Russell and Logical Ontology

Introduction

This paper focuses on an account of implication that Russell held intermittently from 1903 to 1908.[1] On this account, logical propositions are formal truths that are maximally general and consist solely of logical terms. A consequence is that logical implications, which are instances of logical propositions, are distinguished by appealing to extra-linguistic facts. This makes logic substantive. Russell abandoned this view of logical propositions and the corresponding account of implication, never to replace it. In *Introduction to Mathematical Philosophy* (*IMP*) and in other places he acknowledges that the propositions of logic are fully general and, following Wittgenstein, tautologous. But, as he confesses in *IMP*, he doesn't completely understand what Wittgenstein means by *tautology*, with its connotations of decidability and triviality. In the introduction to the second edition of *The Principles of Mathematics* Russell writes,

> I confess, however, that I am unable to give any clear account of what is meant by the phrase "true in virtue of its form." But this phrase, inadequate as it is, points, I think, to the problem which must be solved if an adequate definition of logic is to be found" (p. xii).

Russell has reason to reject his account of logical implication because he cannot make it reflect his conception of logic as universal. On this conception logic is, in the words of Gödel, "a science prior to all others, which contains the ideas and principles underlying all sciences" (1944, p.125). The Russellian view of logic as both substantive and universal requires that logic have its own ontology. Unable to provide a rationale within the framework of type theory for the needed logical ontology, Russell is forced to abandon his account of logical implication. In this paper, I shall sketch a rationale within a type-theoretic framework for an ontology of purely logical entities that supports Russell's conception of logic as both substantive and universal. My interest in such an ontology is partly due to recent criticism of the model-theoretic characterization of implication in first-order logic. The criticism is that since the model-theoretic characterization fixes the extension of first-order logical implication on the basis of non-logical, set-theoretic states of affairs, it fails to reflect that logic is prior to all other sciences. I am sympathetic to this criticism since I too conceive of logic as universal and independent of set theory. On the other hand, I think that logic is substantive. Hence, my motivation for considering a return to the pre-Wittgensteinian notion of a distinctly logical ontology that is in the early work of Russell (and Frege).

I begin with a *precis* of the Russellian account of logical implication, focusing on those features relevant to the purposes of this paper. Next, I highlight how this view makes logic substantive in exactly the same way as the standard model-theoretic account. After considering Russell's notion of logic as universal, I offer a motive for Russell's rejection of his account of implication, noting the tension between treating logic as both substantive and universal against the backdrop of Russell's type-theoretic framework. I end by sketching a rationale for an ontology of logical entities that maintains Russell's view of implication within the framework of type theory, and preserves the substantiality and universality of logic.

Russell's account of implication[2]

At the heart of Russell's account of implication are the notions of material and formal implication. According to Russell, an inference from one proposition to another is valid (i.e., one is deducible from another) if and only if (iff) the relation of material implication holds between them (*Principles*, p. 33). Material implication is a basic relation that "…holds for nothing except propositions, and holding between any two propositions of which either the first is false or the second is true" (*Principles*, p.34). For example, the inference from the proposition *Bill Clinton is human*, to *Bill Clinton is mortal* is valid because the former materially implies the latter. Since *Bill Clinton is an author* does not materially imply *Bill Clinton is a Republican*, the latter is not deducible from the former. For propositions *p*, *q*, we let $\ulcorner p \rightarrow q \urcorner$ abbreviate *p* materially implies *q*.

Russell writes, "It seems to be the very essence of what may be called a *formal* truth, and of formal reasoning generally, that some assertion is affirmed to hold of every term; and unless the notion of *every term* is admitted, formal truths are impossible [all italics are Russell's] (*Principles*, p. 40; also see p. 105). In a formal truth, a propositional function is true for all values of its free variables. For example, the proposition, *for all terms x, x=x*, is a formal truth. A formal implication is "a relation which holds between propositional functions when one implies the other for all values of the variable" (*Principles*, p. 14 and see p. 93). For example, the formal implication

(x) (x is human→x is mortal)

is derived from

Bill Clinton is human→ Bill Clinton is mortal

by replacing the term *Bill Clinton* with the variable *x*, adding parentheses, and then prefixing the result with a universal quantifier that ranges over all terms. The formal implication, *(x) (x is human→x is mortal)*, tells us that the propositional function *x is human* materially implies *x is mortal* for all values of the variable *x*. So a formal implication determines a class of material implications, each an instance of the formal implication. Also, Russell remarks that a material implication "as a rule may be regarded as a particular instance of some formal implication" (*Principles* p. 34). For simplicity, we treat formal implications as formal truths and so, for example, treat *(x is human→ x is mortal)* as a complex propositional function (composed of, in part, other propositional functions) true for all values of *x*.

Russell writes that the "fundamental importance of formal implication is brought out by the consideration that it is involved in all rules of inference. This shows that we cannot hope to wholly define it in terms of material implication, but that some further element or elements must be involved" (p.40). In order to elaborate, consider the following pairs of propositions.

(A)	Bill Clinton is human Bill Clinton is mortal	(B)	Bill Clinton is human Bill Clinton is human or he is mortal

For each pair, the bottom proposition is deducible from the top one because the following material implications hold.

(A') Bill Clinton is human→Bill Clinton is mortal
(B') Bill Clinton is human→(Bill Clinton is human or he is mortal)

A principle of inference is a formal implication that legitimizes an inferential move. The following formal implications represent principles of inference that validate the move from the top proposition to the bottom one in each of the pairs (A) and (B).

(A") (x) (x is human→ x is mortal)
(B") (x) (F)(F')(F(x) →(F(x) or F'(x)))

I shall call a material implication a *logical implication* if it is an instance of a formal implication that is a proposition of logic (i.e., a law of logic). Correspondingly, a proposition *q* is *logically* deducible from *p* solely in accordance with a law of logic iff *p*→*q* is a logical implication. Russell takes propositions of logic to be truths consisting only of variables and logical constants (*Principles*, pp. 10-11). Since formal implications embody principles of inference and not all formal implications are propositions of logic, there are non-logical principles of inference. For example, the material implication (B') is a logical implication because it is an instance of (B"), which is a proposition of logic. However, the material implication (A') is not a logical implication (i.e., *Bill Clinton is mortal* is not *logically* deducible from *Bill Clinton is human*) because the formal implication (A") is not a proposition of logic, given, as Russell thinks, that *human* and *mortal* are not logical constants. Rather, *Bill Clinton is mortal* may be correctly inferred from *Bill Clinton is human* in accordance with the general (non-logical) principle (A"), according to which being mortal is necessary for being human.

In short, Russell understands the notion of deducibility in terms of the concept of material implication. That a proposition *q* is deducible from *p* turns on whether or not *p*→*q*. The principle by which one may infer one proposition from another may be represented in terms of the relevant formal implication. A proposition *q* is logically deducible from *p* just in case $p \rightarrow q$ is a logical implication, i.e., an instance of a formal implication that is a logical proposition. To be clear, nowhere in *Principles* does Russell explicitly acknowledge the class of material implications that I have called logical implications (however a logical implication is akin to what Russell calls an analytic proposition in his 1905a).[3] Rather, his account of implication is committed to there being what I am calling logical implications, and it is these material implications that I shall focus on in the remainder of this paper.

I now summarize Russell's account of logical implication, starting with the following equivalence.

(1) A proposition *p* is a logical implication if and only if (iff) there exists a propositional function *PF* such that *p* is an instance of *PF*, only variables and logical constants appear in *PF*, and *PF* is necessary.

Let's call a propositional function with just variables and logical constants a formal propositional function. For each proposition there corresponds a formal propositional function; the former is an instance of the latter. The notion of a propositional function being necessary is understood in terms of the notion of a propositional function being always true.[4]

(2) A propositional function *PF* is necessary iff *PF* is always true.

