Group Lending, Matching Patterns, and the Mystery of Microcredit: Evidence from Thailand

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Abstract

How has the microcredit movement managed to push financial frontiers? Theory shows that if borrowers vary in unobservable risk, then group-based, joint liability contracts price for risk more accurately than individual contracts, provided that borrowers match with others of similar project riskiness (Ghatak, 1999, 2000). This more accurate risk-pricing can attract safer borrowers and rouse an otherwise dormant credit market. We extend the theory to include correlated risk, and show that borrowers will seek to undo joint liability by matching to anti-diversify risk within groups. We use unique data on Thai microcredit borrowing groups to test for homogeneous matching by project riskiness and for intra-group diversification of risk, against a null of random matching. Multidimensional matching analysis is also carried out using Fox's (forthcoming) matching maximum score estimator. Evidence largely supports the theory, lending credence to the idea that group lending improves risk-pricing by embedding a discount for safe borrowers, and thus can plausibly explain part of the unprecedented rise in financial intermediation among the world's poor. However, the anti-diversification results point to a potential pitfall of voluntary group formation, and suggest strategies for lender intervention.

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1 Introduction

Recent impact studies have called into question the "miracle" of microcredit – i.e. transformative impacts of new formal credit access on well-being of poor households.[1] The question is not fully settled,[2] however, and there remains a strong prima facie case for some degree of net positive impacts from microcredit: the apparently large number of microcredit institutions lending sustainably to poor borrowers without needing subsidies[3] suggests that gains from trade are being realized.

Whatever microcredit's net impact may be, there is little doubt about how widespread it is and how rapidly it has grown in recent years. Maes and Reed (2012) report that over two hundred million people have borrowed from nearly four thousand microfinance institutions throughout the world. Forty years ago, any prediction of this development would likely have been greeted with skepticism. As the 2006 Nobel Peace Prize Press Release puts it, "Loans to poor people without any financial security had appeared to be an impossible idea."[4] This unprecedented expansion of microcredit gives rise to the following puzzle: how has this growth in intermediation and financial services among the world's poor been possible? How have lenders managed to overcome the obstacles involved in lending to borrowers without using collateral?

The current paper is focused on this "mystery" of microcredit.[5] Specifically, it tests one candidate theory, due to Ghatak (1999, 2000), whose answer is based on group lending and borrower matching. The context is a standard adverse selection environment in which there

[1]See discussion in Banerjee et al. (2015b), and the studies cited there.

[2]Some studies do find significant immediate impacts, e.g. Kaboski and Townsend (2011, 2012). Also, the studies cited can reject large impacts on the average villager, but typically cannot rule out large impacts on villagers who actually borrow (Banerjee et al., 2015b), leaving unsettled the cost-benefit question. Finally, inframarginal and longer-run impacts may be bigger, but remain largely unmeasured (exceptions are Breza and Kinnan, 2018, and Banerjee et al., 2015a).

[3]For example, see Cull et al. (2009).

[4]This Prize was given to Muhammad Yunus and the Grameen Bank for pioneering efforts in microcredit.

[5]This paper is not the first to do so. A growing literature has explored innovative practices and contract forms associated with the microcredit movement that may underpin its unprecedented success in lending among the poor. Armendariz and Morduch (2010), Ghatak and Guinnane (1999), and Morduch (1999) provide introductions to the topic; see the next section for further elaboration.

is limited liability and no collateral, and borrowers' projects have identical expected values but different degrees of risk. In this environment, a lender that cannot observe project risk offers all borrowers the same terms; its inability to price for risk results in cross-subsidization of riskier borrowers by safer borrowers and can cause a large portion of the potential market (safer households) to avoid borrowing. This market breakdown is the key inefficiency: good projects go unfunded due to the lender's inability to price for risk.

Ghatak adds to this context local information – borrowers know each other's risk, though the lender does not – and shows that group-based, joint liability lending contracts can harness this local information to improve the lender's ability to price for risk. The idea is as follows. First, joint liability induces borrowers of similar risk levels to match with each other. Second, given this matching pattern (which we will call "homogeneous matching"), the lender can use joint liability contracts to increase efficiency. Consider the pooling case.[6] Even though contract terms are the same for all borrowers, an *implicit discount* is built in for safer borrowers: they have safer partners, due to homogeneous matching, and thus when they succeed the joint liability clause is less costly in expectation for them. That is, joint liability plus homogeneous matching helps to undo the cross-subsidization and equalize the repayment burden across borrowers.[7] This can draw into the market safer borrowers who would have been inefficiently excluded under standard, individual loans.

