Dynamic microlending under adverse selection:
Can it rival group lending?

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Abstract

We derive an optimal lending contract in a two-period adverse selection model with limited commitment on the borrower side. The contract involves “penalty” interest rates after default, and favorable rates after success. Under some conditions, it also charges first-time borrowers higher rates than repeat borrowers, as in “relationship lending”, because the lender is constrained to keep borrowing attractive while using revealed information to price for risk. We compare the efficiency of a group lending contract (of the kind popularized by the microcredit movement) to the dynamic, individual contract. Both types of contracts reveal the same information, but the contracts face different constraints on using the information to improve risk-pricing. As a result, either type of contract can lead to greater efficiency depending on specifics of the environment – opening the possibility that dynamic lending has played a role comparable to that of group lending in the success of microcredit. We also characterize the optimal dynamic group contract when both lending techniques are feasible, and find that it combines both approaches, but with varying emphases. A recurrent theme is that in more marginal environments, dynamic lending performs relatively better than, and is prioritized over, group lending. We also discuss a number of extensions, including (spatially and serially) correlated risk and the effect of competition.

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1 Introduction

Despite seeming impossibility several decades ago, financial intermediation among the world’s poor has grown at unprecedented, rapid rates, to the extent that now an estimated two hundred million people have borrowed from a microfinance institution (“MFI”) (Maes and Reed, 2012). Many of these MFIs sustain operations while covering costs, raising a question of interest to economists: how has lending been successful in these environments where use of collateral is nearly impossible due to low wealth and/or weak institutions?

Economic research on this question has naturally focused on the innovative techniques of MFIs, especially group lending. Group lending typically involves group members mutually co-signing each other’s loans (or some other form of liability). It has been shown theoretically to increase efficiency relative to standard individual loans in a number of contexts, including adverse selection (Ghatak, 1999, 2000, Van Tassel, 1999). However, group lending is at best part of the answer, since a number of successful MFIs have never used it. And while group lending is still widely used by micro-lenders, there is speculation that its popularity is waning in favor of individual loan contracts.

Though not as novel, dynamic individual lending is also a common strategy among MFIs, probably the chief alternative to group lending. This paper explores the effectiveness of these two lending strategies – both in isolation and in combination – in a well-known adverse selection setting. To do so, we provide a first analysis of optimal dynamic individual contracts in the adverse selection environment in which group lending has been studied most heavily. Our next step is to compare the effectiveness of each approach, which highlights interesting similarities and differences, and clarifies how well (relatively) a lender with only one of these two tools in its toolbox may expect to do. Finally, we show how a lender will optimally combine both approaches if both are feasible, and discuss what contextual features push an

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1 According to the 2006 Nobel Peace Prize press release: “Loans to poor people without any financial security had appeared to be an impossible idea” (www.nobelprize.org).

2 See also the evidence of Gine and Karlan (2014).

3 De Quidt et al. (2012) provide evidence that group lending remains quite prevalent throughout the world.
optimal contract to rely more heavily on one or the other approach.

Dynamic lending can take a number of forms, including adjustment of interest rates, adjustment of loan sizes, and exclusion. It can also be used in a number of contexts, including moral hazard and adverse selection. This paper does not address all these variants, but focuses on a standard adverse selection context, in part for straightforward comparison with previous work, and in part to emphasize that dynamic lending can be useful beyond moral hazard contexts.\(^4\)

Following most literature on the topic, the paper also abstracts from loan size adjustment, though this does admittedly appear to be a common dynamic feature of microlending contracts. Instead, the paper features interest rates that are optimally adjusted depending on past performance.\(^5\) This lending technique features prominently in a number of microlenders’ strategies. A classic example is the Thai BAAC, a state-backed agricultural lender (see Townsend and Yaron, 2001, and BAAC annual reports, 2007, 2011, etc.). The BAAC offers individual microloans and after each year of repayment with no problems, the borrower’s rate is lowered. Interestingly, at least until 2007 new borrowers were charged the same rates as borrowers with past repayment problems. Another example is the well-known Bolivian MFI Banco Sol, which improved various loan terms, including effective interest rates and loan sizes, after good repayment performance (Gonzalez-Vega et al., 1996). One of the largest MFIs in South Africa, studied by Karlan and Zinman (2009), also uses this strategy, extending small consumer loans at rates that depend on past repayment performance. Indonesia’s BRI charged fixed interest rates, but gave refunds for borrowers that repaid on time (Maurer, 1999) – akin to lowering interest rates based on good performance. While we lack systematic data on MFI contract terms, and hence cannot quantify an overall tendency of MFIs to use interest rate adjustments – or other contracting strategies, for that matter – these examples make clear that it is one of the key tools in some prominent MFIs’ arsenals.

\(^4\)Dynamic lending is often referred to as “dynamic incentives”, which seems to imply some form of moral hazard – but a simple, unoriginal point of this paper is that dynamic contracts can also be useful in solving adverse selection.

\(^5\)This allows for exclusion, including partial exclusion, if rates are set high enough.
How does this form of dynamic lending help overcome adverse selection? When lenders do not know borrowers well initially, repeated lending allows the lender to gather information on the borrower over time as evidence from realized behavior accumulates. Use of this information allows the lender to tailor contract terms to fit initially unobserved borrower risk characteristics, and thus improve risk-pricing.

However, the lender can be constrained in its use of borrower information as it is revealed over time. First, even if lenders can commit to sequences of loan terms (potentially contingent), borrowers can typically drop out of the relationship at any time – at least after having repaid the current loan. This places limits on how high interest rates can be raised without losing borrowers. Second, lenders may be limited in how much they can reward repayment, if risky borrowers can at least temporarily pretend to have done well.

To formalize these ideas, we first analyze a simple two-period adverse selection setting in which borrowers have no extra information about each other, and thus group lending is not useful. The lender learns about borrowers’ fixed types as repayment/default episodes accumulate, and seeks to use this information to overcome a lemons problem caused by unobserved risk heterogeneity in the credit market – constrained, however, by borrower limited commitment and by a monotonicity constraint that limits the rewards to success.

Limiting attention first to a simple, standardized (pooling), two-period individual loan contract, which consists of three interest rates – an initial rate, and second period rates conditional on first-period success or failure – we find that the dynamic contract achieves first-best efficiency over more of the parameter space than one-shot loans. This is because the information revealed over time allows the lender to price for risk more accurately, reducing the cross-subsidy from safe to risky borrowers. The lender does this by charging more in states of the world where risky borrowers predominate (after failure) than where they do not (after success).

We also find that contracts tend to be back-loaded from the borrower’s perspective – rates for first-time borrowers are high, as high as (or even higher than) rates for borrowers
with a default – reminiscent of the BAAC lending schedule where new customers received the same rates as borrowers with some repayment problems. The model thus rationalizes a key feature of relationship lending, that terms get better for the borrower over time in lender-borrower relationships.\textsuperscript{6} Here, the ability of borrowers to end the relationship pushes the lender to charge more upfront so that the later, information-differentiated rates can be low enough to promote continued borrowing.

We also show that in some cases when full efficiency is not attainable, nearly efficient lending can be achieved by giving up on unlucky safe borrowers, i.e. by charging such high rates for the loan after failure that safe agents opt out of borrowing. Interestingly, safe borrowers often prefer this contract to all others, since it extracts more surplus from risky borrowers and results in lower rates for first-time borrowers. Thus there can be an equity-efficiency tradeoff, with equity favored by “punishing” failure more aggressively.

These results restrict attention to simple pooling contracts. However, we show that a menu of contracts that screens borrowers cannot improve efficiency in our setting of risk neutrality. The lender can also do no better with forced savings or collateral (given borrowers have no pledgeable wealth to start with), since it can replicate those contracts by adjusting interest rates. Thus this framework gives rise to a very simple, standardized contract as optimal, without having to assume away a much broader class of (menus of) contracts.

We turn next to a comparison with static group lending, in particular joint liability lending, studied in this context by Ghatak (1999, 2000). A first observation is that two-period dynamic lending and two-person group lending reveal the same amount of information to the lender. In the dynamic case, the lender gets two time series observations of project success/failure, from observing the same borrower twice. In the group case, the lender gets two cross-sectional observations, from observing a borrower and his partner – both are equally informative about the borrower, given that groups form homogeneously. Further, without any constraints on the contracts, the two kinds of contracts can achieve the same outcomes.

\textsuperscript{6}See further discussion in the literature review, section 2.
But with the imposed constraints they do differ: group lending achieves *fully efficient* lending over more of the parameter space than dynamic lending. The intuition is that dynamic lending is more constrained in its use of information to price for risk. Ideally, the second-period rate after failure should be high, to put more of the repayment burden on risky borrowers – but this “penalty” rate cannot be too high without inefficiently causing unlucky safe borrowers to drop out. Thus, limited borrower commitment constrains the lender’s use of information by hampering its ability to vary the interest rate with repayment record.

However, when group lending fails to improve lending outcomes, dynamic lending may outperform it, by pricing safe borrowers who fail out of the market. This strategy allows the lender to vary interest rates more freely based on information, which eliminates more of the cross-subsidy and attracts safe borrowers most of the time – i.e., initially and after success – in cases where group lending would exclude safe borrowers altogether.

In short, group lending or individual lending can be the more effective lending strategy, depending on context. We find that it is in the most marginal environments – where repayment affordability is low and/or value-added from outside capital is not very high – that dynamic lending outperforms group lending, by giving up on full efficiency. Overall, the results suggest that dynamic lending may have played as significant a role as group lending in the success of microcredit, and that its greatest relative usefulness comes in more marginal lending environments.\(^7\)

Of course, the assumptions required for each kind of contract to operate (as modeled) are more plausible in some contexts than others. Group lending requires that borrowers have a significant informational advantage about each other’s risk, relative to the lender, and can form groups relatively freely. Dynamic lending requires that the lender can commit to a two-period contract, and that borrowers are endowed with two similar projects over two periods. When the context allows for both types of lending, lenders need not choose between but may combine the strategies. We analyze optimal dynamic group contracts for

\(^7\)Baland et al. (2013) make a similar point on the limits of group lending, in an ex post moral hazard setting.
such settings, and find a related result: where borrower affordability is low, the contract emphasizes dynamic features – in particular, discounts after success – at the expense of joint liability – no liability is imposed on the first loan; while at higher affordability levels, joint liability is stressed – full liability is imposed on all loans – at the expense of dynamic features – penalty rates after partial failure are eliminated.

Several extensions are discussed, including the effects of competition, risk aversion, discounting, and length of the relationship on the dynamic individual contract. The analysis also points to correlated risk as important in these contexts. Where spatial correlation is significant, for example in some agricultural contexts, information revelation under group lending is reduced; serial correlation, on the other hand, which may be relatively more prevalent in small business contexts, hampers information revelation in a dynamic setting.

Section 2 discusses related literature. The basic model is presented in section 3. Optimal dynamic individual lending is derived in section 4 and compared with static group lending in section 5. Optimal dynamic group lending is derived in section 6. Extensions are discussed in section 7, and proofs are in the Appendix.

2 Related Literature

The paper contributes along three dimensions. First, it provides novel analysis and characterization of a dynamic adverse selection lending model. Second, it compares the effectiveness of dynamic and group lending strategies in the adverse selection context. Third, it shows how dynamic and joint liability features are both optimally used, and traded off, when both are available.

The first contribution fits into a very large literature on dynamic adverse selection; see, e.g., Bolton and Dewatripont (2005). Three features of our model distinguish it from the vast majority of this literature, including the literature covered in this textbook. First, agent types are fixed over time, an assumption that separates it from a large literature on insurance
and risk-sharing (e.g. Townsend, 1982, Thomas and Worrall, 1990, Phelan, 1995, etc.). Second, the principal (lender) can commit to the contract, which distinguishes it from literature focusing on the ratchet effect (e.g. Laffont and Tirole, 1988) and relationship lending with informational capture (see below). Third, the agent (borrower) cannot commit to remaining in the contract. Fixed types with one-sided, lender commitment seem applicable in many credit market contexts, especially microcredit, yet this setting remains largely unexplored.

Similar versions of one-sided commitment are analyzed in a few other papers, including Harris and Holmstrom (1982) in the labor market context, Phelan (1995) and Cooper and Hayes (1987) in the insurance context, and Boot and Thakor (1994) in the lending context. These papers tend to find that contract terms improve over time for the agent, as is the case here. Of these papers, only Cooper and Hayes examine a dynamic adverse selection model with fixed types, and they also find that contract terms depend on revealed information, though only for safe customers. However, their focus is on screening heterogeneous risk-averse agents with insurance contracts, in constrast to our focus on solving a lemons problem caused by unobserved risk heterogeneity in the credit market. With the exception of Boot and Thakor (1994), discussed more below, we know of no other dynamic lending models that explore the implications of this one-sided commitment.

Ghosh and Ray (2001) develop a lending model with some similar predictions, in that loan sizes improve over time, though not necessarily interest rates. In their model, borrowers must be given incentives to repay through the prospect of future loans. However, a fraction of the population cares nothing for the future. This can lead to a costly “testing” period for new borrowers, involving small loans, on which the myopic borrowers will always default; but the prospect of facing this testing period again with a new lender is what keeps good borrowers faithful to their current lenders. Their paper differs from ours in focusing on

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8 Backloading has also been derived for reasons other than one-sided commitment of the kind we assume. Ray (2002) provides a general result on contract back-loading, in a hidden action setting where neither the principal nor the agent can commit to long-term contracts. Webb (1992) and Monnet and Quintin (2005) find that contract terms tend to improve over time in dynamic models of costly state verification.

9 Harris and Holmstrom (1982) model imperfect but symmetric information; Phelan (1995) considers types that are i.i.d. over time rather than fixed; and Boot and Thakor (1994) study a hidden effort problem.
exclusion as a way to provide incentives for repayment, and how this can be aided by an adverse selection problem. Also, their lender is unable to commit, so the interest rate can rise as the relationship progresses, as in the “relationship lending” literature discussed next.

Much of the extensive theoretical and empirical literature on “relationship lending” – reviewed by Boot (2000) – has a different focus from ours: competition between asymmetrically informed lenders. Representative, and closest to our paper, is Sharpe’s (1990) two-period hidden-type model. There, the repeat lender has superior information in the second period and can charge some borrowers more than competitors. The result is then the opposite of ours, that loan terms get worse for the borrower over time as he becomes informationally captured. Key to the differences in our results is that in Sharpe, the lender develops superior information but cannot commit to a two-period contract.

Our model thus provides a new rationalization for relationship lending that does not revolve around the inside lender acquiring superior information. Here, what looks like a developing relationship with improving contract terms is simply the lender’s best way of pricing for risk without driving away borrowers as information is revealed over time. In this, it is similar to Boot and Thakor (1994). They rationalize what look like relationship lending contracts in an infinitely repeated moral hazard lending situation where lenders can commit to long-term contracts but borrowers can leave. They find that borrowers face relatively expensive, collateralized loans until they repay a loan, and from then on relatively cheap, uncollateralized loans that induce efficient effort.\textsuperscript{10,11} We view our papers as complementary since the fundamental problems being solved are quite different: inducing effort provision vs. pricing for risk as information is revealed over time.

\textsuperscript{10}A number of empirical contributions test whether repeat loans from the same bank come with higher or lower rates. (The theoretical result that terms should get better over time is typically attributed to Boot and Thakor, 1994; that terms should get worse is often attributed to Sharpe, 1990, and Rajan, 1992.) More seem to find improvements in terms over time, (e.g. Berger and Udell, 1995, Bharath et al., 2011), though this is not without exception (Degryse and Van Cayseele, 2000). In all the microfinance examples cited in the introduction, interest rates improve as borrowers continue to borrow and repay.

\textsuperscript{11}Stiglitz and Weiss (1983) study a related moral hazard model, which is a two-period extension of Stiglitz and Weiss (1981). They also find that interest rates are high initially and drop to the efficient level after success. Their main focus, however, is on exclusion from the credit market as an incentive device that is robust to competition but potentially inefficient.
Webb (1991) has perhaps the closest model to ours. He shows in a two-period adverse selection lending model that borrowers can be screened by the degree of conditionality of the second-period contract on first-period performance.\textsuperscript{12} Our paper differs from his in imposing limited borrower commitment and monotonicity constraints on the contracts, which limits what dynamic contracts can achieve and also results in the back-loaded contract structure. We also explore in more detail the efficiency properties of the optimal contract and show that in this risk neutral setting, simple pooling contracts suffice.

In short, the current paper analyzes a new problem in dynamic lending under adverse selection, and derives new results on efficiency and contract structure. Some results have parallels elsewhere – back-loaded contracts have been derived in a number of settings – but the analysis of constrained credit-risk pricing as information is revealed over time is new, as best we can tell, and provides an alternative explanation for what look like relationship contracts. It is also particularly relevant in the microcredit context, a first analysis of dynamic interest rate variation as a key tool at lenders’ disposal, even for improving selection rather than providing incentives.

The second contribution of the paper is the comparison of dynamic individual lending contracts with static group lending contracts. Most of the group lending theory, particularly in the context of adverse selection, explores how group lending can outperform one-shot individual loan contracts; this includes the seminal contributions of Ghatak (1999, 2000), Van Tassel (1999), and Gangopadhyay et al. (2005). Our goal is to extend this literature by working out a plausible \textit{dynamic}, non-group based alternative lending strategy, and then to compare this dynamic strategy to a group-based strategy in a standard framework (essentially, a simplified Stiglitz-Weiss, 1981, model) – highlighting how and when two potentially very different approaches work. To our knowledge, this paper is the first to point out the similarities in information revelation between group and dynamic lending, and to explore how constraints on use of information and features of the lending environment make one or

\textsuperscript{12}This is related to what Cooper and Hayes (1987) show in the insurance context.
the other more effective.

