Truth Simpliciter and Logical Consequence

Introduction

According to a truth simpliciter account of logical consequence, the conclusion of an argument is a logical consequence of its premises, i.e., the argument is valid, just in case truth is preserved from the premise(s) to the conclusion, the world remaining as it is. Definitive of such accounts, which do differ, is that logical consequence is explained by appealing to truth simpliciter, i.e., truth in the actual world, rather than to truth in possible worlds that represent alternative ways the world might have turned out. Bolzano, Frege, Russell, and Tarski all advocated truth simpliciter accounts of logical consequence. More recent proponents include Quine, Maddy, Putnam, Sher, and Shapiro.¹

Criticism of the appeal to truth simpliciter in a characterization of logical consequence goes back to at least the early Wittgenstein. In this paper, I defend the appeal to truth simpliciter by responding to two recent criticisms. The first criticism, due to William Hanson, claims that no truth simpliciter account is extensionally correct in first-order logic with identity. The second criticism, derived mainly from John Etchemendy, is that a truth simpliciter account is intensionally incorrect because it fails to reflect the modal and epistemic features of the intuitive concept of logical consequence.

I shall argue that the first criticism is false and that it misrepresents the nature of a truth simpliciter account of logical consequence. With respect to the second criticism, I shall argue that it fails to offer a plausible clarification of the modal feature of logical consequence which shows that it cannot be captured by a truth simpliciter account. Also, I shall argue that the second criticism fails because it misconceives the epistemic feature of the concept of logical consequence. I don’t see why a truth simpliciter account misrepresents this feature, properly understood. Since both criticisms are framed in standard first-order predicate logic with identity, that logic shall serve as the framework for my responses.
I begin with a preliminary sketch of a truth simpliciter account of logical consequence, sketching substitutional and interpretational versions of it. Both versions are then developed in response to the first criticism. Finally, I consider the second criticism and conclude the paper.

**Truth Simpliciter Accounts of Logical Consequence**

According to a truth simpliciter account of logical consequence, an argument is valid if and only if (iff) there is no way to understand the argument’s non-logical elements so that the premises and negated conclusion are, in fact, true, i.e., are true simpliciter. A truth simpliciter account is a formal account of logical consequence. All that matters to the validity of an argument is its structure in terms of logical words. Furthermore, we don’t leave this world to do logic; modal ontology is eschewed in fixing the extension of logical consequence. Rather than considering the truth values of sentences in merely possible worlds, truth simpliciter accounts build counterexamples to arguments by playing language off actual states of affairs in securing true premises and false conclusions.

There are substitutional and interpretational varieties of truth simpliciter accounts of logical consequence. Given an identification of the logical terminology, the substitutional version says that an argument is valid iff there are no substitutions for its atomic formula(s) which yield an argument with true premises and a false conclusion. The substitutions may involve complex (possibly open) wffs replacing atomic wffs, the same complex (possibly open) wff replacing the atomic wff throughout the sentence. The interpretational version says that an argument is valid iff there are no possible uses (or meanings) for its non-logical elements according to which the premises are true and the conclusion false. So, on the substitutional and interpretational truth simpliciter accounts, logical consequence is reduced to truth simpliciter and either a generalization over substitutions or a generalization over possible interpretations. The reason why an argument is substitutionally valid is that uniformly substituting sentences (possibly complex and open) in place of the component atomic sentences (open or otherwise) in the argument never results in true premises and a false conclusion. An argument is interpretationally valid because
there is no possible interpretation of the argument’s non-logical terminology according to which the premises are true and the conclusion is false. For example, we may establish that the argument,

$$\exists x \text{ Male}(x) \& \exists x \text{ Nurse}(x)$$

$$\therefore \exists x (\text{Male}(x) \& \text{Nurse}(x))$$

is substitutionally invalid by replacing the atomic formula ‘Nurse(x)’ with the complex one ‘~Male(x)’. This produces a structurally similar argument whose premise is actually true, and whose conclusion is, in fact, false. The interpretational invalidity of the argument may be established by reinterpreting ‘Male’ to mean ‘Frog’. On this reinterpretation of ‘Male’, worldly states of affairs make the premise true and the conclusion false. On both accounts we look to the actual world with its actual extensions to construct counterexamples to arguments.

**Are all truth simpliciter accounts extensionally correct in first-order logic?**

The first criticism of truth simpliciter accounts of logical consequence that I entertain makes use of two model-theoretic properties that I call $\aleph_0$-invalidity and existential invalidity. I first introduce them, then present the criticism and my response. Consider the following two arguments.

(A) $$\forall x \forall y \forall z [(\text{Greater}(x,y) \& \text{Greater}(y,z)) \rightarrow \text{Greater}(x,z)]$$

$$\forall x \exists y \text{Greater}(x,y)$$

$$\therefore \exists x \text{Greater}(x,x)$$

(B) $$\exists x \exists y x \neq y$$

$$\therefore \exists x \exists y \exists z (x \neq y \& y \neq z \& x \neq z)$$

Argument (A) is an $\aleph_0$-invalid argument. A quantificational argument is $\aleph_0$-invalid iff no interpretation serves as a counterexample to it unless its domain is comprised of at least a denumerable infinity of elements ($\aleph_0$-invalidity will be generalized below in considering the second criticism of truth simpliciter accounts). Argument (B) is an existentially invalid argument. Letting $n$ be any positive integer, existentially invalid arguments are of the form,

There exists $n$ things

So, there are $n+1$ things.
I shall call any sentence of the form, ‘there are \( n \) things’, an existential sentence. Note that existential sentences, and thus the arguments they compose, consist only of logical terminology. Also, for each positive integer \( n \), an interpretation is a counterexample to the corresponding existentially invalid argument only if its domain consists of exactly \( n \) individuals.

In a recent article, William Hanson argues against truth simpliciter accounts of logical consequence (he refers to them as formal accounts of logical consequence) as follows.

Our logical intuitions seems to tell us that arguments \([A]\) and \([B]\) (and all arguments similar to \([B]\)) are invalid, yet the formal account is incapable of delivering all [italics are Hanson’s] these results. This inadequacy is clearly due to the fact that both versions of the formal account make use of truth simpliciter, that is truth in the actual world, rather than truth in all possible worlds or truth in all models. It is unfortunate that this is the source of the difficulty, for it is the formal account’s reliance on truth simpliciter that is responsible for much of its appeal (Hanson, 1997, 370-371).

Hanson’s criticism may be put in the form of the following argument.

1. Either there are infinitely many individuals or there are, in fact, only finitely many.
2. If the former, then all existentially invalid arguments are truth simpliciter valid.
3. If the latter, then all \( \aleph_0 \)-invalid arguments are truth simpliciter valid.
4. All \( \aleph_0 \)-invalid and existentially invalid arguments are intuitively invalid.
5. Some truth simpliciter valid arguments are intuitively invalid, i.e., truth simpliciter accounts are extensionally incorrect in first-order logic.

Since I believe that there are infinitely many individuals, I believe that an appeal to truth simpliciter is enough to secure the invalidity of \( \aleph_0 \)-invalid arguments. So I shall assume that there is an existent infinity of first-order individuals (i.e., the individuals that first-order variables range over). The premise and conclusion of argument (B) are both true. But then, according to Hanson’s criticism, no argument with the same form as (B) has a true premise and a false conclusion, since every such argument just is argument (B). More generally, since no non-logical elements occur in any given existentially invalid argument and since the premise and conclusion are, in fact, true, they remain true on all substitutions for and on all interpretations of the non-logical elements occurring in the argument. Each existentially invalid argument seems to be both substitutionally and interpretationally valid. Hanson’s diagnosis of the problem: its “reliance on
truth in the actual world, rather than truth in all possible worlds or truth in all models” is why no truth simpliciter account makes invalid all $\aleph_0$-invalid and existentially invalid arguments.

The criticism supposes that since existentially invalid arguments are composed of sentences consisting of only logical terminology, there are no non-logical components of such arguments to replace or re-interpret in order to generate the needed counterexamples. In what follows, I defend truth simpliciter accounts of logical consequence against the criticism by attacking the second premise of the above argument. I shall highlight overlooked representations of existentially invalid arguments according to which they have non-logical components. I sketch Quine’s substitutional account of logical consequence and show that by not treating the identity symbol as a primitive logical constant, his account portrays existentially invalid arguments so that they contain atomic sentences. This allows Quine’s substitutional account to generate counterexamples to all existentially invalid arguments. I take this to show that the second premise of the above argument against truth simpliciter accounts is false. Then I develop an interpretational truth simpliciter account of logical consequence that claims there is a non-logical component in the meaning of first-order quantifiers. This grounds the quantifier domain restrictions needed to secure counterexamples to existentially invalid arguments on the assumption that identity is a primitive logical constant. The developed interpretational account of logical consequence is not only further evidence that the above argument against truth simpliciter accounts is not cogent, but it also reveals that Hanson’s diagnosis of the alleged problem with truth simpliciter accounts is misleading because it falsely opposes “truth in the actual world” and “truth in all possible worlds” or “truth in all models”. I shall argue that the interpretational account appeals to (semantically) possible worlds, yet it is still a truth simpliciter account for it does not call upon changes in the makeup of the extra-linguistic world.

**Quine’s substitutional version of a truth simpliciter account of logical consequence**

Hanson explicitly identifies Quine’s substitutional account of logical consequence as an example of a truth simpliciter account that cannot secure the invalidity of both $\aleph_0$-invalid and existentially
invalid arguments (1997, 370). This is somewhat surprising given Quine’s (1986). There Quine gives a substitutional characterization of logical truth and derives logical consequence from it in the standard way. He tells us that for a first-order language $L$ with identity that is rich enough to talk about the positive integers (and express the notions “$x+y=z$” and “$x \cdot y = z$”), a sentence $S$ remains true under all substitutions for its simple sentences (it is a substitutional logical truth) iff $S$ is true on each interpretation of its non-logical terminology in every non-empty domain. Since Quine’s substitutional definition and the model-theoretical definition of logical truth in $L$ do not diverge, Quine prefers to capture standard first-order logic in terms of the substitutional definition of logical truth for a first-order language adequate for number theory rather than in terms of the model-theoretic definition because the former offers savings on ontology. “Sentences suffice, sentences even of the object language, instead of a universe of sets specifiable and unspecifiable” (Quine 1986, 55).

