

## **What if the Principle of Induction is Normative?**

### **Formal Learning Theory and Hume's Problem**

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#### **Abstract**

This essay argues that a successful answer to Hume's problem of induction can be developed from a sub-genre of philosophy of science known as formal learning theory. One of the central concepts of formal learning theory is logical reliability: roughly, a method is logically reliable when it is assured of eventually settling on the truth for every sequence of data that is possible given what we know. I show that the principle of induction (PI) is necessary and sufficient for logical reliability in what I call simple enumerative induction. This answer to Hume's problem rests on interpreting the PI as a normative claim justified by a non-empirical epistemic means-ends argument. In such an argument, a rule of inference is shown by mathematical or logical proof to promote a specified epistemic end. Since the proof concerning the PI and logical reliability is not based on inductive reasoning, this argument avoids the circularity that Hume argued was inherent in any attempt to justify the PI.

## 1. The Problem

David Hume's infamous problem of induction is an argument that can be concisely stated as follows:<sup>1</sup>

1. The Principle of Induction (PI) is a premise in any inductive argument.
2. The PI is a statement concerning either relations of ideas or matters of fact.
3. If the PI concerns relations of ideas, then its denial is a contradiction.
4. But the denial of the PI is not a contradiction.
5. If the PI concerns matters of fact, then it must be justified by an inductive argument.
6. Therefore, there is no non-circular justification of the PI.

Hume formulated the PI as the statement that the future will resemble the past or that the course of nature is uniform. There have been a number of critiques of Hume's problem,<sup>2</sup> for example, several authors have argued that the first premise of Hume's problem is false, since many inductive arguments rely upon domain specific background knowledge rather than the PI.<sup>3</sup> Nevertheless, I agree with Marc Lange<sup>4</sup> that Hume's problem does pertain to enumerative inductions in which detailed background knowledge indicating what to expect in light of past observations is substantially incomplete, a situation that often occurs in science and ordinary life.

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<sup>1</sup> See the *Enquiry* (Section IV, part II) and the *Treatise* (Book I, Part III, Section VI). Of course, the label "Hume's Problem" is not intended to suggest that the argument is a problem *for* Hume's own position. The label indicates that the argument is a problem posed *by* Hume for those who claim that inductive inference has a rational basis in something other than habit or custom.

<sup>2</sup> See A. Hayek and N. Hall, "Induction and Probability," in P. Machamer and M. Silberstein, *The Blackwell Guide to the Philosophy of Science* (Oxford: Blackwell Publishers 2002), pp. 149-172, J. Ladyman, *Understanding Philosophy of Science* (New York: Routledge 2002), pp. 40-52, C. Howson, *Hume's Problem* (Oxford: Oxford University Press 2000), and W. Salmon, *The Foundations of Scientific Inference* (Pittsburgh: University of Pittsburgh Press 1967) for surveys.

<sup>3</sup> See J. Norton, "A Material Theory of Induction," *Philosophy of Science*, LXX (2003): 647-70, and S. Okasha, "Does Hume's Argument against Induction Rest on a Quantifier-Shift Fallacy?" *Proceedings of the Aristotelian Society*, CV (2005): 237-255.

<sup>4</sup> Lange, "Okasha on Inductive Skepticism," *Philosophical Quarterly*, LII (2002): 226-32.

In this essay, I argue that a successful answer to Hume's problem of induction can be developed from a sub-genre of philosophy of science known as formal learning theory.<sup>5</sup> One of the central concepts of formal learning theory is logical reliability: roughly, a method is logically reliable when it is assured of eventually settling on the truth for every sequence of data consistent with our background knowledge. Thus, logical reliability is an epistemically desirable feature for a method to have, since a logically reliable method will (if given sufficient data) eventually permanently settle on the correct hypothesis, no matter which one that turns out to be. I show that the PI is necessary and sufficient for logical reliability in what I call simple enumerative induction, a type of inference commonly used to illustrate Hume's problem. Given that the PI is interpreted as a normative claim, this result becomes the basis for a *non-empirical epistemic means-ends* justification of the PI. The justification is *non-empirical* because the link between means and end—the necessity and sufficiency of the PI for logical reliability in simple enumerative induction—is established entirely by a logical proof. And it is *epistemic* because the end—logical reliability—concerns the attainment of truth rather than such things as power, fame, or fortune.

I claim that this reasoning shows Hume's problem to be a false dilemma. Hume's problem asserts that the PI must either be a relation of ideas whose denial is a contradiction or an empirical hypothesis that could only be justified by inductive reasoning. The third possibility that the PI is a normative statement justified by a non-empirical epistemic means-ends argument is not considered by Hume. In this case, the denial of the PI is not a contradiction, since it could be consistently rejected by a person

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<sup>5</sup> The classic text in this sub-genre is K. Kelly, *The Logic of Reliable Inquiry* (Oxford: Oxford University Press 1996); also see E. Martin and D. Osherson, *Elements of Scientific Inquiry* (Cambridge: MIT Press 1998).

who valued other epistemic ends (such as never holding a false belief) more highly than logical reliability. And since the link between the PI and logical reliability is forged by a logical deduction, there is no threat of circularity. I consider and respond to several objections that might be made to this critique of Hume's problem, including some related to Nelson Goodman's new riddle of induction.