Russell remarks that, "In every proposition of logic, some expression containing only variables is said to be always true or sometimes true. The question "what is logic?" is the question what is meant by such propositions" (Russell 1912, p.56). Russell's understanding of a propositional function being always true that is reflected in *On Denoting* ((1905b), p. 416-417) and in his discussion of formal implication in the *Principles* (e.g., pp.36-39) may be captured as follows:

(3) A propositional function *PF* is always true iff the quantification that results from binding all free variables in *PF* with universal quantifiers is true.

From (1)-(3) and the definition of a formal propositional function, we derive Russell's characterization of logical implication.

A proposition *p* is a logical implication iff the quantification that results from binding all free variables in the corresponding formal propositional function with universal quantifiers is true.

Russell's recipe for determining whether *q* is logically deducible from *p* is to determine the formal propositional function for $p \rightarrow q$ by replacing its non-logical terminology with first-order and second-order variables, bind all variables, and then ascertain whether or not the resulting second-order universal closure is true. For example, to show that the true material conditional

(C) If George W. Bush is an elephant, then everything is an elephant

is not a logical implication we transform it into the formal propositional function: $F(x) \rightarrow (y)F(y)$. Then by (1), we say that (C) is a logical implication iff the propositional function $F(x) \rightarrow (y)F(y)$ is necessary, i.e., by (2), is always true. By (3), $F(x) \rightarrow (y)F(y)$ is always true iff

(C ') $(F)(x)(F(x) \rightarrow (y)F(y))$.

Since there exists a value for *F* which makes $(x)(F(x) \rightarrow (y)F(y))$ false, $F(x) \rightarrow (y)F(y)$ is not necessary, and (A) is not a logical implication. This is Russell's explanation for why *everything is an elephant* does not logically follow from *George W. Bush is an elephant* even though (C) is a material implication by virtue of the fact the proposition *George W. Bush is an elephant* is false. (C) is a material implication that Russell would say has no practical utility (1905a, p. 517), since we know that the antecedent is false.

Consider proposition (D).

If (nothing is taller than itself, and if x is taller than y and y is taller than z, then x is taller than z), then there is a tallest object (i.e., an object that is at least as tall as everything else)

(D) essentially tells us that if the *Taller than* relation is irreflexive and transitive, then there is something which is a minimal element of the *Taller than* relation. A minimal element of the *Taller than* relation is an individual (possibly more than one) which is at least as tall as everything else. On the Russellian account, (D) is a logical implication iff (D') is true.

(D') $(F)((x){\sim}F(x, x) \;\&\; (x)(y)(z)((F(x, y) \;\&\; F(y, z)) \rightarrow F(x, z))) \rightarrow (\exists x)(y) {\sim}F(y, x))$

(D') is false and (D) is not a logical implication because there exists a relation *F* such that *F* is irreflexive, transitive, and does not have a minimal element.[5] Since the extension of such a relation must be denumerably infinite, that (D) is not a logical implication turns on the fact that there is an existent infinity. Consequently, to know that (D) is not a logical implication (i.e., to know that (D') is false) requires knowing that an infinity actually exists.

In order to later highlight the connection between ontology and logic, as conceived by Russell, I now say something about Russell's notion of a term and Russell's realism. This also will help avoid any distortion of Russell's view of the domain of the quantifiers in logical propositions arising from the use of modern notation in the above examples. Russell writes that,

Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*.…A man, a moment, a number, a class, a relation, a chimera, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false. (1903, p. 43)

Everything is a term. And according to Russell,

Being is that which belongs to every conceivable term, to every possible object of thought—in short to everything that can possibly occur in any proposition, true or false, and to all such propositions themselves…Numbers, the Homeric gods, relations, chimeras, and four-dimensional spaces all have being for if they were not entities of a kind, we could make no propositions about them (1903, p. 449).

Whatever can be thought has being, and its being is a pre-condition, not a result of its being thought" (1903, p. 451).

For Russell, every term has being. But not every term that *is*, *exists.* He writes that, "*Existence*, on the contrary, is the prerogative of some only amongst beings" (1903 p.449). Existents are beings that are either mental or physical (Hylton (1990a), p. 172-173).[6] Russell says that,

Misled by the neglect of being, people have supposed that what does not exist is nothing. Seeing that numbers, relations, and many other objects of thought, do not exist outside the mind, they have supposed that the thoughts in which we think of these entities actually create their own objects. Every one except a philosopher can see the difference between a post and my idea of a post, but few see the difference between the number 2 and my idea of the number 2. Yet the difference is as necessary in one case as in the other (1903, pp. 450-451).

According to Russell, terms can be distinguished on the basis of the roles they can play in a proposition.

Among terms it is possible to distinguish two kinds, which I shall call respectively *things* and *concepts*. The former are the terms indicated by proper names, the latter those indicated by all other words. Here proper names are to be understood in a somewhat wider sense than usual (1903, p. 44).

The general idea is that concepts can occupy either the subject or predicate positions of a proposition, but things can only play the role of subject.

Socrates is a thing, because Socrates can never occur otherwise than as a [subject] in a proposition: Socrates is not capable of that curious twofold use which is involved in *human* and *humanity* (1903, p. 45).

On the Russellian view, the variables, *x*, *y*, and *z*, in the above logical propositions (C') and (D') range over all terms (things and concepts) and *F* ranges over concepts.[7] So, for example, the proposition, *if unicorns are elephants, then everything is an elephant* is an instance of (C'). The domain of discourse for propositions of logic is the realm of being. We now turn to Russell's notion of necessity.

As is clear from the above, the notion of necessity has no role to play in explaining the nature of a logical proposition since it is reduced to the notion of generality captured in logical propositions. Russell, in *Principles*, has no notion of *ways the world might have been*. He writes that, "…there seems to be no true proposition of which there is any sense in saying that it might have been false. One might as well say that redness might have been a taste and not a colour. What is true, is true; what is false, is false; and concerning fundamentals there is nothing more to be said" (p. 454, see also (1904), p. 482)). And after concluding that there are several senses of logical necessity and possibility, Russell writes in "Necessity and Possibility" that

If this conclusion is valid, the subject of modality ought to be banished from logic, since propositions are simply true or false, and there is no such comparative and superlative of truth as is implied by the notions of contingency and necessity (1905a, p. 520).

One rationale for highlighting a notion of necessity in the above development of Russell's account of logical implication is that it is a notion that might be amended (and Russell is tempted to do so) in response to the challenge, presented below, to Russell's account that is posed by type theory.

Logic is Substantive

The notion of logic as substantive is the notion that logic has metaphysical implications, i.e., that truths of logic have implications about what the world must be like. Taking truths of logic to be Russell's propositions of logic, it isn't hard to see that logic is substantive by the lights of Russell's account of logical implication. As Russell acknowledges in the second edition of *Principles* (vii-viii), the account there makes propositions of the form *there are at least n entities* propositions of logic for they are true propositions that consist solely of logical constants and variables. For example, *there are at least two entities* may be construed as $\exists F \exists x \exists y(Fx \& {\sim}Fy)$.[8] The second order closure of $\exists F \exists x \exists y(Fx \& {\sim}Fy)$ just is $\exists F \exists x \exists y(Fx \& {\sim}Fy)$. This proposition is true and does not contain any non-logical terms. Hence, it is a proposition of logic and it is a synthetic *a priori* proposition given that Russell thinks that our knowledge of the existence of at least some beings (e.g., classes) is *a priori*. Indeed, according to Russell, all propositions of logic are synthetic (*Principles* p.457) and *a priori* (*Principles* p. 8).

There are different vantage points from which to see the metaphysical character of Russellian logic (see, for example, Klemke (1970), Hylton (1990a) p. 205ff, (1990b)), Linsky (1999), Chapters 1 and 2, and Landini (2003)). For example, the objects of logical analysis are propositions, which are non-linguistic complexes composed of the relevant terms, e.g., the proposition *Plato loves Socrates* consists of Plato, Socrates, and the universal *Loves*. Also, logical constants, terms in propositions of logic, are beings. Hence, the truth of any proposition of logic requires distinctly logical beings. This is a far cry from the linguistic view of logic according to which the subject matter of logic is linguistic entities and logic tells us nothing about extra-linguistic reality. In what follows, I highlight the substantiality of Russell's logic in a way that crystallizes its connection with the standard model-theoretic account of logical truth, which also makes logic substantive.