The beauty of this result is that the lender is improving risk-pricing – and with it the efficiency and size of the market – by offering all borrowers the same contract, without learning their riskiness. This is appealing in practical terms. It implies that even a very passive or unsophisticated lender that offers a single, standardized group contract is giving implicit discounts to safe borrowers, and hence more accurately pricing for risk than if it used individual contracts. Thus, this theory can help explain the popularity of group lending in

[6] Ahlin (2015) shows that pooling works just as well as screening in this context, i.e. a single contract can achieve the same efficiency as any menu. What matters is not the lender's ability to screen borrowers, but its ability to improve risk-pricing through joint liability, with or without screening.

[7] Optimal joint liability plus homogeneous matching plus asymptotically large groups *fully* equalizes the repayment burden across borrowers, as long as typical joint liability scenarios are affordable (Ahlin, 2015).

microcredit – lenders that use it may be reversing partial market breakdown – as well as the growth of credit markets among the poor as this contract form is discovered and diffused.

The lynchpin in this theory is that borrowing groups match homogeneously by project risk; this is what provides the implicit discounts for safe borrowers.[8] To our knowledge, however, matching patterns of microcredit groups have yet to be empirically tested.[9] A main contribution of the current paper is to test directly for homogeneous matching by project risk among microcredit groups in Thailand.

The paper also extends the theory on matching for credit to consider *correlated* risk, asking whether borrowers will match with other borrowers exposed to similar, or different, risks. The result derived is that groups match homogeneously in both dimensions: they match with borrowers of similar riskiness, and among those, with partners exposed to similar types of risk. The intuition for the latter result is that by inducing correlated risk within their borrowing group, borrowers reduce the odds of facing liability for their fellow group members. This points to a potentially negative consequence of voluntary group formation, since correlated risk within groups limits the effectiveness of group lending (Debrah, 2017).

To test empirically whether groups are homogeneous in both riskiness and type of risk exposure, the Townsend Thai dataset is used. It includes information on borrowing groups from the Bank for Agriculture and Agricultural Cooperatives (BAAC). The BAAC is the predominant rural lender in Thailand. It offers joint liability contracts to self-formed groups of borrowers with little or no collateral. Importantly, this unique dataset includes multiple groups from each of a number of villages – taking the village as the matching market, this allows matching patterns to be tested using a number of independent matching markets.

To analyze matching along one dimension, we develop two related approaches. The *pattern approach* focuses on matching patterns, using several metrics to measure how homo-

[8] Random matching makes group lending no better than individual lending in this context (Ahlin, 2015).

[9] The literature has recognized this as an important open question. For example, it is first on the microfinance mechanisms empirical research agenda Morduch (1999, p. 1586) lays out: "Is there evidence of assortative matching through group lending as postulated by Ghatak (1999)?" See next section for complementary and related work.

geneously groups within each village have matched. The *payoff approach* instead specifies a match payoff function, and seeks to identify the key features of the payoff that determine matching behavior, i.e. complementarity vs. substitutability (similar to Fox, 2010, forthcoming).[10] Having specified the payoff function and homogeneity metrics, we proceed nonparametrically to test the null hypothesis of random matching, against alternatives of homogeneous matching (pattern approach) and complementarity-based matching (payoff approach). First, a given village's homogeneity metric or calculated payoff is compared to those of all permutations of observed borrowers into groups of the sizes observed; this delivers a percentile ranking of the village's observed grouping on a homogeneity or payoff scale. Second, we show that if matching is random, these percentile rankings are distributed uniformly. Finally, the distributions of villages' percentile rankings are compared to the uniform distribution using the Kolmogorov-Smirnov test.