## **2. Non-Empirical Epistemic Means-Ends Arguments**

Means-ends arguments recommend a course of action on the grounds that it will contribute to the attainment of a desirable end. For example, you should eat your broccoli because it contains anti-oxidants that lower your LDL cholesterol level. As this example illustrates, means-ends arguments often depend on causal knowledge, for instance, about the effects of diet upon health. Consequently, a means-ends argument might not seem like a very promising line of response to Hume's problem. As many instructors of introductory philosophy courses have had occasion to explain, it is no answer to Hume to argue that we should believe the PI because we wouldn't live long otherwise. For any connection between longevity and believing the PI could only be established by inductive generalization from past experience, which presupposes the very thing that Hume's problem calls into question.

But not all means-ends arguments rest on empirical generalizations about cause and effect. Consider this example.

1.  $611 \div 13 = 47$ .
2. Sue wants to divide \$611 equally among 13 people.
3. Therefore, Sue should give \$47 to each person.

This is an example of what I call a *non-empirical means-ends argument*. In a non-empirical means-ends argument, the connection between means and end is established solely by a logical or mathematical proof. No inductive generalization from past observations is required to show that giving \$47 to each person is the means for Sue to attain her goal of dividing \$611 evenly among 13 people. All that is needed is a little arithmetic.

None of the means-ends arguments we have considered so far have concerned an epistemic aim. I follow Alvin Goldman<sup>6</sup> in characterizing epistemic ends as those having to do with the attainment of truth. For example, the goal of discovering the true theory of gravitation would count as epistemic, but so would the aim of making accurate predictions. After all, accurate predictions are true statements about the future, or statements that are within some specified distance of the truth (e.g. tomorrow's temperature plus or minus one degree Celsius). An epistemic means-end argument, then, recommends a rule of inference on the grounds that it contributes to the attainment of truth. In the next section, I present a non-empirical epistemic means-ends argument for the PI with regard to an inductive problem that I call simple enumerative induction. In this argument, the PI is shown, by reasoning that relies solely on logical deduction, to further an epistemic end, namely, logical reliability. I claim that this argument constitutes a non-circular justification of the PI in this context, and hence that Hume's problem is unsound.

The previous answer to Hume's problem that comes closest to what I propose is perhaps Hans Reichenbach's<sup>7</sup> pragmatic "vindication of induction." But unlike the

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<sup>6</sup> Goldman, *Knowledge in a Social World* (Oxford: Oxford University Press 1999).

<sup>7</sup> Reichenbach, *Experience and Prediction* (Chicago: University of Chicago Press 1938).

approach advocated here, Reichenbach's vindication attempts to justify the descriptive generalization that relative frequencies quickly converge to limits, which he regarded as a formulation of Hume's principle of the uniformity of nature.<sup>8</sup> Reichenbach suggested that we have much to gain and nothing to lose by accepting this proposition. Yet since a generalization about relative frequencies quickly converging to limits is an empirical hypothesis, it is unclear how Reichenbach's vindication can avoid the circularity that Hume's problem emphasizes. And the suggestion that we may as well accept the PI since we are lost otherwise amounts to little more than conceding the case to Hume. The analysis of Hume's problem I propose is also not found in the literature on formal learning theory, in large part I suspect because those working in this genre have tended to misinterpret Hume's argument. As Samir Okasha points out,<sup>9</sup> there are two main readings of Hume's problem. According to the first of these, Hume's problem merely amounts to the assertion that deductive arguments are the only legitimate kind. I call this the *deductive chauvinist* interpretation of Hume's problem. The deductive chauvinist interpretation characterizes Hume's problem as asserting that, since inferences about the future based on past experience are not deductive, no such inferences can be rationally justified. According to the other interpretation, Hume does not assume that deductive arguments are the only legitimate form of justification. Instead, Hume attempts to show that no non-circular justification can be given for the PI. I call this the *circularity* interpretation of Hume's problem.

The circularity interpretation is certainly the more charitable one, since Hume's problem of induction would amount to little more than a colossal instance of begging the

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<sup>8</sup> Reichenbach, *The Theory of Probability* (Berkeley: University of California Press 1949), p. 473.