Call a proposition *p* a *conditional logical implication* if *p* is a logical implication on the assumption that there are no more than *n* entities. The fact that a conditional logical implication is not a logical implication depends on there being more than *n* entities. Since I wish to construe the notion of a conditional logical implication in a way that is neutral between the metaphysics of *Principles* and that of model-theory, I use the term *entities* to signify *values of first-order variables*. For Russell, these would simply be terms. Let's call an account of logical implication a *conditional logical implication* account if it entails the existence of a class of conditional logical implications.

Russell's account of logical implication is a conditional logical implication account. For example, recall that

(C) If George W. Bush is an elephant, then everything is an elephant

is not a logical implication (i.e., *everything is an elephant* is not logically deducible from *George W. Bush is an elephant*) because

(C') $(\exists F)(\exists x)(\exists y)(F(x) \,\&\!\sim F(y))$

is true. If there is no more than one entity (i.e., no more than one term), (C') is false and (C) is a logical implication. So (C) is a conditional logical implication. Another example: the proposition

(E) If G.W. Bush is a Republican senator and Hillary Rodham Clinton is not a senator, then Al Gore is not a senator or he is a Republican

is not a logical implication because

(E') $(\exists F)(\exists G)(\exists x)(\exists y)(\exists z)\,(G(x) \,\&\, F(x) \,\&\, \sim F(y) \,\&\, F(z) \,\&\!\sim\! G(z))$

is true. However, (E') is false and (E) is a logical implication if there are no more than two entities. So, (E) is another conditional logical implication.

If we think that logic is non-substantive (i.e., that matters of logic do not turn on what obtains in the world), then for each proposition *p*, the truth of *p is a logical implication* is invariant across the range of assumptions about how many entities there are. On this view, there are no conditional logical implications. The fact that *p* is a logical implication is conditional on there being no more than *n* entities, suffices to establish that *p* is not a logical implication.[9] Any account of logical implication that allows a class of conditional logical implications is an account that makes logic substantive. According to the *logic is substantive* view, the fact that a proposition *p* is a conditional logical implication does not by itself rule *p* out as a logical implication. In order to establish that *p* is not a logical implication, one has to do more than show that whether or not *p* is a logical implication is conditional on there being no more than *n* entities. One also has to show that there are more than *n* entities.

Since the Russellian account is a conditional logical implication account, adopting it requires the *logic is substantial* view. The fact that the truth of, say,

(D') $(F)((x){\sim}F(x, x) \,\&\, (x)(y)(z)((F(x, y) \,\&\, F(y, z)) \rightarrow F(x, z))) \rightarrow (\exists x)(y) {\sim}F(y, x))$

is conditional on the domain of *x*, *y*, and *z* being finite suffices to establish that (D) is not a logical implication on the *logic is not substantial* view. However, by Russell's conditional logical implication account, to establish that

(D) If (nothing is taller than itself, and if x is taller than y and y is taller than z, then x is taller than z), then there is a tallest object (i.e., an object that is at least as tall as everything else)

is not a logical implication one must establish that (D') is false, i.e. one must establish that there is an infinity of terms. More generally, according to the *logic is substantial* view, we establish that no conditional logical implication is a logical implication by establishing that there is an existent (as opposed to a merely possible or potential) infinity.[10] Since whether or not any conditional logical implication qualifies as a logical implication is a logical question, so too is the number and, therefore, existence of entities, i.e., existence of elements of the range of what we moderns call first-order variables.

Like Russell's account, the model theoretic account of logical implication is a conditional logical implication account.[11] For example, we say that a proposition *p* is a logical implication just in case it is true on all interpretations of its non-logical terminology. Since proposition (D), symbolized as

$$((x){\sim}F(x, x) \mathbin{\&} (x)(y)(z)((F(x, y) \mathbin{\&} F(y, z))\rightarrow F(x, z)))\rightarrow(\exists x)(y) {\sim}F(y, x)),$$

is false on some interpretation with an infinite domain, it is not a logical implication. However, if there is not an existent infinity (i.e., if it is true that there exists a finite *n* such that there are no more than *n* entities), then there is no such interpretation and by the lights of the model-theoretic account this would make (D) a logical implication. This reflects the fact that if the world contained less, then there would be more model-theoretic logical implications. The model-theoretic account is a species of the *logic is substantive* view. Etchemendy's well-known (1999) criticizes the account as such.

According to the *logic is substantive* view, fixing the extension of *logical implication* presupposes an ontology. On the model theoretic account, belief in the plurality of entities from a set-theoretic ontology justifies holding that no conditional logical implication is a logical implication. Russell's account appeals to an ontology that Russell derives from Moore (*Principles* p. xviii). Hence, on both accounts, my knowledge of the extension of what logically implies what is founded on my metaphysical knowledge, to wit my knowledge of how much exists up to a countable infinity.

Logic is Universal

In *Principles*, Russell writes that "Symbolic logic is essentially concerned with inference in general, and is distinguished from various special branches of mathematics mainly by its generality" *Principles* (p.11). The idea that the variables in logical propositions are unrestricted, i.e., they range over all entities whatsoever, is central to Russell's view that an essential characteristic of logic is its unrestricted generality or universality.[12] If the variables in a logical proposition range over everything, then they are obviously true of the entities studied by any particular branch of science. The universality of logic, a view attributed to Frege as well, is nicely characterized by Ricketts (see also Goldfarb (1979), p.352).

Logic is a science; but in contrast to the laws of special sciences like geometry or physics, the laws of logic do not mention this or that thing. Nor do they mention properties whose investigations pertain to a particular discipline. Basic logic laws contain variables to the exclusion of vocabulary idiosyncratic to this or that subject matter. It is by dint of the generality conferred by these variables that logical laws "abstract" from the differences that distinguish the claims of the special sciences. Thus Frege thinks of logic as the maximally general science (Ricketts (1985), p. 4).

The topic-neutrality of logic is a function of its universality; since the propositions of logic are true of everything, they are topic-neutral. One consequence of Russell's many-faceted conception of logic as universal is that logic takes an absolute epistemic priority over all other knowledge. We rely on logic to help formulate our scientific theories, and to determine their consequences. Hence, in order for logic to be the important tool that it is in deciding what to believe about the world, knowledge of what is and is not formally true must be prior to the results of any particular branch of science.[13] The laws of logic are more fundamental than laws from other fields and are not subject to repeal because of extra-logical investigations of the world. But then, if logic is substantive, the entities required to do logic must be distinctly logical ones in order to reflect the universality of logic. This brings us to the class of logical terms in Russell's logic.

Russell does not offer necessary and sufficient criteria of logicality in *Principles*. He tells us that logical constants can only be defined by enumeration, since any attempt to define the class of logical constants will be circular, presupposing some element of the class (p. 8). He offers a list: formal implication, material implication, the relation of a term to a class of which it is a member, the relation of *such that*, the notion of a relation, and truth (p.11). He later adds propositional functions and classes (p. 13 and 18). On the contemporary view, the topic neutrality of logic requires that logic have no ontology (see Linsky (1999) p. 5 and Godwyn and Irvine (2003) p. 172). However, as noted above, this is not the case on the Russellian conception of logic according to which extra-linguistic entities do not lie outside the boundary of logical inquiry.

Logical terms are topic neutral in that their being is presupposed in some way by any branch of inquiry. For example, classes are the extensions of concepts (i.e., universals) and thus are resident in the ontology studied by any field of inquiry to the extent that particular, topic-specific concepts are at play. Since truth is the primary aim of inquiry, and the bearers of truth are propositions, propositions are ontological furniture germane to any branch of inquiry. This is reason to treat a proposition as a logical term. Also, since correct reasoning is essential for any inquiry, the extension of the relation of formal implication is relevant to any branch of inquiry. In short, the fact that logic has its own ontology does not conflict with either the notion that logic is universal or the claim that its propositions are *a priori* because we know of logical terms *a priori*, and such knowledge is prior to scientific investigation of the empirical world.