The data reject random matching with respect to riskiness, against the alternative of homogeneous matching and complementarity-based matching. The data also reject random matching with respect to types of risk exposure, against the alternative of anti-diversified groups, when measured based on clustering of bad income years. These findings support the theory: groups are more similar in riskiness, and in timing of bad income years, than random matching would predict. The one finding counter to theory is that random matching can be rejected in favor of diversified groups when this is measured by occupational similarity; that is, groups look more occupationally diversified than random. Possibly the lender encourages diversification in an observable attribute like occupation, but borrowers are still able to achieve some anti-diversification by matching on lender-unobserved characteristics.

We next carry out multidimensional matching analysis to check whether both modeled dimensions of matching are independently predictive for matching patterns. Fox's (forthcoming) matching maximum score estimator is used; it chooses parameter values that maximize the frequency with which observed groupings yield higher payoffs than feasible, unobserved

[10] The two approaches need not coincide; as Ahlin (2017) shows, homogeneous matching can be driven by complementarity or substitutability in the payoff function.

groupings. The results are supportive of earlier conclusions.

In sum, Ghatak's theory receives support from the data: within-group homogeneity of project risk is significantly greater than random matching would predict. Evidently, group lending is successfully embedding a non-negligible discount for safe borrowers because their equilibrium partners are safer; if so, this can partly explain how microcredit has successfully awakened previously dormant credit markets. However, results on anti-diversification caution of a potentially negative aspect of voluntary group formation, and suggest that lenders may benefit from increasing the incentives to match for diversification, if this can be done cleanly.

The paper does not decisively establish causal determinants of group formation. However, we argue that to assess whether group lending enables better risk-pricing by targeting discounts to safe borrowers, this is not necessary (Section 5.3). As is clear from the theory, whether risk-homogeneity results from purposeful matching or as a byproduct of other constraints or objectives, it is by itself sufficient for the improvement in risk-pricing that enables group lending to revitalize markets.

In what follows, related literature is discussed in Section 2. The model setup and theoretical matching results are in Section 3. Data are described and variables defined in Section 4. Section 5 presents the methodology behind the univariate tests (Section 5.1), the empirical results (Section 5.2), and a discussion of causality (Section 5.3). Section 6 presents the multivariate estimation. Section 7 concludes. Proofs are in the appendix.

2 Relation to the Literature

This paper contributes to framing and unraveling a key mystery of microcredit, that is, how and why institutional lending has grown so dramatically among low-asset households across the world in the past several decades. It does so by highlighting a plausible mechanism through which credit markets can be revived, and finding empirical evidence for it.

Of course, it does not fully resolve the puzzle. For one, not all successful microlenders use group lending contracts. Also, the paper focuses on one mechanism, in an adverse selection environment, rather than testing across multiple mechanisms or environments. However, given that the puzzle's solution is likely to be multi-faceted, this paper makes the significant contribution of providing empirical backing to one key theory.

A number of other papers also shed light on this puzzle empirically or theoretically. Among other topics, they examine the innovations that gave rise to microcredit's expansion,[11] the underlying credit market frictions,[12] and the types of contracts that work best.[13] Relative to this literature, this paper is the first to focus empirically on matching combined with group lending as a key mechanism for repairing credit markets, and to offer direct evidence on a specific mechanism that may help explain the rise of microcredit.

The substantive focus of the paper is an empirical assessment of matching patterns in microcredit groups. To our knowledge this has not been done before, though related and complementary work exists. Eeckhout and Munshi (2010) study commercial ROSCAs[14] in India and show that changes in group composition and characteristics, in response to new regulation capping interest rates, are in line with predictions of their matching model. We differ in characterizing microcredit groups rather than ROSCAs, which in theory display quite different equilibrium matching patterns. ROSCAs group together both borrowers and lenders, i.e. agents of heterogeneous types, while microcredit groups are composed of homogeneous borrowers. The empirical approaches are also different: we characterize matching patterns of borrowing groups, while they test comparative statics of group composition in response to changes in the environment. Although it is not their main focus, Gine et al. (2010) study group formation in a microcredit-inspired field laboratory game, and find evidence that participants with similar levels of risk aversion group together to play the game.

[11] See for example Ghatak and Guinnane (1999), Armendariz and Morduch (2000), and Cull et al. (2009).