<sup>9</sup> Okasha (*op. cit.*), pp. 254-7.

question if the deductive chauvinist interpretation were correct. Clearly, deductive chauvinism cannot be a premise in a non-circular argument that aims to call inductive inference into question. Moreover, the non-circularity interpretation is more faithful to Hume's text in the relevant passages of the *Treatise* and *Enquiry*, and with the fact that Hume frequently indulged in inductive generalizations himself. So, I agree with Okasha and others<sup>10</sup> that the circularity interpretation of Hume's problem of induction is the correct one. Yet the deductive chauvinist interpretation of Hume's problem is common in the formal learning theory literature. For example, this interpretation occurs in some crucial passages of Reichenbach's pragmatic vindication of induction, which is one important source of inspiration in formal learning theory.<sup>11</sup> According to Reichenbach:

Hume believed that any justified application of the inductive inference presupposes a demonstration that the conclusion is true. It is this assumption upon which Hume's criticism is based.<sup>12</sup>

One of the leading exponents of formal learning theory, Kevin Kelly, similarly characterizes Hume as a deductive chauvinist.

In the *Meno*, Plato seems to assume that inquiry must be logically guaranteed to terminate with certainty that its answer is correct. ... This very strict version of logical reliabilism is still evident in the work of Descartes, *Hume*, and Kant, over two thousand years later.<sup>13</sup>

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<sup>10</sup> See Howson (*op. cit.*), Salmon (*op. cit.*), and B. Stroud, *Hume* (London: Routledge 1977).

<sup>11</sup> For example, see C. Juhl, "The Speed-Optimality of Reichenbach's Straight Rule of Induction," *British Journal for the Philosophy of Science*, XLV (1994): 857-63, and Kelly, "Reichenbach, Induction, and Discovery," *Erkenntnis*, XXXV (1991): 123-49.

<sup>12</sup> Reichenbach (1938), p. 348.

<sup>13</sup> Kelly (*op. cit.*), p. 3; italics added.

Kelly's formulation of Hume's problem straightforwardly reflects this interpretation of Hume.

This is precisely Hume's argument for inductive skepticism. No matter how many sunrises we have seen up to stage  $n - 1$ , the sun may or may not rise at stage  $n$ .<sup>14</sup>

That is, according to Kelly, Hume's problem of induction is merely the observation that the conclusion of an inductive argument may be false even if its premises are true. The deductive chauvinist reading of Hume is also suggested in the writings of some of Kelly's former students. For instance, according to Oliver Schulte, "David Hume observed that all such [inductive] generalizations are uncertain and hence involve a risk of error, and concluded that none has normative justification over and above our customs or inferential habits."<sup>15</sup> Of course, the finer points of Hume exegesis are not a central concern of formal learning theory. But a correct interpretation of Hume's problem of induction is important if one's aim is to diagnose the point at which Hume's reasoning goes awry.

### 3. Logical Reliability and Simple Enumerative Induction

A simple type of inductive problem has the following form. All objects of a particular type, call it A, observed so far also have the property B, and the question is whether all A's *that will ever be observed will have B*. Moreover, since there is no upper limit on the number of A's that can be observed, there are always more yet to be seen. Hence, no matter how many A's are observed to have B, it is always possible that the next A to be seen will not have B. I call inductive problems of this type *simple enumerative induction*,

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<sup>14</sup> Kelly (*op. cit.*), p. 46-7.

<sup>15</sup> Schulte, "Means-Ends Epistemology," *British Journal for the Philosophy of Science*, L (1999): 1-31.



and many standard examples given to illustrate Hume's problem (e.g. will fire always burn?) are of this form. The claim that the charge of every electron is  $-1.602 \times 10^{-19}$  coulombs provides a more scientifically interesting illustration of simple enumerative induction. This is true of all electrons observed so far, but is it true of all that will ever be observed? In this section, I show that in simple enumerative induction following the PI is necessary and sufficient for logical reliability.

In formal learning theory, an inductive problem is defined by a set of alternative hypotheses  $H$  and a set of infinite sequences of data  $D$ . In simple enumerative induction,  $H$  consists of a universal generalization and its negation, for instance, in the electron example  $H = \{\text{every electron has a charge of } -1.602 \times 10^{-19} \text{ coulombs; not every electron has a charge of } -1.602 \times 10^{-19} \text{ coulombs}\}$ . The set  $D$  consists of all sequences of data consistent with our background knowledge. A sequence of data can be represented by an infinite string of 1s and 0s. For convenience, I will let 1s denote positive instances of the universal generalization and 0s denote counter-instances, for example, 1 could represent a the observation of an electron whose charge is  $-1.602 \times 10^{-19}$  coulombs and 0 the observation of an electron with a different charge. The sequence of data in which the charge of every electron is  $-1.602 \times 10^{-19}$  coulombs, then, would consist of an infinite sequence of 1s, while any sequence of which this generalization is false would contain one or more 0s. Although there are many sequences of data in  $D$ , only one is the *actual sequence*. In simple enumerative induction, we do not know at the beginning of inquiry whether the universal generalization is true or false, and the task is to learn whether it is true of the actual sequence. Since the truth or falsity of the universal generalization is being considered only in relation to the actual sequence, it would not be falsified by an

eternally unobserved counter instance. However, since data sequences are infinitely long, the learner never observes the entire actual sequence but only a continually lengthening initial segment of it that we can call the *data so far*. Simple enumerative induction, then, falls within the scope of Hume's problem: we can make as many observations as we like, but no matter how many positive instances we see it is always possible that the next observation will refute the universal generalization.