Several papers do compare individual lending to group lending in a dynamic framework. Chowdhury (2007) shows that dynamic group lending is no better than dynamic individual lending at solving a hidden-type/hidden-action problem unless loans are made to group members sequentially rather than simultaneously. This is because, under some assumptions, sequential loans can harness social enforcement to raise repayment, while simultaneous group loans (and individual loans) cannot. Bhole and Ogden (2010) and de Quidt et al. (2013) compare dynamic individual lending to dynamic group lending – where joint liability may be explicit or implicit, and full or partial – in a strategic default setting. The insights of these papers are clearly different, but complementary to the work here.

Finally, this paper contributes an analysis of the optimal dynamic group contract. The papers cited in the preceding paragraph also analyze dynamic group contracts, but none in the adverse selection context with its focus on optimal risk-pricing. In addition, the context of this paper gives rise to interesting tradeoffs, when constraints on the contract force the lender to choose whether to rely more heavily on dynamic or group-based contracting strategies. This generates predictions on the relative prevalence of these lending styles that, to our knowledge, are novel.

### 3 Baseline Model

There is a continuum of risk-neutral agents, of measure one. Each is endowed with no capital, one unit of labor, an outside subsistence option, and a project. Both the outside option and the project require one unit of labor. In addition, the project requires one unit of capital. The outside option gives net return $u \geq 0$.

Agents’ projects differ in risk, indexed by $p \in \mathcal{P}$. The project of a type-$p$ agent pays $R_p$ with probability $p$, and 0 otherwise. Project risk is the agent’s private information. As in Che (2002) also compares dynamic individual and dynamic group loans, under hidden effort. However, the individual loans are assumed to be repeated one-shot loans, which do not take advantage of information on borrower effort provision as it is revealed over time.
Stiglitz and Weiss (1981), assume that all projects have the same expected value:

\[ p \cdot R_p = \overline{R}, \quad \forall p \in \mathcal{P}. \tag{A1} \]

This implies that \( R_{p'} < R_p \) when \( p' > p \), i.e. safer projects pay less when successful.

Agents require outside funding to carry out their projects. We assume limited liability, in particular that agents’ exposure in any contract is limited to project returns. It follows that an agent who fails owes nothing to an outside financier. We also assume that output can be publicly verified, but only coarsely: anyone can distinguish between \( Y = 0 \) (fail) and \( Y > 0 \) (succeed), but not between different levels of \( Y > 0 \). This assumption along with limited liability makes debt contracts the only feasible financial contracts.\(^{14}\) We also restrict attention to deterministic contracts.

There is a single non-profit lender with access to capital at market gross rate \( \rho > 0 \). The lender’s objective is to maximize total borrower surplus subject to earning, in expectation, the market rate of return \( \rho \) on all capital lent.\(^{15}\)

There are three rate-of-return parameters: \( \overline{R}, \mu, \) and \( \rho \). It is often more convenient to think in terms of an equivalent set of three parameters, \( \rho, \mathcal{G}, \) and \( \mathcal{N} \), where

\[ \mathcal{G} \equiv \frac{\overline{R}}{\rho} \quad \text{and} \quad \mathcal{N} \equiv \frac{\overline{R} - \mu}{\rho}. \tag{1} \]

The numerator of \( \mathcal{N} (\mathcal{G}) \) is the net (gross) return to a unit of capital invested in these projects, while the denominator is the return to a unit of capital invested elsewhere; hence, \( \mathcal{N} (\mathcal{G}) \) can be interpreted as the net (gross) excess return to capital embodied in the agents’ projects. Of course, \( \mathcal{G} \geq \mathcal{N} \), since \( \mu \geq 0 \). We assume

\[ \overline{R} > \rho + \mu, \quad \text{equivalently,} \quad \mathcal{N} > 1. \tag{A2} \]

\(^{14}\)There are no enforcement issues by assumption: borrowers who can repay, do.

\(^{15}\)Outcomes under a competitive market are discussed in section 7.
This implies that all projects have higher expected return than the cost of their inputs, capital and labor. Thus, social surplus is strictly increasing in the number of projects funded, and fully efficient lending is equivalent to lending to all agents. If the lender exactly breaks even, borrower surplus equals social surplus and hence increases in number of projects funded.

The analysis considers the two-type case: \( \mathcal{P} = \{p_r, p_s\} \), with \( 0 < p_r < p_s < 1 \) and \( R_s < R_r \), where \( R_\tau \equiv R_{p_\tau}, \tau \in \{r, s\} \). Let \( \theta \in (0, 1) \) be the proportion of risky, and denote the population average of any function \( g(p) \) as \( \overline{g(p)} \).\(^{16}\) For example, \( \overline{p} \) is the mean risk-type and \( \overline{p^2} \) the mean squared-type.

**Static individual lending.** One-shot contracts are loans involving \( r \) paid after success and 0 paid after failure. As is well-known, a lemons problem can arise: loans are priced based on average risk in the borrowing pool, but this can be too expensive for safe borrowers.

An agent of type \( \tau \in \{r, s\} \) will prefer to borrow and undertake the project iff

\[
\overline{R} - p_\tau r \geq \overline{u} \iff r \leq \hat{r}_\tau \equiv \frac{\overline{R} - \overline{u}}{p_\tau}.
\]

The first inequality requires expected project revenue less expected repayment to exceed the subsistence payoff. The second inequality rearranges to obtain the reservation interest rate \( \hat{r}_\tau \), above which a type-\( \tau \) agent will opt for subsistence. Clearly, \( \hat{r}_s < \hat{r}_r \), i.e. safe agents are harder to attract, since they repay with higher probability. Hence, \( r \leq \hat{r}_s \) is necessary for fully efficient lending. Also necessary and sufficient for the contract to be affordable by both types is \( r \leq R_s \) (since \( R_s < R_r \)).

If all agents borrow, the lender expects to earn \( \overline{pr} \) on a unit of capital lent. Equating this with \( \rho \) gives the break-even interest rate \( r = \rho/\overline{p} \). This rate attracts all borrowers iff

\[
r \leq \hat{r}_s \iff \frac{\rho}{\overline{p}} \leq \frac{\overline{R} - \overline{u}}{p_s} \iff \mathcal{N} \geq \mathcal{B}_{1,1} \equiv \frac{p_s}{\overline{p}}.
\]

\(^{16}\)Note that \( \overline{g(p)} = \theta g(p_r) + (1 - \theta)g(p_s) \).
It\textsuperscript{17} is affordable by all borrowers iff

\[ r \leq R_s \iff \rho \frac{\overline{R}}{\overline{p}} \leq \frac{p_s}{\overline{p}} \iff G \geq B_{1,1}, \]

which is guaranteed by \( N \geq B_{1,1} \) since \( G \geq N \). Efficient lending can thus be achieved if the net excess return to capital in this market, \( N \), is greater than the degree of asymmetric information as captured by \( p_s/\overline{p} \). If so, the borrowers get all the surplus from the projects, but risky borrowers earn more than safe.

The lender cannot break even and attract all agents if instead \( N < B_{1,1} \). The next best option is to give up on safe agents and lend only to risky. Thus, when \( N \in (1, B_{1,1}) \), one-shot individual loans fund only a fraction \( \theta \) of the efficient projects. Inefficiency arises from the lender’s inability to price for risk. This gives rise to a cross-subsidy from safe to risky borrowers, since safe are more likely to repay; and the anticipated excess repayment burden can keep safe borrowers out of the market.\textsuperscript{18}

\section{4 Dynamic, Individual Lending}

The model is now extended to assume agents receive the same endowment in each of two periods: a unit of labor, a subsistence option, and a project. An agent’s project type is fixed over time, and outcomes are independently distributed. To focus better on risk-pricing and efficiency, discounting and consumption smoothing motives are ignored, so that borrower payoffs are just the sum of their two-period expected payoffs.

In section 4.1, attention is restricted to simple two-period pooling contracts. In section 4.2 we show that more complex contracts cannot improve on this simple one.

\begin{footnotesize}
\textsuperscript{17}Notationally, the subscripts of cutoff \( B_{1,1} \) are since the loan is for one agent and one period.

\textsuperscript{18}Under full information, the lender can perfectly price for risk by charging \( \rho/p_s \) to safe borrowers and \( \rho/p_r \) to risky. This achieves efficient and equitable lending, with all surplus going to the borrowers.
\end{footnotesize}
4.1 Simple pooling contracts

A “simple pooling contract” is a single contract offered in period 1 to all borrowers that offers one unit of capital in each period at pre-specified, potentially history-dependent interest rates, in which nothing is paid by borrower or lender after a borrower failure, and which does not allow first-time borrowing in period 2. A simple pooling contract boils down to three parameters: \((r_\emptyset, r_1, r_0)\), where \(r_\emptyset\) is the gross interest rate on the first loan and \(r_1\) (respectively, \(r_0\)) is the gross interest rate offered on the second loan for a borrower who has succeeded (respectively, failed) in his first-period project.

In addition to borrower limited liability and the lender breaking even, several constraints are imposed on the contract. First, we assume the lender can commit to the two-period contract, but the borrower cannot commit to taking a second loan. Hence, borrowers drop out at period two if the loan terms are such that the outside option is more attractive. This assumption seems appropriate since typically borrowers can end a borrowing relationship at will, at least after settling existing debts, while lenders may often have the reputational incentives, mission-based integrity, and/or credible legal consequences not to renege from long-term commitments.\(^{19}\)

Second, we follow Innes (1990), Che (2002), and Gangopadhyay et al. (2005) in imposing monotonicity constraints. These ensure borrowers are not required to pay more when they fail than when they succeed. The argument is that feigning success might be relatively easy, e.g. via very short-term loans from relatives or moneylenders, so rewarding success with lower payments is not feasible.\(^{20}\) The second-period monotonicity constraints are

\[
\begin{align*}
  r_0, r_1 &\geq 0 \\
\end{align*}
\]

These ensure that the amount due after second-period success \((r_0 \text{ or } r_1)\) is no less than the

\(^{19}\)See section 2 for other contributions assuming one-sided commitment of this form.

\(^{20}\)This constraint can also be motivated as a reduced-form constraint from a costly state verification problem in which the lender only audits when a failure is reported. Since reports of success go unverified, the constraint ensures there is no incentive to falsely report success.
The left-hand side is the payoff from claiming success: paying $r_\emptyset$ and enjoying the option of a second-period loan at rate $r_1$ (which will be exercised if it gives a better payoff than $\bar{u}$). The right-hand side is from claiming failure: paying nothing upfront but facing a second-period loan option at rate $r_0$. It is not obvious whether to impose this dynamic monotonicity constraint – even if claiming success and paying $r_\emptyset$ is worthwhile, a failed borrower may not be able to pay $r_\emptyset$.\(^{21}\) It will be imposed, but makes no difference here for efficiency.

Define a type-$\tau$ agent’s two-period payoff from taking the loan in period one and, only if optimal, in period two, as $\Pi_\tau(r_\emptyset, r_1, r_0)$; then

\[
\Pi_\tau(r_\emptyset, r_1, r_0) = \bar{R} - p_\tau r_\emptyset + p_\tau \max \{\bar{R} - p_\tau r_1, \bar{u}\} + (1 - p_\tau) \max \{\bar{R} - p_\tau r_0, \bar{u}\} = 2\bar{R} - p_\tau r_\emptyset - p_\tau^2 \min \{r_1, \hat{r}_\tau\} - p_\tau (1 - p_\tau) \min \{r_0, \hat{r}_\tau\}.
\]

The second equality uses the fact that $\bar{R} - p_\tau \hat{r}_\tau = \bar{u}$ (see equation 2).

In the dynamic case also, including safe borrowers is the hard part:

**Lemma 1.** If safe agents prefer to borrow in period one under contract $(r_\emptyset, r_1, r_0)$, so do risky agents.

Hence, as usual, surplus maximization is highly related to attracting safe agents to borrow. Our approach will thus be to maximally tilt contract parameters in favor of safe borrowers.

First we analyze the feasibility of fully efficient lending, i.e. all borrowers taking loans

\(^{21}\)The bank cannot force a failed borrower to pay $r_\emptyset$, by assumption. However, borrowers may be able to come up with the money voluntarily, perhaps at some cost, e.g. from relatives or moneylenders.
in all periods. With the lender exactly breaking even, this ensures the borrowers maximal surplus – thus the lender will implement it whenever feasible.

Under fully efficient lending, the limited commitment and limited liability constraints guarantee that no borrowers want to drop out in period 2 and that all required payments are affordable (tighter for safe borrowers):

\[ r_1, r_0 \leq \hat{r}_s \quad \text{and} \quad r_\emptyset, r_1, r_0 \leq R_s . \]

The period-2 and dynamic monotonicity constraint 3, and the lender zero-profit constraint (ZPC), are

\[ r_1, r_0 \geq 0 , \quad r_\emptyset \geq p_\tau(r_0 - r_1) , \quad \tau \in \{r, s\} , \quad \text{and} \quad p r_\emptyset + \overline{p} r_1 + p(1 - p) r_0 \geq 2 \rho . \]

Since safe borrowers are harder to attract, consider maximizing the safe-borrower payoff,

\[ 2 \overline{R} - p_\emptyset r_\emptyset - p_s^2 r_1 - p_s(1 - p_s) r_0 , \quad \text{(5)} \]

subject to the above constraints. A key observation is that lowering \( r_1 \) and raising \( r_\emptyset \) along the ZPC raises the safe-borrower payoff, as does raising \( r_0 \) and lowering \( r_\emptyset \) along the ZPC.\(^{22}\)

The intuition is that \( r_1 \) is encountered relatively more frequently by safe borrowers and \( r_0 \) by risky borrowers, while \( r_\emptyset \) is encountered equally by both. Thus the best contract for safe borrowers pushes \( r_0 \) as high as possible, \( r_1 \) as low as possible, and adjusts \( r_\emptyset \) to satisfy the ZPC. Incorporating these optimal interest rates and checking whether \( \Pi_s(r_\emptyset, r_1, r_0) \geq 2 \overline{\Pi} \)

leads to the following result, for which we first define

\[ B_{1,2} \equiv \frac{2 p_s}{p_s \overline{p} + p(2 - p)} = \frac{p_s}{\overline{p} + (p_s - \overline{p}) \frac{p}{2}} , \quad C_{1,1} \equiv \frac{p_s}{\overline{p}} = B_{1,1} , \quad \text{and} \quad C_{1,2} \equiv (1 + p_s) B_{1,2} . \]

\(^{22}\)Equivalently, the safe borrower’s indifference curve is less steep than the bank’s isoprofit line in \( (r_\emptyset, r_1) \) space (slopes: \(-1/p_s \) vs. \(-\overline{p}/p^2\)), but steeper in \( (r_\emptyset, r_0) \) space (slopes: \(-1/(1 - p_s) \) vs. \(-\overline{p}/p(1 - p)\)).
Proposition 1. Under assumptions A1-A2, a simple pooling contract that maximizes borrower surplus subject to borrower limited liability, lender breaking even, monotonicity, and limited borrower commitment achieves full efficiency, i.e. maximal outreach, with all surplus going to the borrowers, iff

\[
N \geq \begin{cases} 
B_{1,1} - \frac{b_{1,1} - b_{1,2}}{c_{1,2} - c_{1,1}} (g - c_{1,1}) & g \in [c_{1,1}, c_{1,2}] \\
B_{1,2} & g \geq c_{1,2}
\end{cases}.
\]

Figure 1 illustrates the Proposition:\textsuperscript{23} as \( g \) increases away from \( c_{1,1} \), net returns required for efficient lending decrease linearly from \( B_{1,1} \), hitting a floor at \( B_{1,2} \).\textsuperscript{24} Thus, by varying interest rates based on borrowing history, two-period dynamic lending can achieve full efficiency in some cases where static individual lending cannot. Intuitively, the dynamic contract better prices for risk by varying the second-period interest rate so as to shift the repayment burden toward risky borrowers \( (r_1 < r_0) \).

The dependence on \( g \) here is due to affordability considerations, i.e. limited liability after success. If affordability is not a concern, then \( r_1 \) is set as low as possible (to 0) to provide the maximal discount for success, while \( r_\emptyset \) is set high enough to pay for this discount. But this \( r_\emptyset \) is affordable only if \( g \) is high enough \( (g \geq c_{1,2}) \). If \( g \) is smaller, \( r_\emptyset \) must be lower and \( r_1 \) higher so the lender can break even. This limits the variation in second-period interest rates and thus limits the improvement in risk-pricing, raising the bar for efficient lending. The smaller is \( g \), the more this affordability constraint binds; at the corner where \( g = c_{1,1} \), no variation in interest rates is possible, and dynamic lending offers no improvement over static lending in attaining fully efficient lending. The left panel of Figure 2 displays the variation in optimal interest rates as \( g \) varies.\textsuperscript{25}

\textsuperscript{23}The conditions in this and following Propositions divide up the model’s parameters into two types: the technological return parameters \((\bar{R}, \bar{\pi}, \rho)\), which fully determine \( N \) and \( g \); and the population risk parameters \((p_s, p_r, \theta)\) which exclusively determine the \( B \)’s and \( C \)’s.