In *Principles*, Russell highlights arguments that demonstrate an infinity of classes, and an infinity of propositions or concepts (p. 357). In Russell's (1906), as his confidence in the viability of the notion of *class* for logic waned in light of the paradoxes, he stressed that the paradoxes have no essential reference to infinity (p.197) and he gives a proof that there are ✎$_0$ many propositions (p.203). With such an ontology of logical beings in hand, it follows at once that propositions of the form *there are at least n entities* are propositions of logic. Again, this is compatible with the universality of logic because the relevant entities are presupposed in any branch of inquiry. Since the truth of all such existential propositions is the reason why no conditional logical implication is a logical implication this allows us to demarcate between the two classes of propositions without sacrificing the universality (and *a prioricity*) of logic. For example, recall that

(C) If George W. Bush is an elephant, then everything is an elephant

is not a logical implication because the negation of (C') $(F)(x)(F(x) \rightarrow (y)F(y))$, i.e.,

(C'') $(\exists F)(\exists x)(\exists y)(F(x) \& {\sim} F(y))$,

is true. The conception of logic as universal, as sketched above, requires that my knowledge that (C'') is true (and, therefore, my knowledge that (C) is not a logical implication) does not necessarily depend on, say, what physics tells me about the existence of a plurality of physical entities. Logic is metaphysically and epistemologically prior to physics and so the range of the quantifiers in (C") must include terms that are distinctly logical.

Let's connect the pictures of logic as substantive and universal. Russell's account of logical implication is a conditional logical implication account. Hence, it makes logic substantive. Specifically, that no conditional logical implication is a logical implication turns on there being infinitely many entities. The Russellian view that logic is universal demands that the needed infinite totality be composed of distinctly logical entities whose being is knowable *a priori*. The pluralistic ontology of distinctly logical entities of the *Principles* serves this need and secures the proper extension of logical implication.

Russell's Challenge

By 1908, Russell fully embraces type theory in response to the paradoxes and accepts the required stratified metaphysics (e.g., in his (1908)). Entities (again, the values of what we call first-order variables) are elements of the lowest type, i.e., individuals ((1908), p. 72-76). The impact of this for the Russellian conception of logic is significant. Russell's account of logical implication can no longer maintain the universality of logic in the type theoretic framework of *Principia*. The substantiality of logic, given the constraints of type theory, now makes whether or not a conditional logical implication is a logical implication depend on how many individuals exist. But individuals are non-logical entities. In *Sur les axiomes de l' infini et du transfini* (1911), Russell remarks that the axiom of infinity asserts the existence of classes of *n* individuals for any finite cardinal number *n* and then writes that "Here the word *individual* is opposed to class, function proposition, etc.; in other words, *individual* signifies *beings of the actual world, as opposed to the beings of logic* [italics are Russell's]" (p.44). The axiom of infinity cannot be established by logic alone (p. 43); that there are infinitely many individuals is an empirical hypothesis (p.52) (see also *IMP*, p. 203). It is not, therefore, logic's business to tell us how many individuals there are. But then the universality of logic is lost since it is logic's business to fix the extension of logical implication and, as sketched above, Russell's account of implication requires determining how many individuals there are.

The essence of Russell's problem is that entities (i.e., the values of what we call first order variables) must be of type 0 and there are no logical entities of this type. He can no longer think that true propositions enunciated solely in logical terms are propositions of logic for there are such propositions (e.g., that there are two entities, the axiom of infinity, etc…) that can only be proved or disproved by empirical evidence (*Principles*, p. viii, (1918), p. 240, and *IMP*, Chapter XIII). Therefore, such propositions are not logical propositions, which must be *a priori*. In response to this Russell abandons (1) on p.5 and no longer maintains that the truth of the fully general second-order closure of a proposition suffices to make that proposition a logical implication (*Principles*, p. viii, *IMP*, p. 202-203). He now holds a weaker version of (1): a proposition *p* is a logical implication only if there exists a propositional function PF such that *p* is an instance of PF, only variables and logical constants appear in PF, and PF is necessary ((1918), pp.240-241). As far as I can make out, he holds the weakened version of (1), (2), and (3), the remnants of his earlier account of logical implication as rendered in this paper, through his Logical Atomism period.[14]

To be sure, Russell is no empiricist. Furthermore, during the period from 1908-1913 he still maintains that logic has its own ontology. And in *Problems of Philosophy*, he writes that

> We shall find it convenient only to speak of things existing when they are in time, that is to say, when we can point to some time at which they exist (not excluding the possibility of their existing at all times). Thus thoughts feelings, minds and physical objects exist. But universals do not exist in this sense; we shall say that they subsist or have being, where 'being' is opposed to 'existence' as being timeless. The world of universals, therefore, may also be described as the world of being (p.99-100).

Beings are abstract and, therefore, atemporal entities whose subsistence is knowable a priori ((1997), p. 89-90). Some of these entities are logical beings; even after he abandons his pre-1908 account of logical implication, Russell still regards logic as having its own ontology. In the 1913 manuscript on the theory of knowledge he refers to the beings of logic as logical forms and discusses the manner in which we are acquainted with them.[15] However unclear Russell is on the nature of logical objects, he is clear that they are not entities, and therefore they are not available as values for type-0 variables. Otherwise, there would be values of *n* that would make *there are n entities* a logical proposition. Russell explicitly and consistently rules this out in work ranging from *Mathematical logic as Based on a Theory of Types* (1908) to *Philosophy of Logical Atomism* (1918) p. 240 and *Introduction to Mathematical Philosophy* (1919) Ch. XIII.

In short, Russell's account of logical implication is no longer acceptable to him because Russell does not have available a logical ontology by which the extension of logical implication may be fixed. What are Russell's options for salvaging his account of logical implication? In particular, what totality of entities may we appeal to in order to draw the line between conditional logical implications and logical implications in a way that reflects the universality of logic? In what follows, I sketch three proposals for a logical ontology and assess their viability for maintaining both Russell's account of logical implication and the universality of logic.

Some Responses[16]

In *Introduction to Mathematical Philosophy* (*IMP*), Russell makes an informal appeal to possible worlds in characterizing logical implication. How does this help? Replace (2) (given above on p. 5)

(2) A propositional function *PF* is necessary iff *PF* is always true

with

(2') A propositional function is necessary iff it is always true at all possible worlds.

Then we say that a proposition *p* is a logical implication iff the second-order universal closure of its formal propositional function is true at all possible worlds. Whether or not *p* is a logical implication is knowable *a priori* by virtue of the fact that the truth of the relevant second-order universal closure at a possible world is knowable *a priori*. Furthermore, logical implication now has modal force. A logical implication is true regardless of how many actual individuals there are, because there is no possible distribution of the world's individuals according to which a logical implication is false. Proposition (D),

If (nothing is taller than itself, and if x is taller than y and y is taller than z, then x is taller than z), then there is a tallest object (i.e., an object that is at least as tall as everything else),

is not a logical implication because an existent infinity of individuals is possible and (D) is not true in a possible world with infinitely many individuals. Whether or not there is an existent infinity of individuals is beside the point.

(2') improves on (2) in securing the universality of logic only if the elements of possible worlds are not restricted to (actual) individuals. Obviously, if possible worlds are merely re-arrangements of worldly, individuals, then we do not answer the challenge facing Russell's account of logical implication. Recall that Russell thinks that knowledge of how many individuals there are is non-logical; but then knowledge of collections of them serving as domains of possible worlds is dependent on knowledge that according to Russell is non-logical. If (D) is not a logical implication even if the world is finite because

(D') $$(F)((x)\sim F(x, x) \;\&\; (x)(y)(z)((F(x, y) \;\&\; F(y, z))\rightarrow F(x, z)))\rightarrow(\exists x)(y) \sim F(y, x))$$

is false in a possible world with an infinitely large domain *d*, then clearly *d* must consist of entities other than (actual) individuals. Hence, if understanding *there could logically be n individuals* in terms of there being *n* individuals at a logically possible world (with a domain of at least *n* individuals) helps Russell's account of logical implication reflect the universality of logic, then what is required is an ontology of possible, non-actual individuals. Then we can maintain the universality of logic by holding that knowledge of possible individuals is prior to the results of non-logical investigation.

