T^1 \frac{IV_1}{\mu} - T^1 \frac{IV_0}{\mu} = T^1 \frac{IV_1 - IV_0}{\mu} = W_{UB}$$

(65)
where CV is the correct repeated RUM welfare measure. K: is positive. For price decreases/quality increases, T>0 which implies T $\frac{\partial \pi}{\partial p_k}$ will be positive. M CL must be positive if IV j(pk 0)>0 and negative if IV j(pk 1)<0. Since IV j can be greater than or less than 0, a systematic direction of bias cannot be established. The CL measure will equal the true CV for the repeated RUM only when T $\frac{\partial \pi}{\partial p_k}$ + M CL=0. In general, the terms will not cancel. The above holds for any change in prices or quality at any array of sites. For quality changes, the integral is over dq and the integrand is with respect to the marginal values of quality (the site probabilities adjusted by $\frac{\partial}{\partial p_k}$).

This illuminates what the CL measure would yield within the context of the repeated RUM, which is a familiar framework. One will know if IV j is positive or negative in any
empirical application since it must be calculated to get the CL welfare measure. The point is that it does not necessarily bound the true welfare measure. Since all the information required for the CL measure is the same as required for the Morey bounds, use of the Morey bounds is preferred. Moreover, as shown below, a suitably interpreted exact measure can be obtained with the information required to derive the CL measure.

**Hausman, Leonard, and McFadden:** As mentioned in section 2.1, Hausman, Leonard, and McFadden (HLM) use the negative of the surplus measure from the RUM site choice model as a price index in a count-demand model. Thus, the HLM model is an example of linking a RUM and a separate participation equation. Rather than explaining trips as the outcome of repeated discrete choices, the HLM model is based on two-stage budgeting where total trips are decided at one stage and site choices are made at the next stage, conditioned on the total trips.

The HLM price index, $S$, is defined as follows:

$$
S = \frac{-IV}{\mu} \quad \text{where} \quad IV_j = \ln \left( \sum_{j=1}^{J} e^{y_j} \right)
$$

which is the negative of the expected indirect utility from the RUM site choice model -- ignoring Euler's constant, divided by the marginal utility of income.

Welfare measures for indirect valuation methods are usually based on the concept of weak complementarity and are based on changes in areas under the demand curves for specific sites where quality changes. However, to get a welfare measure, HLM directly integrate the demand function for total trips with respect to changes in the price index $S$. Consider a change in price at site $k$ from $p_k^0$ to $p_k^1$. The HLM welfare measure is

$$
W = \int_{S^6}^{S^1} T(S) dS
$$

where $T(S)$ is the overall trip count as a function of the site choice "price index." The purpose of this section is to show that the HLM measure applied to a repeated RUM will result in exactly the same welfare measure as is usually employed for repeated RUM models where the go/don't go decision is nested separately from the site choice decisions.
**Theorem 4:** In a NL repeated RUM, the HLM measure (integrating total trips with respect to S) is equivalent to the repeated RUM welfare measure, $N_{oc}(IV^1-IV^0)/\mu$; both are tantamount to the sum of the changes in area under the site-specific demand curves for the sites where quality changes.

**Proof:** In the repeated RUM, the overall trip function, $N_{go}$, is given by the number of choice occasions times the probability of going. Suppose price changes at a single site $k$. Consider the integral of this with respect to S:

$$
\int_{S^0}^{S^1} N_{oc} \pi_{go} \, dS = \int_{p_k^0}^{p_k^1} N_{oc} \pi_{go} \frac{dS}{dp_k} \, dp_k
$$

where $B_{jk}$ is the unconditional probability of selecting site $j$. Note that the first step made use of the fact that $B_{go}$ can be expressed as a function of the price index $S$ as follows

$$
\pi_{go} = \frac{e^{\theta IV_j}}{e^{\theta v_s} + e^{\theta IV_j}} = \frac{e^{-\mu S}}{e^{\theta v_s} + e^{-\mu S}}
$$

and the final step in (69) made use of equation (14), i.e., the $MV/MK_k = \delta_k B_k$. Similarly, for evaluating quality changes, one could make the change of variables with respect to $q$ to get an integral of the marginal valuation of $q$. The "change in inclusive value measure" would still result. Price changes at multiple sites are found by summing changes in the areas under the respective site demand curves. Quality changes follow from above by integrating the marginal valuations $\$B_k/\mu$. 

$\text{45}$
Thus, the HLM measure is equivalent to the usual repeated RUM welfare measure. Note that the above HLM measure requires integrating the trip demand function with respect to $S$. However, many attempts to link a participation equation to a separate trip demand function are based on the $IV_j$, and not $S=IV_j$. In these, it would not be appropriate to integrate a total demand function with respect to $IV_j$, because the integral with respect to $IV_j$ would fail to take into account the change of variables adjustment $dS/dIV_j = 1$. Thus, in a repeated RUM, integrating a trip demand function with respect to $IV_j$ would yield a welfare measure that is times too big (since $\mu$ is typically less than one, the measure integrating $IV_j$ would be too small).

Yet another way to think about this is to consider what would happen if the repeated RUM were integrated over $IV$, rather than over $S$.

\[
-IV_j^0 \int_{-IV_j^0}^{p_k^1} \frac{d(-IV_j)}{dp_k} d\pi_k = \int_{-IV_j^0}^{p_k^1} \int_{p_k^0}^{p_k^1} \frac{d(-IV_j)}{dp_k} d\pi_k d\pi_{k|go}
\]

\[
= \int_{p_k^0}^{p_k^1} \mu N_{oc} \pi_{k|go} dp_k
\]

\[
= \mu N_{oc} \int_{p_k^0}^{p_k^1} \pi_k dp_k
\]

\[
= \mu N_{oc} \frac{IV(p_k^1) - IV(p_k^0)}{\mu}
\]

which is exactly what was argued above: \textit{integrating total trips with respect to } $IV_j \text{yields a measure that is } \mu \text{ times the correct measure.}

While the HLM model is distinct from the repeated RUM, when applied to the repeated RUM, the HLM is conceptually measuring the same welfare effect as the repeated RUM. HLM, by integrating the seasonal total demand curve with respect to $S$, are essentially integrating the seasonal site $k$ demand curves, $T_k$. Changes in areas under demand curves are our traditional measures of surplus. Since both the HLM measures and the repeated RUM measures were shown to be equivalent to areas under site specific demand curves, it is no surprise that they are measuring the same thing in principle.
The other price indices proposed in the literature (Feather, Hellerstein, and Tomasi; Parsons and Kealy, 1995; and Shonkwiler and Shaw, 1996b) do not exhibit this correspondence. Therefore, integrating a total trip demand function with respect to these price indices will not map to the repeated RUM measure because these price indices cannot be translated into a conditional site probability as usually formulated in a RUM. Note that while the Feather et al. and Parsons and Kealy (1995) models are based on traditional RUMs at their site choice level, the Shonkwiler and Shaw (1996b) model is based on a MNL model where the site travel costs are weighted by each individual's observed trips. As such, that model is not expected to clearly map to the repeated RUM since it is based on a fundamentally different (though similar looking) site allocation model. However, since the Feather et al. and Parsons and Kealy (1995) models are based on RUMs at their site choice level, they should be used cautiously since they will not result in the same welfare measure as the repeated RUM -- were the true model to be repeated RUM.

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