In his (1986), Quine demurs from treating ‘=’ as a primitive logical term for singling out ‘=’ from other two-place predicates as uniquely logical undermines Quine’s aspiration to analyze logical truth and consequence in terms of grammatical structure. However, Quine thinks that the laws of identity should be represented as logical truths. He resolves this tension by defining ‘=’ in such a way that laws of identity become truths of pure quantification theory, i.e., the logic of quantification and truth functions. In essence, Quine proposes to represent the identity relation in a given language $L$ in terms of ‘$x =_L y$’ (i.e., ‘$x$ and $y$ are indiscernible relative to $L$’) which satisfies the laws of identity. Borrowing from Dummett (1991), we define, for any first-order language $L$, a relation ‘$=_L$’ relative to the finitely many primitive predicates $P^1$, ..., $P^k$ of $L$ where each $P^j$ ($1 \leq j \leq k$) has $n_j$ argument places.

$$x =_L y \leftrightarrow \forall P^{(1 \leq j \leq k)} \forall Z_1 \ldots \forall Z_{i-1} \forall Z_{i+1} \ldots \forall Z_{n_j} (P^j(Z_1 \ldots Z_{i-1}, x, Z_{i+1} \ldots Z_{n_j}) \leftrightarrow P^j(Z_1 \ldots Z_{i-1}, y, Z_{i+1} \ldots Z_{n_j}))$$

The definition, which assumes that the predicates of $L$ are the only non-logical constants, can be adapted to handle functional symbols. The quantification (in the meta-language) over the finite
domain of predicates of L is innocent enough. Given a specification of the values of \(k, n_1, \ldots, n_k\), for a given language the right side of the biconditional can be written out as a conjunction of biconditionals.

For example, consider a first-order language L with just two one-place predicates, ‘P’, ‘Q’, and one two-place predicate ‘R’. Then,

\[
\exists x \exists y \ x = y
\]

becomes,

\[
\exists x \exists y ((Px \leftrightarrow Py) \land (Qx \leftrightarrow Qy) \land \forall z((Rxz \leftrightarrow Ryz) \land (Rzx \leftrightarrow Rzy)))
\]

On this construal, ‘\(\exists x \exists y \ x = y\)’ tells us that \(x\) and \(y\) are indistinguishable by the three predicates; that they are indistinguishable from each other even in their relations to any other object \(z\), insofar as these relations can be represented as simple sentences in the object language L. Since Quine restricts a language’s lexicon of predicates to a finite list, his simulation of identity is a recipe for turning the sentence ‘\(x = y\)’ from any first-order language L into a quantification stating that \(x\) and \(y\) are indistinguishable with respect to the predicates of L’s lexicon. Using this recipe, Quine can assign logical forms to existential arguments that allow substitutions that invalidate them.

For example, consider a first-order language L’ with just two one-place predicates, ‘P’ and ‘Q.’ The formal representation in L’ of,

There are two things, so there are three

is not

\[
(B) \quad \exists x \exists y \ x \neq y \therefore \exists x \exists y \exists z \ (x \neq y \land y \neq z \land x \neq z).
\]

but

\[
(B') \quad \exists x \exists y [(\neg (Px \leftrightarrow Py) \lor \neg (Qx \leftrightarrow Qy)]
\]

\[
/ \therefore \exists x \exists y \exists z [(\neg (Px \leftrightarrow Py) \lor \neg (Qx \leftrightarrow Qy)) \land (\neg (Py \leftrightarrow Pz) \lor \neg (Qy \leftrightarrow Qz)) \land (\neg (Px \leftrightarrow Pz) \lor \neg (Qx \leftrightarrow Qz))]
\]
Suppose that $L''$'s domain of discourse is the collection of people. If ‘P’ is ‘is a republican’ and ‘Q’ is ‘is larger than itself’, $(B')$ is substitutionally invalid because the premise is true and the conclusion false. The premise is true since there exists an $x$ and $y$ such that one is a republican and one is not. The conclusion is false, i.e., it’s negation,

\[
\forall x \forall y \forall z [(P_x \leftrightarrow P_y) \land (Q_x \leftrightarrow Q_y)] \lor [(P_x \leftrightarrow P_z) \land (Q_x \leftrightarrow Q_z)] \lor [(P_y \leftrightarrow P_z) \land (Q_y \leftrightarrow Q_z)],
\]

is true. For all persons $x$, $y$, and $z$, each of ‘$(Q_x \leftrightarrow Q_z)$’, ‘$(Q_y \leftrightarrow Q_z)$’ and ‘$(Q_x \leftrightarrow Q_y)$’ is true given the replacement of ‘is larger than itself’ for ‘Q’ (no person satisfies ‘x is larger than itself’), and at least one of ‘$(P_x \leftrightarrow P_y)$’, ‘$(P_x \leftrightarrow P_z)$’, or ‘$(P_y \leftrightarrow P_z)$’ is true for all persons $x$, $y$, and $z$.

But suppose ‘Q’ is ‘is a male’ and ‘P’ is as before; now the premise and conclusion are true. We may establish the substitutional invalidity of (B') as follows. Letting $v$ be any of the variables occurring in (B'), for each occurrence of ‘Qv’, substitute ‘(Qv & ¬Qv)’. E.g., put ‘(Qx & ¬Qx)’ in for each occurrence of ‘Qx’, ‘(Qy & ¬Qy)’ for each occurrence of ‘Qy’, etc. Then the premise remains true, but now the conclusion, thus transformed, is false, because its negation,

\[
\forall x \forall y \forall z [(P_x \leftrightarrow P_y) \land ((Q_x \leftrightarrow Q_x) \leftrightarrow (Q_y \land ¬Q_y))] \lor [(P_x \leftrightarrow P_z) \land ((Q_x \leftrightarrow Q_z) \leftrightarrow (Q_y \land ¬Q_y))] \lor [(P_y \leftrightarrow P_z) \land ((Q_y \leftrightarrow Q_z) \leftrightarrow (Q_z \land ¬Q_z))]
\]

is true. At least one of ‘$(P_x \leftrightarrow P_y)$’, ‘$(P_x \leftrightarrow P_z)$’, or ‘$(P_y \leftrightarrow P_z)$’ is true for all $x$, $y$, and $z$ and each conjunction embedded in the disjunction contains an open sentence ‘$(Qv \land ¬Qv) \leftrightarrow (Qv' \land ¬Qv')$’ which is satisfied by all objects $v$ and $v'$. Regardless of the quantity of predicates in $L''$’s lexicon, we may secure a counterexample to each expressible existentially invalid argument with the true statement that there are $n$ individuals as a premise.

More generally, for any first-order language L, no matter how sparse the lexicon, Quine’s treatment of ‘x=y’ allows us to derive a false instance of any true existential sentence simply by uniformly replacing the atomic formula with open sentences that either every element of the domain satisfies or none do. Hence, no existential sentence—construed as above—is a substitutional logical truth.
In sum, Quine would not construe the logical form of existentially invalid arguments in the way that the criticism requires. The identity symbol is not treated as a primitive logical term. Nevertheless, I believe that Quine offers a characterization of logical consequence in first-order logic with identity. On my view, a first-order logic with identity needn’t be a logic that counts some symbol referring to identity as a logical constant. For example, Mendelson (1987, 74) regards a first-order logic with identity as a first-order logic that represents the laws of identity as logical truths. Quine’s account does represent the laws of identity as logical truths.

To be clear, I am not a fan of Quine’s substitutional approach to logical consequence. Recall that the goal here is not to defend Quine’s account per se, but to show that a substitutional version of a truth simpliciter account is extensionally correct in standard first-order logic. If there is a problem with Quine’s account it is not, contra Hanson, that by appealing to the plain truth its yield in first-order logic with identity is incorrect. Assuming that there is an existent infinity, a substitutional version of a truth simpliciter account of first-order logical consequence has the resources to invalidate all existentially invalid arguments. The second premise of the above argument given above in criticism of truth simpliciter accounts of logical consequence is false.

_An interpretational truth simpliciter account of logical consequence_

I now develop an interpretational version of a truth simpliciter account of logical consequence that I shall call the possible meaning (PM) account. It is my favorite truth simpliciter account. Figuring out whether an argument is valid involves imagining ways sentences could be true or false. For example, in order to think that an argument with ‘Al Gore is President of the U.S.’ as a premise is invalid, one must think that ‘Al Gore is President of the U.S.’ could logically be true. On the PM way of understanding possible truth values to sentences we show that a sentence could be true or false not by imagining that the world changes, but by only changing the meaning of at least some of its terms. ‘Al Gore is President of the U.S.’ would have been true had, say, ‘Al Gore’ referred to Tony Blair and ‘is President of the United States’ meant is Prime Minister of Great Britain. We establish the (logically) possible truth of ‘Al Gore
is President of the U.S.’ ignoring the fact that it would have been true had things gone Gore’s way. Similarly, ‘Somebody is over 7 feet tall’ is true if ‘is over 7 feet tall’ has its ordinary interpretation and the domain of discourse is comprised of all human beings. This sentence could be false not in the sense that it is false in a possible world whose inhabitants are short, but in the sense that ‘Somebody is over 7 feet tall’ is false if the domain is changed to, say, my daughter’s fourth grade classmates. Here we appeal to a possible use or meaning for the quantifier ‘somebody’, which is stipulated by specifying my daughter’s fourth grade classmates as the domain.

On the PM account, the conclusion of an argument is a logical consequence of its premises iff there is no possible use or meaning of its constituent words under which the premises and negated conclusion are true simpliciter. The notion of a possible meaning for a term is central here. As previously illustrated, by appealing to extraordinary interpretations of ordinary terms, we may establish the logical contingency of a sentence without moving from the actual world to a merely possible one. On this way of understanding possible truth and falsity, we appeal to the fact that the connection between a word and its use is contingent. Possible uses or meanings for terms aren’t that strange. The connection between a word and its ordinary use is, after all, a matter of convention, e.g., the word ‘married’ didn’t have to mean married. Also, many words have more than one use that is possible relative to the conventions of English. For example, indexicals like ‘you’ and ‘here’ change their referent with occasion of use; there are many people named ‘Bill’; and ‘married’ can be used to mean was married to or performed the marriage of. The idea that there are possible uses for words leads to the PM notion of logical possibility. The sentence, ‘two is less than one’ could logically be true if the words ‘two’, ‘one’, and ‘less than’ were used differently (imagine that ‘two’ and ‘one’ refer to Bill and Hillary Clinton respectively, and ‘less than’ to the marriage relation.)