Russell himself never develops a theory of possible worlds or of possible individuals. In the period after *Principia*, Russell explicitly rejects unreal individuals (i.e., *possibilia*).[17] He writes in his 1913 manuscript *Theory of Knowledge*, "It may be laid down generally that *possibility* always marks insufficient analysis: when analysis is completed, only the *actual* can be relevant, for the simple reason that there is only the actual, and that the merely possible is nothing" (p. 27). Perhaps his anti-modal metaphysical views are compatible with being a modal actualist. Russell can then represent non-actual, possible individuals in terms of actual non-individuals (ersazt possible individuals) which would be entities of higher types than type 0. Such possible individuals might have represented actual individuals had things been different, but as is they represent nothing.[18] Supposing the actual world has just *n* individuals, an unactualized ersatz possible world with a domain of *n+1* entities would have represented the world correctly had the world consisted of *n+1* many individuals. It is far from clear, however, that this proposal preserves the universality of logic given Russell's position that knowledge of the cardinality of the collection of individuals is non-logical.

As Russell acknowledges (e.g., *Principia* V. II, p. 281, *Mathematical Logic as Based on a Theory of Types* (1908), p. 97, and *IMP* p. 133ff., among other places), it is impossible to manufacture an infinite number of entities of a given type if there is only a finite number of individuals, i.e., entities of type 0. The axiom of infinity is needed precisely because if there are only finitely many individuals, then there will be only finitely many entities of each higher type. So, there will not be enough ersatz possible individuals to represent a possible world with infinitely many individuals unless the axiom of infinity is true. But then Russell would have to know that the axiom of infinity is true prior to thinking that it is possible. Given that knowledge of the former is non-logical, then so too is knowledge of the later. In other words: ersazt possible individuals must have a home in the type-theoretic hierarchy and thus are entities that are dependent ontologically and epistemologically, *via* Russell's technique of logical constructivism, on the collection of individuals. Hence, knowledge of the existence of an infinite number of actual individuals is prior to knowledge of the *possibility* of there being infinitely many individuals. Since the former knowledge is non-logical, the universality of logic is not preserved.

Given the type-theoretic framework, I am claiming that the modal actualist cannot allow for the possibility of there being infinitely many individuals without allowing that there actually are infinitely many individuals. Apart from type-theoretic metaphysics, it seems that on any version of modal actualism (e.g., for an overview of versions of modal actualism see Divers (2002), Ch. 13) that does not resort to modal primitivism (see below), the possibility of the axiom of infinity can only be accounted for on the basis of its actual truth. For according to modal actualism the set of ersatz possible individuals (subsets of which serve as domains of possible worlds) is either the collection of actual individuals or is supervenient upon (but distinct from) the totality of actual individuals. However, in either case, if the world is finite, there will not be enough actual surrogates to represent the possible individuals needed for a possible world that makes the axiom of infinity true and implication (D') (above on p. 18) false. Of course, the modal actualist can argue that there is a denumerable infinity of ersatzers (she can argue that there is even more; for discussion see Lewis (1986), p. 143-144). But this grounds the possibility of the axiom of infinity on the basis of its actual truth.

Another option in response to the challenge to Russell's account of logical implication is to adopt a version of modal primitivism[19] by accepting irreducible modal properties that hold of propositions of logic. Again, replace

(2) A propositional function *PF* is necessary iff *PF* is always true

with

(2'') A propositional function *PF* is necessary iff it is not possible that ~PF is sometimes true.

Leave the possibility mentioned in (2'') undefined, treating it as a modal primitive. The (logical) possibility that a propositional function is always true or sometimes true is a brute, primitive fact, not to be explained in terms of other facts. The modal primitivist grants logical intuitions epistemic primacy over intuitions about sets.[20] That no conditional logical implication is a logical implication is not grounded on an ontology of individuals, actual or otherwise. Rather, it is grounded on the primitive fact, knowable *a priori*, that the axiom of infinity is logically possible. Logical ontology includes the ontology of these irreducible modal properties. The plausibility of modal primitivism aside, it is doubtful that Russell would find it acceptable for he rejects the existence of irreducible modal properties. In *Theory of Knowledge*, he writes that "When we were discussing relations, we said that, with a given relation and given terms, two complexes are 'logically possible'. But the notion of what is 'logically possible' is not an ultimate one, and must be reduced to something that is *actual* [italics are Russell's] before our analysis can be complete (p. 111)."

It is not my purpose here to argue against modal actualism, modal realism or modal primitivism in accounting for logical necessity. Obviously, the thesis that logic is universal constrains the modal actualist in accounting for the domains of logically possible worlds only if she follows Russell and accepts the thesis. Also, that modal realism and modal primitivism conflict with Russellian metaphysics doesn't show that they are implausible. Again, I am interested here in exploring Russell's options for maintaining his account of logical implication and his view that logic is universal.

In this regard, the above two responses are wanting for they require a significant change in Russell's account of logical implication, which is the account that we are trying to make compatible with the thesis that logic is universal. Like the accounts of implication proposed by Bolzano, Tarski, and Quine,[21] Russell's account yields a reductive analysis of logical necessity. This is reflected in (2) and (3), first given above on p. 5.

(2) A propositional function *PF* is necessary iff *PF* is always true.

(3) A propositional function *PF* is always true iff the quantification that results from binding all free variables in *PF* with universal quantifiers is true.

The philosophically difficult notion of logical necessity is reduced to the well-understood notion of truth simpliciter, and whatever is involved in the associated generalization described in (3). The above attempts to reflect the universality of logic abandon this essential feature of Russell's account of logical implication—the reductive analysis of logical necessity—by revising (2) and making modality prominent. So it is desirable in defense of Russell's account to provide a rationale for believing that the worldly collection of entities includes *logical beings* at the lowest level of types which preserves Russell's reductive explanation of logical necessity while doing the least amount of mutilation to Russell's epistemology and metaphysics. Consider the following.

Suppose we agree with the modal primitivists that intuitions about what is/isn't a logical implication is prior to results from inquiries outside of logic. In Russellian terms, we agree that logic is universal. So, we assert the primacy of logical intuition over intuitions about, say, what pure sets there are (sets are creatures of mathematics and not logic). We disagree with the modal primitivists by claiming that what is logically possible turns on an ontology of entities. Let *PF* be a propositional function consisting of nothing but variables and logical constants. We say that my perception that *PF* is possible justifies my belief that there is the required number of entities to make it the case that *PF* is sometimes true.

Russell's account of logical implication grounded belief that a propositional function *PF* is possible on the belief that *PF is sometimes true* which in turn is justified, in part, on the prior belief that the required number of entities exist. Against this, we assert the epistemic primacy of the intuition that *PF* is possible and use it to ground the belief that *PF* is sometimes true. The idea here is that I know, *a priori*, that *PF is possible* and this grounds my belief that there are the entities needed for the truth of *PF is sometimes true*. These individuals are *logical* entities by virtue of the way that I come to know of them. I come to know of actual, logical entities by reflection on the possibility of propositional functions consisting of nothing but variables and logical constants.

To elaborate, on Russell's account of logical implication, knowledge that the propositional function $F(x) \& \sim F(y)$ is possible is based on *prior* knowledge that there are values for *F*, *x*, and *y* according to which $F(x) \& \sim F(y)$ is true i.e., on the prior knowledge that

(C'') $(\exists F)(\exists x)(\exists y)(F(x) \& \sim F(y))$.

Knowing that (C'') is true prior to knowing that the propositional function $F(x) \& \sim F(y)$ is possible makes knowledge of the possibility of $F(x) \& \sim F(y)$ incapable of justifying

(C ') $(F)(x)(F(x) \rightarrow (y)F(y))$.