\textsuperscript{24}The subscript (1, 2) is used since the contract is for one borrower over two periods. It is straightforward to verify that \( 1 < B_{1,2} < B_{1,1} = c_{1,1} < c_{1,2} \).

\textsuperscript{25}See equations 11 and 13 in the proof of Proposition 1. By optimal interest rates, we mean the unique contract that achieves fully efficient lending for all values of \( N \) for which this is possible; this is also the
Note: Dynamic individual lending achieves fully efficient lending above the higher solid border, and nearly efficient lending between the two solid borders.

What happens if \((N, G)\) are too low for fully efficient lending? Unlike in the static case, there is an alternative to giving up on safe borrowers altogether: the contract can price them out of the market in a subset of states of the world. Consider next “nearly efficient” lending: all agents borrow at all histories except safe borrowers in period 2 after default in period 1. In other words, \(r_0 \in [\hat{r}_s, \hat{r}_r]\). This is the best alternative to fully efficient lending under the standard assumption that

\[
p_s + p_r > 1. \tag{A3}
\]

This guarantees that \(p_s > 1/2\), which implies that giving up on safe borrowers who fail costs best-for-safe contract at the lowest \(N\) for which fully efficient lending is achievable. (Above this minimum \(N\), safe borrowers are inframarginal at the best-for-safe contract, so it can be perturbed in all directions while still attracting them and achieving the same borrower surplus; in this sense, there is typically no unique optimal contract for a given \((G, N)\). But there is a unique optimal contract at this minimum \(N\) and this contract also achieves fully efficient lending for all \(N\) greater.)
Figure 2: Optimal Contract Interest Rates

FULLY Efficient Lending

NEARLY Efficient Lending

Note: Full-information interest rates for safe and risky types – $r_{FB}^s$ and $r_{FB}^r$ – are graphed with dash-dotted lines. Effective average interest rates over the two periods for safe and risky types in the optimal contract – $\tilde{r}_s$ and $\tilde{r}_r$ – are graphed with dashed lines. Parameter values are the same as in Figure 1.

less surplus than giving up on safe borrowers who succeed, since the majority succeed.

Consider maximizing the safe-borrower payoff in the case of $r_0 \in (\hat{r}_s, \hat{r}_r]$,

$$2\bar{R} - p_s r_0 - p_s^2 r_1 - p_s(1 - p_s)\hat{r}_s ,$$

subject to modified limited commitment and limited liability constraints consistent with only risky agents taking loans at rate $r_0$,

$$r_1 \leq \hat{r}_s , \quad r_0 \in (\hat{r}_s, \hat{r}_r] \quad \text{and} \quad r_0, r_1 \leq R_s , \quad r_0 \leq R_r ,$$

and subject to monotonicity constraints and a modified lender ZPC reflecting the fact that
only risky agents borrow after a failure:

\[ \bar{p}r_0 + \bar{p}^2 r_1 + \theta(1 - p_r)p_r r_0 \geq [1 + \bar{p} + \theta(1 - p_r)] \rho. \]

An immediate observation is that since safe borrowers never face \( r_0 \) (due to dropping out), the solution is to set it as high as possible to raise funds for the lender, i.e. \( r_0 = \hat{r}_r \). This maximally extracts surplus from risky borrowers on their loan after failure. Also, as before lowering \( r_1 \) and raising \( \bar{p} \) along the ZPC raises the safe-borrower payoff. Thus the best contract for safe borrowers sets \( r_1 \) as low as possible, and adjusts \( \bar{p} \) to satisfy the ZPC.

Incorporating these optimal interest rates and checking whether \( \Pi_s(r_0, r_1, r_0) \geq 2\bar{p} \) leads to the following result, for which we first define

\[
B_{1,2}^* \equiv \frac{p_s[1 + \bar{p} + \theta(1 - p_r)]}{\bar{p} + p_s[\bar{p} + \theta(1 - p_r)]} = \frac{p_s}{\bar{p} + (p_s - \bar{p}) \frac{\bar{p} + \theta(1 - p_r)}{1 + \bar{p} + \theta(1 - p_r)}}, \quad C_{1,2}^* \equiv (1 + p_s)B_{1,2}^*,
\]

\[
B_{1,1}^* \equiv \frac{p_s[1 + \bar{p} + \theta(1 - p_r)]}{\bar{p} + \bar{p}^2 + p_s\theta(1 - p_r)} = \frac{p_s}{\bar{p} + (p_s - \bar{p}) \frac{\bar{p} - p_r + \theta(1 - p_r)}{1 + \bar{p} + \theta(1 - p_r)}}, \quad \text{and} \quad C_{1,1}^* \equiv B_{1,1}^*.
\]

**Proposition 2.** Under assumptions A1-A3, a simple pooling contract that maximizes borrower surplus subject to borrower limited liability, lender breaking even, monotonicity, and limited borrower commitment achieves near efficiency, i.e. outreach to all borrowers in all states except safe borrowers after failure, with all surplus going to the borrowers, iff

\[
N \geq \begin{cases} 
B_{1,1}^* - \frac{B_{1,1}^* - B_{1,2}^*}{C_{1,2}^* - C_{1,1}^*} (G - C_{1,1}^*) & G \in [C_{1,1}^*, C_{1,2}^*] \\
B_{1,2}^* & G \geq C_{1,2}^*
\end{cases}
\]

but the analogous condition in Proposition 1 is not met.

Similar to fully efficient lending, as \( G \) increases away from \( C_{1,1}^* \), nearly efficient lending is possible for lower and lower levels of \( N \), with a floor reached at \( B_{1,2}^* \). Again, the dependence of nearly efficient lending on \( G \) is due to affordability considerations. If affordability is no issue, then \( r_1 \) can be set at its lower bound (0) with \( r_0 \) set high enough to ensure the
lender zero profits. But if $\mathcal{G}$ is too small ($\mathcal{G} < \mathcal{E}^*_1, 2$), $r_0$ must be lower and $r_1$ higher so the lender can break even. Again, this limits the variation in interest rates and thus limits the improvement in risk-pricing, raising the bar for nearly efficient lending; see Figure 1.

It is straightforward to verify that $\mathcal{B}^*_1, 1 < \mathcal{B}_{1,1}$ and $\mathcal{E}^*_1, 1 < \mathcal{E}_{1,1}$, $\mathcal{B}^*_1, 2 < \mathcal{B}_{1,2}$, and $\mathcal{E}^*_1, 2 < \mathcal{E}_{1,2}$, implying that the conditions for nearly efficient lending are strictly weaker than those required for fully efficient lending. Thus, lenders may be able to include safe borrowers most of the time when always including them is out of reach – by giving up on failed safe borrowers. The intuition is that giving up on failed safe borrowers enables even better risk-pricing, since quite a few risky borrowers can be perfectly targeted with quite high (reservation) rates. This is clear from Figure 2.\(^{26}\) The dashed lines in both panels give the respective average rates paid in the optimal contract by safe and risky borrowers over the two periods, while the dash-dotted lines give the respective full-information rates. In both the fully efficient and nearly efficient cases, higher $\mathcal{G}$ allows the expected rates to move toward the full-information rates; however, in the nearly efficient case, risky and safe borrowers pay quite different effective rates even for low $\mathcal{G}$, since $r_0$ can still be quite high when $\mathcal{G}$ is low. Thus, charging failed borrowers the risky reservation rate dramatically improves risk-pricing.

An interesting corollary is that safe borrowers often prefer being priced out of the market, i.e. prefer the nearly efficient contract to the fully efficient contract.\(^{27}\) The reasoning is that safe borrowers who fail get the same payoff under both contracts in period 2, their reservation payoff – whether they borrow and pay $\hat{r}_s$ (as under the fully efficient contract), or drop out instead of paying $\hat{r}_r$ (as under the nearly efficient contract). From this narrow standpoint, there is no cost of being excluded. What matters, then, is how much the lender can extract from failed borrowers so as to lower other rates, $r_0$ and $r_1$; and charging the risky borrowers’ reservation rate extracts the most profits when $N$ is not too high.

One implication of this counterintuitive result is that sometimes when fully efficient lending is achievable, there is a tradeoff between equity and efficiency (if $N \leq \mathcal{B}_{1,1}$). The

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\(^{26}\)See equations 15 and 17 in the proof of Proposition 2 for the nearly efficient interest rates.

\(^{27}\)Sufficient for this is $N \leq \mathcal{B}_{1,1}$. 

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efficient, low “penalty” rate \( r_0 = \hat{r}_s \) creates more surplus by funding all projects, but results in a larger cross-subsidy from safe to risky; the high “penalty” rate \( r_0 = \hat{r}_r \) sacrifices some projects’ surplus but reduces the cross-subsidy even more. Exclusion from the credit market turns out to be a good thing here for those excluded.

The contracts derived above tend to feature back-loaded incentives, with borrowers paying relatively high initial rates and lower rates over time, especially as they develop good borrowing records. Safe agents borrow in period 1 even though they may earn less than their outside option, in anticipation of cheaper future loans. This back-loaded structure entices borrowers to keep borrowing while allowing the lender some ability to vary rates as information is revealed. By contrast, starting with a neutral rate and hiking it after failure would run the risk of driving away unlucky safe borrowers.\(^{28}\)

What can the lender achieve when neither efficient nor nearly efficient lending is feasible? Interestingly, in some cases when \( G \) is low, it can be optimal to exclude safe borrowers after success but not failure, or after both success and failure, as a way to extract sufficient funds from risky borrowers in period 2 to attract safe agents to borrow in period 1. If \( G \) is high enough, these options are never optimal; but when affordability is low, they may be optimal in subsets of the parameter space. Thus, even when nearly efficient lending is not possible, other contracts can sometimes include safe borrowers some of the time.\(^{29}\) The main point, however, is that when static individual lending breaks down, a simple dynamic contract can in some cases achieve efficient lending, or at least nearly efficient lending. It does this by using information revealed over time to improve risk-pricing by targeting higher rates to borrowers who have defaulted and lower rates to borrowers with good records.

\(^{28}\)The variation in interest rates called for by the optimal contracts can be quite high for high \( G \). In reality, some lenders may be constrained for reasons outside the model not to vary their interest rates too widely – if so, this would limit the effectiveness of dynamic contracting in this context. However, as discussed in the next section, charging a gross interest rate \( r_1 \) at or near zero is not necessary if we move beyond a simple pooling contract. One may equivalently implement the optimum by “collecting” some of the period-1 rate as forced savings that collateralizes the period-2 loan after success, and charging a moderate, positive rate on the period-2 loan after success.

\(^{29}\)Details are available from the authors on request.
4.2 More complicated contracts

Given the constraints under which many micro-lenders operate and their apparent preference for simple products – standardized contracts seem quite commonplace, perhaps because borrowers and bank officers often lack significant quantitative expertise – the pooling contracts analyzed thus far may be interesting in their own right. Here we argue that the simple pooling contracts cannot be improved upon by a range of more complicated contracts. Thus, this model provides a context in which simple contracts are optimal, and screening borrowers is not key to efficiency.

First, consider forced savings, collateral, and self-financing. In general, each of these instruments could be valuable by raising the amount paid by borrowers after failure, thus shifting more of the repayment burden onto risky borrowers. However, here they are only available after period-1 success, since limited liability implies that the borrower has no pledgeable wealth initially and after period-1 failure. In the end, these instruments offer nothing that cannot be accomplished by a simple contract of the form \((r_0, r_1, r_0)\).

In the case of collateral, consider a contract with rates \(r_0\) and \(r_1\), with required collateral \(\kappa > 0\) on the period-2 loan after success.\(^{30}\) The monotonicity constraint requires \(\kappa \leq r_1\); otherwise a borrower who failed would be tempted to claim success to pay the lower amount.\(^{31}\) Thus the successful borrower pays \(r_0\) at the end of period one; pays \(r_1\) if he succeeds in period two; and pays \(\kappa\) if he fails in period two. In terms of total two-period payoffs, this is equivalent to the simple contract \(r'_0 = r_0 + \kappa, r'_1 = r_1 - \kappa \geq 0\), and nothing due after failure in period two. This modified contract satisfies the same constraints as the previous one. Thus the bank can mimic any collateral policy with a simple contract already analyzed, by using the period-1 rate essentially to collect the collateral upfront.\(^{32}\)

\(^{30}\)We assume no deadweight loss associated with pledging collateral.

\(^{31}\)Without the monotonicity constraint here, collateral could indeed improve on the simple contract derived earlier, but only by stipulating more-than-full collateralization of loans. This would make borrowers pay more after failure than success, further shifting repayment burden toward risky borrowers. However, the optimal contract would be vulnerable to the failed borrower claiming success and paying up rather than lose the more valuable collateral \(\kappa\).

\(^{32}\)The same is also true in reverse: one can implement the optimal simple pooling contract derived earlier,
Nearly identical arguments show that a forced savings policy, where the savings is used as collateral, or a period-2 self-financing requirement can be replicated by a simple pooling contract. The idea is the same: the lender can collect the savings or self-financing amount upfront through the period-1 interest rate without adversely affecting any constraints. It is also clear that hidden savings poses no problem for the optimal pooling contract: after succeeding, the borrower gets a free loan, which he will take whether or not he saves.

We also assumed that simple pooling contracts cannot involve negative interest rates or first-time borrowing in period 2. One can show that these also would offer no improvement. Regarding interest rates, it is true that the lender may want to subsidize success by charging a negative interest rate on the period-2 loan after success, if possible – but monotonicity forces it to subsidize failure at least as strongly. Thus negative interest rates offer no increased scope for differentially subsidizing success. Regarding first-time borrowing in period 2, note that the same payoffs can be achieved by giving the would-be first-time period-2 borrowers the same deal in period one and an unattractive deal in period two. Thus, contracts that induce first-time period-2 borrowing offer nothing that cannot be accomplished without allowing it.

In sum, savings, collateralization, and self-financing are allowed for by simple contracts; and negative interest rates and first-time period-2 borrowing offer no efficiency gains. A final question is, can a menu of contracts do better by screening?

**Lemma 2.** Under assumptions A1-A3, if an optimal incentive compatible menu of contracts of the form \( \{(r_0^r, r_1^r, r_0^s)\}_{\tau \in \{r,s\}} \) achieves fully efficient (resp., nearly efficient) lending while satisfying borrower limited liability, lender breaking even, monotonicity, and limited borrower commitment, there exists a simple pooling contract satisfying the same constraints that also achieves fully efficient (resp., nearly efficient) lending, with all surplus going to the borrowers. Thus, screening offers no improvement. This is due to the fact that payoffs of one type of borrower and lender profits from that type of borrower are zero-sum. By incentive which involves \( r_0^r \) high and \( r_1 = 0 \), with a more moderate \( r_0^s \), a strictly positive rate \( r_1 > 0 \), and a collateral or forced savings stipulation fully guaranteeing the second loan.

Thus the result would not obtain if borrowers were risk-averse. Payoffs of the two parties could even
compatibility, the risky borrower’s payoff is not higher under the safe borrower’s contract; the zero-sum property implies that the lender’s profits are not lower if the risky borrower chooses the safe borrower’s contract instead. Thus, offering only the safe borrower’s contract does not adversely impact the safe borrower or lender profits; it also attracts the risky agents to borrow, due in part to cross-subsidization (Lemma 1). Thus, offering both risky and safe only the safe borrower’s contract – i.e. a simple pooling contract – does as well as the menu.

5 Comparing Group Lending and Dynamic Lending

Group lending has been analyzed extensively in this context of low-wealth borrowers of unknown risk, and has been shown to improve efficiency relative to static individual contracts. But, it is unknown how group lending fares here compared to dynamic individual lending, and further, whether dynamic lending offers similarly large efficiency improvements that could plausibly have underpinned part of the success of the microcredit movement.

In this section, existing results on group lending in this context are reviewed first. Second, the efficiency properties of group and dynamic lending are formally compared. Third, additional factors that could affect the comparison are discussed.

5.1 Group lending results

Here we review results from the model of Ghatak (1999, 2000) and Gangopadhyay et al. (2005). The environment is the one studied above, except that there is only one period, and agents are now assumed to know each other’s risk; risk is still unobservable to the lender.

The theory considers a symmetric “group contract”, written with pairs of homogeneously-matched borrowers. There are now two parameters: $r$ is the amount paid by an agent who succeeds, and $c$ is the additional amount paid by an agent who succeeds and whose partner move in the same direction with parameters, due to the gains from selling insurance. Diametrically opposed preferences are the key feature here making the standard problems with pooling inapplicable.
fails. Without loss of generality for efficiency, we restrict attention to pooling contracts. The expected payoff of a borrower of type $\tau \in \{r, s\}$ is then

$$R - p_r r - p_r (1 - p_r) c = R - p_r [r + (1 - p_r) c].$$

The beauty of joint liability in this context is that it causes the effective interest rate (the bracketed term above) to vary positively with borrower risk, just as in the full-information contract, even though the lender has no information on risk.

Several constraints are put on the contract, in addition to lender break-even and limited liability. First, homogeneous matching must be stable even when side payments are possible. This is equivalent to $c \geq 0$, as Ghatak (1999, 2000) shows. Second, as in Gangopadhyay et al. (2005), the contract must involve group payments that increase with the number of successes:

$$0 \leq r + c \leq 2r \iff - r \leq c \leq r.$$

If this were not true, a successful borrower would have the incentive to claim his partner succeeded when he failed, in order to pay $2r$ rather than $r + c$.