If there were no restrictions on possible use, then every argument would be PM-invalid, which is, of course, undesirable. We do not want things arranged so that ‘∀x Fx’ fails to be a
logical consequence of ‘∀y Fy’ because ‘x’ can be used to range over a domain where everything is an F and ‘y’ can be used to range over a domain where nothing is. And we shouldn’t think that the argument, ‘A and B, so A’ is invalid because ‘and’ could be used to mean or.

In specifying a use for a first-order language L, we specify both a domain of discourse and an interpretation of the non-logical elements of L in the domain. The domain of discourse serves as a parameter for a possible use of the sundry variables in L; a possible use for the L variables is given by an assignment which makes the variables range over the domain. Hence, domain restrictions are treated as an essentially global part of the interpretation of the language as a whole. So, for example, there is no possible use for ‘x’ and ‘y’ according to which ‘∀x Fx’ is false and ‘∀y Fy’ true because, in part, a possible use for all variables from the relevant language is fixed in terms of the one collection selected as the domain of discourse for the entire language. Relatedly, the domain of discourse for L is a determinant of a possible use of L’s names and predicates because the extension of these lexical items on a given use of them must come from the elements of the domain. In general, the domain of discourse serves to coordinate the possible uses of a language L’s non-logical elements as part of the global interpretation L. For example, although a possible use for ‘happy’ is to refer to any set, its use must be coordinated in the right way with a use of ‘x’ in order for the sentence ‘some x is happy’ to receive a truth value on a given use of it, i.e., ‘happy’ must be used to pick out a subset of the domain as this is fixed by a given use of ‘x’ and the other variables.

The PM account, as a formal account of logical consequence, takes structural constraints as basic in characterizing possible uses. Briefly, we imagine that an ordinary language sentence has a logical structure (or form) as this is determined by the logical constants that appear, if any, and the pattern of the remaining non-logical elements. The logical form of a sentence determines how the truth or falsity of the sentence depends on a use of its non-logical elements as this is coordinated by a domain of discourse. For example, taking ‘and’ to be the logical constant in ‘A
and B’ determines that it is only true on possible uses for the components of A and B according to
which they both are true. A possible use for a sentence will be any coordinated use of its non-
logical elements consistent with its logical structure. Hence, there is no possible use for ‘A and
B’ which treats ‘and’ as an or, because such a use ignores the sentence’s logical structure and
allows uses for ‘A’ and ‘B’ according to which ‘A and B’ is true when ‘B’ is false, which is
inconsistent with the fact that ‘A and B’ has a conjunctive form.

Appealing to a standard (uninterpreted) first-order formal language L to represent the logical
forms of ordinary language sentences and given a domain of discourse for L, possible uses for
non-logical elements (i.e., variables, individual constants, predicates, functional expressions, and
primitive sentences) are to be constrained only by the type of expression they are (e.g., predicates
must be used to pick out a subset of the domain, individual constants must refer to elements of the
domain, variables must range over the domain, etc.). By allowing truth value assignments to
quantificational sentences that are relative to any non-empty subset of the world’s first-order
individuals, the PM account represents the fact that there are different ways of assigning portions
(i.e., fragments) of the actual world as meanings for quantifiers. For example, we needn’t think
that the only way for the sentence, ‘Everybody is large’ to be true while keeping the meaning of
‘large’ fixed is for it to be interpreted in a merely possible world whose residents are all
mesomorphs. Rather we imagine a context in which an utterance of the sentence would make it
actually true. For example, while looking at a picture of NFL football players a person comments
to another, ‘Everybody is large’. The context here determines that the quantifier ‘everybody’
refers to those pictured, not to all human beings. Clearly, ‘Everybody is large’ has at least as
many uses as there are domains for ‘Everybody’.

On the PM account, like on Quine’s substitutional account, we look to the actual world with
its actual extensions to construct counterexamples to arguments. However, unlike Quine’s
account, the PM account does not endorse the total expurgation of modality from logic. This is
one reason why I favor the PM account over Quine’s. As is clear from above, the PM account
recognizes possible uses for quantifiers, individual constants, and predicates in fixing the extension of validity. For example, it is PM-possible for ‘Bill Clinton is a bird’ to be true because there is a possible use for ‘Bill Clinton’ and ‘is a bird’ according to which the sentence is true. A possible world in which ‘Bill Clinton is a bird’ is true represents the coordinated possible uses for the non-logical terminology according to which the sentence is actually true. Hanson remarks that, “if the modal element of the logical consequence relation is to play the crucial role in invalidating [existentially invalid arguments], it must allow … for each positive integer $n$, that there are or might have been just $n$ things” (1997, 379). On the PM account, the modal element of the logical consequence relation does play a crucial role in invalidating existentially invalid arguments, but it is construed differently.

The idea of a possible use for an ordinary language quantifier leads to the idea that it is logically possible for each existential sentence to be false by highlighting the fact that (assuming an existent infinity) for each positive integer $n$, a possible use of the variables and therewith the quantifiers in the corresponding existential sentence is to range over just $n-1$ of the things that, in fact, exist (as applied to classical logic, $n>1$). By assigning a truth value to an existential sentence relative to a subset of worldly objects, we are not considering different ways earth, air, fire, and water could have been distributed. Rather, we represent one way the variables of our language can range over what, in fact, exists. The PM theorist has a philosophical rationale for her model-theoretic use of any non-empty set as a domain in order to invalidate existentially invalid arguments. Domains of different sizes are not introduced as an ad hoc device merely to secure the invalidity of existentially invalid arguments. Even though there is no room for possible worlds that portray the world with finitely many individuals on an interpretational account of logical consequence, it does not follow that, “Logical consequence [on an interpretational truth simpliciter account] is then determined on the basis of the truth or falsity of the variously ‘interpreted’ premises and conclusion in the actual world, the domain of individuals being simply the entire collection of things in the actual world” (Hanson 1997, 371, note 12).
Again, the different quantifier ranges countenanced by the PM theorist do not represent the world with different numbers of individuals, but rather possible contexts in which the quantifiers range over a subset of what, in fact, exists. The PM account has the theoretical resources for explaining why no existentially invalid argument should be valid without abandoning its status as a truth simpliciter account of logical consequence.

I don’t believe that the conception of the universal quantifier as a logical constant is seriously threatened by the PM approach. In fact, its treatment of the quantifiers parallels the model-theoretic treatment. For example, Hodges tells us that in the first-order languages of model theory, the quantifier symbols “…always mean ‘for all individuals’ and ‘for some individuals’; but what counts as an individual depends on how we are applying the language. To understand them, we have to supply a domain of quantification [italics are Hodges’]” (1986, 144). Correspondingly, a possible use for the quantifier symbols of a language is L determined by a particular domain of discourse for L. As L may have different domains of discourse (i.e., may have different applications), there are different uses for the quantifier symbols of L. The fact that the quantifiers are logical constants means that the part of their meaning that is fixed is the portion of the domain they pick out (e.g., the existential quantifier refers to at least one member of the domain and the universal quantifier refers to all of it), and the part of their meaning that can be varied (i.e., the non-logical element of the quantifiers) in order to make a sentence true or false is the size of the domain. So, the PM theorist may regard the existential quantifier as a logical constant and still think that a sentence like ‘∃x∃y(x≠y)’ should be logically contingent since a possible use of the variables is to range over exactly one thing, and when the sentence is used this way it is false.9

One might still be suspicious about my claim that the PM account does not abandon its use of truth simpliciter by appealing to any non-empty set of objects in making existentially invalid arguments invalid. Is there good reason for holding that the truth of an interpreted sentence relative to a domain that is less than the totality of the world’s first-order individuals is truth non-
simpliciter? I say no. Recall that I take ‘truth simpliciter’ to be truth in the actual world as opposed to truth in other possible worlds. On my view, to say that a sentence is true simpliciter is just to say that it is, in fact, true. If one treats truth in the actual world as an unrelativized notion of truth, i.e., as truth with respect to the domain of all first-order individuals, as Hanson does (1997, 370), then this supports thinking that the PM account abandons its status as a truth simpliciter account of logical consequence because it appeals to truth relative to a context in which the domain of quantification is a restriction of the totality of the world’s first-order individuals. But this identification of truth in the actual world with an unrelativized notion of truth is unmotivated.

Clearly, the PM’s contextual approach to truth differs from Tarski’s “absolute concept of truth”, which he characterizes in his well-known (1933) as truth in the domain of all individuals. The contextual concept of truth, however, is at home with the relativized concept of truth that Tarski countenances in his (1933). He writes,

This is the concept of correct or true sentence in an individual domain a. By this is meant (quite generally and roughly speaking) every sentence which would be true in the usual sentence if we restricted the extension of the individuals considered to a give class a, or –somewhat more precisely—if we agreed to interpret the terms ‘individual’, ‘class of individuals’, etc., as ‘element of the class a’ ‘subclass of the class a’ etc., respectively (1933, 199).

The implication here, as I read Tarski, is that the quantificational sentence, ‘For some individual x and some individual y, ~(x=y)’ true in a restricted domain a, is true in the usual (truth simpliciter) sense because we interpret ‘individual’ as ‘element of the class a’. Writing ‘∃A’, and ‘u’, ‘v’ when absolute quantification is at issue, and ‘∃R’, and ‘x’, ‘y’ when restricted quantification is at issue, consider the following two quantifications.

\[
\begin{align*}
(1) & \quad \exists_A u \exists_A v \neg (u=v) \\
(2) & \quad \exists_R x \exists_R y \neg (x=y)
\end{align*}
\]
(1) is true and (2) is true in domain D which, let’s say, consists of exactly three individuals. As I understand Tarski, (1) and (2) are true in the usual, truth simpliciter sense for we may transfer the R-quantification (2) into something like A-quantification (3) (I say “something like” because I don’t want to go into messy but essentially irrelevant issues about exactly how the presupposition of non-emptiness of domains is to be represented) where ‘D(u)’ means ‘u is an element of D’.

\[(3) \exists_x \exists_y \exists_z (D(u) \& D(v) \& \sim u=v)\]

Interestingly, Tarski says that relativizing quantifiers to a particular class is “not essential for the understanding of the main theme of [his] work” (1933, 199). Given the above, the merits of the contextual, relativized concept of truth over the absolute concept are clear enough. If the concept of truth in a formalized language L must be absolute then the existentially invalid arguments formulated in L are PM-valid and Hanson’s criticism against the PM account goes through.