A rule, then, is a procedure for conjecturing or concluding hypotheses from  $H$  given the data so far. More precisely, I will say that a *strict rule* is a function from the data so far to subsets of  $H$ . Given the data so far, a strict rule outputs a subset of  $H$ , the disjunction of whose members the rule is said to *indicate*. In the electron example, if the data so far consist of a segment of one thousand 1s, a strict rule might output {every electron has a charge of  $-1.602 \times 10^{-19}$  coulombs} and hence *uniquely indicate* that hypothesis (i.e. indicate it and no others). If the rule suspended judgment, it would indicate the uninformative disjunction "every electron has a charge of  $-1.602 \times 10^{-19}$  coulombs or not every electron does." I consider only two types of indicating here: conjecturing and concluding. To conjecture is to indicate in a tentative manner, while to conclude is to indicate while stating that no further data are required to decide which hypothesis is true. Rules in general often involve open-ended expressions that can be interpreted in a number of ways in particular cases, and rules for making inferences from data are no exception. For example, formulations of the PI often make reference to a "large number" of instances, where the specific number that qualifies as large can vary from one context to another. A *rule*, then, is a set of strict rules. Typically, a rule is a schematic version of a strict rule in which some parameters, such as

the number of observations required before the universal generalization will be conjectured, are left free.

There are a number of different inference rules that one could follow in simple enumerative induction. One such rule is the PI.

Principle of Induction (PI): If a sufficient number of positive instances and no negative instances have been observed, conjecture that the universal generalization is true; if a counter instance is observed, conclude that the universal generalization is false.

Thus, the PI prescribes that one conjecture that the charge of every electron is  $-1.602 \times 10^{-19}$  coulombs if a sufficient number of electrons have been observed and all have that property. The PI as formulated here is very similar to familiar statements of the principle,<sup>16</sup> although the PI is often, though not always, formulated as a descriptive claim. Notice that the PI does not say how many positive instances are sufficient nor does it specify what to do prior to receiving a sufficient number of solely positive instances. An inductivist might think one positive instance is sufficient or she might think that ten thousand are. And prior to receiving a sufficient number of positive instances, she might conjecture that the generalization is true, conjecture that it is false, or suspend judgment. The PI, therefore, is an example of a rule. It marks out a set of strict rules that all share two features in common: (1) there is some  $n$  such that if  $n$  positive instances and no negative instances have been observed, then the universal generalization is conjectured, and (2) if a counter-instance is observed, then it is concluded that the universal generalization is false.

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<sup>16</sup> For example, see A. F. Chalmers, A. F., *What is this thing called Science?* (Indianapolis: Hackett Publishing 1999), 3<sup>rd</sup> edition, p. 47, B. Russell, *The Problems of Philosophy* (New York: H. Holt & Company 1912), and Ladyman (*op. cit.*), pp. 28-29.

The PI is far from the only rule that could be followed in cases of simple induction. Here is another.

Skeptic's Rule: If a counter instance is observed, conclude that the universal generalization is false; otherwise, suspend judgment.

Notice that the skeptic's rule is a strict rule, since it gives unambiguous guidance as to which subset of  $H$  to indicate for any data. The question, then, is whether there is some reason why one should follow the PI rather than the skeptic's rule, or indeed any other inference rule applicable to simple enumerative induction. The answer to this question is *yes*, provided that one accepts that it is desirable that a rule be *logically reliable*. Logical reliability is a property that a rule may have with respect to an inductive problem, defined by a set of hypotheses  $H$  and a set of sequences of data  $D$  consistent with background knowledge. Let  $d|n$  denote the initial segment of the data sequence  $d$  of length  $n$ . Then logical reliability can be defined as follows.

Logical Reliability: A strict rule  $r$  is said to be logically reliable with respect to an inductive problem  $\{H, D\} \Leftrightarrow$  for every sequence of data  $d$  in  $D$ , if  $h$  is the member of  $H$  true of  $d$ , then there is an initial segment of  $d$ ,  $d|n$ , such that  $r$  uniquely indicates  $h$  given  $d|m$ , for any  $m \geq n$ .

A set of strict rules (i.e., a rule full stop) is logically reliable just in case every strict rule in the set is. The important thing to notice about logical reliability is that it requires that the rule be assured of settling on the correct answer for *every* sequence of data that is possible given what we know. In contrast, a rule that settles on the correct answer for some data sequences but not others is not logically reliable, even if it happens to get things right for the actual sequence. Luck is not the same as reliability.