[12] See for example Ahlin and Townsend (2007a, 2007b), who find evidence consistent with the adverse selection context studied here; and Karlan and Zinman (2009), who do not find strong evidence for adverse selection, but rather for moral hazard. The current paper studies the same geographic setting as Ahlin and Townsend (2007a, 2007b), raising our expectation that adverse selection may be an issue.

[13] See for example Gine and Karlan (2014) and Ahlin and Waters (2016).

[14] "ROSCA" stands for rotating savings and credit associations.

A key difference is that we use data on active microcredit groups; this avoids the concern that a specific lab game may differ from practiced microcredit in important ways. There is also a literature on matching for risk-sharing, in the lab (e.g. Attanasio et al., 2012 and Barr and Genicot, 2008) and using household data (e.g. Fafchamps and Gubert, 2007). While sharing some features in common with microcredit group formation, these settings lack key features of credit, so it is not clear that results are applicable to a microcredit context.[15,16]

The paper also proposes a new statistical test for homogeneous or heterogeneous matching, and matching based on complementarity or substitutability, as alternatives to a null hypothesis of random matching. This test applies to one-sided matching when data on matches in multiple markets is available. It shares in common with independent work by Fox (forthcoming) the idea of comparing observed and unobserved matches in multiple markets, but takes this in a new direction using permutation testing combined with a result linking the uniform distribution to random matching. Both approaches also share significant nonparametric components. Unlike Fox's estimator, however, this test is not equipped to estimate matching fundamentals in a multi-characteristic matching setting.

Finally, the paper contributes to the theory of matching for microcredit by introducing a second dimension of heterogeneity of borrowers, the *type* of risk they are exposed to. This is the first multi-dimensional matching analysis we know of in the microcredit context, and it uncovers a new result: that matching based on type of risk exposure may lead to anti-diversification, as borrowers form groups so as to undo joint liability. The novel implication is that voluntary matching need not work in favor of efficiency, at least not in all dimensions.

Group lending with joint liability is a fundamental building block of this paper. In one field experiment, however, no significant difference in repayment rates between group and individual lending was found (Gine and Karlan, 2014); other studies have documented a trend toward declining use of group lending among microfinance institutions (MFIs) (e.g.,

[15]Indeed, Schulhofer-Wohl (2006) finds equilibrium matching to be negative assortative in his model of matching to share risk, while the microcredit model of this paper finds positive assortative matching.

[16]An even more different, but interesting, setting in which matching has been analyzed is in the formation of Community-Based Organizations – e.g., Arcand and Fafchamps (2012) and Barr et al. (2015).

de Quidt et al., 2016). These findings may cast doubt on group lending as a key to unlocking dormant credit markets. However, we argue that the debate on group lending is far from settled. For one, the experimental evidence cited came from an MFI that was willing to abandon group lending at its own risk, and thus potentially unrepresentative of a typical MFI using group lending. Further, there is evidence that, to the extent that group lending is on the decline, this is better explained by MFI aging than by an industry-wide movement away from group lending (Ahlin and Suandi, 2017). If so, this appears compatible with the conclusions of the current paper. Risk-pricing may become less difficult as MFIs age, thereby gaining experience with particular clients or locations; this can make their reliance on group lending less necessary. Still, their earlier reliance on group lending may have been instrumental in the initial opening up of credit markets, when asymmetric information was more systemic.

In sum, this paper advances the theoretical understanding of how microcredit groups form, and provides a first empirical characterization of matching patterns of existing groups. These results underpin a plausible (partial) explanation for the recent explosion in microlending, and point to the necessity for more work unraveling this mystery.

3 Theoretical Framework

3.1 Baseline model and results

The model here follows Ghatak (1999, 2000), which builds on work of Stiglitz and Weiss (1981). Risk-neutral agents are each endowed with no capital and one project. Each project requires one unit of capital and has expected value $\overline{R}$. Agents and their projects differ in riskiness, indexed by $p \in \mathcal{P}$, where $\mathcal{P} = [\underline{p}, \overline{p}]$ and $0 < \underline{p} < \overline{p} < 1$. The project of an agent of type p grosses R_p ("succeeds") with probability p and grosses 0 ("fails") with probability $1 - p$. Thus, $p \cdot R_p = \overline{R}$, for all $p \in \mathcal{P}$. The higher p, the lower the agent's riskiness.