Similar to dynamic lending, a monotonic group lending contract that attracts safe borrowers also attracts risky. Thus, group lending achieves fully efficient lending, or only attracts risky borrowers. The analysis then focuses on whether fully efficient lending is feasible. If so, the constraints that must be satisfied by $(r, c)$ simplify to

$$0 \leq c \leq r, \quad r + c \leq R_s, \quad \text{and} \quad \bar{p} \cdot r + p(1 - p) \cdot c \geq \rho.$$

Consider maximizing the safe-borrower payoff subject to the above constraints. A key observation is that lowering $r$ and raising $c$ along the ZPC raises the safe-borrower payoff.

\footnote{Ahlin (2013) shows this is without loss of generality in a more complicated context. Lemma 2 could also easily be adapted to the model of this section.}

\footnote{The safe borrower’s indifference curve in $(r, c)$ space is steeper (slope: $-1/(1 - p_s)$) than the bank’s isoprofit line (slope: $-\bar{p}/p(1 - p)$).}
intuition is that a higher $c$ relative to $r$ puts more repayment burden on states of the world with more failures, and thus on risky borrowers. Thus the best contract for safe borrowers has full liability $c = r$ if affordable, and the maximum affordable level otherwise. Incorporating these parameters and checking whether the safe-borrower payoff exceeds $\pi$ leads to the following result, for which we first define

$$B_{2,1} \equiv \frac{p_s(2 - p_s)}{p(2 - p)} = \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r}{2-p_s}}$$

and

$$C_{2,1} \equiv \frac{2}{2-p_s} B_{2,1}$$

**Proposition 3.** Under assumptions A1-A2, a group contract that maximizes borrower surplus subject to homogeneous matching, borrower limited liability, lender breaking even, and monotonicity achieves full efficiency, i.e. maximal outreach, with all surplus going to the borrowers, iff

$$N \geq \begin{cases} 
B_{1,1} - \frac{B_{1,1} - B_{2,1}}{C_{1,1} - C_{2,1}} (G - C_{1,1}) & G \in [C_{1,1}, C_{2,1}] \\
B_{2,1} & G \geq C_{2,1}
\end{cases}$$

Otherwise, only risky agents borrow.

That is, as $G$ increases away from $C_{1,1}$, net returns required for efficient lending decrease linearly from $B_{1,1}$, hitting a floor at $B_{2,1}$.\footnote{The subscript $(2,1)$ is used since the contract is for two borrowers in one period. It is straightforward to verify that $1 < B_{2,1} < B_{1,1} = C_{1,1} < C_{2,1}$.} The dependence on $G$ here is again due to affordability considerations. The contract specifies full liability if this is affordable by successful borrowers; if not, $c$ must be less than $r$. This limits the improvement in risk-pricing that group lending offers, raising the bar for efficient lending.

### 5.2 Model-based comparison

We next compare maximal surplus achievable by the two types of contracts. The goal is to derive insight into the strengths and weaknesses of two seemingly popular contract forms in
improving lending markets.

Of course, if the economic environment is such that both types of contracts are feasible, lenders may not need to choose between them; the optimal contract can combine elements of both dynamic and group-based strategies, as shown in Section 6. However, it is easy to imagine environments in which one of the two types of contracts is not workable. The dynamic contract assumes two periods of project endowment and lender commitment; it therefore may not work well for a new lender that has not yet gained the trust of borrowers, or if borrowers have only sporadic need for capital. The group contract assumes borrowers know each others’ types and match frictionlessly; it therefore may not work well in relatively anonymous or mobile urban contexts. Hence, the following comparisons shed light on surplus attainable across different environments, one in which dynamic lending is feasible but group lending is not, and vice versa. For example, if group lending is not available or ineffective in some environment, how good a substitute is dynamic lending?

In sum, while the comparison may not be considered useful as a comparison of two different tools in a lender’s toolbox – since if they are both in the toolbox, they can both be used – it is informative about how well two different lenders, with two different tools available, can expect to do. If it is true that quite a few lenders have not had both tools in their toolbox, then the comparison sheds light on the relative significance group lending and dynamic lending may have had in the microcredit revolution.

We first compare the ability to achieve fully efficient lending.

**Corollary 1.** Under assumptions A1-A2, static group lending achieves fully efficient lending whenever dynamic individual lending does; and for any \( G > C_{1,1} \), there exists a range of \( N \) for which static group lending achieves fully efficient lending but dynamic individual lending does not.

Thus, group lending dominates dynamic lending in achieving full efficiency; see Figure 3.

Why? Note that both types of lending reveal the exact same information, namely two draws from the borrower’s distribution. The two draws are time series observations in the
dynamic case. In the group case, the two draws are cross-sectional observations, just as informative of each borrower’s type since groups contain identical borrowers.\textsuperscript{37}

Note also that without constraints on the parameters, the two types of contracts are equally effective at fully efficient lending. The group lending payoff is

$$
\bar{R} - p_r r - p_r (1 - p_r)c = \bar{R} - p_r [(r + c) - p_r c],
$$

while the per-period dynamic lending payoff is

$$
\frac{2\bar{R} - p_r r_\emptyset - p_r^2 r_1 - p_r (1 - p_r)r_0}{2} = \bar{R} - p_r \left[ \frac{(r_\emptyset + r_0)}{2} - p_r \frac{(r_0 - r_1)}{2} \right].
$$

\textsuperscript{37} Thus the lender’s Bayesian posterior assessment that a borrower is safe, e.g., would be the same after an individual succeeded in both projects under a dynamic contract as after both group members succeeded under a group contract $\left( \frac{(1-\theta)p_r^2}{\theta p_r^2 + (1-\theta)p_r^2} \right)$.
All average payoffs (including the lender’s) resulting from dynamic contract \((r_0, r_1, r_0)\) can be replicated by group contract \(r = (r_0 + r_1)/2\) and \(c = (r_0 - r_1)/2\), and any group contract \((r, c)\) can be replicated by dynamic contract \(r_0 = 2r, r_1 = 0,\) and \(r_0 = 2c\). Both contracts give payoffs quadratic in risk-type, and with two types, two contract instruments can position the quadratic so as to eliminate any cross-subsidy and attract all borrowers.

Thus, the contracts do not differ in information revelation and, unconstrained, in ability to tailor payoffs by risk-type. All differences come from constraints. As it turns out, group lending is less constrained in its ability to target discounts to safe borrowers. To see this, note that the bracketed terms in the above payoffs can be considered the “effective interest rate” under each type of contract.\(^{38}\) Embedded in each of the effective interest rates is a “discount” term (in bold), multiplying risk-type and thus differentially enjoyed by safe borrowers: \(c\) for group lending, \((r_0 - r_1)/2\) for dynamic lending.\(^{39}\) Both contracts are constrained in how high this discount can be. With group lending, the monotonicity constraint forces \(c \leq r\). With dynamic lending, the monotonicity constraint forces \(r_1 \geq 0\) and the limited commitment constraint keeps \(r_0 \leq \hat{r}_s\); the overall discount is then bounded by \(\hat{r}_s/2\). It turns out that the dynamic lending bound is more restrictive, and thus the dynamic contract has a harder time targeting discounts to safe borrowers.

In sum, with dynamic lending, the need to keep safe borrowers in the market over time critically limits the amount by which the lender can vary the interest rate to target discounts to safe borrowers. When “penalty” rates need to attract unlucky safe borrowers, they cannot be too severe.

However, when fully efficient lending is impossible for dynamic contracts, nearly efficient lending may still be possible, while group contracts only attract risky borrowers if fully efficient lending is not possible. This leaves open the possibility that dynamic lending can

\(^{38}\) The effective interest rate is the analog to the interest rate from a static individual loan contract, where the payoff is \(\Pi - p_t r\).

\(^{39}\) Thus, under the group contract the safe borrower’s effective interest rate is lower than the risky’s by \((p_s - p_r)c\): the safe borrower has a safer partner (by \(p_s - p_r\)) so, conditional on success, is less likely to owe \(c\). The comparable figure under the dynamic contract is \((p_s - p_r)(r_0 - r_1)/2\): the safe borrower is more likely to succeed (by \(p_s - p_r\)) and get an interest rate discount \((r_0 - r_1)\) on \(1/2\) its loans.
outperform group lending in some cases. Indeed, defining

\[ p_B \equiv \frac{(2 - p_s)[p_s + \theta(1 - p_s)]}{1 + p_s + \theta(1 - p_s)} \quad \text{and} \quad p_A \equiv \frac{(2 - p_s)[p_s + \theta(1 - p_s)]}{2 + \theta(1 - p_s)}, \]

one can show that \( B_{1,2}^* < B_{2,1} \) iff \( p_r < p_B \).

Further:

**Corollary 2.** Under assumptions A1-A3, for any \( G > C_{1,1}^* \) but not too large, there exists a range of \( N \) for which dynamic individual lending achieves nearly efficient lending but static group lending attracts only risky borrowers. If in addition \( p_r < p_A \), then for any \( G > C_{1,1}^* \), there exists a range of \( N \) for which dynamic individual lending achieves nearly efficient lending but static group lending attracts only risky borrowers.

That is, dynamic lending can achieve near-efficiency in some cases when group lending attracts only risky. These cases always exist when \( G \) is relatively low – see Figure 3 – and if \( p_r < p_A \) (unlike in Figure 3), they exist for any value of \( G \), i.e. the cutoff for nearly efficient dynamic lending is always below the one for group lending.

Thus, dynamic lending in some cases achieves greater efficiency than group lending, but only when it prices failed safe borrowers out of the market to lower the new-borrower rate. Giving up on failed safe borrowers allows the lender to vary the interest rate more greatly based on record, thus targeting greater discounts to safe borrowers.

In general, then, the efficiency comparison between group and dynamic lending is ambiguous, and neither style of lending outperforms the other in every parameter configuration. More specifically, it is the most marginal regions – relatively low returns \((N, G)\) – that would benefit most from dynamic lending, which could revive a market to nearly efficient levels while group lending would fail to make an impact. This finding is consistent with the idea that dynamic lending may have done more to revive credit markets in more marginal environments than group lending.\(^{41}\)

\(^{40}\)Two sufficient conditions for \( p_r < p_B \) are \( p_r \leq 1/2 \) and \( p_s \leq [1 - 4\theta + \sqrt{1 + 8\theta}]/(4 - 4\theta) \). The latter condition is equivalent to \( p_s \leq p_B \), and is sufficient because \( p_r < p_s \) by assumption; its bound ranges from 1/2 when \( \theta \approx 0 \) to 2/3 when \( \theta \approx 1 \).

\(^{41}\)Baland et al. (2013) make a related point in a different context, that group lending may have difficulty
5.3 Factors that alter the comparison

Other factors within and beyond the model may of course alter the comparison. Already mentioned is the obvious point that different assumptions are required for each type of contract. Group lending assumes good local information and low-friction, purposeful matching. Realistically, these are only available in degrees, and may be especially lacking in more anonymous urban contexts. Dynamic lending assumes commitment by the lender to adjust rates in a pre-announced way based on borrowing history. This may be easier said than done – lenders may not follow through in pricing borrowers out of the market when outreach is the main goal, and they may not follow through with large discounts if financial sustainability is at stake. Lenders may also be constrained by political or religious concerns not to vary interest rates too widely, if at all. Further, even if the lender can commit, it may be difficult to signal this credibly to the borrowers. Also, borrowers’ need for capital may be sporadic, so that future loans cannot be used reliably to price for risk after information has been revealed. In short, the relative performance of the two types of contracts in many cases depends more on which strategy is feasible to implement – based on political, informational, and commitment frictions – than on the specific risk and return parameters focused on in the previous section.

The previous section also points to an interesting parallel between group lending and dynamic lending: the analog between size of group in group lending and number of periods in a dynamic lending relationship, both of which determine the amount of information revelation. Specifically, a \( k \)-person group contract reveals \( k \) draws from the distribution of a borrower’s type (under homogeneous matching), as does a \( k \)-period dynamic contract. One

\[ \text{reach} \] low-wealth areas.

However, in support of this assumption in one context, Ahlin (2009) finds evidence consistent with moderately homogeneous risk-matching among rural/semi-rural microcredit groups in Thailand.

This perspective points to a potential explanation for microfinance shifting away from group lending as it matures (though this pattern is not obvious in the data, see de Quidt et al., 2012). As microlenders continue to lend, they may become more able to credibly commit, making dynamic lending more feasible; and they accumulate information on borrowers, reducing the informational advantage borrowers had regarding each other, and thus the value of group lending.
might conjecture that larger groups and longer contracts reveal more information and are better able to price for efficiency. While a formal analysis is outside the scope of this paper, this suggests that contextual limits on either group size or relationship length would shift the comparison toward one or the other type of contract.

Correlated risk also affects information revelation. Consider serial correlation. In the extreme case of perfect correlation, a two-period dynamic contract reveals to the lender not two draws from the borrower’s distribution, but one. The dynamic contract is no better than a static one. Consider spatial correlation. Again under perfect correlation, a two-person group contract gives the lender only one draw from the borrower’s distribution, and the group contract is no better than an individual contract. While a detailed analysis is beyond the scope of the paper, it appears that serial correlation works against dynamic lending while spatial correlation works against group lending. Plausibly, spatial correlation is more prevalent in agricultural endeavors, while serial correlation could be more relevant to small enterprise start-ups, if early shocks to a firm have long-term effects.

6 Dynamic Group Contracts

Assume now that the conditions for both types of contracts are met – dynamic endowments, local information, frictionless matching, and lender commitment. Following on the previous analysis, we restrict attention to homogeneous-matching, two-person, two-period pooling contracts (i.e. all borrowers sign identical contracts) in which nothing is paid by borrowers or lender immediately after both borrowers fail, and which does not allow first-time borrowing in period 2. We also restrict attention to contracts that are symmetric even ex post – that is, the rates borrowers face depend only on group performance, not individual performance. Thus

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44Ahlin (2013) analyzes group size under static group lending and finds that larger groups generally allow for more efficient lending. (This no longer holds if the quality of local information deteriorates sufficiently with group size.) Section 7 states a similar conjecture for relationship duration in individual dynamic lending.

45Ex post symmetry is without loss for efficiency. Intuitively, it might appear optimal to charge different rates to two borrowers after one succeeded and the other failed. However, in this homogeneous-matching adverse selection context, differentiating between co-grouped borrowers is pointless, since their types are
the “dynamic group” contract can be written \((r_\emptyset, c_\emptyset, r_0, c_0, r_1, c_1, r_2, c_2)\), where \((r_\emptyset, c_\emptyset)\) are the interest and joint liability rates at the null history (period 1), and \((r_\sigma, c_\sigma)\) for \(\sigma \in \{0, 1, 2\}\) are the interest and joint liability rates faced by both group borrowers in period 2 after \(\sigma\) successes in the group in period 1.

When and how is fully efficient lending achievable? If so, the safe-borrower payoff is

\[
\Pi_{ss} = 2R - p_s r_\emptyset - p_s (1 - p_s) c_\emptyset - p_s^3 r_2 - p_s^3 (1 - p_s) c_2 - 2p_s^2 (1 - p_s) r_1 - 2p_s^2 (1 - p_s)^2 c_1 - p_s (1 - p_s)^2 r_0 - p_s (1 - p_s)^3 c_0 ,
\]

and analogously for risky borrowers. The contract must satisfy the following ZPC,

\[
\bar{p} r_\emptyset + p(1 - p) c_\emptyset + \bar{p}^2 r_2 + \bar{p}^2 (1 - p) c_2 + 2p^2 (1 - p) r_1 + 2p^2 (1 - p)^2 c_1 + p(1 - p)^2 r_0 + p(1 - p)^3 c_0 \geq 2 \rho ,
\]

limited liability (more binding for safe borrowers),

\[
r_\emptyset, r_0, r_1, r_2, r_\emptyset + c_\emptyset, r_0 + c_0, r_1 + c_1, r_2 + c_2 \leq R_s ,
\]

limited commitment\(^{46}\) (again more binding for safe borrowers),

\[
r_0 + (1 - p_s) c_0 \leq \hat{r}_s , \quad r_1 + (1 - p_s) c_1 \leq \hat{r}_s , \quad r_2 + (1 - p_s) c_2 \leq \hat{r}_s ,
\]

monotonicity in period 2,

\[
2r_0 \geq r_0 + c_0 \geq 0 , \quad 2r_1 \geq r_1 + c_1 \geq 0 , \quad 2r_2 \geq r_2 + c_2 \geq 0 ,
\]

identical. The formal argument is omitted for brevity.