I concur with Tarski that a relativized notion of truth is truth in the usual sense, i.e. truth in the actual world. The only rationale for thinking otherwise is the view that a relativized notion of truth must call upon changes in the makeup of the world. But, as I have argued above, this is false. Indeed, the model-theoretic characterization of logical consequence makes use of a relativized notion of truth and yet on my view it is a truth simpliciter account. In fixing the extension of model-theoretic validity for a first-order language, the range of quantifiers varies from interpretation to interpretation of the language. For example, to show that an argument with \(\exists x \exists y \forall z(x \neq y \& (z=x \lor z=y))\) as a premise is model-theoretically invalid we invoke an interpretation of the language whose domain has only two elements. The premise is true relative to such an interpretation, and false relative to any interpretation with a domain of more than two elements. Model-theoretic truth is a relativized conception of truth, e.g., \(\exists x \exists y \forall z(x \neq y \& (z=x \lor z=y))\) is true relative to some interpretations, false relative to others. However, the truth of an
interpreted sentence in a domain smaller than that of the collection of all first-order individuals is
nevertheless truth simpliciter, i.e., truth in the actual world.

To see this clearly, the model-theoretic characterization may be portrayed as follows. Consider a first-order argument with finitely many premises. The antecedent of the corresponding conditional to this inference is the conjunction of its premises, while the consequent is its conclusion. Generate a new sentence S by uniformly replacing each individual constant occurring in this conditional by a first-order variable, each one-place predicate ‘Fv’ by ‘v∈F’ (i.e., ‘v is a member of F’), each two-place predicate ‘Rvu’ by ‘<v,u>∈R’ (choosing new variables each time) and so on. Then restrict all variables to a new second-order variable ‘X’, which ranges over the universe of sets. To do this, make sentence S the consequent of a new conditional in whose antecedent put ‘v∈X’ for each individual variable ‘v’, and ‘V⊆X’ (i.e., ‘V is a set of ordered n-tuples of members of X’) for each n-adic second-order variable ‘V’, and then universally close. The general form of the resulting sentence will be:

∀X,∀v₁, ...,∀vₙ,∀V₁, ...∀Vₙ[(X≠∅→(v₁∈X, &...& vₙ∈X &V₁∈X & ... &Vₙ∈X))→S],

where the vᵢ and Vᵢ are the free first-order and second-order variables occurring in S. Then an argument is model-theoretically valid iff its corresponding set-theoretic sentence is true simpliciter. For example, the argument

\[ \exists x \forall x \exists \neg \exists y \exists \neg \exists F \neg (X \notin \emptyset \land x \in X \land y \in X \land x \neq y \land F \subseteq X \land x \in F \land y \notin F) \]

is model-theoretically invalid because ‘\[ \exists x \exists y \exists \neg \exists F (X \notin \emptyset \land x \in X \land y \in X \land x \neq y \land F \subseteq X \land x \in F \land y \notin F) \]’ is true where the quantifiers range over all the entities of the appropriate type, i.e., ‘x’ and ‘y’ range over the totality of first-order elements, ‘X’ and ‘F’ range over the totality of sets of first-order elements. Since no non-logical terminology appear in an existentially invalid argument, there exists n things, so there are n+1 things, it is invalid simply because of the truth of the proposition which claims that there exists a set with just n elements. For example,
existentially invalid argument (B) is model-theoretically invalid because ‘∃X∃x∃y∀z[(X≠∅ & (x∈X & y∈X & x≠y)) & (z∈X →(z=x ∨ z=y))]’ is, in fact, true.

In sum, the technical, model-theoretic account of logical consequence is a truth simpliciter account. The truth of an interpreted sentence in a domain smaller than that of the collection of all first-order individuals is truth in the actual world. This shows, by analogy, that the PM account does not abandon its use of truth simpliciter by appealing to any non-empty set of objects in making existentially invalid arguments invalid. There is no good reason for holding that the truth of an interpreted sentence relative to a domain that is less than the totality of the world’s first-order individuals does not suffice to make it true in the actual world. Since model-theoretic validity makes use of truth simpliciter, there is no reason to think that it cannot represent PM-validity just because PM-validity makes use of truth simpliciter.11

Contra Hanson, the appeal to truth simpliciter does not make an account of logical consequence incapable of securing the invalidity of both ℵ₀-invalid and existentially invalid arguments. Furthermore, an account of logical consequence such as the PM account can countenance a modal notion in the concept of validity without losing its status as a truth simpliciter account. There is no need for an extensionally correct account of first-order logical consequence to represent alternative ways the world might be. The first criticism fails. Of course, the criticism could be significantly weakened to something along the lines of the following.

If the terms expressing existential quantification, negation, and identity are all treated as primitive logical terms, then a truth simpliciter account (assuming that it makes use of just a single domain of individuals) cannot make all ℵ₀-invalid and existentially invalid arguments invalid.

I accept this. But then the issue is no longer the adequacy of the appeal to truth simpliciter in a characterization of logical consequence, but rather the adequacy of certain versions of a truth
simpliciter account. As such, it is not all that interesting. Why should the inadequacy of some truth simpliciter accounts motivate incorporating an appeal to possible worlds? Certainly, I am not defending all truth simpliciter accounts of logical consequence; I am motivated to see how far we can go in logic without having to appeal to ways the world might have turned out. Quine’s substitutional account and the PM account demonstrate that there is no reason to assume that a truth simpliciter account must treat identity as a primitive constant or that such an account must appeal to a single domain of individuals in fixing the extension of first-order logical consequence. Both accounts make all $\aleph_0$-invalid and existentially invalid arguments invalid.

Again, there is advantage in not fixing the extension of logical consequence in terms of mysterious modal notions. To establish the invalidity of an argument whose conclusion is, say, ‘Bill Clinton is a human’ we are not required to search out possible worlds where this is false (arguably such a world cannot be found). According to the PM account, we merely appeal to, say, the fact that ‘Bill Clinton’ could be used to refer to the number six, and ‘human’ could have meant ‘prime number’. Similarly, to establish the logical possibility of ‘there exists exactly one thing’ we do not appeal to modal considerations pertaining to whether the world could have been Parmenidean. Rather, we appeal to the fact that the quantifiers could be used to range over just one of the individuals that, in fact, exist. Admittedly, this familiar story generates a high comfort level. A logically possible situation does not turn on the mysterious notion “ways the world could be”. This results in a better explanation of validity and a stronger epistemological foundation for our judgments of validity since we have more to say about what uses for non-logical terms are possible than about the modal features of the world. However, by making what is logically possible depend on the actual make-up of the world, a potential problem arises for the PM account and all other truth simpliciter accounts

**Are truth simpliciter accounts of logical consequence intensionally correct?**

In order to be intensionally correct, an analysis of logical consequence must reflect the essential features of the pre-theoretic, ordinary concept of logical consequence. I now consider a criticism
of truth simpliciter accounts, derived from the work of John Etchemendy, which claims that they are intensionally inadequate. Etchemendy writes that the extension of the logical consequence relation “should not depend on substantive, extra-logical facts, whether historical or physical or mathematical” (1990, 112). He also remarks that “if the assessment of [validity] rests on substantive facts about the size of the universe, then … it would have been wrong had the facts in question been otherwise. This is damaging to the claim to have captured the ordinary concept of [validity] (1990, 109).” In particular, by making the extension of the logical consequence relation turn on claims about the extra-linguistic world, truth simpliciter accounts fail to reflect the modal and epistemic features of the ordinary concept of logical consequence. We may put the criticism in the form of the following argument.

(1) The yield of a truth simpliciter account is not logically independent of consistent claims about the extra-linguistic world.

(2) In order to reflect the modal and epistemic features of the concept of logical consequence, the extension of an adequate analysis of the concept should be logically independent of all (consistent) claims about the extra-linguistic world.

(3) So, truth simpliciter accounts are intensionally incorrect (i.e., they are inadequate as analyses of the concept of logical consequence).

In order to illustrate why (1) is true, I will describe a set T of sentences as satisfiable in a domain D iff some interpretation whose domain is a subset of D satisfies T. An argument \( P_1, \ldots, P_k / \therefore C \) is valid in D iff \( \{P_1, \ldots, P_k, \neg C\} \) is not satisfiable in D. As previously noted, argument

\[
\text{(B)} \quad \forall x \forall y \forall z [(\text{Greater}(x,y) \& \text{Greater}(y,z)) \Rightarrow \text{Greater}(x,z)] \\
\forall x \exists y \text{Greater}(x,y) \\
\therefore \exists x \text{Greater}(x,x)
\]

is valid in every finite domain but invalid in every infinite domain, while argument

\[
\text{(C)} \quad \exists x \text{Happy}(x) \\
\therefore \forall x \text{Happy}(x)
\]

is valid in domains with one member (or none, if we permit empty domains) and is invalid in other domains.
Let an \( n \)-invalid argument be one which is invalid in domains with \( n \) or more members and valid in domains with fewer. Like existentially invalid arguments, \( n \)-invalid arguments consists of those that turn out valid or invalid depending on the restrictions we place on the cardinalities of the domains of interpretations. Argument (B) is \( \aleph_0 \)-invalid, (C) is 2-invalid. On a truth simpliciter account, the invalidity of \( n \)-invalid arguments is conditional. For each \( n \)-invalid argument if there are at least \( n \) objects, it is truth simpliciter invalid, otherwise it is truth simpliciter valid. By the Lowenheim-Skolem theorem, all \( n \)-invalid arguments are invalid in a denumerably infinite domain. Hence, all \( n \)-invalid arguments are truth simpliciter invalid only if the collection of the world’s individuals is denumerably infinite. So, to move to categorical judgments about the truth simpliciter invalidity of all \( n \)-invalid arguments, we need to know that there exists an actual (countably) infinite totality of individuals. Clearly, the yield of a truth simpliciter account is not logically independent of a claim about the upper bound of the collection of worldly individuals. This fact makes truth simpliciter accounts vulnerable to the objection that they fail to reflect the modal and epistemic features of validity. However, I have yet to see a development of the objection that I find plausible.