I now show that following the PI is both necessary and sufficient for a strict rule to be logically reliable in simple enumerative induction. As before, let  $D$  consist of all infinite sequences of 1s and 0s, where 1s represent positive instances and 0s negative instances of the universal generalization. Recall that in simple enumerative induction, the universal generalization is true if the actual data sequence consists of an infinite series of 1s and false otherwise. Now we can show that following the PI is *sufficient* for logical reliability in this case. If the universal generalization is true, then for any  $n$ ,  $n$  positive instances and no negative instances will eventually be observed. Hence, in this case, any strict rule consistent with the PI will eventually conjecture that the universal generalization is true and not change her conjecture thereafter. On the other hand, if the universal generalization is not true, then at least one 0 occurs at some point in the data sequence, and hence any strict rule consistent with the PI will eventually conclude that the universal generalization is false. Thus, all strict rules falling within the compass of the PI are assured of eventually settling on the true hypothesis in cases of simple enumerative induction.

Following the PI is also *necessary* for logical reliability in simple enumerative induction. To see this, notice that at least one of the following must be true of any strict rule that is inconsistent with the PI: (1) there is no  $n$  such that the strict rule conjectures that the universal generalization is true if at least  $n$  positive instances and no negative instances have been observed; (2) the strict rule does not reject the universal generalization when a counter instance has been observed. The skeptic's rule is an example of (1). Rules in this category are not logically reliable because they never uniquely indicate the universal generalization when it is true. For example, consider any

data stream in which the charge of every electron is  $-1.602 \times 10^{-19}$  coulombs. In any such data stream, the skeptic suspends judgment forever, and hence never uniquely indicates the true hypothesis. Moreover, it is clear that rules falling under (2) are also not logically reliable, since they might never conclude that the universal generalization is false even after it has been refuted by the evidence. So, the PI demarcates all and only those strict rules that are logically reliable in simple enumerative induction.

Thus, following the PI is necessary and sufficient for logical reliability in simple enumerative induction. Provided that logical reliability is an epistemically desirable feature for a rule to have, this provides a justification for the PI. Moreover, since the link between the PI and logical reliability is established by a logical deduction rather than inductive reasoning, there is no threat of circularity. It is also interesting to note that the above argument does not show that a person who rejects the PI is guilty of a contradiction. For example, the skeptic values avoidance of error above all else, even above learning the truth. The skeptic's absolute fear of being mistaken is surely idiosyncratic and perhaps even pathological, but it does not involve any logical contradiction. After all, a person who only believes what is known with complete certainty would never believe a contradiction or any other false claim. Thus, the argument proposed here not only avoids the circularity highlighted in Hume's problem, it also accommodates Hume's insight that no logical contradiction follows from denying the PI. What the argument *does* show is that, in simple enumerative induction, a person who rejects the PI is not very good at discovering the truth. Or to put it differently, the argument shows that the risk of being temporarily wrong is sometimes a price you must pay for the assurance of eventually being right.

#### **4. Hume's Problem is Unsound**

I claim that the justification of the PI presented in the foregoing section shows that Hume's problem of induction is an unsound argument, in particular, that it is a false dilemma. Hume claims that the PI must either be a relation of ideas whose denial is a contradiction, or a matter of fact that could only be justified by inductive generalization from experience. The discussion above shows that there is a crucial third possibility that Hume did not consider: the PI might be a normative statement justified by a non-empirical epistemic means-ends argument. In this section, I consider and respond to several objections that might be made to this critique of Hume's problem.

One might respond that Hume regarded the PI as inherently descriptive, so that the dilemma is genuine given his way of conceiving the matter. However, this move does not save Hume's problem from unsoundness because it makes its first premise—that the PI must be assumed by any inductive argument—false. Questions about what to infer from a set of data can be, and often are, answered by appeal to a rule, which is to say, a normative claim. Indeed, given this interpretation, the first premise is false even of the simple enumerative inductions to which Hume's problem seems most pertinent. Another possible objection is that simple enumerative induction is not the only inductive problem that falls within the scope of Hume's argument, and that for many inductive problems there is no logically reliable method. These claims are undeniably true, but they do not rescue Hume's problem. The discussion in this section shows that there is an inductive problem for which Hume's problem is a false dilemma. Hence, Hume's problem is unsound. The fact that there may be other inductive problems which satisfy the premises

of Hume's problem does nothing to change this. To demonstrate that an argument is unsound, it suffices to show that its premises are false in *some* cases that fall within its intended scope. And simple enumerative induction is not just any old case, but a type of example commonly used to illustrate Hume's problem. Furthermore, logical reliability is not the only epistemic aim worth considering, so non-empirical epistemic means-ends justifications may be useful for inductive problems for which no logically reliable method exists. The discussion of simple enumerative induction and logical reliability is just one example of a more general approach to justifying inductive rules.