According to Russell's account, the intuition that it is possible that there are values for *F*, *x*, and *y* according to which *F(x) &~ F(y)* is true cannot be used to criticize the Parmenidean about whether or not there is more than one entity. The debate with the Parmenidean has to be decided beforehand. Against this, what I am suggesting here is an epistemology of logic which makes sense of, in Russellian terms, using the intuition that the propositional function *F(x) &~ F(y)* is possible as a reason against a monistic metaphysics since the intuition is an insight into the cardinality of the world's entities, i.e., the values of first-order variables. On this view, the intuition that, say,

(D) If (nothing is taller than itself, and if x is taller than y and y is taller than z, then x is taller than z), then there is a tallest object (i.e., an object that is at least as tall as everything else)

is not a logical implication does not presuppose that Finitism is false, it is a reason to think that Finitism is false.

The intuition that the axiom of infinity is possible is not used to ground a belief in the existence of an infinite number of unreal entities. Rather it grounds the belief that there is an infinity of actual, abstract entities, i.e., an existent infinity of logical entities.[22] This grounds the *a priori* determination that no conditional logical implication is a logical implication. That no conditional logical implication is a logical implication is grounded on an ontology that I come to know by virtue of the perception of the possibility of truths about propositional functions consisting of just logical terminology and variables.

One consequence of this approach is that claims to the effect that there are at least *n* entities become true propositions of logic.[23] Here we return to Russell's view of logic as asserting the existence of abstract entities, which is espoused in the *Principles*. However, we do not argue for them in the style of the *Principles* arguments. Those allegedly *a priori* arguments turn on considerations dealing with the nature of propositions, the fact that an idea of a thing is different from the thing, etc… considerations independent of and prior to considerations about the possibility of propositional functions being sometimes true, always true etc… Here intuitions about the possibility of propositional functions are the epistemic pathways to belief that there are logical entities.

The conflict with the empiricist strand in Russell's type-theoretic metaphysics is somewhat mitigated in that logic is not telling us anything about empirical individuals. The above is an attempt to make room for logical beings at the lowest type level. The epistemic pathway to the required ontology is a kind of transcendental argument, which can be put thusly. I know that the axiom of infinity is logically possible. How is this knowledge possible? We can ground it on the belief that logical possibility is an irreducible modal property, the belief that there are infinitely many *possibilia*, or on the belief that there are infinitely many actual, logical entities. Given the universality of logic, each option assigns logic an ontology. Russellian metaphysics, like my own, is anti-modal and so does not accommodate the first two options. This leaves the third option, which (again) is justified on the basis of intuitions about the possibility of propositional functions being sometimes true (i.e., intuitions about the possible truth of second-order generalizations).

While I can only be suggestive here, it is not obvious to me that the metaphysics and epistemology required in defense of Russell's account of logical implication is far removed from the corpus of Russell's evolving views espoused between 1903 and 1913. As is acknowledged by Klement (forthcoming) and Landini (1998, Ch. 10), during this period Russell allows universals as values of the individual quantifiers.[24] Recall that in Chap. IV of *Principles of Mathematics* Russell uses "individual", "term", "entity", "logical subject", etc. interchangeably, and explicitly argues that concepts/predicates/verbs (by which he means universals) are terms. Also, in "Mathematical Logic as Based on the Theory of Types" (1908) an individual is defined as any entity that can occur as term or as logical subject. In both "Analytical realism" (1911) and the "The Basis of Realism" (1911), Russell claims that relations can themselves occur as terms of a relation. So for Russell there seems to be no metaphysical reason to bar universals, which Russell calls the subject matter of the abstract sciences, from being type 0 entities and the values of first-order variables.[25] On this scenario, that no conditional logical implication is a logical implication would turn on there being infinitely many universals. So, the existential proposition (C'') $(\exists F)(\exists x)(\exists y)(F(x) \& \sim F(y))$, the truth of which rules (C) *If George W. Bush is an elephant, then everything is an elephant* out as a logical implication, is concerned with type 0 entities which are not empirical and which do not, "properly speaking, *exist*, either in the mental or in the physical realm" ((1997), p. 89).

Turning to the epistemological front, recall that what makes Russell's account of logical implication a version of the *logic is substantial* view is that facts relevant to the extension of logical implication, logical facts, include existential facts about the cardinality of the world's (first-order) contents, which according to Russell's (1911) includes universals. Russell says in his (1914) that an object of acquaintance may be an abstract logical fact (p. 127) which, like many mathematical facts, includes things that are in some sense necessarily outside my present experience (e.g., that there are prime numbers greater than any that we shall have ever thought of ((pp.134-135), see also (1997) pp. 107-108). Russell argues ((1918) p.235) that there must be primitive knowledge of general propositions (knowledge of propositions not obtained by inference). So too, according to my defense of Russell's account of logical implication, for existential propositions such as (C'') $(\exists F)(\exists x)(\exists y)(F(x) \& \sim F(y)$ and the negation of (D') $(F)((x)\sim F(x, x) \& (x)(y)(z)((F(x, y) \& F(y, z)) \rightarrow F(x, z))) \rightarrow (\exists x)(y) \sim F(y, x))$.

Just as I can know the truth of a general proposition without knowing any of its instances (1997, p. 108), I can know the truth of the existential propositions required to dismiss conditional logical implications as logical implications without knowing of any of it's instances. This is in sync with Russell's view that existence propositions do not say anything about the instantiating elements but only about the relevant propositional function ((1918), p. 234 and pp. 252-254).

In brief, my defense of Russell's account of implication maintains that the intuition that the axiom of infinity is possibly true is the logical insight that grounds belief in an infinite totality of universals. Such an insight qualifies the totality as logical and preserves the universality of logic. To be sure, the Russell of 1919 views *Principia*'s assumption of the existence of at least one individual as a defect in logical purity (*IMP*, p. 203). However, to the best of my knowledge, Russell nowhere acknowledges the *prima facie* tension between maintaining (1) the *a priori* logical possibility of the axiom of infinity and therefore the *a priori* logical possibility of propositions of the form *there are n entities* for each natural number *n* ((1911a), p. 52) ((1919), p. 141), (2) a reductionist explanation of logical necessity, and (3) the truth of the axiom of infinity is knowable (if at all) only *a posteriori* ((1911a) pp. 52-53, (1919), pp. 140-141). As previously mentioned, after 1908 Russell does not have a full blown account of logical implication nor a theory of logical necessity and possibility, and he writes nothing that indicates an inclination to give up (3) in order to maintain (1) and (2). As is implicit in the previous two paragraphs, my defense of Russell's account of logical implication favors the subset of Russell's metaphysical and epistemological views that are amenable to rejecting (3) and maintaining (1) and (2).

Conclusion

Whitehead once remarked that, "No science can be more secure than the unconscious metaphysics which it tacitly presupposes." We believe that no conditional logical implication is a logical implication. The model-theoretic account justifies this belief in terms of belief in the existence of infinitely many pure sets. Critics such as John Etchemendy (1999) and Hartry Field (1989, 1991) complain that by marrying logic to set theory we lose, in Russellian terms, the universality of logic according to which logic is prior to set theory. As Etchemendy writes, the extension of logical concepts such as logical implication or validity "should not depend on substantive, extra-logical facts, whether historical or physical or mathematical" ((1999), p. 112). What I find persuasive here is the implicit claim that logic is universal. However, Etchemendy's claim that the size of the universe is a non-logical feature of the world (e.g., pp. 111 and 115, among others) is not obvious. Furthermore the idea that logic has an ontology and that the size of the universe is a logical state of affairs has historical precedent in the early work of Russell (and others such as Frege). Why should it be the case that because a fact is substantive it is non-logical? If it is logic's business to demarcate logical implications from conditional logical implications and this requires an appeal to an existent infinity, then the universality of logic, which Etchemendy seems to accept, requires that the size of the universe is a logical matter. I eschew modal ontology, and I am in the company of Maddy, Quine, Sher, Shapiro, and others in thinking that it is because there is an existent infinity that no conditional logical implication is a logical implication (see Shapiro (1993) for discussion and references).