More specifically, I don’t know of a non-question begging way of developing the modal and epistemic features of the concept of logical consequence that supports premise (2). In what follows, looking primarily at Etchemendy’s work, I’ll clarify what I take to be the picture of the modal feature of validity that drives the above criticism of truth simpliciter accounts. Then I argue that the fact that an account is truth simpliciter does not bar it from reflecting the modal feature so understood. Next, I clarify the view of the epistemic feature of logical consequence that I think is presupposed by the criticism. Although truth simpliciter accounts do not reflect this view of the epistemic feature, the fact that it does not seem to be reflected by truth non-simpliciter accounts indicates that it is wrong. First the modal feature.

That \( n \)-invalid arguments are invalid cannot be due to an existent infinity as a truth simpliciter account would have it for, intuitively, no \( n \)-invalid argument would be valid even if the universe
were finite (Etchemendy 1990, 113 and 115). The problem is that truth simpliciter validity does not remain invariant on alternative configurations of actual states of affairs. More specifically, if there are fewer than \( n \) objects in a world \( w \), then all \( n \)-invalid arguments will be truth simpliciter valid in \( w \). But, according to Etchemendy, the modal feature of the ordinary notion of validity demands not only that all \( n \)-invalid arguments are invalid in the actual world (which a truth simpliciter account can meet, given the existence of an infinite totality), but also that this hold in each possible world. Presumably this is part of the rationale for thinking that “when we ask whether a conclusion follows from another we are interested in how things could have been, not how they actually are or how we believe them to be” (1990, 120-121). I now elaborate.

If an argument is invalid, then this is necessarily the case. The proof is relatively straightforward. Let \( A \) be an arbitrary invalid argument.

(1) An argument is invalid iff the conjunction of true premises and the negation of the conclusion is logically possible.

Let \( A' \) be the conjunction of \( A \)’s premises and negated conclusion. From the hypothesis and (1) we get

\[
\Box \neg A'
\]

where ‘\( \Box \)’ is the logical possibility operator. A truth regarding the modality inherent in the concept of logical consequence is encapsulated by the logical modality thesis: for any proposition \( P \), if \( \Box P \), then \( \Box \Box P \) (‘\( \Box \)’ is the logical necessity operator). By instantiation,

(3) if \( \Box A' \), then \( \Box \Box A' \).

From (2) and (3) it follows that

(4) \( \Box \Box A' \).

From the necessity of (1) and from (4), it follows that it is necessary that \( A \) is invalid. So, for any invalid argument \( A \), if \( A \) is invalid then it is necessary that \( A \) is invalid. Since \( n \)-invalid arguments
are invalid, it is necessary that they are invalid. In terms of possible worlds (where a possible world is a way the actual world could be),

(M) For each possible world \( w \) with \( n \) individuals, no \( n \)-invalid argument is valid in \( w \).

The truth of (M) requires that an existent infinity is necessarily possible. As far as I can see, the only way that (M) functions as a criticism of truth simpliciter accounts is if there is a finite possible world at which \( n \)-invalid arguments are truth simpliciter valid (e.g., argument (C) is PM-valid in a possible world which contains exactly one individual). This reflects the fact that if the world contained less, then, contra (M), more arguments would be truth simpliciter valid.

I believe that there is an existent infinity of mathematical entities and that this collection exists in all possible worlds. But if the mathematical contents of the world are necessary, as I believe, then there is no work for an appeal to merely possible worlds to do in securing (M). Hence, a truth simpliciter account such as the PM one satisfies (M) by virtue of the fact that in no possible world is there only a finite number of things. In order to elaborate, I borrow from an argument due to McGee (1992b).

For any possible use of the non-logical elements of first-order sentences, there exists an isomorphic one with a domain of pure sets. Pure sets necessarily exist, and their structural features are essential. So, in any possible world, we get possible uses for first-order sentences that secure standard first-order logic. Hence, what is PM-valid in first-order logic is invariant on different ways the universe could be, and there is no way the world could be which would turn a PM-invalid argument into a PM-valid one. So, by appealing to actual (set-theoretic) states of affairs, we get the modal characteristic of logical consequence for free on the PM account.

Consider the following 3-invalid argument.

\[ \text{(D)} \quad \text{Bill Clinton is President and a Democrat.} \]
\[ \text{Hillary Rodham Clinton is not President.} \]
\[ \therefore \text{Al Gore is not President or he is a Democrat.} \]

Taking ‘and’, ‘not’, and ‘or’ to be the only logical elements, we may show that (D) is PM invalid by appealing to the fact that ‘Bill Clinton’ could be used to refer to George W. Bush, and
President’ and ‘Democrat’ could be used to designate *male* and *Republican*, respectively. However, this meaning assignment fails to establish the PM-invalidity of (D) in a possible world in which the relevant states of affairs fail to obtain, e.g., a world in which Gore doesn’t exist, or a world in which Bush is not a Republican, but Gore is. Can we secure a counterexample to (D) on the PM approach that makes D’s premises and negated conclusion true at each possible world? Yes. Appealing to the above rendition of McGee’ argument, we know that there is a possible use of the non-logical elements of (D), isomorphic to the one above, that makes D’s premises and negated conclusion truths about actual, set-theoretic states of affairs (as described by a standard theory like Zermelo-Fraenkel set theory), which hold in all possible worlds. By the axiom of infinity, there exist distinct things (pure sets) $\alpha, \beta$, and $\delta$. The pair-set axiom tells us that for any $x$ and $y$, there is a set that has $x$ and $y$ as elements. So, there exists the sets: $\{\alpha\}$, and $\{\alpha, \delta\}$. We appeal to these set-theoretic facts to show the PM-invalidity of (D). Consider the following possible use of ‘Democrat’ and ‘President’: let the former denote $\{\alpha\}$ and the latter $\{\alpha, \delta\}$. Let ‘Bill Clinton’, ‘Hillary Rodham Clinton’, and ‘Al Gore’ refer to $\alpha, \beta$, and $\delta$, respectively. The premises and negated conclusion of (D) are necessarily true relative to this use of its non-logical elements. The PM account respects the fact that argument (D), as well as all other $n$-invalid arguments, is necessarily invalid.

One could adopt some version of logical primitivism and hold that there is a special logical modality according to which it is logically possible for there to be finitely many individuals even though this is metaphysically impossible. From this perspective, the claim against truth simpliciter accounts that they fail (M) can be made. Etchemendy writes that the set-theoretic structures used in model theory are useful as mathematical models of *logically possible ways the world might have been* (e.g., 1999, p.25), but, as far as I can make out, he never develops the notion of *logically possible ways the world might have been*. In my (2001), I interpret Etchemendy as a logical primitivist who understands this notion in terms of irreducible logically
possible worlds, i.e., states of affairs that do not supervene on actual or metaphysically possible
states of affairs. I am not a fan of logical primitivism for I, along with others, think that there is a
story to tell about logical possibility. I argue against logical primitivism in my (2001) and (2005,
forthcoming) along lines similar to the ones in Shapiro (1993).

Turning to the relevant epistemic feature of logical consequence, Etchemendy’s view is not
very clear. He remarks that, “If you accept the premises of a valid argument, you must also
accept the conclusion (to which we sometimes add ‘on pain of irrationality’)” (1990, 89). And he
writes, “A logically valid argument must, at the very least, be capable of justifying its conclusion.
It must be possible to come to know that the conclusion is true on the basis of knowledge that the
argument is valid and that its premises are true” (1990, 93). Following Hart (1991, 491),
Etchemendy’s conception of the epistemic feature of validity may be that knowledge of what is
valid is knowledge that is certified from the essence of rationality independently of (“prior to”) all
other issues and is guaranteed to be infallible in such applications as it has to such subsequent
issues. On this interpretation of Etchemendy, logical knowledge is epistemically privileged to the
extent that it is not justified in any way on an investigation of worldly states of affairs. There is no
interaction between knowledge of logic and knowledge of, say, set theory, because logic is prior
to all fields of study.

On this view, the source of the intuition that \( n \)-invalidity entails invalidity is not some
conviction about the number of individuals in the world. For example, a finitist can rationally
hold both that there is necessarily a finite number of worldly individuals and that the conclusion
of an \( \aleph_0 \)-invalid argument such as (B) is not a logical consequence of its premises. At least part
of Etchemendy’s logical epistemology seems to be captured by Wittgenstein’s remark in the
Tractatus that, “Whenever a question can be decided by logic at all it must be possible to decide it
without further ado. (And if we get into a position where we have to look at the world for an
answer to such a problem, that shows that we are on a completely wrong track)” (1922, 5.551).
The spirit of Wittgenstein’s remark is reflected in the following metaphor, borrowed from Bencivenga (1999, 6-7).

*The locked room metaphor*

Suppose that you are locked in a dark windowless room and you know everything about your language but nothing about the world outside. A sentence $X$ and a class $K$ of sentences are presented to you. If you can determine that $X$ is true if all the sentences in $K$ are, $X$ is a logical consequence of $K$.

Ignorant of US politics, I couldn’t determine the truth of ‘Kelly is not US President’ solely on the basis of ‘Kelly is a female’. However, behind such a veil of ignorance I would be able to tell that ‘Kelly is not US President’ is true if ‘Kelly is female and Kelly is not US President’ is true. How? Based on my understanding of the semantic contribution of ‘and’ to the determination of the truth conditions of a sentence of the form $\neg (P \land Q)$. For any sentences $P$ and $Q$, I know that $\neg (P \land Q)$ is true just in case $P$ is true and $Q$ is true. So, I know, *a priori*, if $\neg (P \land Q)$ is true, then $Q$ is true. As noted by Fodor, “This really is remarkable since, after all, it’s what they mean, together with the facts about the non-linguistic world, that decide whether $P$ or $Q$ are true” (Fodor 2000, 12).

The idea that logical knowledge does not require knowledge of extra-linguistic fact falls out of the logical positivist line on the nature of logic. In recounting the influence of Wittgenstein of the *Tractatus*, Carnap writes that, “Logical statements are true under all conceivable circumstances; thus their truth is independent of the contingent facts of the world. On the other hand, it follows that these statements do not say anything about the world and thus have no factual content” (1963, 25). From the modal feature of logical truth, we derive that it says nothing about the world. This, in turn, yields an epistemic feature of logical truth: knowledge of what is and isn’t a logical truth does not turn on knowledge of the extra-linguistic world. We formulate the following argument.