A more serious objection is that proofs about reliability in the long run tell us nothing about what conclusions should be drawn from any finite set of data. For example, the PI allows one to wait for an arbitrarily large number of positive instances before conjecturing that the generalization is true. Moreover, one might suspect that judgments about how long to wait can only be based on substantive background knowledge, which threatens to reintroduce the circularity highlighted in Hume's problem. There are two responses that are pertinent to this objection. First, concerns about the practical relevance of the long run notwithstanding, logical reliability is an epistemically desirable feature for a rule to have. It is a good thing that an inference rule will be assured of eventually settling on the correct answer. What the objection shows is that logical reliability is not the *only* feature that matters: it is also important to think about the rule's performance in the short to medium term. Thus, so long as logical reliability is *one* epistemically desirable characteristic of a rule, the reasoning in the foregoing section has demonstrated that Hume's problem of induction is unsound.



Secondly, there are non-empirical epistemic means-ends arguments that have implications for what inferences should be drawn in the short term. Epistemic ends relevant to short term judgments can be specified and logical or mathematical demonstrations can be provided to show what is needed to achieve these ends. Although it is a characteristic feature of inductive problems that no method is guaranteed to settle on the true hypothesis in the short term, there might be other worthy epistemic ends that pertain to this situation. For instance, one might try to show that, among those methods that are logically reliable in a given case, some get to the truth more quickly or more efficiently than others. This approach has been pursued by several authors working in the formal learning theory genre,<sup>17</sup> and will be discussed in greater detail in the subsequent section in connection with Goodman's new riddle of induction.

Of course, prior empirical knowledge typically is relevant to our assessment of a hypothesis, and in some cases may give us good reason to suspect that it is false. For example, consider universal generalizations about personal characteristics associated with racial or ethnic groups (e.g., they are all lazy; they are all good at math, etc). Given the enormous variability of personal characteristics within human groups, there is excellent reason for skepticism about such claims. There are a number of ways that one might take background information into account consistent with the approach advocated here. Within the formal learning theory genre, background knowledge is typically interpreted as a restriction on the set of possible data streams. But in many cases, background knowledge favors some possibilities over others without unequivocally eliminating the less favored ones. For example, background knowledge might suggest that exceptions to

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<sup>17</sup> For example, see Juhl (op. cit.), Schulte, "The Logic of Reliable and Efficient Inquiry," *Journal of Philosophical Logic*, XXVIII (1999): 399-438, and Kelly, "A New Solution to the Problem of Simplicity," *Philosophy of Science*, LXXIV (2007): 561-573.

a universal generalization are probable, but not certain. Bayesian inference is one approach for addressing such cases. Bayesian inference can be treated as an inductive rule as discussed in section 3 given the following conventions. A Bayesian agent can be said to *conjecture* a hypothesis if her personal probability for that hypothesis given the evidence is strictly greater than .5. She *concludes* that a hypothesis is correct if her probability for that hypothesis given the evidence equals 1. As in section 3, evidence would consist of initial segments of data sequences. Given this set up, it is easy to show that a Bayesian agent is logically reliable in simple enumerative induction as long as she does not assign a prior probability of zero to the true hypothesis.<sup>18</sup> And since she is logically reliable, the Bayesian agent must follow the PI. Moreover, how many positive instances are sufficient to make her conjecture that the universal generalization is true will depend on her prior probability distribution, which in turn is presumably guided by her background knowledge. The less probable the universal generalization is in light of that background knowledge, the greater the number of positive instances that are required.

### 5. The New Riddle of Induction

It is sometimes claimed that Nelson Goodman's<sup>19</sup> "new riddle of induction" shows that the PI is inconsistent.<sup>20</sup> In Goodman's famous example, past observations of many green emeralds are positive instances of the hypothesis that all emeralds are green, but also are positive instances of the hypothesis that all emeralds are grue, where that predicate "applies to all things examined before  $t$  just in case they are green but to other things just

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<sup>18</sup> Howson (*op. cit.*), pp. 71-2.

<sup>19</sup> Goodman, *Fact, Fiction and Forecast* (Cambridge: Harvard 1955), pp. 69-75.

<sup>20</sup> Howson (*op. cit.*), p. 30.

in case they are blue.”<sup>21</sup> Thus, although both hypotheses agree that all emeralds observed before  $t$  are green, they disagree about whether emeralds first observed after  $t$  are green or blue. It might seem, then, that the PI would lead one to infer that future emeralds will be both green and blue. In this section, I argue that the new riddle of induction does not in fact demonstrate any inconsistency in the PI, but instead is another way of showing that reliability in the long run does little to constrain predictions in the short term.