\(^{46}\)These constraints ensure the effective interest rate does not exceed safe agents’ reservation rate; they are equivalent to period-2 payoffs at each history being at least \(\bar{r}\).
Values for the outreach, with all surplus going to the borrowers, iff even, monotonicity, and limited borrower commitment achieves full efficiency, i.e. maximal rower surplus subject to homogeneous matching, borrower limited liability, lender breaking

\[ 2r + 2p_r[r_2 + (1 - p_r)c_2] \geq r + c + 2p_r[r_1 + (1 - p_r)c_1] \geq 2p_r[r_0 + (1 - p_r)c_0], \quad \tau \in \{r, s\}, \]

and the constraint guaranteeing stable homogeneous matching, \( \Pi_{ss} + \Pi_{rr} \geq \Pi_{sr} + \Pi_{sr} \), which simplifies to

\[
c + (p_s + p_r)(p_s + p_r - 1)c_2 - (p_s + p_r)r_2 + [2(p_s + p_r)(2 - p_s - p_r) - 1]c_1
\]

\[ + [2(p_s + p_r) - 1]r_1 - (p_s + p_r - 1)(2 - p_s - p_r)c_0 - (p_s + p_r - 1)r_0 \geq 0.\]

Maximizing the safe-borrower payoff subject to these constraints gives rise to:

**Proposition 4.** Under assumptions A1-A3, a dynamic group contract that maximizes borrower surplus subject to homogeneous matching, borrower limited liability, lender breaking even, monotonicity, and limited borrower commitment achieves full efficiency, i.e. maximal outreach, with all surplus going to the borrowers, iff

\[
N \geq \begin{cases}
B_{1,1} - \frac{b_{1,1} - b_s^a}{c_{2,2} - c_{1,1}} (g - c_{1,1}) & g \in [c_{1,1}, c_{2,2}] \\
B_{2,2}^a - \frac{b_{2,2} - b_s^a}{c_{2,2} - c_{2,2}} (g - c_{2,2}^a) & g \in [c_{2,2}^a, c_{2,2}] \\
B_{2,2}^b - \frac{b_{2,2} - b_s^b}{c_{2,2} - c_{2,2}} (g - c_{2,2}^b) & g \in [c_{2,2}^b, c_{2,2}] \\
B_{2,2}^c - \frac{b_{2,2} - b_s^c}{c_{2,2} - c_{2,2}} (g - c_{2,2}^c) & g \in [c_{2,2}^c, c_{2,2}] \\
B_{2,2} & g \geq c_{2,2}
\end{cases}
\]

Values for the \( B \) and \( C \) cutoffs are defined in the proof. It is easy to verify that \( 1 < B_{2,2} < B_{2,2}^b < B_{2,2}^a < B_{2,2}^c = c_{1,1} < c_{2,2}^a < c_{2,2}^b < c_{2,2}^c < c_{2,2} \). Thus, the net return required for fully efficient lending is decreasing and piecewise linear in \( g \), hitting a floor at \( B_{2,2} \) – see Figure 4. Not surprisingly, these conditions for fully efficient lending are strictly weaker

\footnote{These constraints guarantee that total expected group payments are monotonic in number of successes. An alternative approach would take the perspective of an individual borrower in the group, and thus not take into account current repayment choice on the partner’s future expected repayment. Both seem to have merit, but we opt for the more cooperative view of the group here.}
Note: Fully efficient lending is achieved above the respective lines by dynamic group lending, static group lending, and dynamic individual lending.

than those for static group lending and dynamic individual lending – here the lender can use both joint liability and record-dependent interest rates to improve risk-pricing.

The nature of the optimal contract\textsuperscript{48} varies interestingly with $\varpi$, which determines affordability. To see how, first consider the “benchmark contract” from lending without the limited liability and dynamic monotonicity constraints. The benchmark contract takes a very simple form: full liability on all loans; high period-2 rates (jointly at safe reservation levels) for groups with any failures in period 1; low period-2 rates (at zero) for groups with no failures in period 1; and period-1 rates high enough to satisfy the ZPC. In other words, the benchmark contract is a perfect hybrid of joint liability lending (full liability), and dynamic individual lending (discounts after success, high rates after failure, back-loading) – with no tradeoffs, both group and dynamic strategies are used to the fullest. However, affordability

\textsuperscript{48}As before, we mean the unique contract that achieves fully efficient lending for all $N$ above the cutoff.
Figure 5: Optimal Dynamic Group Contract Interest Rates

Note: Parameter values are the same as in Figure 4.

and dynamic monotonicity concerns introduce tradeoffs between the two strategies.

Contract terms as a function of $G$ are displayed in Figure 5 (expressions are in the proof of Proposition 4). For $G$ at $C_{1,1}$, joint liability and temporal variation in interest rates are completely unaffordable – so, all interest rates are the same (all $r$'s equal to $R_s$) and there is no joint liability (all $c$'s equal to zero). Dynamic group lending offers no improvement over static lending in achieving full efficiency. As $G$ increases away from $C_{1,1}$, the contract can begin to use some of these strategies. Interestingly, in the first two intervals of $G$ in condition 9 above, $G \in \left[ C_{1,1}, C_{2,2}^{a} \right]$, the optimal contract uses slackening in affordability constraints (i.e. increases in $G$) to provide greater discounts for success, i.e. to lower $r_2$. 
(and the effective interest rate \( r_2 + (1 - p_s)c_2 \)) rapidly. By contrast, \( c_\emptyset \) stays at zero over this interval, i.e. joint liability on the first loan is not utilized. This is because affordability constraints give rise to a tradeoff – greater affordability can be used to raise joint liability, i.e. \( c_\emptyset \), or provide discounts for success, i.e. lower \( r_2 \). Evidently, discounts for success are the more effective way to price for risk, and thus \( r_\emptyset \) is kept at the maximum affordable level in order to raise money to lower \( r_2 \), even though this precludes joint liability in period 1. At \( G = C_{2,2}^b \), the contract achieves the maximal discount for success, i.e. \( r_2 = c_2 = 0 \), but still no joint liability in period 1 (\( c_\emptyset = 0 \)). In sum, the first priority at low levels of affordability is not joint liability, but dynamic discounts for success – a point similar to Section 5’s.\(^{49}\)

In the third and fourth intervals of \( G \) in condition 9, \( G \in [C_{2,2}^b, C_{2,2}] \), the optimal contract uses further slackening of affordability to increase joint liability. In particular, \( c_\emptyset \) increases from zero to \( r_\emptyset \) as \( G \) increases from \( C_{2,2}^b \) to \( C_{2,2} \), and remains there for \( G > C_{2,2} \). This would leave the contract identical to the benchmark contract described above – full liability on all loans, free loans after two successes, reservation-rate loans after any failure – except for the dynamic monotonicity constraint, which forces another tradeoff. In particular, consider the benchmark contract after one success and one failure in period 1. It imposes full liability, so the group pays for two loans in period 1, as if there had been no failure; but then in period 2, the group pays penalty rates, as if there had been no success. Clearly, the group would prefer to avoid this double jeopardy by claiming two successes, paying the same amount in period 1, but then getting discount rates in period 2. In short, the benchmark contract fails to satisfy dynamic monotonicity. The dynamic monotonicity constraint forces the lender to moderate the penalty for one group failure, and in particular to choose between joint liability or dynamic interest rate penalties (or some mix of each). Joint liability wins out as the more effective approach, so that in the fourth interval of \( G \) in condition 9, \( G \in [C_{2,2}^e, C_{2,2}] \),

\(^{49}\)It is true that period-2 joint liability is in some cases rising as \( G \) increases in these two intervals. However, this is essentially costless: affordability is slackening, but limited commitment is not, so the contract is not free to raise \( r_0 \) and \( r_1 \) to fund discounts for success, as it does \( r_\emptyset \). What it does instead is to achieve similar levels of profits and payoffs at each period-2 history in a way that is less tilted against safe borrowers – i.e. more through joint liability and less through the direct interest rate – thus improving risk-pricing.
and \( r_1 \) and \( c_1 \) decrease from reservation-rate levels all the way down to zero as \( G \) increases. In short, joint liability is prioritized in these higher ranges of \( G \), even though the lender must consequently use suboptimal dynamic interest rate adjustments – awarding preferential rates to groups with one failure, even though these groups are more likely to be risky (under assumption A3).  

Overall, the optimal contract prioritizes dynamic pricing – particularly discounts for success – at low levels of affordability and places greater priority on joint liability pricing at higher levels of affordability. This matches the insights of the earlier analysis, that joint liability lending may not work as well in more marginal markets or where affordability of loan repayment after success is an issue.

7 Extensions

Competition. The contracts derived maximize borrower surplus, which may correspond best to the case of a single non-profit lender. This was a common scenario in microcredit markets around the world, but for-profit lending and multiple lenders are on the rise as microcredit becomes more commonplace.

While a full analysis is beyond the scope of the paper, assume there is no local information (and thus no group lending) and consider competition in the case of \( G \geq c_{1,2} \) and \( N \in [B_{1,2}, B_{1,1}] \). Propositions 1 and 2 show that the optimal contract here delivers nearly efficient lending if \( N \in [B_{1,2}, B_{1,2}] \) and fully efficient lending if \( N \in [B_{1,2}, B_{1,1}] \).

Competition results in two changes. First, one can show that fully efficient lending is not an equilibrium under competition anywhere in this parameter region. This follows here because any fully efficient contract that a) both safe and risky prefer to their outside option

\footnote{Matching is not as straightforward as in static group lending, since the dynamic interest rate penalty works against homogeneous matching. To see this, consider a contract similar to the optimal one for \( G > c_{2,2} \), but without joint liability at any date or history: \( r_0, r_0 > 0 \) and \( r_1 = r_2 = c_0 = c_1 = c_2 = c_0 = 0 \). By inspection, this violates the homogeneous matching constraint. (See also Guttman, 2008.) However, this effect is not enough to overcome the incentives for homogeneous group formation provided by joint liability. Similar statements hold for the optimal contract in all phases of \( G \).}
and b) that covers costs assuming both types borrow, is vulnerable to a nearly efficient contract that attracts only safe borrowers and makes positive profits. Safe borrowers are willing to give up the loan after failure in order to reduce their cross-subsidy to risky, while risky borrowers are not tempted because the cross-subsidy in the original contract is too attractive.51

Second, one can show that nearly efficient lending is an equilibrium when \( G \geq c_{1,2} \) if \( N \in [B_{1,2}^{*C}, B_{1,1}] \) but not if \( N \in [B_{1,2}^{*}, B_{1,2}^{*C}] \), where

\[
B_{1,2}^{*C} \equiv \frac{p_s(1 + \bar{p})}{\bar{p}(1 + p_s)} = \frac{p_s}{\bar{p} + (p_s - \bar{p})\frac{\bar{p}}{1 + \bar{p}}}
\]

and thus \( B_{1,2}^{*} < B_{1,2}^{*C} < B_{1,2} \). The reason that nearly efficient lending is no longer possible for \( N \in [B_{1,2}^{*C}, B_{1,1}] \) is that the optimal nearly efficient contract involves lending to borrowers after failure at the risky reservation rate, \( r_0 = \hat{r}_r \); but extracting all the risky borrowers’ surplus at this history would not survive competition, since risky borrowers could always get loans at the competitive full-information rate by declaring themselves risky. Competition thus forces the lender to lower the “penalty” rate to the break-even level, \( r_0 = \rho/p_r \), and to raise the initial rate, \( r_0 \). As a result, safe borrowers are harder to attract, and the cutoff \( N \) for nearly efficient lending rises to \( B_{1,2}^{*C} \). However, when nearly efficient lending is achievable, i.e. for \( N \in [B_{1,2}^{*C}, B_{1,1}] \), it also survives competition, since there is no other profit-making contract that can lure away only risky borrowers (since they earn more than two loans’ full surplus), only safe borrowers (since there is no other contract that reduces the cross-subsidy more without sacrificing too much efficiency, or that raises efficiency enough to counteract the increased cross-subsidy), or both risky and safe borrowers (since raising both payoffs would require an increase in efficiency, but fully efficient contracts earn less for safe).

Interestingly then, at least in this parameter region, competition makes dynamic lending harder to sustain, and pushes the equilibrium toward the equity end of the equity-efficiency

51 This is somewhat similar to probabilistic screening (ignored here under the assumption of deterministic contracts), under which safe borrowers are offered cheap loans at some probability, while risky borrowers prefer more expensive loans granted with certainty.
tradeoff involved in fully efficient versus nearly efficient lending.

In terms of the comparison between dynamic and group lending, a modified version of Corollary 2 would still obtain, though requiring a stronger assumption than $p_r < p_A$.

**Risk Aversion.** Risk aversion would change optimal contracting in two main ways. First, borrowers would disprefer variation in interest rates over time, so the lender would have to trade off varying interest rates by revealed risk against keeping borrower payoffs smooth. Risk aversion would thus work against the improvements in risk-pricing that are otherwise achievable. Second, a pooling contract would no longer be optimal. The risky borrower could be given a contract that gives him the same utility as the safe borrower’s contract, but with less risk – this would result in greater revenue for the lender, which could then be used to lower interest rates. Risk aversion would similarly affect group lending, hampering risk pricing, so it is not clear without further analysis how the comparison between the two contracts would be affected.

**More than two periods.** As discussed in section 5.3, longer relationships allow for more information revelation and more opportunity for the lender to improve risk-pricing. Our conjecture is that in the $T$-period problem, efficient lending can be achieved over more of the parameter space the larger is $T$, and can be achieved for any $N > 1$ for $T$ and $\mathcal{F}$ large enough. This analysis is left for future work.

**Discounting.** If borrowers discounted the future, this would make future discounted loans less attractive and thus make safe borrowers harder to attract. Conditions required for fully and nearly efficient lending would tighten. The quantitative impact would be small at standard discount rates, but more significant in a model with more than two periods. However, if we make the assumption that project sizes and outside options are growing at a rate equal to the discount factor, the conditions for efficient lending would remain unchanged. Thus, while ignoring discounting may have a quantitative impact on the effectiveness of

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52 Details are available upon request. The optimal group contract derived in the proof of Proposition 3 is robust to competition; one can show that no other contract breaks even while luring away both safe and risky, risky only, or safe only.

dynamic lending, its impact would be counterbalanced in a setting of economic growth.

8 Conclusion

When borrower types are unknown but fixed, dynamic loan contracts can effectively draw safe borrowers into the market. However, they can be limited in their effectiveness by borrowers’ ability to drop out, which constrains the lender’s ability to price for risk as information is revealed over time. As a result, initial loans from a lender will tend to come with higher rates, while repeat loans will often be cheaper – as in “relationship lending”.

Dynamic loan contracts share some remarkable similarities in this context to the group lending contracts popularized by the microcredit movement. There is a close connection between number of periods in a dynamic contract and number of borrowers per group in a group contract – both determine the amount of information revelation. However, the contracts face different constraints on the use of information. And, while the two kinds of contracts require different assumptions to be met, neither dominates the other in terms of overall ability to revive a dormant credit market, providing increased intermediation and lower interest rates. The theory is thus consistent with dynamic lending playing a role of similar magnitude to that of group lending in the recent unprecedented rise in financial intermediation among the poor, particularly in more marginal environments, where net and gross returns are relatively low. Similar insights come from the form of the optimal contract when both group and dynamic lending strategies are possible.
References


Proof of Lemma 1. One can derive the reservation interest rate in period 1 given \( r_1 \) and \( r_0 \), call it \( \hat{r}_r \) for a type-\( \tau \) agent, from the condition \( \Pi_\tau(\hat{r}_r, r_1, r_0) = 2\bar{\mu} \). Using payoff 4 and the definition of \( \hat{r}_r \) (see equation 2), \( \hat{r}_r \) can be written

\[
\hat{r}_r = 2\hat{r}_r - p_r \min\{r_1, \hat{r}_r\} - (1 - p_r) \min\{r_0, \hat{r}_r\}.
\]

We then have

\[
\hat{r}_r - \hat{r}_s = 2(\hat{r}_r - \hat{r}_s) + [p_s \min\{r_1, \hat{r}_s\} - p_r \min\{r_1, \hat{r}_r\}] - [(1 - p_r) \min\{r_0, \hat{r}_r\} - (1 - p_s) \min\{r_0, \hat{r}_s\}].
\]

It can be verified that the bracketed term on the first line is positive, and that the bracketed term on the second line is positive and maximized at \( r_0 = \hat{r}_r \). It follows that

\[
\hat{r}_r - \hat{r}_s \geq 2(\hat{r}_r - \hat{r}_s) - (1 - p_r)\hat{r}_r + (1 - p_s)\hat{r}_s = \hat{r}_r - \hat{r}_s + p_r\hat{r}_r - p_s\hat{r}_s = \hat{r}_r - \hat{r}_s > 0.
\]

Thus, \( \hat{r}_s < \hat{r}_r \), so if at some contract \((r_0, r_1, r_0)\) safe borrowers choose to borrow in period one \((r_0 \leq \hat{r}_s)\), so do risky \((r_0 \leq \hat{r}_r)\). □

Proof of Proposition 1. We consider two phases, corresponding to \( G \) greater or less than \( C_{1,2} \). First, consider maximizing safe-borrower payoff 5 subject to three of the constraints applicable to the full-efficiency case, the full-efficiency ZPC, limited commitment for safe at \( r_0 \), and monotonocity for \( r_1 \) (multipliers in brackets):

\[
\begin{align*}
[\mu] & \quad \bar{p}r_0 + \bar{p}^2r_1 + p(1 - p)r_0 \geq 2\rho \\
[\gamma] & \quad r_0 \leq \hat{r}_s \\
[m] & \quad r_1 \geq 0.
\end{align*}
\]

The first-order conditions are

\[
\begin{align*}
[r_0] & - p_s + \mu\bar{p} = 0 \\
[r_0] & - p_s(1 - p_s) + \mu(p(1 - p) - \gamma) = 0 \\
[r_1] & - p_s^2 + \mu\bar{p}^2 + m = 0.
\end{align*}
\]

One can solve for the multipliers as

\[
\mu = \frac{p_s}{\bar{p}} > 0 \quad \text{and} \quad \gamma = m = \frac{\mu(p_s\bar{p} - \bar{p}^2)}{p_s\bar{p}} > 0,
\]

all strictly positive. Thus the best contract for safe borrowers subject to these constraints involves \( r_1 = 0, r_0 = \hat{r}_s \), and \( r_0 \) from the ZPC at equality, which can be written as

\[
(r_1, r_0, r_0) = \left(0, \frac{N}{p_s}, \frac{2p_s - p(1 - p)N}{p_s\bar{p}}\right).
\]

(10)

Some algebra gives that the safe-borrower payoff 5 under contract 10 equals or exceeds \( 2\bar{\mu} \).
iff $N \geq \mathcal{B}_{1,2}$. Now consider contract 10 at $N = \mathcal{B}_{1,2}$:

$$\left(r_1^*, r_0^*, r_\emptyset^*\right) = \left(0, \rho \frac{2}{p_s \overline{p} + p(2 - p)}, \rho \frac{2(1 + p_s)}{p_s \overline{p} + p(2 - p)}\right).$$

(11)

It is already established that at $N = \mathcal{B}_{1,2}$, safe borrowers prefer to borrow in period 1 under this contract. It is easy to see this is also true for $N > \mathcal{B}_{1,2}$. Thus, contract 11 attracts all safe borrowers in period 1 if $N \geq \mathcal{B}_{1,2}$, and hence all risky too (by Lemma 1). It remains to verify that it satisfies all omitted constraints when $N \geq \mathcal{B}_{1,2}$. ZPC, limited commitment, period-2 monotonicity, and period-2 affordability are all already imposed or straightforwardly checked. Dynamic monotonicity requires $r_\emptyset^* \geq r_r r_\emptyset^*$, clearly true. Finally, affordability of $r_\emptyset^*$, i.e. $r_\emptyset^* \leq R_s$, holds iff $\emptyset \geq \mathcal{C}_{1,2}$. Thus, if $\emptyset \geq \mathcal{C}_{1,2}$ and $N \geq \mathcal{B}_{1,2}$, fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If $\emptyset \geq \mathcal{C}_{1,2}$ and $N < \mathcal{B}_{1,2}$, fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to (only) three imposed constraints cannot attract them if $N < \mathcal{B}_{1,2}$, as established above.