(1) From the fact that a logical truth is true under all conceivable circumstances it follows that these statements say nothing about the world.
(2) If logical truths say nothing about the world, then knowledge of extra-linguistic fact is not necessary in order to fix the extension of logical truth (e.g., logical truth is *a priori* precisely because it is knowable prior to investigation of extra-linguistic fact).

(3) So, knowledge of extra-linguistic fact is not necessary in order to fix the extension of logical truth.

Transferring the considerations from logical truth to logical consequence, we may say that since n-invalid arguments are invalid regardless of the actual make-up of the world, the alleged epistemic feature that drives the above criticism of truth simpliciter accounts is as follows.

(E) Knowledge that n-invalid arguments are invalid does not require knowledge about substantive (i.e., extra-linguistic) facts.

A substantive fact is any fact about the language-independent world. In contrast, facts about language are non-substantive. Any fact about the semantic functioning of the logical and non-logical terminology of the object language will count as non-substantive. Etchemendy does not always make it clear that semantic truths will count as non-substantial. But he acknowledges the point when he accepts that a demonstration of the truth in all models of a sentence that appeals only to “the logic of the meta-theory and the semantics of our language” really does establish its logical truth (1990, 141). So, the semantic fact that ‘p & q’ is true iff ‘p’ is true and ‘q’ is true is non-substantive, as is the fact in standard first-order semantics that ‘=’ and ‘Bill Clinton’ have different semantic roles (e.g., the fact ‘=’ cannot be used to refer to a first-order particular). According to Etchemendy, an analytic truth is non-substantial because “it is true in virtue of the meanings of its terms, one that is true independent of the facts...For this reason its truth is obvious to anyone who fully understands the language” (1988a, 92).

Clearly, truth simpliciter accounts do not reflect (E). However, it isn’t obvious to me that this motivates moving to a truth non-simpliciter account of logical consequence, because it isn’t clear to me that (E) is true (I suspect that (2), in the above argument, is false). To see why, consider the striking asymmetry between the criteria for validity and invalidity, which has been remarked on before.¹⁴ To establish the validity of an argument, one uses a logical deduction that proceeds in accordance with rules grounded solely on the meanings of the relevant logical terminology. To
establish invalidity one uses a counterargument or, equivalently, a countermodel. To prove that an argument is valid we rely on nothing but semantic facts pertaining to the relevant logical terminology, worldly matters are irrelevant. However, to prove invalidity, we need a contribution from reality. Of course, this is because to prove invalidity we must establish that the premises of an argument and the denial of its conclusion are true relative to a state of affairs, modal or otherwise, on a given interpretation of the occurrent non-logical terminology. Hence, to know that there exists a counterexample to a given argument (i.e., to know that it is invalid) I must know that the relevant state of affairs obtains. Prima facie, this is knowledge of substantive, extra-linguistic fact. Hence, (E) is problematic. I believe that parallel reasoning with respect to logical truth shows a similar problem with premise (2) in the above argument.

As was previously stated, that the n-invalidity of an argument makes it invalid requires the possibility of an existent infinity. Any semantic account of logical consequence in first-order logic must fix the extension of validity with respect to the class of n-invalid arguments by appealing to a logically possible situation in which there exists an infinite totality. Truth simpliciter accounts are supposedly defective because by reducing “the size of the universe could be infinite” to “the size of the universe is infinite” they violate (E). The actual size of the universe is a substantive fact. What is far from obvious is how an appeal to an irreducibly modal state of affairs generates an account of validity that is any less substantive than a truth simpliciter one. The question arises as to how we are to understand a “logically possible situation” in a way which does not make this appeal to the possibility of an existent infinity a substantive one, and, therefore, one that violates (E). This suggests a tension between (M) and (E). Since I accept (M), I think that (E) is false. In order to crystallize this tension, I sketch a truth non-simpliciter account and show that it is far from clear how it satisfies (M) without violating (E).

Suppose we say that it is logically possible for a sentence p to be true iff there is a possible meaning for the non-logical terms in p which makes it come out true in some possible world. Then, on the PM+possible world (PW) account, an argument is valid iff there are no possible
meanings for the non-logical terms which make all the premises and negated conclusion come out true in some possible world. Unlike on the PM account, we do not appeal to the way the world is in imposing an upper bound on the cardinalities of the domains we may consider in determining whether or not a given argument is PM+PW valid. So, the combined account, unlike the PM-account, makes possible the invariance of the validity of an argument under different considerations as to ways the world could be. Hence, it is coherent to both think that all \( n \)-invalid arguments are PM+PW-invalid and believe that the world is finite.

In order for the (PM+PW)-account of validity to satisfy both (M) and (E), what is required is a modal assumption that ensures that all \( n \)-invalid arguments are necessarily invalid, while itself being non-substantial. Recall that (M) is true only if for each finite \( n \), if there were \( n \) things, then it would be possible for there to be infinitely many, i.e., for each world \( w \) with \( n \) things, there is a world \( w' \) possible relative to \( w \) and \( w' \)'s domain is infinite. How exactly are we to underwrite this structure of possible worlds without violating (E), i.e., without appealing to any substantive (modal) assumptions about the non-linguistic world?

The modality in the ordinary notion of validity has been unpacked via the PM+PW approach as: the conclusion of a valid argument holds in all possibilities in which the premises hold under every interpretation of the non-logical vocabulary. One models a set of sentences by choosing a possible world and then re-interpreting the non-logical terms in that world. The range of worlds registers the possibilities. The concern here is that the PM + PW account fails (E) because the possibilities register modal facts about the non-linguistic world, and, therefore, knowledge of the yield of the PM+ PW account is not independent of knowledge of consistent modal claims about the non-linguistic world.

Etchemendy’s tentative suggestions that the requisite modality might be metaphysical (1988a, 102; 1990, 82) or epistemological (1990, 88-89, and 120) clearly result in making PM+PW-validity turn on substantive facts, i.e., facts about the non-linguistic world, thereby violating (E). Very quickly, if the domains of possible worlds represent metaphysically possible collections,
(M) is true only if from the perspective of each metaphysically possible world, it is (metaphysically) possible that there be a denumerably infinite totality of individuals. So, the PM+PW theorist might argue for a modal system in which accessibility is the universal relation as the correct logic of metaphysical modality, thus ensuring the existence of an accessible infinite world at each world (granted the existence of one such world.) But this is based on a flat denial of the finitist’s claim of the necessary non-existence of a completed infinity; we are doing modal metaphysics and PM+PW validity violates (E).

Note that we need more than the assumption that there is a possible world with an infinite domain. For example, it could be argued that the actual world is infinitely large and that there are finite possible worlds, but that the cardinality of their domains is necessary from the point of view of such finite worlds. The finite worlds are possible from the point of view of the infinite world, but not vice versa. If the domain of a finite world \( w \) consists of \( n \) individuals, then all \( n \)-invalid arguments will be PM+PW valid at \( w \). For example, on a Parmenidean view, it is metaphysically necessary that there be exactly one thing (“The One”). If this view represents a metaphysically possible world \( w \), then all 2-invalid arguments turn out PM+PW valid at \( w \). So we need to argue that from the point of view of each metaphysically possible world, an infinite one is metaphysically possible. But then it is knowledge of this metaphysical modal fact (knowledge about the extra linguistic world) which grounds knowledge of (M) and violates (E).

If we take the modality to be epistemological, then in order to underwrite (M) it must be the case that from the perspective of each view about the size of the world’s domain, the view that the world’s domain is denumerably infinite is epistemically possible. From the finitist’s point of view, the existence of an actual infinity is incoherent. Nevertheless, a counterexample to argument (B) may be derived from the (epistemically) possible world that represents the case in which the finitist is wrong, and there does exist a denumerably infinite totality. But the appeal to potential error in the finitist’s position must be based on some claim as to the fallibility of our cognitive faculties with respect to the “clear and distinct” perception of modal facts.
The skepticism required to justify (M) is a skepticism with respect to the veracity of basic intuitions about what is metaphysically possible. The challenge for this view is to ground a non-ad hoc restriction of the required skepticism to claims of the form, \( n \) things exist, for all positive integers \( n \). But suppose that the finitist denial is based on (a priori) intuitions she finds clear and distinct. If strong intuitions that \( p \), don’t make \( \neg\negp \) epistemically impossible, then very little, if anything, turns out PM+PW-valid. If the finitist must countenance, as logically possible, situations which she regards as unintelligible, then why not also the classical logician who grounds the logical necessity of ‘Jack is at home or he is not’ in terms of the intuition that there is no way the world could be which would make this false? The concern arises that there is no natural way of restricting the relevance of the possibility of error to the determination of what is logically possible. Obviously, appealing to what is logically possible in order to delimit what is epistemically possible will not work since the notion of epistemic possibility is being developed here precisely to get at the notion of logical possibility. So a substantial claim about the nature of our epistemic abilities is offered as the basis for accepting that no claim of the form “\( n \) things exist” is immune from doubt. Is this really any less substantive than the claim relied by the PM account that there is an existent infinity? I think not.