Recall that the PI required that, in a case of simple enumerative induction, there be a finite number  $n$  of positive instances sufficient to make one conjecture the universal generalization. Goodman’s riddle illustrates that one might consider several universal generalizations at once. In this case, the PI requires that, for each universal generalization, there be a finite number of positive instances that suffice for it to be conjectured. *But the PI does not require that this number be the same for each universal generalization.* There is no requirement that one must, for instance, conjecture every universal generalization consistent with the data after 100 observations have been made. In fact, there are logically reliable rules for Goodman’s riddle that are consistent with the PI and which never conjecture more than one universal generalization at a time. I give an example of such a rule below.

Let  $\text{grue}_t$  be defined as before but with the subscript indicating the year when the switch from green to blue occurs. For example, one could generate a list of hypotheses of the form: all emeralds are  $\text{grue}_{2015}$ ; all emeralds are  $\text{grue}_{2016}$ , all emeralds are  $\text{grue}_{2017}$ , and so on. Then the set of hypotheses would consist of the following: all emeralds are green; for every future  $t$ , a hypothesis that all emeralds are  $\text{grue}_t$ , and finally the hypothesis that emeralds are neither all green nor all  $\text{grue}$ . “Neither green nor  $\text{grue}$ ” covers cases in

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<sup>21</sup> Goodman (*op. cit.*), p. 74.

which a green emerald is observed after blue one, as well as cases in which there is an emerald that is neither green nor blue. It is easy to devise a logically reliable rule for deciding among these alternatives. First, list the universal generalizations. For example, “all emeralds are green” might be at the top of the list followed by the grue hypotheses in chronological order. The order in which the universal generalizations are listed does not matter for logical reliability, though order does matter for efficient convergence as we will see below. Then proceed by the following rule:

Logically Reliable Rule for Goodman’s Riddle: Conjecture the first universal generalization on the list that is consistent with the data observed so far; if none of the universal generalizations are consistent with the data, conclude that they are all false.

This rule is assured of eventually settling on the true hypothesis in Goodman’s riddle, whichever one it may be. If one of the universal generalizations is true, the rule is eventually driven to it by the data and subsequently conjectures it forevermore. If emeralds are neither all green nor all grue, then decisive evidence for this will eventually be encountered (i.e., either a green emerald after a blue one, or an emerald that is neither green nor blue). Moreover, the above rule is consistent with the PI, because for each universal generalization there are a finite number of positive instances that will result in it being conjectured, though that number is distinct for each generalization. And a skeptic who refused to conjecture a universal generalization no matter what would not be logically reliable. In short, the only difference between this case and simple enumerative induction is that there is more than one universal generalization under consideration.

However, the new riddle of induction does raise interesting questions about what inferences to draw from the data in the short term. In the setup just described, this amounts to asking how to rank the universal generalizations. For example, instead of “all emeralds are green” at the top of the list followed by the grue hypotheses in chronological order, one could put “all emeralds are green” in 578<sup>th</sup> place. Placing a hypothesis near the top of the list amounts to favoring it over others that are farther down. The question, then, is what grounds there could be for such preferences. Two possible answers to this question were discussed in the previous section: background knowledge might render some alternatives more probable than others, or efficient convergence might be enhanced by conjecturing some alternatives before others. Let us consider these two possibilities in turn.

Given ordinary background knowledge, there are straightforward reasons to judge it very improbable that all emeralds are grue.<sup>22</sup> Note that the grue hypotheses do not say that green emeralds will *turn* blue after the crucial time  $t$ . Rather, those emeralds observed before  $t$  would stay green, but all of the emeralds excavated after  $t$  would be blue. In other words, all of the green emeralds will be unearthed before  $t$ , and blue emeralds, although always there in the ground, will not be dug up until after  $t$ .<sup>23</sup> Put this way, the grue alternatives are highly implausible. For if there were a mixture of green and blue emeralds buried in the earth, the overwhelmingly likely scenario is that extensive digging in the earth’s crust would have revealed some of both colors. Thus, “all emeralds are grue” is not a hypothesis worth taking seriously unless some plausible

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<sup>22</sup> Indeed, since by the standard mineralogical definition emeralds are green beryls, there is good case for claiming that “all emeralds are green” is an analytic truth. However, I set aside that point for the purposes of this discussion.

<sup>23</sup> For a defense of this reading of Goodman’s riddle, see R. Israel, “Two interpretations of grue—or how to misunderstand the new riddle of induction,” *Analysis*, LXIV (2004): 335-339.

scenario is specified to explain why all and only the green emeralds might be unearthed before  $t$  and only blue ones dug up after that.<sup>24</sup>

