Formal Reasoning

Philosophy 330  
Spring 2005  
M & W: 10:20-12:10p  
Berkey Hall, Rm. 217

Instructor Information

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Required Text


Course Objectives

- Introduce you to the concept of logical consequence, and other notions central to logic such as argument, consistency, and logical truth.
- Introduce you to the basic methods of proof (both formal and informal), and teach you how to use them.
- Introduce you to the formal structure of the language or languages that we use daily in order to deepen your understanding of the meaning of words and of the nature of truth.
- Provide those of you who will encounter formal logic outside of this course with a firm foundation in symbolic logic. (This includes students of philosophy, especially analytic philosophy, as well as students of mathematics, computer science, and other technical disciplines.)
- To give you a sense of what a logician does for a living and to jumpstart your thinking about an answer to the historically significant and philosophically important question: What is logic?

Please let me know at any point if you think we aren’t making sufficient progress toward these goals, or if there are goals that you think we should have. The following course requirements measure the extent to which these goals are attained.

Course Requirements

- Six homework assignments. Your average grade will make up 30% of your final course grade. Each assignment consists of problems from ten or so text exercise sets. Typically, homework is assigned in class on a Wednesday and is due by the beginning of the next class on Monday. An assignment not turned in when I collect the homework at the end of class counts as late. **Late homework will be excused only in the event of hospitalization (yours not someone else’s) with the proper documentation.** There will be no make-up for late homework.
A mid-term and a final examination. Each exam may consist of some combination of the following: problems similar in kind to those encountered in the text, T/F questions, short-essay questions, and problems that require you to extrapolate from what you have learned in class and from the text. Your average exam grade will comprise 70% of your final grade. The first exam will be given in class on Wednesday, March 2\textsuperscript{nd}. \textbf{Note well: this exam will not be rescheduled unless a student is hospitalized and makes available the required documentation.} The final exam is scheduled for Tuesday, May 3\textsuperscript{rd} from 10:00-12:00 noon. This exam will be rescheduled only if the student is hospitalized during the exam time or there is a confirmed conflict with another final exam.

\section*{Course Overview}

\begin{itemize}
  \item \textbf{Content (see the attached handout)}
\end{itemize}

This course is a rigorous introduction to formal logic. The courseware package includes several software tools that students will use to complete homework assignments and the self-diagnostic exercises from the text. The software, easy to learn and use, makes some of the more technical aspects of symbolic logic accessible to introductory students and, just as importantly, it is fun to use. However, since students cannot use a computer to complete the two in-class exams, the software should be used as a learning aid and not as a crutch.

The central question of the course is: what is the nature of logical consequence? In other words: what makes the truth of one sentence follow from the truth of others? In particular, we want to know what role the meanings of words plays in guaranteeing the truth of one sentence given the truth of others. This will require, among other things, that we get clear on the related notions of meaning and truth. We are not only after conceptual clarification here, but also seek to reveal and highlight reasoning processes engaged in outside of the classroom.

We shall develop an account of logical consequence for a symbolic language (i.e., the language of first-order predicate logic or FOL). This will allow us to develop rigorous methods for determining when one FOL sentence follows from others and develop the corresponding notion of \textit{proof}, a notion central to mathematics, logic and other deductive disciplines. Proofs are not only epistemologically significant as tools for slaying Descartes’ evil demon and securing knowledge. The formal derivations that make up a proof serve as models of the informal reasoning that we perform in our native languages. This is of importance in, among other areas, computer science and artificial intelligence. Furthermore, the study of proofs allows us to follow Socrates and better know ourselves as reasoners. After all, like Molière’s M. Jourdain, who spoke prose all his life without knowing it, we reason all the time without being aware of the principles underlying what we are doing.

From time to time we shall step back from our inquiry into the concept of logical consequence and consider questions about the framework of the inquiry itself. For example, we shall suppose that there are only two truth-values (True, False). Is this correct? Also, the logic we shall study makes it the case that the meaning of a proper noun is the thing it refers to (so, \textit{somebody is jolly} follows from \textit{Santa Claus is jolly}). Is this correct? Should we think that in order for the term \textit{Santa Claus} to be meaningful it must have a referent?

\begin{itemize}
  \item \textbf{Class Time}
\end{itemize}
Class time will primarily be spent going over text exercises, and reviewing or expanding on key points from the reading. It is imperative that students keep up with the rigorous pace of the course by doing the assigned readings in a timely manner, and by doing enough of the relevant practice exercises to get a feel for one’s level of understanding BEFORE coming to class. Class time is your opportunity to clear up those things that you find mysterious or troublesome. So, coming to class unaware of what you don't know is not the best way to use class time.

- **Attendance**

As Woody Allen says, 80% of success is just showing up. Regular class attendance is critical to being successful in this course. In general, it is my experience that the majority of those who frequently miss class are less successful in the course than those who attend regularly. I consider any more than two absences excessive. Students that are absent from class are responsible for missed announcements and for getting class notes.