Thus my incentive to seriously consider a return to the pre-Wittgensteinian notion of a non-set-theoretic, logical entity that is in the early work of Russell. This means a return to the Russellian view of logic as a science that makes substantive claims about non-linguistic matters of fact. In particular, the perception that no conditional logical implication is a logical implication is a pre-mathematical insight into the existence of purely abstract, logical entities. What I have proposed here, reflecting on Russell's abandonment of his early logic, is that our modal intuitions about the possibility of sentences of the form *there are n entities*, are epistemically prior to and ground our ontological intuitions about how many (logical) entities there are. With the rationale for a logical ontology in place, an account of the nature of "logical beings" is desirable (e.g., are they properties or individuals that have properties?). This motivates future work in which I will provide the needed metaphysical theory.[26]

---

[1] In this paper I ignore Russell's substitutional account of propositional functions, which Russell considers in several papers beginning in 1906. The resulting logic requires the existence of propositions and individuals, but not propositional functions. The latter are treated as "incomplete symbols". Statements about a given propositional function are reducible to statements about the various propositions that result from the substitution of one term for another

in a given proposition. The notion of one proposition coming from another by substitution for terms is taken as basic. Russell abandons this account when he sees that it generates its own version of the paradoxes. For further details see Landini (1998) and the discussion in Linsky (1999), Chapter 5 in particular.

2 Since the aims of the paper can be accomplished without getting into Russell's propositional logic, I develop Russell's account of logical implication discussing its application to propositional logic.

3 Russell writes the following: "We may…usefully define as *analytic* those propositions which are deducible from the laws of logic" (p.516). The material implication (A') *Bill Clinton is human*→*Bill Clinton is mortal* is not an analytic proposition but (B') *Bill Clinton is human*→*(Bill Clinton is human or he is mortal)*, what I am calling a logical implication, is an analytic proposition. Russell goes on to write that, "It is noteworthy that, in all actual valid deductions, whether or not the material is of a purely logical nature, the relation of premiss to conclusion, in virtue of which we make the deduction, is one of those contemplated by the laws of logic or deducible from them" (p.517). Above, I identified the non-logical proposition

(A") (x) (x is human→ x is mortal)

as a principle of inference legitimizing the inferential move from *Bill Clinton is human* to *Bill Clinton is mortal*. But the deduction of *Bill Clinton is mortal* from *Bill Clinton is human* requires in addition to the non-logical proposition (A"), the logical principles of universal instantiation and modus ponens. Compare: only logical principles validate the inference of *Bill Clinton is human or he is mortal* from *Bill Clinton is human*. So, both inferences represented by (A) and (B) above on p. 3 draw on principles of logic, but (B) draws exclusively on such principles.

4 In *Principles*, Russell adopts a Moorean notion of necessity, which is defined in terms of implication: "a proposition is more or less necessary according as the class of propositions for which it is a premise is greater or smaller" (p. 454). Russell abandoned this notion of necessity

soon after the publication of *Principles* (see Griffin (1980), p. 121-122) in favor of a notion which predicates necessity of propositional functions instead of propositions (see Russell (1905a), pp.518-520). Russell regarded necessity as an attribute of propositional functions and not propositions well into his post-Principia period (e.g., see Russell (1919) p. 165).

[5] An existential quantifier occurs in (D'), and they occur in the logical proposition (C'') below on p. 14. However, Russell does not countenance the notion of an existential quantifier as a primitive in *Principles* because it can be defined. He does use the existential quantifier shortly thereafter in, among other places, *On Denoting.* See Byrd (1989).

[6] In *Principles*, Russell uses "existence" in two distinct senses. One sense is the spatio-temporal notion described above. Another is a predicate that is only true of all nonempty classes (p. 21). In this paper, I am primarily concerned with the former notion.

[7] I take *concepts* to be universals.

[8] For Russell, statements of the form *there are at least n entities* are second-order propositions. Following *IMP*, (i) *there are at least n entities* is in the language of propositional functions: (ii) "the propositional function 'a is a class of individuals and a member of the natural number n' is sometimes true". Proposition (ii) amounts to the second-order proposition (iii) $(\exists F)(\{z{:}Fz\} \in n)$ & $n \in N$.

[9] In possible worlds terminology the kernel of the view may be put thusly. For each possible world *w* with *n* individuals, no conditional logical implication is a logical implication at *w*. Conditional logical implications are necessarily not logical implications because it is logically possible for there to be an existent infinity whatever the composition of the world may be.

[10] By one of the Lowenheim-Skolem results.

[11] This is not to deny that there are significant differences between the two accounts. See Hylton (1990a) and (1990b). For a different opinion see Landini (1996).

[12] For discussion see, among others, Goldfarb (1979), Pelham (1993), Hylton (1990a) Chapter 5, and Landini (2003).

[13] A closely related view is espoused in a recent logic text. "What do the fields of astronomy, economics, finance, law, mathematics, medicine, physics, and sociology have in common?… these fields all presuppose an underlying acceptance of basic principles of logic. For that matter *all* rational inquiry depends on logic...There is an overwhelming intuition that the laws of logic are somehow more fundamental, less subject to repeal, than the laws of the land, or even the laws of physics" (Barwise and Etchemendy 1999, pp.1-2).

[14] See (1918) section V, and (1919) p. 165.

[15] (1984), Chapter IX. Russell acknowledges that he does not have a theory of logical objects. He writes that,

> Such words as *or*, *not*, *all*, *some*, plainly involve logical notions; and since we can use such words intelligently, we must be acquainted with the logical objects involved. But the difficulty of isolation is here very great, and I do not know what the logical objects involved really are. (P.99)

Also, Russell says that the logical form (which is a non-linguistic being) of the proposition "Socrates precedes Plato" may be represented by "*xRy*" and, although he confesses that he does not have an account of logical form, he is confident that we are acquainted with this form for otherwise we would not understand (as we do) the concept of a relation (p.98-99). Russell appears to still be in the grip of his earlier *Principles* views.

[16] Two responses not commented in the body of the paper are *logical if then-ism and empirical logic*. *Logical if then-ism* drives a wedge between Russell's conditional logical implication account and the *logic is substantial* view, by claiming that logic cannot tell us that no conditional logical implication is a logical implication, but only that this is so on the hypothesis of an existent infinity. This option is not promising for those of us who think that no conditional logical implication is, in fact, a logical implication. I take Russell to be included in this group. Another response, I've entitled *empirical logic*, holds that logic is empirical and assert that no conditional

logical implication is a logical implication by affirming that the axiom of infinity is an empirical hypothesis. But then we lose the distinction between propositions of logic and other kinds of propositions. I can find no indication in anything that Russell writes that he would be willing to give up this distinction.

[17] For example, see *IMP,* p. 169.

[18] These formulations of modal actualism are due to Lewis (1986) who characterizes the position as ersatz modal realism (see pp. 136-142). The relevant feature of the position for purposes of this paper is the commitment to an actualist ontology in accounting for (merely) possible situations.

[19] A view held by Hartry Field and, at one time, Hilary Putnam.

[20] E.g., Field (1991) writes,

The idea that the notion of implication is primitive, rather than equivalent in meaning to a claim about models seems plausible on the basis of more simple-minded considerations. Suppose someone were to assert each of the following:

(a) 'Snow is white' does not logically imply 'Grass is green'.
(b) There are no mathematical entities such as sets.

Such a person would not appear to the untutored mind to be obviously contradicting himself or herself, in the way he or she would be obviously in contradiction in asserting both "Jones is a bachelor' and 'Jones is married'; but of course (a) and (b) would be in obvious contradiction if 'logically implies' was defined in terms of sets in the Tarskian manner. (p. 7). Field goes on to reject the idea that the model-theoretic definition is a reductive account of implication and not intended to define *implication.* Field thinks that his not-so-simple-minded argument (derived from Kriesel) shows that "the Tarskian model-theoretic account does not *in any important sense* [italics Field's] give the 'essence' of logical implication". (p. 8)

[21] For a detailed overview and defense of Quine's reductive portrayal of necessity in his account of logical truth see McKeon (2004). McKeon (forthcoming) defends the Tarskian model-

theoretic account of logical consequence against recent criticism of its treatment of logical necessity.