But one might hope for a more general answer to the new riddle of induction that is not dependent on background knowledge specifically relevant to mining gems.<sup>25</sup> Oliver Schulte<sup>26</sup> shows that one minimizes the maximum number of retractions of conjectures, or “mind changes,” by placing “all emeralds are green” at the top of the list of universal generalizations. Suppose that one lists the universal generalizations with all green at the top. Then the maximum of number of mind changes is two: from green to grue, and then from grue to neither green nor grue. For example, this would occur if one encountered, after a series of green emeralds, a blue emerald followed by another green one. In contrast, suppose that one begins by conjecturing that all emeralds are grue <sub>$t$</sub> , for some  $t$ . Then the maximum number of mind changes is at least three: from grue <sub>$t$</sub>  to green, from green to grue <sub>$t+n$</sub> , and finally from grue <sub>$t+n$</sub>  to neither green nor grue. For suppose that a sufficiently long sequence of green emeralds is observed to refute grue <sub>$t$</sub> , and make the agent conjecture that all emeralds are green. But then if a blue emerald is subsequently observed at  $t + n$ , the agent switches to conjecturing that all emeralds are grue <sub>$t+n$</sub> . And if a green emerald is observed after that point, then the agent makes a third switch from all grue <sub>$t+n$</sub>  to neither all green nor all grue. In fact, if the grue hypotheses are listed in chronological order, then the farther down on the list one places “all emeralds are green”, the greater the maximum number of mind changes. For example, suppose the

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<sup>24</sup> The argument in this paragraph is similar to the treatment of Goodman’s riddle proposed by Peter Godfrey-Smith, “Goodman’s Problem and Scientific Methodology,” this JOURNAL, C (2003): 573-590: knowledge of the process by which the data are generated can provide a basis for judging “grue-like” hypotheses improbable.

<sup>25</sup> A response to Godfrey-Smith (op. cit.) by R. Schwartz, “A Note on Goodman’s Problem,” this JOURNAL, CII (2005): 375-379, raises a similar concern, namely, that a solution to Goodman’s riddle should be relevant to related issues such as curve fitting and simplicity.

<sup>26</sup> Schulte, “Means-Ends Epistemology,” *British Journal for the Philosophy of Science*, L (1999): 1-31.

grue hypotheses are listed chronologically and that “all emeralds are green” is at 1,001<sup>st</sup> place. Then the agent could make 1,000 retractions of conjecture prior to getting to “all emeralds are green,” and she might still make two more mind changes after that.

However, Schulte’s treatment of Goodman’s riddle is dependent on framing the problem in terms of a particular set of alternative hypotheses. For example, it is possible to devise a set of alternative hypotheses, *distinct* from those mentioned in Goodman’s riddle, such that efficient convergence favors conjecturing grue before green.<sup>27</sup> Since the set of alternative hypotheses is part of what defines an inductive problem, this would effectively amount considering a distinct inductive problem than that posed by Goodman. I think it is an interesting question why inquiry should focus on one set of alternative hypotheses rather than another, but I also think that issue lies outside the scope of Hume’s problem of induction. It is useful to distinguish three general questions about inductive inference. (1) Why make inductive generalizations at all? (2) How to select a generalization from a set of alternatives when many are consistent with the data? (3) Which set of alternative generalizations should inquiry focus on? I take (2) and (3) to be challenges raised by Goodman’s riddle and (1) to be that posed by Hume.

In sum, the important points regarding the new riddle of induction for the present purposes are the following. First, the new riddle of induction does not demonstrate any inconsistency in the PI. Goodman’s riddle is merely a more general version of simple enumerative induction, and the same arguments made in section 3 apply, suitably generalized, to the new riddle. Secondly, while Goodman’s riddle illustrates that reliability in the long run does little to constrain inferences in the short term, it is

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<sup>27</sup> See D. Chart, “Schulte and Goodman’s Riddle,” *British Journal for the Philosophy of Science*, LI (2000): 147-149, and Schulte, “What to Believe and What to Take Seriously: A Reply to David Chart Concerning the Riddle of Induction,” *The British Journal of the Philosophy of Science*, LI (2000): 151-153.

susceptible to treatment in the manner discussed in the previous section. If our ordinary background knowledge is granted, there are straightforward reasons for judging it improbable that all emeralds are grue, and in the absence of any background knowledge, efficient convergence favors conjecturing green before grue in Goodman's riddle. And while unresolved issues concerning Goodman's riddle remain, they are distinct from Hume's problem.

## **6. Conclusion**

In this essay, I have argued that Hume's famous problem of induction is a false dilemma. The PI might be neither a relation of ideas whose denial is a contradiction nor a matter of fact that could only be justified by inductive generalization from experience, but instead a normative claim justified by a non-empirical epistemic means-ends argument. An argument of this kind shows by logical or mathematical proof that a rule of inference promotes a particular epistemic aim. I showed that, in simple enumerative induction, following the PI is necessary and sufficient for logical reliability. Roughly, a rule is logically reliable when it is assured of eventually settling on the correct answer. Since this argument does not attempt to justify the PI by inductive means, it avoids circularity. Moreover, the argument does not show that a person who rejects the PI—for example, an inductive skeptic—is thereby guilty of a contradiction. Instead, it shows that the skeptic's refusal to take a chance of being mistaken seriously impedes his ability to discover the truth: if you want to be assured of eventually getting it right, you may have to risk being temporarily wrong.