MIND THE TEMPO: CHILDREN'S CONCEPTIONS OF SYMMETRY IN TIME

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Introduction

Testing the current conceptual/cultural boundaries of symmetry may result in a more nuanced, and arguably messy view of the concept. Studies examining basic understanding of symmetry are few and far between, and attaching symmetry to the larger idea of time is simply not done within the general confines of mathematical education research. Therefore, a non-traditional, tactile approach for examining children’s thinking about symmetry is pursued as it goes beyond traditional conceptualizations of symmetry into an embodied, temporal one.

A Framework for Temporal Symmetry

Symmetry is an example of a concept that has strong common usage outside of the mathematical community. As a result of it too-often being “viewed as a collection of disconnected concepts,” Leikin and colleagues (1998) define symmetry as “a triplet (S, Y, M) consisting of an object (S), a specific property (Y) of the object, and a transformation (M) satisfying the following two conditions: i) The object belongs to the domain of the transformation; ii) Application of the transformation to the object does not change the property of the object” (p.4). This is the basis on which I discuss the overall concept of symmetry; however, the type of symmetry examined herein is not one that is readily found in textbooks. The notion of temporal symmetry, where time is the medium through which the property of an object is transformed, is only one of many variations on the theme of symmetry: consider Polya’s (1973) use of symmetry to explain the interchangeability of variables in some algebraic expressions, or Dhombres (1993) invocation of the concept to describe the relation between any two distinct mathematical proofs of one statement.

Yet, the messiness of this idea (or perhaps the decided shift away from the Gestalt-like, holistic view of symmetry experiences) does raise questions about the nature of its domain: is this a primitive conception of symmetry, or some sort of metaphor by which symmetry can be understood. Lakoff and Nuñez (1999) would suggest that even the most abstract mathematical concepts arise from basic human experience – from the way the body interacts with the world. In inventing mathematics, they contend, humans use metaphors to connect sets of ideas. It is my contention that children’s conceptions of the traditional, static symmetry can be understood through their use of the metaphors of time and tempo.

Methodological Considerations

This study examines the conceptions of temporal symmetry through the use of linking metaphors of two elementary school children (Abe, age 8, and Ben, age 10), both of similar ethnic and socioeconomic background. The participants were selected due to their attendance in a non-school related tutorial program, and were volunteers. These particular boys attend the same school (different by two grades) in a small Midwestern city, and are of particular interest for one main reason. They come to the study with different formalized mathematics training in the concept of symmetry (Ben has explicitly studied the concepts of translation, reflection and rotation), which allows for commentary on how the primitive metaphors of time and tempo operationalize the symmetry concept for each participant.

Participants were interviewed in an individual setting. The interviews were both video and audio taped so that repeated observations and careful discursive analysis of their responses, as well as of their physical motions (in itself, a discourse), was permitted. Participants were given a
small, one-octave keyboard with two mallets to explore and play with for a few minutes prior to
the tasks. After this time, they were introduced to each picture and asked to play the shapes
(Figures 1-9, with varying levels of traditional and temporal symmetry) on the keyboard; a brief
interview followed, with probing follow-up questions. Both participants had at least 1 year of
piano lessons, which made them conversant with the basic keyboard layout. Rather than use a
hierarchical, level-based framework such as van Hiele levels or the SOLO taxonomy, I choose a
more naturalistic, ethnographic analysis of the words and actions used by participants to
represent the figures, as well as a simple measurement of any consistent tempo in what they play.

**Findings**

Overall, a subconscious awareness of symmetry was present. Tempo seems to be
subconscious for the two participants; which is supported by the strong presence of a consistent
tempo (16 out 18 tasks had less than 5% overall variance in speed), and their lack of concern for
whether changes in tempo alter the “sameness” of two interpretations. The fact that both used
language to evoke equality of some aspects within the same Figures suggests that there is a latent
awareness of symmetry within them – perhaps the use of tempo is a subconscious externalization
of this awareness. If that is the case, then referring to this awareness as a kind of temporal
symmetry may be extraordinarily valuable in allowing the explicit inclusion of dynamic, holistic
and kinesthetic discourse into the typically static geometry found in textbooks.

In terms of temporal symmetry, Abe presented more references to concrete objects, time, and
motion as reasons for his playing. This certainly could be due to a lack of technical vocabulary,
yet I find it heartening for the future of temporal symmetry that both participants were quick to
play and describe tasks 8 and 9 (‘bubbles’ and ‘feet’) in those same types of terms. Using the
language of motion may simply be something that is increasingly reserved for ‘real’ objects as
students of geometry get older – increasing abstraction calls for decreasing realism. Yet, more
evidence is certainly required for making any broad claims.

**Figures**

![Figure 1: “stairs”](image1)

![Figure 2: “crown”](image2)

![Figure 3: “star”](image3)

![Figure 4: “flower”](image4)

![Figure 5: “wave”](image5)

![Figure 6: “semi-symmetry”](image6)

![Figure 7: “non-symmetry”](image7)

![Figure 8: “bubbles”](image8)

![Figure 9: “footsteps”](image9)

**References**

Burger, W. F., & Shaughnessy, J.M. (1986). Characterising the van Hiele levels of development


cognitive foundations for a mind-based mathematics, in L. English (Ed.), *Mathematical
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