I believe that there is a serious question about whether a truth non-simpliciter account such as the PM+PW account does any better than a truth simpliciter account such as the PM account in satisfying (E). If the invalidity of \( n \)-invalid arguments either requires the truth of a substantive claim dealing with either how many first-order individuals there are or how many there could be, then it is unclear why the epistemic feature of validity is not respected by a truth simpliciter account of logical consequence which ignores worlds alternative to the actual one in characterizing logical consequence. In such a case, it is hard to see what can be gained epistemologically by making an account of first-order logical consequence depend on the much more obscure question of the correct representation of the metaphysical or epistemological modalities.
Is there a way to interpret the modality in the PM+PW account in such a way that an infinite domain is necessarily possible (i.e., (M) is true) but that this fact is a non-substantial one? Note that if we appeal to a primitive notion of logical possibility, then there is no need to appeal to possible meaning assignments and the PM+PW account ought to reduce to a PW account according to which the premises and negated conclusion of a valid argument are true in no logically possible world. Another approach is to appeal to a notion of semantic possibility, which is already implicit in the PM account.\textsuperscript{16}

Recall that on the PM account, a possible use of the non-logical terminology of a language is an assignment of actually existing objects and sets of objects to the language’s singular terms and predicates. To quantify over all possible uses of the non-logical terminology in this narrow sense is to consider all possible semantic values of expressions, given the way the world actually is. Suppose it is fact about ‘greater’ that given the type of expression it is, it is capable of having an infinite extension. If there isn’t an existent infinity, then the narrow notion of possible use will not reflect this fact about the possible uses of expressions of that type. To capture it, we must quantify over all possible uses in the broad sense, and consider all possible semantic values of non-logical expressions in all (semantically) possible worlds. To say that it is semantically possible for ‘greater’ to have an infinite extension is to say that if the world contained an infinite number of individuals, then ‘greater’ could have an infinite set as its semantic value. Even if the world were necessarily finite, this subjunctive would still state a fact about the semantic functioning of ‘greater’. On this line of thinking, the modal fact required to underwrite (M) is a semantic one, not a substantive one. To say that there is an infinite semantically possible world is just to say that nothing about the semantic functioning of the language rules out a possible use for it according to which it has an infinite domain. If (E) is to be respected, truths about semantically possible worlds must be independent of commitments about metaphysical possibility. For example, whether ‘greater’ could have an infinite extension must be independent of whether an
existent infinity is metaphysically possible. I reject this. I do not see that a semantic notion of possibility is sufficient to carve out the space of possibilities needed to ground (M).

On my view, a characterization of logical consequence in terms of meaning assignments in possible worlds is substantive, because facts about the possible semantic values of expressions are not independent of ontological commitments. As mentioned earlier, the possible uses of terms are assignment of objects and sets of objects to the languages singular terms and predicates. That these meaning assignments are successful depends on the existence of these objects and sets of objects. So, the very notion of a possible use or meaning of a term relies on substantive claims.

More precisely, letting $\alpha$ be a first-order particular or a set of first-order particulars, there is a possible use for a term $t$ according to which it designates $\alpha$ only if

(i.) nothing about the semantic functioning of $t$ rules out it referring to $\alpha$, and
(ii.) $\alpha$ could exist (if $\alpha$ exists, then (ii.) is satisfied).

Only if both (i.) and (ii.) are satisfied can $t$ be used to pick out $\alpha$. (i.) is a semantic claim that is independent of metaphysical commitments and is non-substantive.\textsuperscript{17} There is nothing about the semantic functioning of ‘Bill Clinton’ qua individual constant that rules out it designating an impossible thing like a circular square. But ‘Bill Clinton’ cannot be used to refer to a circular square or to the greatest prime number because such things cannot exist.\textsuperscript{18}

So on my view, the claim that a term $t$ can be used to pick out $\alpha$ is partly semantic (i.) and partly metaphysical (ii.) Since, possible uses for linguistic items are limited by possibilities for real change, the possibility of using a predicate to pick out an existent infinity is parasitic on the possibility of an existent infinity. Accordingly, it is inconsistent for the finitist to maintain that there is necessarily a finite number of objects in the world and hold that argument (B) is invalid because ‘greater’ could be used to pick out a relation with an infinite extension. Again, whether, ‘greater’ could have had an infinite extension is not independent of whether an infinite number of objects could exist. So, since she thinks that such a totality is impossible, the finitist cannot countenance a possible use for ‘greater’ which designates such a totality.
In sum, since the mathematical contents of the actual world are necessary, all n-invalid arguments are truth simpliciter invalid in each possible world. Therefore, the criticism that truth simpliciter accounts fail (M) and thereby miss the modal feature of the concept of logical consequence doesn’t get off the ground. Furthermore, I don’t see a way of uncovering the modal feature in way that is in sync with (E). My knowledge of the possible size of the universe seems just as non-linguistic as knowledge of its actual size. This makes me skeptical of (E) and the logical positivist picture of logical epistemology that is illustrated by the locked room metaphor.

Etchemendy writes that the relation [of logical consequence] emerges from the semantic characteristics of the relevant language (1988b, 10). I believe that this is wrong if it requires that the extension of validity not turn on non-linguistic facts. I’ve distinguished between linguistic facts and non-linguistic facts and taken (E) to claim that only the latter are non-logical. I’ve argued that both truth simpliciter and truth non-simpliciter accounts of logical consequence appeal to non-linguistic facts. This seems to undercut the import of (E). However, one might argue that while some non-linguistic facts are logical (i.e., some modal facts), the actual size of the universe is not one of them. I am not confident about the success of such an argument because I don’t know of a non-question-begging way of marking the required distinction between those extra-linguistic facts that are logical and those that are not. For example, one might say that logical facts are those that determine the extension of validity (and of the other logical properties). But “validity” according to which account? Presumably, the demarcation of logical from non-logical facts presupposes an account of validity. I don’t see the force of the claim that although some logical facts are extra-linguistic, the actual size of the universe is not one of them, because such a claim presupposes the inadequacy of truth simpliciter accounts.

It is unclear to me why an account of logical consequence that makes knowledge of the invalidity of n-invalid arguments turn on knowledge of the size of the universe is intensionally inadequate. Perhaps, it is that an account that makes the extension of the logical consequence relation turn on facts about the extra-linguistic world misses features of the intuitive, pre-theoretic
construal of validity, not previously touched on here but which are often highlighted by Etchemendy (e.g., 1990, 85, 89, and 97).

1. The validity of an argument is knowable \textit{a priori};  
2. The connection between the true premises of a valid argument and its conclusion is analytic; and  
3. The truth of the premises in a valid argument guarantees the truth of the conclusion.

Granting that 1-3 characterize the intuitive concept of logical consequence, it seems to me that they can be understood so that a truth simpliciter account such as the PM account captures all three.\textsuperscript{19} It may be plausibly argued that knowledge of possible meanings and knowledge of the existence of sets is \textit{a priori}, hence the extension of PM-validity is knowable \textit{a priori}. I take the import of (2) to be the idea that just by knowing the meanings of the logical-terminology occurring in a valid argument one can know that the conclusion is true on the basis of knowing only that the premises are true.

But this rules out the PM account of logical consequence only if knowing the meanings of logical terminology does not require knowing whether non-linguistic states of affairs obtain. This may be questioned. The quantifier counts as a logical term, and at least part of its meaning is the domain over which it ranges. Since the elements of such domains must be items of the world, if there is necessarily just one thing then ‘At least one x is a P’ means ‘for all x, x is a P’ in which case the connection between the premise of argument (C)

(C)  \[ \exists x \text{ Happy}(x) \]

\[ \therefore \forall x \text{ Happy}(x) \]

and its conclusion is analytic. It is because there exists (or could exist) more than one thing that the meaning of ‘at least one’ does not include ‘all’, and (C) is invalid. For those non-Quineans who balk at making analyticity turn on non-linguistic matters, note that Etchemendy’s view (as presented in this paper) denies an analytic connection between the premises and conclusions of all \( \neg \)-invalid arguments on the basis of an intuition that an existent infinity is (necessarily) possible. But this intuition does not seem to be about the meanings of words.
Etchemendy does not fully develop (3), and so when he faults an account of logical consequence for missing it, it isn’t clear what exactly has been missed. The strength of the guarantee turns on the underlying modality operative in the notion of logical consequence: the less likely the possibility of true premises and false conclusion, the stronger the guarantee of a true conclusion given true premises. It is not clear to me why the yield of truth simpliciter accounts of logical consequence would increase if the world were finite *per impossible* shows that a truth simpliciter account such as the PM one does not offer the proper guarantee. I guess the idea is that we cannot have confidence in the account’s determination of the validity of any argument if it can be wrong about the invalidity of n-invalid arguments.

But even if we grant that logic does take an absolute epistemic priority over all other knowledge, why think that this strengthens the guarantee offered by a valid argument? Suppose that the intuition that the axiom of infinity could logically be true is basic and non-substantial. Couldn’t the intuition still be erroneous? The ancient Greeks and Intuitionists have questioned the coherence of the notion of an actual infinity. Many others think that truth of the logical possibility of the axiom of infinity is not self-evident, and that the required evidence is less than conclusive. Field, for example, points to the fact that the inability to derive a contradiction from fairly rich mathematical theories in which the axiom of infinity or (its equivalent) is a theorem (e.g., Zermelo-Fraenkel or Von Neumann-Godel-Bernays) counts as evidence for the possible truth of the axiom of infinity (1984, 520-521). But this is inductive evidence about what is derivable in a theory, and the claim that the axiom of infinity is logically possible has the epistemic status of a hypothesis. Etchemendy assumes, I think, that the epistemic transparency of the possibility of an existent infinity is entailed by the fact that it is non-substantial. But this is not obvious, and it is not acknowledged by others (e.g., Field) who agree with Etchemendy that knowledge of logical possibility is prior to knowledge of mathematical existence claims. If we grant—as it is becoming increasingly common to grant—that calling the intuition of a possible infinity “logical” does not *ipso facto* make it epistemically privileged *vis-à-vis* intuitions about
the existence of an infinite set, then one will be comfortable with the guarantee offered by truth simpliciter validity, and, perhaps, wonder what all the fuss is about with thinking that logic is substantial.

**Conclusion**

The two criticisms of truth simpliciter accounts considered here do not motivate incorporating an appeal to alternative ways the world might be in an account of first-order logical consequence. The appeal to truth simpliciter does not render an account of first-order logical consequence extensionally incorrect. Furthermore, I do not see how the intensional inadequacy of such an account can be established in the context of first-order logic with identity.

Of course, issues arise when we consider the adequacy of using truth simpliciter to characterize the logical consequence relation outside of first-order logic. One issue is what qualifies a term as logical. There is greater pressure on a truth simpliciter account than on a truth non-simpliciter one not to add, say, *is a US president* and *is a female* to the list of standard first-order logical constants. If these terms are treated as logical ones, then, since there is no way to understand “Kelly” that will yield a true premise and a false conclusion in the argument,

Kelly is a female
\[ \therefore \text{Kelly is not US President}, \]

the argument is truth simpliciter valid since an appeal to worlds alternative to the actual one is ruled out. But this doesn’t seem right for it is just an empirical, contingent truth that there has only been male US presidents (see Etchemendy 1990 for discussion). When it comes to considering possible extensions of first-order logic, a proponent of a truth simpliciter characterization of logical consequence should have some story about what qualifies a term as logical. The challenge is to tell the story so that the extensional adequacy of the resulting truth simpliciter account is preserved. Nothing close to universally accepted criteria for logical constancy has been produced.
Also, a truth simpliciter characterization of second-order logical consequence makes highly abstract set-theoretic facts logical facts. That the ontological implications of logic are so robust makes some uncomfortable. Quine (1986) thinks that this is evidence that second-order logic isn’t logic, Etchemendy (1999) thinks that it shows that a truth simpliciter characterization of second-order consequence is bankrupt. Shapiro (1991) defends the status of second-order logic, characterized in the truth simpliciter way, as “logic”. The issues are complicated and cannot be explored here.