**Second**, consider maximizing safe-borrower payoff 5 subject to the full-efficiency ZPC, limited commitment for safe at $r_0$, and affordability for $r_\emptyset$:

$$\begin{align*}
[\mu] & \quad \overline{p} r_\emptyset + \overline{p}^2 r_1 + p(1 - p)r_0 \geq 2p \\
[\gamma] & \quad r_0 \leq \hat{r}_s \\
[\lambda] & \quad r_\emptyset \leq R_s.
\end{align*}$$

The first-order conditions are

$$\begin{align*}
[r_\emptyset] & \quad - p_s + \mu \overline{p} - \lambda = 0 \\
[r_0] & \quad - p_s(1 - p_s) + \mu p(1 - p) - \gamma = 0 \\
[r_1] & \quad - p_s^2 + \mu \overline{p}^2 = 0.
\end{align*}$$

One can solve for the multipliers as

$$\mu = \frac{p_s^2}{\overline{p}^2} > 0 \quad \text{and} \quad \gamma = \lambda = \mu (p_s \overline{p} - \overline{p}^2)/p_s > 0 ,$$

all strictly positive. Thus the best contract for safe borrowers subject to these constraints involves $r_\emptyset = R_s$, $r_0 = \hat{r}_s$, and $r_1$ from the ZPC at equality, which can be written as

$$\left(r_1, r_0, r_\emptyset\right) = \left(\rho \frac{2p_s - p(1 - p)N - \overline{p} \emptyset}{p_s \overline{p}^2}, \rho \frac{N}{p_s} / \rho \frac{\emptyset}{p_s}\right).$$

(12)

Some algebra gives that at some $\emptyset$, the safe-borrower payoff 5 under contract 12 equals or exceeds $2\overline{p}$ iff $N \geq \mathcal{B}_{1,1} - \frac{21,1 - \mathcal{B}_{1,2}}{\mathcal{C}_{1,2} - \mathcal{C}_{1,1}} (\emptyset - \mathcal{C}_{1,1}) \equiv \mathcal{N}_{1,2}(\emptyset)$. Now consider contract 12 at
\( N = N_{1,2}(g) \):

\[
(r^*_1, r^*_0, r^*_0) = \left( \frac{2p_s(1 + p_s) - \left[ p_s \bar{p} + p(2 - p)\right] g}{p_s(p_s \bar{p} + p^2)}, \frac{2p_s^2 - (p_s \bar{p} + p^2) g}{p_s(p_s \bar{p} + p^2)}, \frac{g}{p_s} \right). \tag{13}
\]

By arguments used above, at some \( g \), this contract attracts all safe borrowers in period 1 if \( N \geq N_{1,2}(g) \), and hence all risky (by Lemma 1). It remains to verify that it satisfies all omitted constraints when \( N \geq N_{1,2}(g) \). It can be verified that contracts 11 and 13 coincide at \( g = c_{1,2} \), and \( N_{1,2}(c_{1,2}) = B_{1,2} \); thus, the previous case guarantees that all constraints are satisfied by contract 13 at \( g = c_{1,2} \) for \( N \geq N_{1,2}(c_{1,2}) \). One can also verify that at \( g = c_{1,1} \), contract 13 reduces to \( (r^*_1, r^*_0, r^*_0) = (\rho/\bar{p}, \rho/\bar{p}, \rho/\bar{p}) \), and that all constraints are satisfied by this contract for \( N \geq N_{1,2}(c_{1,1}) = B_{1,1} \). Note that the contract is linear in \( g \), and that all constraints except limited commitment can be written as nonnegativity of linear functions of the contract rates (normalized by \( \rho \)) and \( g \) (since \( R_s = \rho g/p_s \)). Thus these constraints are all satisfied for \( g \in [c_{1,1}, c_{1,2}] \), since they are satisfied at both endpoints. Finally, limited commitment is satisfied by \( r^*_0 \) at \( N = N_{1,2}(g) \) by construction, and a fortiori at \( N > N_{1,2}(g) \); it is also satisfied by \( r^*_1 \), since \( r^*_1 < r^*_0 \) for \( g \geq c_{1,1} \). Thus, if \( g \in [c_{1,1}, c_{1,2}] \) and \( N \geq N_{1,2}(g) \), fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If \( g \in [c_{1,1}, c_{1,2}] \) and \( N < N_{1,2}(g) \), fully efficient lending is not achievable, since the best contract for safe borrowers subject to (only) three imposed constraints cannot attract them. If \( g < c_{1,1} \), fully efficient lending is also not achievable, since necessary for attracting safe borrowers is \( N \geq N_{1,2}(g) \); but this is impossible when \( g < c_{1,1} \), for then \( N_{1,2}(g) > B_{1,1} (= c_{1,1} > g \geq N) \).

**Proof of Proposition 2.** If full efficiency cannot be attained, which is true by Proposition 1 under the assumptions here, the next greatest outreach (and thus social surplus) available from a simple pooling contract is near efficiency. This is because some group of borrowers must be excluded, and this is the smallest safe group under assumption A3 (which implies \( p_s > 1/2 \), so the successful safe outnumber the unsuccessful safe); further, excluding any type in period 1 means excluding them in period 2, and excluding only a risky group is impossible. Thus, near-efficiency is the maximum social surplus possibly attainable when full efficiency is not; and if the lender exactly breaks even, as it will in the contracts derived below, this is also the maximum borrower surplus possibly attainable. Thus, nearly efficient lending will be chosen whenever possible as the next best alternative to fully efficient lending.

It remains to show nearly efficient lending is possible under the conditions of the Proposition. We consider two phases, corresponding to \( g \) greater or less than \( c_{1,2}^* \). **First**, consider maximizing safe-borrower payoff 6 subject to three of the constraints applicable to the near-efficiency case, the near-efficiency ZPC, limited commitment for risky at \( r_0 \), and monotonocity for \( r_1 \) (multipliers in brackets):

\[
\begin{align*}
\mu & \quad \bar{p} r_0 + \bar{p} r_1 + \theta(1 - p_r) p_r r_0 \geq [1 + \bar{p} + \theta(1 - p_r)] \rho \\
\gamma & \quad r_0 \leq \tilde{r}_r \\
\mu & \quad r_1 \geq 0.
\end{align*}
\]
The first-order conditions are

\[
\begin{align*}
[r_0] & \quad -p_s + \mu \overline{r} = 0 \\
[r_0] & \quad \mu \theta p_r(1 - p_r) - \gamma = 0 \\
[r_1] & \quad -p_s^2 + \mu \overline{r}^2 + m = 0.
\end{align*}
\]

One can solve for the multipliers as

\[
\begin{align*}
\mu &= \frac{p_s}{\overline{p}} > 0, \\
m &= \mu(p_s \overline{p} - \overline{r}^2) > 0, \quad \text{and} \quad \gamma = \mu \theta p_r(1 - p_r) > 0,
\end{align*}
\]

all strictly positive. Thus the best contract for safe borrowers subject to these constraints involves \( r_1 = 0, \ r_0 = \hat{r}_r, \) and \( r_\emptyset \) from the ZPC at equality, which can be written as

\[
(r_1, r_0, r_\emptyset) = \left(0, \frac{N}{p_r}, \frac{1 + \overline{p} + \theta(1 - p_r) - \theta(1 - p_r)N}{\overline{p}} \right). \tag{14}
\]

Some algebra gives that the safe-borrower payoff \( \delta \) under contract 14 equals or exceeds \( 2\overline{p} \) iff \( N \geq B_{1,2}^+. \) Now consider contract 14 at \( N = B_{1,2}^+ \):

\[
(r_1^*, r_0^*, r_\emptyset^*) = \left(0, \frac{p_s}{p_r} \frac{[1 + \overline{p} + \theta(1 - p_r)]}{[p_s + \overline{p} + \theta(1 - p_r)]}, \rho \frac{(1 + p_s)[1 + \overline{p} + \theta(1 - p_r)]}{\overline{p} + p_s[\overline{p} + \theta(1 - p_r)]} \right). \tag{15}
\]

It is already established that at \( N = B_{1,2}^+ \), safe borrowers prefer to borrow in period 1 under this contract. It is easy to see this is also true for \( N > B_{1,2}^+ \). Thus, contract 15 attracts all safe borrowers in period 1 if \( N \geq B_{1,2}^+ \), and hence all risky too (by Lemma 1). It remains to verify that it satisfies all omitted constraints when \( N \geq B_{1,2}^+ \). ZPC, the relevant limited commitment constraints, period-2 monotonicity, and the relevant period-2 affordability constraints are all already imposed or straightforwardly checked. Affordability of \( r_\emptyset^* \), i.e. \( r_\emptyset^* \leq R_s \), holds iff \( \mathcal{G} \geq \mathcal{C}_{1,2}^+ \). Dynamic monotonicity for risky requires \( r_\emptyset^* \geq p_r r_0^* \), true by inspection. Dynamic monotonicity for safe requires \( r_\emptyset^* \geq p_s \hat{r}_s = \rho N \), or equivalently, \( N \leq B_{1,2}^+(1 + p_s)/p_s \). Thus contract 15 satisfies all constraints and achieves nearly efficient lending if \( \mathcal{G} \geq \mathcal{C}_{1,2}^+ \) and \( N \in [B_{1,2}^+, B_{1,2}^+(1 + p_s)/p_s] \). If \( N > B_{1,2}^+(1 + p_s)/p_s \), which is compatible with the maintained assumptions on \( N \) under some parameter values, dynamic monotonicity is violated by contract 15. In that case, consider modifying contract 15 by raising \( r_\emptyset \) and lowering \( r_0 \) along the ZPC until dynamic monotonicity holds with equality; i.e. the contract is \( r_\emptyset = \rho N, \ r_1 = 0, \) and \( r_0 \) from the ZPC. All constraints are straightforwardly verified, and it is clear that this contract attracts all borrowers in period 1 and thus achieves nearly efficient lending (\( r_0 \) remains above \( \hat{r}_s \), since otherwise fully efficient lending would be achievable, a contradiction). Thus, if \( \mathcal{G} \geq \mathcal{C}_{1,2}^+ \) and \( N > B_{1,2}^+ \), nearly efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If \( \mathcal{G} \geq \mathcal{C}_{1,2}^+ \) and \( N < B_{1,2}^+ \), fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to (only) three imposed constraints cannot attract them if \( N < B_{1,2}^+ \), as established above.

**Second**, consider maximizing safe-borrower payoff \( \delta \) subject to the near-efficiency ZPC,
limited commitment for risky at \( r_0 \), and affordability for \( r_0 \):

\[
\begin{align*}
[\mu] & \quad \overline{p}r_0 + \overline{p}^2r_1 + \theta(1-p_r)p_r r_0 \geq [1 + \overline{p} + \theta(1-p_r)]\rho \\
[\gamma] & \quad r_0 \leq \hat{r}_r \\
[\lambda] & \quad r_0 \leq R_s .
\end{align*}
\]

The first-order conditions are

\[
\begin{align*}
[r_0] & \quad -p_s + \mu \overline{p} - \lambda = 0 \\
[r_0] & \quad \mu \theta(1-p_r)p_r - \gamma = 0 \\
r_1 & \quad -p_s^2 + \mu \overline{p}^2 = 0.
\end{align*}
\]

One can solve for the multipliers as

\[
\mu = \frac{p_s^2}{\overline{p}^2} > 0 , \quad \lambda = \mu(p_s\overline{p} - \overline{p}^2)/p_s > 0 , \quad \text{and} \quad \gamma = \mu \theta p_r(1-p_r) > 0 ,
\]

all strictly positive. Thus the best contract for safe borrowers subject to these constraints involves \( r_0 = R_s \), \( r_0 = \hat{r}_r \), and \( r_1 \) from the ZPC at equality, which can be written as

\[
(r_1, r_0, r_0) = \left( \frac{p_s[1 + \overline{p} + \theta(1-p_r) - \theta(1-p_r)N] - \overline{p} \mathcal{G}}{p_s p_r^2}, \rho \frac{N}{p_r}, \rho \frac{\mathcal{G}}{p_s} \right) .
\]

Some algebra gives that at some \( \mathcal{G} \), the safe-borrower payoff \( \mathcal{G} \) under contract 16 equals or exceeds \( 2\pi \) iff \( N \geq \mathcal{B}^*_{1,1} - \frac{b_{1,1} - b_{1,2}}{e_{1,2} - e_{1,1}}(g - \mathcal{C}^*_{1,1}) \equiv \mathcal{N}^*_{1,2}(\mathcal{G}) \). Now consider contract 16 at \( N = \mathcal{N}^*_{1,2}(\mathcal{G}) \):

\[
(r_1^*, r_0^*, r_0^*) = \left( \frac{p_s[1 + \overline{p} + \theta(1-p_r)] - \overline{p} \mathcal{G}}{p_s[\theta(1-p_r)p_s^2 + (1+p_s)p_r^2]}, \frac{p_s^2[1 + \overline{p} + \theta(1-p_r)] - (p_s\overline{p} - \overline{p}^2)\mathcal{G}}{p_r[\theta(1-p_r)p_s^2 + (1+p_s)p_r^2]}, \rho \frac{\mathcal{G}}{p_s} \right) .
\]

By arguments used above, at some \( \mathcal{G} \), this contract attracts all safe borrowers in period 1 if \( N \geq \mathcal{N}^*_{1,2}(\mathcal{G}) \), and hence all risky too (by Lemma 1). It remains to verify that it satisfies all omitted constraints when \( N \geq \mathcal{N}^*_{1,2}(\mathcal{G}) \). It can be verified that contracts 15 and 17 coincide at \( \mathcal{G} = \mathcal{C}^*_{1,2} \), and that \( \mathcal{N}^*_{1,2}(\mathcal{C}^*_{1,2}) = \mathcal{B}^*_{1,2} \). Thus, if the constraints other than limited commitment are satisfied at \( \mathcal{G} = \mathcal{C}^*_{1,2} \) and at \( \mathcal{G} = \mathcal{C}^*_{1,1} \), continuity arguments as in the proof of Proposition 1, based on linearity in \( \mathcal{G} \) of the contract and constraints, guarantee that they are satisfied for all \( \mathcal{G} \in [\mathcal{C}^*_{1,1}, \mathcal{C}^*_{1,2}] \). At \( \mathcal{G} = \mathcal{C}^*_{1,1} (= \mathcal{B}^*_{1,1}) \), contract 17 reduces to \( (r_1^*, r_0^*, r_0^*) = (\rho \mathcal{B}^*_{1,1}/p_s, \rho \mathcal{B}^*_{1,1}/p_r, \rho \mathcal{B}^*_{1,1}/p_s) \). It is straightforward to verify this contract satisfies affordability, period-2 monotonicity, and ZPC. It also satisfies the relevant limited commitment constraints, which guarantees these constraints for all \( \mathcal{G} \in [\mathcal{C}^*_{1,1}, \mathcal{C}^*_{1,2}] \), since \( r_0^* \) and \( r_1^* \) are decreasing in \( \mathcal{G} \). Finally, dynamic monotonicity for risky requires \( r_0^* + p_r r_1^* \geq p_r r_0^* \), true by inspection. Dynamic monotonicity for safe may not be satisfied over this entire
interval for $G$, but if not, it is because $N$ is too high (specifically, $N > \frac{N_{1,2}^*}{1}(1 + p_s)/p_s$). But as above, contract 17 can be modified by raising $r_0$ and lowering $r_0$ along the ZPC, until dynamic monotonicity exactly holds. This modified contract satisfies all constraints and attracts all borrowers except safe after failure. We have thus shown that if $G \in [C_{1,1}', C_{1,2}']$, and $N \geq \frac{N_{1,2}^*}{1}(G)$, nearly efficient lending is achievable.