[22] To think that from the logical possibility of the axiom of infinity, we may derive its truth is not unusual from a reductionist explanation of logical possibility. Compare with model theory: *it is logically possible that* is characterized by *there exists a model of*. Then the axiom of infinity has a model iff it is true. So, by the model-theoretic analysis of logical possibility, the axiom of infinity is logically possible only if it is true.

[23] This is not anachronistic. See Almog (1989) and Williamson (1999) for accounts of formal truth according to which true propositions about the cardinality of the world's contents are formal truths.

[24] Thanks to Kevin Klement for advice on Russell's position regarding universals as the values of type 0 entities.

[25] Can the predicate version of the Russell paradox be avoided? *Being a universal* (i.e., *universalhood*) is a universal that some individuals have and others lack. If *universalhood* is an individual, then we can ask if it applies to itself, which it does. But then, isn't there a universal *non-self-applicable* of which we can ask whether or not it applies to itself? Following Klement (forthcoming), Russell is not committed to the universal *non-self-applicability*, and by denying that there is such a universal Russell can avoid the predicate version of Russell's paradox.

[26] An earlier version of this paper was read at the 2003 eastern divisional meeting of the APA. This paper was improved by comments from the audience. In particular, thanks to Ed Boedeker and especially Kevin Klement. The paper also benefited from the comments of Rich Hall, Barbara Abbott and Larry Hauser. Also thanks to Gregory Landini from whom I learned much during a two-year email conversation on Russell's logic. I am solely responsible for any remaining defects.

References

Almog, J. (1989): "Logic and the World," pp. 43-65 in *Themes From Kaplan* ed. by J. Almog, J. Perry, H. Wettstein, Oxford: Oxford University Press, (1989).

Barwise, J. and Etchemendy J., (1999): *Language Proof and Logic* New York: Seven Bridges Press and CSLI Publications.

Byrd, M. (1989): "Russell, Logicism, and the Choice of Logical Constants" *Notre Dame Journal of Formal Logic*" Vol. 30 pp. 343-361.

Coffa, J. (1991): *The Semantic Tradition from Kant to Carnap*, Cambridge: Cambridge University Press.

Divers, J. (2002): *Possible Worlds*, London: Routledge.

Dreben, B., and Floyd, J., (1991): "Tautology: How Not To Use A Word," *Synthese* vol. 87, pp. 23-49.

Etchemendy, J., (1999): *The Concept of Logical Consequence*, Stanford: CSLI Publications.

Field, H., (1989): "Is Mathematical Knowledge Just Logical Knowledge?", in H. Field *Realism, Mathematics and Modality*, Oxford: Basil Blackwell.

Field, H., (1991): "Metalogic and Modality," *Philosophical Studies* vol. 62, pp. 1-22.

Gödel, K. (1944): "Russell's Mathematical Logic," pp. 125-153 in *The Philosophy of Bertrand Russell*, ed. by P.A. Schlipp, Evanston: The Library of Living Philosophers, (1944).

Godwyn, G. and Irvine, A., (2003): "Bertrand Russell's Logicism," pp.171-201 in *The Cambridge Companion to Bertrand Russell*, ed. by N. Griffin, Cambridge: Cambridge University Press, (2003).

Goldfarb, W., (1979): "Logic in the Twenties: The Nature of the Quantifier," *The Journal of Symbolic Logic* vol. 44 pp. 351-368.

Grattan-Guinness, I. (1977): *Dear Russell—Dear Jourdain*, New York: Columbia University Press.

Griffin, N., (1980): "Russell on the Nature of Logic (1903-1913)," *Synthese* 45 pp. 117-188.

Hylton, P., (1990a): *Russell, Idealism and The Emergence of Analytic Philosophy*, Oxford: Clarendon Press.

Hylton, P., (1990b): "Logic in Russell's Logicism," pp. 137-172 in *Meaning, Thought and Knowledge*, ed. by D. Bell and N. Cooper, Oxford: Blackwell, (1990).

Klement, K., (forthcoming): "Putting Forms Before Function: Logical Grammar in Frege, Russell, and Wittgenstein."

Klemke, E., (1970): Logic and Ontology in Russell's Philosophy," pp. 416-444 in *Essays on Bertrand Russell*, ed. by Klemke, E., Urbana: University of Illinois Press, (1971).

Landini, G., (1998): *Russell's Hidden Substitutional Theory*, Oxford: Oxford University Press.

Landini, G., (1996): "Logic in Russell's *Principle of Mathematics*," *Notre Dame Journal of Formal Logic* 37, pp.554-584.

Lewis, D., (1986): *On the Plurality of Worlds*, Oxford: Basil Blackwell.

Linsky, B., (1999): *Russell's Metaphysical Logic*, Stanford: CSLI Publications.

McKeon, M., (forthcoming): *Truth Simpliciter and Logical Consequence*.

McKeon, M., (2004): "On the Substitutional Characterization of First-Order Logical Truth," *History and Philosophy of Logic* 25, pp. 195-214.

Pelham J., (1993): "Russell's Early Philosophy of Logic," pp. 325-341 in *Russell and Analytic Philosophy* ed. by A. Irvine and G. Wedeking, Toronto: University of Toronto Press (1993).

Ramsey, F., (1950): *The Foundations of Mathematics*, New York: Humanities Press.

Ricketts, T., (1985): "Frege, The Tractatus, and the Logocentric Predicament" *Nous* vol. 19 pp. 3-15.

Russell, B., (1997): *Problems of Philosophy*, New York: Oxford University Press (first published in 1912).

Russell, B., (1996): *Principles of Mathematics*, 2nd ed., New York: W.W. Norton & Company (first published in 1903).

Russell, B., (1994): *The Collected Papers of Bertrand Russell*, vol. 4, ed. by Urquhart, A., London: Routledge.

Russell, B., (1992): *The Collected Papers of Bertrand Russell*, vol. 6, ed. by Slater, J., London: Routledge.

Russell, B., (1984): *The Collected Papers of Bertrand Russell*, vol. 7, ed. by Eames E. R., London: George Allen & Unwin.

Russell, B., (1956): *Logic and Knowledge*, ed. by R.. Marsh, New York: Capricorn Books.

Russell, B., (1919): *Introduction to Mathematical Philosophy*, London: Routledge.

Russell, B., (1918): "Philosophy of Logical Atomism," pp. 175-281 in Russell (1956).

Russell, B., (1914): "On the Nature of Acquaintance," pp. (127-174) in Russell (1956).

Russell, B., (1912): "What is Logic?," pp. 54-56 in Russell (1992).

Russell, B., (1911a): "*Sur les axiomes de l' infini et du transfini* " pp. 41-53 in Russell (1992).

Russell, B., (1911b): "Analytic Realism" pp. 132-146 in Russell (1992).

Russell, B., (1908): "Mathematical Logic as based on a Theory of Types," pp. 59-102 Russell (1956).

Russell, B., (1906): "On 'Insolubilia' and their Solution by Symbolic Logic" pp. 190-214 in pp. 57-102 in *Essays in Analysis* ed. by D. Lackey, New York: George Braziller, Inc. (1973).

Russell, B., (1905a): "Necessity and Possibility," pp.507-520 in Russell (1994).

Russell, B. (1905b): "On Denoting," pp. 414-427 in Russell (1994).

Russell, B. (1904): "Non-Euclidean Geometry," pp. 482-485 in Russell (1994).

Shapiro, S. (1993): " Modality and Ontology" Mind 102, pp. 455-481.

Van Heijenoort, J., "Logic as Calculus and Logic as Language," *Synthese* vol. 17 (1967), pp. 324-330.

Whitehead, A. and Russell, B. (1927): *Principia Mathematica*, Vols. 1 and 2, 2nd ed., Cambridge:

Cambridge University Press.

Williamson, T., "Existence and Contingency," *Proceedings of the Aristotelian Society*, Supplementary Vol. 73 (1999), 181-203.