Surely, first-order logic with identity is logic if anything is. So, if defects of truth simpliciter accounts emerge here, then the case for bringing in worlds alternative to the actual one is sealed without having to move to more complex scenarios. We would have to grant that our explanation of logical consequence turns on the difficult task of spelling out the requisite non-reducible modal notion. What I am pointing to in this paper is that when it comes to characterizing first-order logical consequence the necessity of accepting this cost and abandoning the plain truth is far from clear. In further work, I shall discuss the interplay between criteria of logical constanthood and accounts of logical consequence, and make the case for appealing to truth simpliciter in characterizing logical consequence in logical settings outside of first-order logic.\textsuperscript{21}

\textsuperscript{1} See Quine (1986), Maddy (2002), Putnam (1971), Sher (1991), and Shapiro (1993).

\textsuperscript{2} Here I use the examples Hanson uses (1997, 368) to support his claim that truth simpliciter accounts are extensionally incorrect in first-order logic with identity.

\textsuperscript{3} For a sketch of how Quine establishes the extensional equivalence of the substitutional and model-theoretic definitions of logical truth and for a general defense of Quine’s substitutional approach to logical truth against published criticism see my (2004).

\textsuperscript{4} While Hanson ignores this, other critics of Quine’s substitutional account acknowledge it. See, among others, Shapiro (2000, 339) and Boolos (1975, 525-526). In fairness to Hanson, he focuses on Quine’s presentation of first-order logic with identity as given in Quine’s (1972). There, in the
introduction (1972, 4), Quine asserts that ‘=’ is a logical term and remarks that statements of such form as ‘x=x’ are logically true and that logical truths are true in virtue of their form.

Hanson identifies what I am calling the PM account as the interpretational version of the formal account of logical consequence (1997, 367 and note 5). It is also known, via Etchemendy (1990), as interpretational semantics. My first acquaintance with what I am calling the PM account is in Lehmann (1980) where it is so named.

Note that this is not incompatible with Kripke’s thesis that proper nouns necessarily tag their referents. For example, acknowledging possible uses for ‘Bill’ does not rule out that one of them is to necessarily pick out Bill Clinton.

That the context determines that ‘everybody’ means all those pictured and not everybody in the universe is controversial. For example, according to Bach (2000), the utterance ‘Everybody is large’ semantically expresses the proposition that everybody in the universe is large, and conveys as—what he calls—an impliciture the proposition that everyone in the picture is large. Contrary to the story I tell, on Bach’s view the context does not contribute to the meaning of ‘Everybody’, and what is literally meant by ‘Everybody is large’ (which is false on Bach construal of the meaning of ‘Everybody’) is not what the speaker intends to convey. The claims made in my paper that a possible use of an ordinary language quantifier is a possible meaning for it and that a use (and, therefore, meaning) of an ordinary language quantifier is determined by context puts me in the camp with those who, against Bach, believe in some kind of context-dependent domain restrictions on ordinary language quantifiers, e.g., Neale (1990), and Stanley & Szabo (2000) who differ on how the restriction is expressed in a quantification—either with some kind of tacit syntactic element or with a semantic element.

There is reason to think that PM-validity is not co-extensive with classical first-order validity, which rules out the empty domain thereby making, for example, ‘∃x(x=x)’ a logical truth. On the PM approach, the possible uses for variables are to range as widely as possible. Since ranging
over the empty set is a possible use for variables, ‘∃x(x=x)’ could (logically) be false because it is in fact false when the quantifier is so used. Clearly, the legitimacy of this use is independent of the issue of whether the universe could (in a metaphysical sense) have been empty. See Lehmann (1980) who uses the PM account to motivate a free logic.

9 This feature of the PM account doesn’t seem to be universally acknowledged. For example, Read (1994) argues against the account as follows. If we allow substitutions on ‘taller than’, we can’t explain why ‘Iain is taller than Bill and Bill is taller than Mary’ entails ‘Iain is taller than Mary’. But if ‘taller than’ is held constant, then from ‘∀x∃y¬(x is taller than y)’ we can infer the truth of ‘∃x∃y (x≠y & x is taller than y)’. Either way, the PM approach generates counterintuitive results. But this is overly harsh, because the PM approach can account for the invalidity of the last argument by appealing to the fact that the quantifiers could be used to range over a domain consisting of, say, two equal sized twins. Again, the PM account allows restrictions on the world’s domain in order to change the truth values of sentences in fixing the extension of validity and logical truth. Read may reply that his dilemma is aimed at the Tarskian approach to validity (which he does discuss.) However Read’s remarks (ibid.) on extensions of the Tarskian approach and on pg. 255 on completeness of interpretational semantics suggest he is also thinking of PM-validity.

10 See Definition 23, p. 195, and Theorem 26, p. 207, in (1933). Thanks to Bill Hanson for the reference.

11 If there is a model of a set of sentences (i.e., an interpretation that makes all the sentences true relative to a domain), then there is a possible use of the non-logical elements of the relevant language according to which the sentences are true. However, possible uses outrun models, at least as standardly construed. For example, it is thought that ‘∀x(x=x)’ can be used to say that absolutely everything is self-identical and that ‘∀x∃y Greater(y,x)’ can be used to say that for each ordinal there is a greater one. But standard models cannot represent such uses of the
quantifiers since their domains must be sets (and not proper classes). However, models are adequate in representing the uses of quantifiers according to which they range over collections that are sets. For useful discussion of how the requirement that domains be sets makes the model-theoretic characterization of validity problematic see Blanchette (2000), McGee (1992a), as well as Etchemendy (1990). Blanchette (2000) distinguishes between models of and readings of uninterpreted sentences. The latter, but not the former, assign propositions to such sentences. Her notion of a reading has certain affinities with what I am calling a possible use of a sentence: a possible use of a sentence is a reading of the sentence which assigns that sentence a proposition true or false relative to actual (non-modal) states of affairs.

12 The main focus of Etchemendy’s criticism is on the Tarski/Bolzano tradition of characterizing logical truth and validity in terms of uniform substitutions to some selected class of terms (e.g., the non-logical constants) while holding the world and also the domain of individuals fixed. His criticism echoes earlier criticism in Kneale (1961). Etchemendy holds that on Tarski’s account of logical truth (as set out in Tarski’s (1936)), sentences such as ‘∃x∃y(x≠y)’ are logically true granting that the quantifiers and identity are logical constants. Based on this, Etchemendy lodges a Hanson-like criticism against Tarski’s account (1990, and 1988b, 64-74). Some recent authors emphasize the relativized concept of truth in Tarski over the absolute one (see p. 16 above) in order to defend the Tarskian account of logical consequence against this criticism (e.g., Ray 1996 and Sher 1996). As is clear from the above, in contrast to Etchemendy’s version of Tarski’s account, on the PM view of logical truth and consequence, the fact that the quantifiers are logical constants implies that the part of their meaning that is constant from interpretation to interpretation is the portion of the domain they pick out (e.g., the existential quantifier refers to at least one member of the domain.) The part of the meaning that varies under interpretation is the size of the domains, so ‘∃x∃y(x≠y)’ is not logically true on the PM approach.
Etchemendy acknowledges that (what I am calling) an interpretational truth simpliciter account can make all existentially invalid arguments invalid by varying the domain of discourse (1990, 112-113). But he regards this “new twist” at best unmotivated and at worst inconsistent with Tarski’s reductive account of logical consequence according to which logical consequence is reduced to facts about how the world actually is and not how it could have been (1999, 11). I believe that my exposition in this paper of the PM approach counters this viewpoint.

The PM approach incorporates the component of the Tarski/Bolzano tradition which fixes the upper bound of the domains we may consider in fixing the extension of validity and logical truth in terms of the upper bound of the collections of the world’s individuals. I believe that Etchemendy would think that this feature of the PM approach renders it intensionally inadequate for the reasons cited by the second criticism of truth simpliciter accounts discussed below.

13 Since x and y can instantiate one individual, the pair-set axiom guarantees the existence of the singleton set.

14 For example, see Tarski (1995, 117-125); for historical discussion see Corcoran and Wood (1980).

15 A possible world would represent one way the world might, for all we know, be.

16 Thanks to John MacFarlane for suggesting this approach to me. The content of next paragraph is heavily indebted to him.

17 There is a problem with fleshing out (i.) as a subjunctive conditional—if α could exist, then α could be the semantic value of t. According to some popular theories, the truth values of such subjunctive conditionals are dependent on the structure of metaphysical possibility. But the account here needs to understand them as making claims that are independent of the metaphysical modal facts.
18 The examples of semantic facts used to illustrate Etchemendy’s notion of non-substantiality are facts about the functioning of non-logical and logical terms, and are what ground specific instances of (i).

19 Amazingly, Etchemendy does not elaborate on his view of “analyticity” despite acknowledging that “it is notoriously difficult to pin down,” (1990, 97) and he tells us that the guarantee offered by a valid argument is “vague and poorly understood” (1990, 85) without ever revealing his explanation of it. But elaboration is required since, as I show below, 1-3 can be understood in such a way that an explanation of validity that makes it substantive accounts for all three.

20 I do not believe that there is anything obviously incoherent in the possibility of an existent infinity. But I agree with Resnik (1985, 173) that statements of the possibility of infinite collections are hypotheses, and, therefore, could turn out false.

21 For discussion of what the PM version of a truth simplicity account of logical truth in first-order modal logic looks like and how it satisfies the logical modality thesis (above on p. 22) see my “A Defense of the Kripkean Account of Logical Truth in First-Order Modal Logic”, (2005) forthcoming.

Bibliography