- **Work Level**

In order to be successful on the exams and homework that come later in the semester, one must understand the earlier material upon which this required work is based. Since later coursework is based on and incorporates earlier work, (again) it is imperative that you keep up with the pace of the course. Furthermore, while the course material is not especially difficult, it does demand quality thinking. Hence, it is vital that throughout the semester you reserve quality time for coursework (doing logic problems when you are rushed or exhausted tends to increase the difficulty of the problems), and that you work diligently and consistently on understanding class material (missing lots of classes and doing the reading for an assignment the night before it is due is a recipe for disaster).

- **Grading**

Grades on homework and tests will be on a 100-pt. scale. Your final grade will be first determined on a 100-pt. scale, and then converted to a 4.0 scale according to the below tabulations. For example, a final grade of an 83% corresponds to a 3.0 and a 77% corresponds to a 2.5.

<table>
<thead>
<tr>
<th>Final Grades</th>
<th>Percentage Range</th>
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<tbody>
<tr>
<td>4.0</td>
<td>90% and above</td>
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<tr>
<td>3.5</td>
<td>85--89%</td>
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<td>65--69%</td>
</tr>
<tr>
<td>1.0</td>
<td>60--64%</td>
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</tbody>
</table>

**Tentative Schedule**

1/10 Introduction
Part I--Propositional Logic

1/12 & 1/19 Introduction—pages 1-10 & 15
Chapter 1, Atomic Sentences
Chapter 2, The Logic of Atomic Sentences—excluding 2.6

1/24 Chapter 3, The Boolean Connectives—excluding 3.8

1/26 Chapter 4, The Logic of Boolean Connectives

1/31-2/9 Chapter 5, Methods of Proof for Boolean Logic
Chapter 6, Formal Proofs and Boolean Logic

2/14-2/28 Chapter 7, Conditionals—excluding 7.4 & 7.5
Chapter 8, The Logic of Conditionals—excluding 8.3

3/2 Midterm exam

3/7-3/11 Spring break

Part II—Quantifiers

3/14-3/21 Chapter 9, Introduction to Quantification—excluding 9.8
Chapter 10, The Logic of Quantifiers

3/23 & 3/28 Chapter 11, Multiple Quantifiers

3/30-4/13 Chapter 12, Methods of Proof for Quantifiers—including 12.5
Chapter 13, Formal Proofs and Quantifiers

4/18-4/25 Chapter 14, More On Quantification

4/27 Conclusion and Review

Homework Schedule

<table>
<thead>
<tr>
<th>Date Assigned</th>
<th>Due</th>
<th>Chapter(s) covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/26</td>
<td>1/31</td>
<td>Chapters 1-3</td>
</tr>
<tr>
<td>2/9</td>
<td>2/14</td>
<td>Chapters 4-6</td>
</tr>
<tr>
<td>2/23</td>
<td>2/28</td>
<td>Chapters 7 &amp; 8</td>
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<td>3/23</td>
<td>3/28</td>
<td>Chapters 9 &amp; 10</td>
</tr>
<tr>
<td>4/6</td>
<td>4/11</td>
<td>Chapters 11 &amp; 12</td>
</tr>
<tr>
<td>4/20</td>
<td>4/25</td>
<td>Chapter 13 &amp; 14</td>
</tr>
</tbody>
</table>
Preliminary Remarks on Proof and Logical Consequence

In order to get a feel for the work that is ahead as well as its significance, consider the following. A proof is a step-by-step demonstration showing that the conclusion of an argument is a logical consequence of its premise(s). Another way of relating logical consequence to proof is to say that each step in a proof is a logical consequence of previous steps and/or the initial premises. Hence, in order to build a proof one must be able to determine whether or not the relation of logical consequence holds between sentences. Typically, informal proofs leave out steps (perhaps because they are too obvious), and do not justify each and every step made in moving towards the conclusion (again, obviousness begets brevity).

In logic, we like to reveal the meat and bones of a proof (i.e., to represent it formally), thereby making each step and justification explicit. We shall do this by first translating the premise(s) and conclusion of an argument into formal or symbolic (i.e., FOL) sentences, and then constructing a proof that appeals to justification rules that operate on FOL sentences. Frequently we shall work on arguments already composed of FOL sentences.

We’ll get into all this later in the semester. For now, check out the following three informal proofs. Each is followed by a commentary highlighting one or two principles of reasoning implicitly used in the proof. These principles would be made explicit in the formal analogs of the proofs. Hence, as mentioned in the syllabus, one value of a formal proof is that it serves as a model of ordinary deductive reasoning which explains the force of such reasoning. Also, the mechanism of proof makes reasoning rigorous and, therefore, safe from error.

I. The Isle of Knights and Knaves

Knights always tell the truth and knaves always lie, and on the Isle of Knights and Knaves where everybody is either a knight or a knave, two natives approach you and one says, “We are not both knights.” Does it follow that the speaker is a knight and his companion a knave? Answer: Yes.

Proof:

Suppose the speaker is a knave. Then what he says would be true. But this contradicts the fact that knaves always lie. So, the speaker must be a knight.

Since the speaker is a knight and what he says is true, it follows that (1) he and his companions are not both knights. But then since (2) he is a knight, it must be the case that (3) his companion isn’t a knight, i.e., he is a knave.