If $G \in [C_{1,1}', C_{1,2}']$ and $N < \frac{N_{1,2}^*}{1}(G)$, nearly efficient lending is not achievable, since the best contract for safe borrowers subject to (only) three imposed constraints cannot attract them. If $G < C_{1,1}'$, nearly efficient lending is also not achievable, since the best contract for safe borrowers subject to (only) three imposed constraints requires $N \geq \frac{N_{1,2}^*}{1}(G)$; but this is impossible when $G < C_{1,1}'$, for then $\frac{N_{1,2}^*}{1}(G) > B_{1,1}'(= C_{1,1}' > G \geq N)$. 

**Proof of Lemma 2.** For the first half of the proof, consider any incentive compatible menu of contracts $C$, $(r_0^s, r_1^s, r_0^r)$ for safe agents and $(r_0^r, r_1^r, r_0^r)$ for risky agents, that achieves fully efficient lending and satisfies the monotonicity, limited liability, and limited commitment constraints,

$$0 \leq r_1^r, r_0^r \leq \hat{r}_r, \quad r_0^r, r_1^r, r_0^s \leq \hat{R}_r, \quad r_0^r + p_r r_1^r \geq p_r r_0^r, \quad \tau \in \{r, s\},$$

as well as the ZPC:

$$\theta [p_r r_0^s + p_r^2 r_1^s + p_r (1 - p_r) r_0^r] + (1 - \theta) [p_s r_0^s + p_s^2 r_1^s + p_s (1 - p_s) r_0^s] \geq 2p .$$

By incentive compatibility,

$$2\hat{R} - p_r r_0^r - p_r^2 r_1^r - p_r (1 - p_r) r_0^r \geq 2\hat{R} - p_r r_0^s - p_r^2 r_1^s - p_r (1 - p_r) r_0^s$$

$$\Rightarrow \quad p_r r_0^s + p_r^2 r_1^s + p_r (1 - p_r) r_0^s \geq p_r r_0^r + p_r^2 r_1^r + p_r (1 - p_r) r_0^r .$$

Combining this latter inequality with the ZPC gives that

$$\theta [p_r r_0^s + p_r^2 r_1^s + p_r (1 - p_r) r_0^s] + (1 - \theta) [p_s r_0^s + p_s^2 r_1^s + p_s (1 - p_s) r_0^s] \geq 2p .$$

Thus, offering only the simple pooling contract $P = (r_0^s, r_1^s, r_0^r)$ allows the lender to break even; attracts safe agents at all histories, since $C$ did; attracts risky agents at all histories, in part by Lemma 1; satisfies all constraints for safe (since $C'$ did) and, by inspection, satisfies all constraints for risky (some of which are slacker because $\hat{r}_s < \hat{r}_r$ and $R_s < R_r$) except perhaps dynamic monotonicity:

$$r_0^s \geq p_r (r_0^s - r_1^s) .$$

Since $r_1^s \geq 0$ and $r_0^s \leq \hat{r}_s$, sufficient for dynamic monotonicity for risky is that $r_0^s \geq p_r \hat{r}_s$. If $r_0^s \geq p_r \hat{r}_s$, then $P$ is a simple pooling contract that attracts all borrowers at all histories and satisfies all constraints. If instead $r_0^s < p_r \hat{r}_s$, define $\tilde{r}_0^s \equiv p_r \hat{r}_s$ and note that simple pooling contract $P' = (\tilde{r}_0^s, r_1^s, r_0^r)$ satisfies all constraints ($\tilde{r}_0^s$ is affordable because $\hat{r}_s \leq R_s$). It also attracts all borrowers at all histories, since no interest rate exceeds $\hat{r}_s$. Thus $P'$ is a simple pooling contract that attracts all borrowers at all histories and satisfies all constraints.

We have thus established existence of a simple pooling contract that achieves fully efficient lending and satisfies all constraints, call it $P$. Note that the lender may earn strictly positive
profits under \( P \), in which case not all surplus goes to the borrowers. We conclude by showing how \( P \) can be modified, if need be, to guarantee that all surplus goes to the borrowers. First, if \( P \) involves \( r_0 < 0 \), then set \( r_0 \) to 0 – all constraints clearly remain satisfied, and all borrowers continue to be attracted at all histories, since no interest rate exceeds \( \hat{r}_s \). Next, if the ZPC is slack, lower first \( r_0 \) toward zero, then if necessary \( r_1 \) toward zero, and finally if necessary \( r_0 \) toward zero, stopping the process at whatever point the ZPC holds with equality. It is straightforward to check that this contract satisfies all constraints (the non-negativity of \( r_0 \) and the order of interest rate reductions guarantee dynamic monotonicity is satisfied), attracts all borrowers at all histories (since lower rates only improves borrower payoffs), and gives the borrowers all surplus (since the lender exactly breaks even).

For the second half of the proof, consider an optimal incentive compatible menu of contracts \( C, (r_0^s, r_1^s, r_0^p) \) and \( (r_0^r, r_1^r, r_0^p) \), that achieves nearly efficient lending and satisfies the monotonicity, limited liability, and limited commitment constraints for safe borrowers,

\[
0 \leq r_1^s \leq \hat{r}_s < r_0^s, \quad r_0^s, r_1^s \leq R_s, \quad r_0^s + p_s r_1^s \geq p_s \hat{r}_s,
\]

the analogous constraints for risky borrowers,

\[
0 \leq r_1^r, r_0^r \leq \hat{r}_r, \quad r_0^r, r_1^r, r_0^r \leq R_r, \quad r_0^r + p_r r_1^r \geq p_r r_0^r,
\]

and the ZPC:

\[
\theta \left[ p_r r_0^r + p_r^2 r_1^r + p_r (1 - p_r) r_0^r \right] + (1 - \theta) \left[ p_s r_0^s + p_s^2 r_1^s \right] \geq [1 + \bar{\rho} + \theta (1 - p_r)] \rho.
\]

By incentive compatibility,

\[
2R - p_r r_0^r - p_r^2 r_1^r - p_r (1 - p_r) r_0^r \geq 2R - p_r r_0^s - p_r^2 r_1^s - p_r (1 - p_r) \min \{r_0^s, \hat{r}_r\}
\]

\[
\Rightarrow \quad p_r r_0^r + p_r^2 r_1^r + p_r (1 - p_r) \hat{r}_r \geq p_r r_0^s + p_r^2 r_1^s + p_r (1 - p_r) r_0^s.
\]

Combining this latter inequality with the ZPC gives that

\[
\theta \left[ p_r r_0^r + p_r^2 r_1^r + p_r (1 - p_r) \hat{r}_r \right] + (1 - \theta) \left[ p_s r_0^s + p_s^2 r_1^s \right] \geq [1 + \bar{\rho} + \theta (1 - p_r)] \rho.
\]

Offering only the simple pooling contract \( P = (r_0^s, r_1^s, \tilde{r}_0^s) \), where \( \tilde{r}_0^s \equiv \hat{r}_r \), attracts safe borrowers at all histories except after failure (since \( C \) did); attracts risky agents at all histories, in part by Lemma 1; allows the lender to break even, by the previous inequality; and satisfies all constraints for safe and risky borrowers (note that \( \hat{r}_r \leq R_r \) except perhaps dynamic monotonicity for risky:

\[
r_0^s \geq p_r (\hat{r}_r - r_1^r).
\]

Since \( r_1^s \geq 0 \), sufficient for dynamic monotonicity for risky is that \( r_0^s \geq p_r \hat{r}_r = p_r \hat{r}_s \). If \( r_0^s \geq p_s \hat{r}_s \), then \( P \) is a simple pooling contract that attracts all borrowers at all histories, except safe borrowers after failure, and satisfies all constraints. If instead \( r_0^s < p_s \hat{r}_s \), define \( \tilde{r}_0^s \equiv p_s \hat{r}_s \) and note that simple pooling contract \( P' = (\tilde{r}_0^s, r_1^s, \tilde{r}_0^s) \) satisfies all constraints. It also attracts all borrowers at all histories, except safe borrowers after failure, since no interest rate exceeds \( \hat{r}_r \), and only \( \tilde{r}_0^s \) exceeds \( \hat{r}_s \). Thus \( P' \) is a simple pooling contract that attracts all borrowers at all histories, except safe borrowers after failure, and satisfies all constraints.
We have thus established existence of a simple pooling contract that achieves nearly efficient lending and satisfies all constraints, call it $P$. As above, the lender may earn strictly positive profits under $P$, in which case not all surplus goes to the borrowers. We conclude by showing how $P$ can be modified, if need be, to guarantee that all surplus goes to the borrowers. $P$ involves $r_0 = \hat{r}$, and satisfies the ZPC. Fix $r_0$ and $r_1$ from $P$, and if the ZPC is slack, lower $r_0$ until it binds. We argue that the ZPC will bind at some $r_0 \in (\hat{r}, \hat{r})$. The reasoning is that profits jump up when $r_0$ is lowered from $\hat{r} + \epsilon$ to $\hat{r}$, for $\epsilon > 0$ small enough, since a mass of safe borrowers enters the market at their reservation rate when $r_0$ is lowered to $\hat{r}$; thus, if profits are positive for all $r_0 \in (\hat{r}, \hat{r})$, they are also positive at $r_0 = \hat{r}$; but if profits are positive at $r_0 = \hat{r}$, then fully efficient lending is achievable, with all constraints satisfied, and (by arguments above) with all surplus going to the borrowers – contradicting optimality of the menu $C$, which only achieved nearly efficient lending. Thus, if $P$ involves a slack ZPC, simply reset $r_0$ to the value in $(\hat{r}, \hat{r})$ that causes the ZPC to bind. This contract satisfies all constraints, achieves nearly efficient lending, and gives the borrowers all surplus (since the lender exactly breaks even).

**Proof of Proposition 3.** Note that with any monotonic contract ($c \leq r$), safe earn less than risky:

$$\bar{R} - p_s r - p_s(1 - p_s)c < \bar{R} - p_r r - p_r(1 - p_r)c \iff (p_s - p_r)(p_s + p_r - 1)c < (p_s - p_r)r,$$

true since $0 \leq c \leq r$ and $p_s + p_r < 2$. Thus, a contract $(r, c)$ attracts either both safe and risky, only risky, or neither. Note that by setting $r = c = \rho/p_r$, the lender can always attract risky borrowers and give them the full surplus from lending. Thus, if the lender cannot attract safe borrowers, it will lend to risky.

The remainder of the proof demonstrates that fully efficient lending, with all surplus going to the borrowers, is achievable iff the conditions outlined hold. We consider two phases, corresponding to $g \geq 1$ greater or less than $g_{21}$. **First**, consider maximizing the safe-borrower payoff subject to the full-efficiency ZPC and a monotonicity constraint (multipliers in brackets):

$$[\mu] \quad \bar{p} r + p(1 - p)c \geq \rho$$

$$[m] \quad c \leq r.$$

The first-order conditions are

$$[r] \quad -p_s + \mu \bar{p} + m = 0$$

$$[c] \quad -p_s(1 - p_s) + \mu p(1 - p) - m = 0.$$

One can solve for the multipliers as

$$\mu = p_s(2 - p_s)/p(2 - p) > 0 \quad \text{and} \quad m = \mu(p_s \bar{p} - \bar{p}^2)/(2 - p_s) > 0,$$

both strictly positive. Thus the best contract for safe borrowers subject to these constraints involves $r = c = \rho/p(2 - p)$, which satisfies the ZPC at equality. The remaining affordability
constraint, \( r + c \leq R_s \), is satisfied iff \( \mathcal{G} \geq \mathcal{C}_{2,1} \), and comparing the safe payoff at this contract to \( \overline{\pi} \) gives that safe prefer to borrow iff \( N \geq \mathcal{B}_{2,1} \). Thus, if \( \mathcal{G} \geq \mathcal{C}_{2,1} \) and \( N \geq \mathcal{B}_{2,1} \), fully efficient lending is achievable, with all surplus going to the borrowers, while if \( \mathcal{G} \geq \mathcal{C}_{2,1} \) and \( N < \mathcal{B}_{2,1} \), only risky are attracted.

**Second**, consider maximizing the safe-borrower payoff subject to the full-efficiency ZPC and the affordability constraint:

\[
\begin{align*}
[\mu] & \quad \overline{\pi}r + \overline{\pi}(1-p)c \geq \rho \\
[\lambda] & \quad c + r \leq R_s .
\end{align*}
\]

The first-order conditions are

\[
\begin{align*}
[r] & \quad -p_s + \mu \overline{\pi} - \lambda = 0 \\
[c] & \quad -p_s (1-p_s) + \mu p(1-p) - \lambda = 0 .
\end{align*}
\]

One can solve for the multipliers as

\[
\mu = p_s^2 / p^2 > 0 \quad \text{and} \quad \lambda = \mu (p_s \overline{\pi} - \overline{\pi}^2) / p_s > 0 ,
\]

both strictly positive. Thus the best contract for safe borrowers subject to these constraints involves \((r, c)\) satisfying the ZPC at equality as well as \( r + c = R_s \):

\[
(r, c) = \left( \frac{p_s - \overline{\pi}(1-p)}{p_s^2}, \frac{\overline{\pi} \mathcal{G} - p_s}{p_s \overline{\pi}^2} \right) .
\]

Some algebra gives that at some \( \mathcal{G} \), the safe-borrower payoff under this contract equals or exceeds \( \overline{\pi} \) iff \( N \geq \mathcal{B}_{1,1} - \frac{\mathcal{B}_{1,1} - \mathcal{C}_{1,1}}{\mathcal{C}_{2,1} - \mathcal{C}_{1,1}} (\mathcal{G} - \mathcal{C}_{1,1}) \equiv \mathcal{N}_{2,1}(\mathcal{G}) \). Remaining constraints are easy to verify for \( \mathcal{G} \in [\mathcal{C}_{1,1}, \mathcal{C}_{2,1}] \). Thus, if \( \mathcal{G} \in [\mathcal{C}_{1,1}, \mathcal{C}_{2,1}] \) and \( N \geq \mathcal{N}_{2,1}(\mathcal{G}) \), fully efficient lending is achievable with all surplus going to the borrowers. If \( \mathcal{G} \in [\mathcal{C}_{1,1}, \mathcal{C}_{2,1}] \) and \( N < \mathcal{N}_{2,1}(\mathcal{G}) \), only risky are attracted. If \( \mathcal{G} < \mathcal{C}_{1,1} \), fully efficient lending is also not achievable, since necessary for attracting safe borrowers is \( N \geq \mathcal{N}_{2,1}(\mathcal{G}) \); but this is impossible when \( \mathcal{G} < \mathcal{C}_{1,1} \), for then \( \mathcal{N}_{2,1}(\mathcal{G}) > \mathcal{B}_{1,1} (= \mathcal{C}_{1,1} > \mathcal{G} \geq \mathcal{N}) \).}

**Proof of Corollary 1.** This is immediate from comparing the lower bounds on \( N \) required for fully efficient lending, derived in Propositions 1 and 3. Both bounds start at \((\mathcal{C}_{1,1}, \mathcal{B}_{1,1})\), but one can check that the bound for group lending decreases more steeply in \( \mathcal{G} \), and reaches a lower plateau: \( \mathcal{B}_{2,1} < \mathcal{B}_{1,2} \).

**Proof of Corollary 2.** This is immediate from comparing the lower bounds on \( N \) required for nearly efficient dynamic lending and for fully efficient group lending, derived in Propositions 2 and 3. For the first statement, note that the bound for nearly efficient dynamic lending starts at \((\mathcal{C}^*_{1,1}, \mathcal{B}^*_{1,1})\), and decreases in \( \mathcal{G} \) from there. Since \( \mathcal{C}^*_{1,1} < \mathcal{C}_{1,1} \) and \( \mathcal{B}^*_{1,1} < \mathcal{B}_{1,1} \), this bound is then lower than the one for efficient group lending, for \( \mathcal{G} \in [\mathcal{C}^*_{1,1}, \mathcal{C}_{1,1} + \kappa] \) for some \( \kappa > 0 \). The second statement follows since \( p_r < p_A \) guarantees that \( \mathcal{B}^*_{1,2} < \mathcal{B}_{2,1} \) (since \( p_A < p_B \)), and guarantees that the nearly efficient dynamic lending bound for \( N \) stays below
the efficient group lending bound for $\mathcal{N}$. (A condition stronger than $p_r < p_B$ is needed, since for $p_r \in (p_A, p_B)$, the two bounds cross—twice—in an intermediate range of $\mathcal{G}$.)

**Proof of Proposition 4.** We consider five phases corresponding to five ranges for $\mathcal{G}$. First, define

$$B_{2,2} = \frac{2p_s(2-p_s)}{[1 + p_s(2-p_s)]p(2-p) + p(1-p)^2(2-p)} \quad \text{and} \quad c_{2,2} = \frac{2[1 + p_s(2-p_s)]}{2-p_s}B_{2,2},$$

and consider maximizing safe-borrower payoff \(7\) subject to eight of the constraints applicable to the full-efficiency case, including ZPC 8, limited commitment for safe and monotonicity of \((r_0, c_0)\), both sets of monotonicity constraints for both \((r_1, c_1)\) and \((r_2, c_2)\), and a dynamic monotonicity constraint:

$$ZPC \quad 8, \quad c_2 \leq r_2, \quad r_2 \geq 0, \quad c_0 \leq r_0, \quad r_0 + (1-p_s)c_0 \leq \hat{r}_s,$$

$$c_1 \leq r_1, \quad r_1 \geq 0, \quad r_0 + 2p_s[r_2 + (1-p_s)c_2] \geq c_0 + 2p_s[r_1 + (1-p_s)c_1].$$

As in the proof of Proposition 1, solving the first-order conditions for the eight multipliers shows that all of them are strictly positive under Assumption A3. Thus, the best contract for safe subject to these constraints involves $r_1 = c_1 = r_2 = c_2 = 0$, $r_0 = c_0 = \hat{r}_s/(2-p_s)$, and $r_0 = c_0$ to satisfy the ZPC at equality. Some algebra gives that the safe-borrower payoff \(7\) under this contract equals or exceeds \(2\mathcal{N}\) iff $\mathcal{N} \geq B_{2,2}$. Consider this contract at $\mathcal{N} = B_{2,2}$:

$$r_0^* = \frac{2[1 + p_s(2-p_s)]}{[1 + p_s(2-p_s)]p(2-p) + p(1-p)^2(2-p)} \quad \text{and} \quad r_0^* = \frac{r_0^*}{1 + p_s(2-p_s)},$$

and \((c_0^*, c_1^*, c_2^*) = (r_0^*, r_0^*, r_1^*, r_2^*). It is straightforward to verify that this contract attracts all borrowers in period 1 for $\mathcal{N} \geq B_{2,2}$, and satisfies all omitted constraints except perhaps affordability. The most stringent affordability constraint, $r_0^* + c_0^* \leq R_s$, holds iff $\mathcal{G} \geq c_{2,2}$. Thus, if $\mathcal{G} \geq c_{2,2}$ and $\mathcal{N} \geq B_{2,2}$, fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If $\mathcal{G} \geq c_{2,2}$ and $\mathcal{N} < B_{2,2}$, fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to a subset of relevant constraints cannot attract them if $\mathcal{N} < B_{2,2}$, as established above.

**Second,** define

$$B_{2,2}^c = \frac{2p_s(2-p_s)}{p_s(2-p_s)p^2 + p(2-p)(2-p^2)} \quad \text{and} \quad c_{2,2}^c = \frac{2}{2-p_s}B_{2,2}^c,$$

and consider maximizing safe-borrower payoff \(7\) subject to eight of the constraints applicable to the full-efficiency case, including ZPC 8, period-1 affordability, limited commitment for safe and monotonicity of \((r_0, c_0)\), both sets of monotonicity constraints for \((r_2, c_2)\), monono-
Consider this contract at 

$$ZPC \ 8, \ r_0 + c_0 \leq R_s \quad \quad c_2 \leq r_2, \ r_2 \geq 0, \quad c_0 \leq r_0, \ r_0 + (1 - p_s)c_0 \leq \hat{r}_s, \quad c_1 \leq r_1, \ r_0 + 2p_s[r_2 + (1 - p_s)c_2] \geq c_0 + 2p_s[r_1 + (1 - p_s)c_1].$$

Solving the first-order conditions for the eight multipliers shows that all of them are strictly positive under Assumption A3. Thus, the best contract for safe subject to these constraints involves $r_2 = c_2 = 0, \ r_0 = c_0 = \hat{r}_s/(2 - p_s)$, and $(r_1, c_1, r_0, c_0)$ to satisfy $r_1 = c_1, \ r_0 + c_0 = R_s$, dynamic monotonicity and the ZPC at equality. Some algebra gives that the safe-borrower payoff 7 under this contract equals or exceeds $2\pi$ if

$$N \geq \frac{B_{2,2} - b_{2,2}}{c_{2,2} - c_{2,2}} (\mathcal{G} - \mathcal{C}_{2,2}) = \mathcal{N}_{N,2,2}(\mathcal{G}).$$

Consider this contract at 

$$N = \mathcal{N}_{N,2,2}(\mathcal{G}):$$

$$r_0^* = \rho \frac{[1 + p_s(2 - p_s)] \left\{ 2p_s^2(2 - p_s) - \left[ p_s(2 - p_s)p(1 - p) - p^2(1 - p)(2 - p) \right] \mathcal{G} \right\}}{p_s \left\{ 2p_s^2(1 - p)(2 - p) + p_s(2 - p_s) \left\{ p[1 + p_s(2 - p_s)] + p(1 - p^2)(2 - p) \right\} \right\}},$$

$$r_0^* = \rho \frac{2p_s^2(2 - p_s)^2 - \left[ p_s(2 - p_s)p(1 - p) - p^2(1 - p)(2 - p) \right] (2 - p_s) \mathcal{G}}{p_s(2 - p_s) \left\{ 2p_s^2(1 - p)(2 - p) + p_s(2 - p_s) \left\{ p[1 + p_s(2 - p_s)] + p(1 - p^2)(2 - p) \right\} \right\}},$$

$$r_1^* = \rho \frac{4p_s(2 - p_s)[1 + p_s(2 - p_s)] - \left\{ [1 + p_s(2 - p_s)]p(2 - p) + p(1 - p^2)(2 - p) \right\} (2 - p_s) \mathcal{G}}{2p_s(2 - p_s) \left\{ 2p_s^2(1 - p)(2 - p) + p_s(2 - p_s) \left\{ p[1 + p_s(2 - p_s)] + p(1 - p^2)(2 - p) \right\} \right\}}.$$

$$r_2^* = 0,$$

and $(c_0^*, c_0^*, c_1^*, c_2^*) = (\rho \mathcal{G}/p_s - r_0^*, r_0^*, r_0^*, r_0^*)$. It is straightforward to verify that this contract attracts all borrowers in period 1 for $N \geq \mathcal{N}_{N,2,2}(\mathcal{G})$. All constraints can also be verified for $\mathcal{G} \in [\mathcal{C}_{2,2}, \mathcal{C}_{2,2}]$. This is aided by noting that this contract is identical to the one of the previous phase when $\mathcal{G} = \mathcal{C}_{2,2}$; further, the interest rates and constraints are linear in $\mathcal{G}$; thus, checking the constraints only at the endpoints $\mathcal{C}_{2,2}$ and $\mathcal{C}_{2,2}$ suffices, and given continuity, work of the previous phase guarantees the constraints are satisfied at $\mathcal{G} = \mathcal{C}_{2,2}$.

In sum, if $\mathcal{G} \in [\mathcal{C}_{2,2}, \mathcal{C}_{2,2}]$ and $N \geq \mathcal{N}_{N,2,2}(\mathcal{G})$, fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If $\mathcal{G} \in [\mathcal{C}_{2,2}, \mathcal{C}_{2,2}]$ and $N < \mathcal{N}_{N,2,2}(\mathcal{G})$, fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to a subset of relevant constraints cannot attract them if $N < \mathcal{N}_{N,2,2}(\mathcal{G})$, as established above.

**Third**, define

$$B_{2,2}^b = \frac{2p_s}{(1 + p_s^2)p(2 - p^2) - p_s p(1 - p^2)}$$

and

$$C_{2,2}^b = (1 + p_s^2)B_{2,2}^b.$$

and consider maximizing safe-borrower payoff 7 subject to eight of the constraints applicable to the full-efficiency case, including ZPC 8, period-1 affordability, limited commitment for safe and affordability for both $(r_0, c_0)$ and $(r_1, c_1)$, and both sets of monotonicity constraints.
for \((r_2, c_2)\):

\[
ZPC\ 8, \quad r_0 + c_0 \leq R_s, \quad c_2 \leq r_2, \quad r_2 \geq 0, \\
r_1 + c_1 \leq R_s, \quad r_1 + (1 - p_s)c_1 \leq \hat{r}_s, \quad r_0 + c_0 \leq R_s, \quad r_0 + (1 - p_s)c_0 \leq \hat{r}_s.
\]

Solving the first-order conditions for the eight multipliers shows that all of them are strictly positive under Assumption A3. Thus, the best contract for safe subject to these constraints involves \(r_2 = c_2 = 0, c_0 = c_1 = \rho(S - N)/p_s^2, r_0 = r_1 = R_s - c_0 = R_s - c_1, \) and \((r_0, c_0)\) to satisfy \(r_0 + c_0 = R_s\) and the ZPC at equality. Some algebra gives that the safe-borrower payoff 7 under \((r_0, c_0)\) to satisfy the ZPC at equality. Some algebra gives that the safe-borrower payoff 7 under

\[
8 \quad \text{implies } \frac{\hat{b}_{2,2}}{c_{2,2}} - \frac{b_{2,2}}{e_{2,2}} (S - C_{2,2}) \equiv N_{2,2}(\mathcal{G}).
\]

Consider this contract at \(N = N_{2,2}(\mathcal{G})\):

\[
r_0^* = \frac{2p_s(1 + p_s^2) - \left[p_s^2 p(1 - p)(2 + p) - p_s p^2 (1 - p^2) + p(1 - p)(2 - p^2)\right]}{p_s \left[p_s^2 p^2 + p^2 (2 - p^2)\right]}, \quad r_1^* = \frac{2p_s - \left[p_s (1 - p_s) p^2 + p(1 - p)(2 - p^2)\right]}{p_s \left[p_s^2 p^2 + p^2 (2 - p^2)\right]}, \quad r_2^* = 0,
\]

and \((c_0^*, c_1^*, c_2^*, c_2^*) = (\rho \mathcal{G}/p_s - r_0^*, \rho \mathcal{G}/p_s - r_0^*, \rho \mathcal{G}/p_s - r_1^*, r_2^*)\). It is straightforward to verify that this contract attracts all borrowers in period 1 for \(N \geq N_{2,2}(\mathcal{G})\). All constraints can also be verified for \(\mathcal{G} \in [c_{2,2}^b, c_{2,2}^e]\), again aided by continuity and linearity arguments which justify checking only at \(\mathcal{G} = c_{2,2}^b\). In sum, if \(\mathcal{G} \in [c_{2,2}^b, c_{2,2}^e]\) and \(N \geq N_{2,2}(\mathcal{G})\), fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If \(\mathcal{G} \in [c_{2,2}^b, c_{2,2}^e]\) and \(N < N_{2,2}(\mathcal{G})\), fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to a subset of relevant constraints cannot attract them if \(N < N_{2,2}(\mathcal{G})\), as established above.

**Fourth**, define

\[
B_{2,2}^a = \frac{4p_s(1 + p_s^2) - 2p_s^4}{p(4 - p^3) + p_s^2 p(4 - p)} \quad \text{and} \quad C_{2,2}^a = \frac{4p_s(1 + p_s^2)}{p(4 - p^3) + p_s^2 p(4 - p)}.
\]

and consider maximizing safe-borrower payoff 7 subject to eight of the constraints applicable to the full-efficiency case, including ZPC 8, both period-1 affordability constraints, limited commitment for safe and affordability for both \((r_0, c_0)\) and \((r_1, c_1)\), and a monotonicity constraint for \((r_2, c_2)\):

\[
r_0 + c_0 \leq R_s, \quad r_0 \leq R_s, \quad c_2 \leq r_2, \quad \text{ZPC } 8, \quad r_1 + (1 - p_s)c_1 \leq \hat{r}_s, \quad r_0 + c_0 \leq R_s, \quad r_0 + (1 - p_s)c_0 \leq \hat{r}_s.
\]

Solving the first-order conditions for the eight multipliers shows that all of them are strictly positive under Assumption A3. Thus, the best contract for safe subject to these constraints involves \(r_0 = R_s, c_0 = 0, c_0 = c_1 = \rho(S - N)/p_s^2, r_0 = r_1 = R_s - c_0 = R_s - c_1, \) and \(r_2 = c_2\) to satisfy the ZPC at equality. Some algebra gives that the safe-borrower payoff 7 under
this contract equals or exceeds $2\pi$ iff $N \geq B_{2,2} - \frac{b_{2,2} - b_{2,2}}{c_{2,2}} (G - C_{2,2}) \equiv N_{2,2}(G)$. Consider this contract at $N = N_{2,2}(G)$:

$$r_0^* = \rho \frac{G}{p_s} + 2p_s(2 - p_s) - \left\{p_s(2 - p_s) \left[\frac{p(2 - p^2)}{p^2(2 - p^2)} - \frac{2}{1 - p^2}\right] + (1 - p_s + p_s^2)p^3(2 - p)\right\} G$$

$$r_1^* = r_0^* \quad r_2^* = \rho \frac{G}{p_s} + 2p_s(1 + p_s^2) - \left\{1 + p_s^2\right\} \left[\frac{p(2 - p^2)}{p^2(2 - p^2)} - (1 - p_s + p_s^2)p^2(1 - p^2)\right] G$$

and $(c_0^*, c_0, c_1^*, c_2^*) = (0, \rho G/p_s - r_0^*, \rho G/p_s - r_1^*, r_2^*)$. It is straightforward to verify that this contract attracts all borrowers in period 1 for $N \geq N_{2,2}(G)$. All constraints can also be verified for $G \in [C_{2,2}, C_{2,2}]$, again aided by continuity and linearity arguments which justifies checking only at $G = C_{2,2}$. In sum, if $G \in [C_{2,2}, C_{2,2}]$ and $N \geq N_{2,2}(G)$, fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If $G \in [C_{2,2}, C_{2,2}]$ and $N < N_{2,2}(G)$, fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to a subset of relevant constraints cannot attract them if $N < N_{2,2}(G)$, as established above.

**Fifth**, consider maximizing safe-borrower payoff 7 subject to eight of the constraints applicable to the full-efficiency case, including ZPC 8, both period-1 affordability constraints, limited commitment for safe and affordability for both $(r_0, c_0)$ and $(r_1, c_1)$, and affordability for $(r_2, c_2)$:

$$r_0 + c_0 \leq R_s \quad r_0 \leq R_s \quad ZPC 8 \quad r_2 + c_2 \leq R_s$$

$$r_1 + c_1 \leq R_s \quad r_1 + (1 - p_s)c_1 \leq \hat{r}_s \quad r_0 + c_0 \leq R_s \quad r_0 + (1 - p_s)c_0 \leq \hat{r}_s$$

Solving the first-order conditions for the eight multipliers shows that all of them are strictly positive under Assumption A3. Thus, the best contract for safe subject to these constraints involves $r_0 = R_s, c_0 = 0, c_0 = c_1 = \rho (G - N)/p_s^2, r_0 = r_1 = R_s - c_0 = R_s - c_1$, and $(r_2, c_2)$ to satisfy $r_2 + c_2 = R_s$ and the ZPC at equality. Some algebra gives that the safe-borrower payoff 7 under this contract equals or exceeds $2\pi$ iff $N \geq B_{1,1} - \frac{b_{1,1} - b_{1,1}}{c_{1,2} - c_{1,2}} (G - C_{1,1}) \equiv N_{2,2}(G)$. Consider this contract at $N = N_{2,2}(G)$:

$$r_0^* = \rho \frac{G}{p_s} \quad r_1^* = \rho \frac{2p_s^3 - \left[p_s^2p(2 - p) - \frac{2}{1 - p^2}\right] G}{p_s^2(p_s^2p^2 + p^4)}$$

$$r_2^* = \rho \frac{2p_s(1 + p_s^2) - \left[p_s^2p(2 - p) + p(2 - p^3)\right] G}{p_s^2(p_s^2p^2 + p^4)}$$

and $(c_0^*, c_0, c_1^*, c_2^*) = (0, \rho G/p_s - r_0^*, \rho G/p_s - r_1^*, r_2^*)$. It is straightforward to verify that this contract attracts all borrowers in period 1 for $N \geq N_{2,2}(G)$. All constraints can also be verified for $G \in [C_{1,1}, C_{2,2}]$, again aided by continuity and linearity arguments which justifi
checking only at $G = \mathcal{C}_{1,1}$. In sum, if $G \in [\mathcal{C}_{1,1}, \mathcal{C}_{2,2}]$ and $N \geq \overline{N}_{2,2}(G)$, fully efficient lending is achievable. Since the ZPC holds with equality, all surplus goes to the borrowers.

If $G \in [\mathcal{C}_{1,1}, \mathcal{C}_{2,2}]$ and $N < \overline{N}_{2,2}(G)$, fully efficient lending is not achievable. This is clear since the best contract for safe borrowers subject to a subset of relevant constraints cannot attract them if $N < \overline{N}_{2,2}(G)$, as established above. If $G < \mathcal{C}_{1,1}$, fully efficient lending is also not achievable, since necessary for attracting safe borrowers is $N \geq \overline{N}_{2,2}(G)$; but this is impossible when $G < \mathcal{C}_{1,1}$, for then $\overline{N}_{2,2}(G) > B_{1,1} (= \mathcal{C}_{1,1} > G \geq N)$. ■