So, the speaker is a knight and his companion a knave.

Commentary/ What's the principle behind the reasoning from (1) and (2) to (3)? Letting A= the speaker is a knight and B= his companion is one. (1) is not (A and B), (2) is A, and (3) is not B. The principle behind this part of the reasoning is: from not (A and B) and A, infer not B. This principle is correct, since not B is a logical consequence of not (A and B) and A. This fact can be established using what is called a truth table.

II. Lots of Love and Too Much Narcissism.
Suppose that (1) everybody loves a lover (a lover is a person who loves somebody or other). Certainly (2) there exists a lover. Does it follow that everybody loves himself or herself? Answer: Yes.

Proof:

Let’s call the unnamed lover, mentioned in (2), x. By (1), everybody loves x, and, consequently, everybody is a lover, i.e., each person is a lover (of x). But then, by (1), each person y is loved by all, (i.e., everybody loves everybody). But then y must love himself or herself since y is one of the all. Hence, everybody loves himself or herself.

Commentary/ Check out the last two sentences in the proof. Note the step from an arbitrarily selected person y must love himself or herself to everybody loves himself or herself. What justifies this move? In deducing that a person y must love himself or herself from the claim that everybody loves everybody, the proof depends on the fact that y is arbitrarily chosen, i.e., we assume no more about y than what we would assume about any other person. Hence, what we prove about y (i.e., y loves himself or herself), we prove about each and every person. The formal principle corresponding to this is often called Universal Generalization or UG for short. Another observation: although the proof demonstrates that everybody loves himself or herself follows from (1) and (2), this by itself does not show that everybody does love himself or herself. In fact, everybody loves himself or herself is probably false. If so, (1) or (2) must be false. Why?

III. A Visit with Euclid

Here is a synopsis of Euclid’s famous proof that there is no greatest prime number.

Suppose that there is a greatest prime number. Let’s call it x. Then

1. x is the greatest prime number.
2. Form the product of all primes less than or equal to x, and add 1 to the product. This yields a new number y, where y=(1 • 3 • 5 • 7 • … x) + 1.
3. y is either prime or composite (i.e., not a prime).
4. If y is a prime, then, x is not the greatest prime (for y>x).
5. But if y is composite, then, x can’t be the greatest prime number (for if y is composite, then it must have a prime divisor z; and z must be different from each of the prime numbers 2, 3, 5, 7, … , x, smaller than or equal to x).
6. Hence, x is not the greatest prime number (from (3),(4), & (5)).

But this contradicts 1. Therefore,

7. There is no greatest prime.

Commentary/ Euclid's informal proof implicitly relies on reasoning principles that a formal representation would make explicit. For example, what’s the principle behind getting 6? Let’s look. Let A, B, and C represent the atomic sentences that are the components of sentences 3,4,5, and 6 (for the moment, ignore the parenthetical remarks in these sentences). The reasoning from 3-5 to 6 may be represented as follows.

3. A or B
4. if A then not C
5. if B then not C
6. not C

This is valid, and arguments of this form are often called constructive dilemmas. Also, note the general structure of the proof: we prove a claim (there is no greatest prime number) by assuming its opposite (there is a greatest prime number), and then derive a contradiction from the assumption. This is taken to show that the assumption is false from which we conclude that the original claim is true. What’s the principle of reasoning behind this? Well, if A implies a contradiction, then we may infer not A (e.g., not A is a logical consequence of if A then (A and not A)). A proof with this structure is called a reductio ad absurdum proof or a proof by contradiction.

Proof Crazy
Page 53 in LPL

The following proofs answer several exercises in text. First check out the exercises and formulate your own proofs and then compare them to the proofs below. Since you haven't read the material, you probably will not understand some of terminology that I use. However, as someone who reasons you should be able to follow the steps and see why these “reasonings” are proofs. In any event, refining your deductive reasoning skills is an important aim of the course. When we get to pg. 53 in LPL refer back to these proofs.

2.8 Invalid…it is possible for the premises to be true and the conclusion false. Proof: create a world in Tarski's World in which there is a large object named by a and a medium-sized object named by c. In this world, the premises are true and the conclusion is false.

2.9 This is a valid argument. From the first premise it follows that RightOf(b,a) because RightOf and LeftOf are inverses. From RightOf(b,a) and the second premise it follows that RightOf(c,a) by the indiscernibility of identicals.

2.10 The conclusion is not a logical consequence of the premises. Proof: construct a world in which two objects named by b and c are the same size and same shape. This makes the premises true and the conclusion false.

2.11 This argument is invalid because the premises are true and the conclusion false with respect to a world in which there is a row such that three objects named by a, b, and c are positioned as follows:

| A | c | b |
2.14 is Valid. The second premise is: \textit{LeftOf}(a,c). If \textit{LeftOf}(a,b) is false, then \textit{Between}(b,a,c) must be false. But we are told that \textit{b} is between \textit{a} and \textit{c}, so \textit{LeftOf}(a,b) can't be false, i.e., it must be true.