An agent's riskiness is observable to other agents, but not to the outside lender. In this context, uncollateralized individual loan contracts can be inefficient. They bear an interest rate based on the average risk in a borrowing pool, a rate at which safer borrowers may find it unprofitable to borrow.[17] Thus, the lending market can (partially) collapse, excluding all but the riskier borrowers due to a failure to price for risk. Efficiency losses in this context result from leaving good projects unfunded – the safer borrowers' – and raising efficiency comes from attracting more borrowers.[18]

In this context, group lending can increase efficiency by improving risk-pricing, by offering implicit discounts to safer borrowers. A lender requires potential borrowers to form groups of size two, each member of which is liable for the other. Without loss of generality (Ahlin, 2015), a single, standardized contract is offered to all borrowers. In the contract, a borrower who fails pays the lender nothing, due to limited liability. A borrower who succeeds pays the lender gross interest rate $r > 0$. A borrower who succeeds and whose partner fails makes an additional liability payment $c > 0$. Thus, a borrower of type p_i who matches with a borrower of type p_j has expected payoff

$$\pi_{ij} = \overline{R} - rp_i - cp_i(1 - p_j), \tag{1}$$

assuming the borrowers' returns are uncorrelated.

Group lending's risk-pricing is clearly seen via comparison to a standard individual loan contract, where the payoff is $\overline{R} - p_i r$ and the interest rate does not vary by risk-type. To compare, rewrite the borrower's payoff under the group lending contract (equation 1) as

$$\pi_{ij} = \overline{R} - p_i \tilde{r}_{ij},$$

where

$$\tilde{r}_{ij} \equiv r + c(1 - p_j). \tag{2}$$

[17]For evidence consistent with this behavior in the Thai context, see Ahlin and Townsend (2007b).

[18]Hence, the term "adverse selection" risks being somewhat misleading here: the goal is to include safe borrowers, not exclude risky, since all have equally good projects (all have expected returns $\overline{R}$).

Here $\tilde{r}_{ij}$ is interpretable as the *implied* interest rate paid by borrower i when successful and matched with borrower j. Two components make up this implied interest rate: the direct interest rate r, and the expected bailout payment for the partner, $c(1-p_j)$.

Because this second component depends on partner quality (p_j), the question of how borrowers match becomes critical. Utility is transferable in this context, and side transfers between borrowers are allowed. Thus, following Ghatak (1999, 2000) and Legros and Newman (2002), the equilibrium includes a) an assignment of borrowing agents into two-member borrowing groups or non-borrowing, and b) payoffs of all borrowing agents such that two co-grouped agents' equilibrium payoffs sum to their total group payoff and such that no two agents can earn strictly higher payoffs by grouping together. It is well known that in such an equilibrium, no two groups can be rearranged to produce a higher sum of group payoffs – a fact that will be used later.

Note that

$$\frac{\partial^2(\pi_{ij}+\pi_{ji})}{\partial p_i \partial p_j} = 2c > 0. \tag{3}$$

That is, the group payoff function exhibits complementarity, and the stable outcome when there is a continuum of agents is that groups are perfectly homogeneous in riskiness, as Ghatak has shown.[19]

A borrower with a safer partner (higher p_j) has a lower implied interest rate (equation 2), because his chance of owing a bailout payment when successful is lower. What homogeneous matching gives is that *safer borrowers* have *safer partners*, and thus, lower implied interest rates. With perfectly homogeneous matching, each group contains identical borrowers, so

$$\tilde{r}_{ij} = \tilde{r}_{ii} = r + c(1-p_i) \qquad \text{and} \qquad \frac{\partial \tilde{r}_{ij}}{\partial p_i} = \frac{\partial \tilde{r}_{ii}}{\partial p_i} = -c < 0\ . \tag{4}$$

Safer borrowers have safer partners, and thus can expect fewer bailout payments when suc-

[19]The intuition is that having a more reliable (safer) partner is worth more to safe borrowers, since a borrower is "on the hook" for his partner only if he succeeds. Thus, even with side payments a riskier borrower cannot lure a safe borrower away from a safe partner.

cessful. As a result, safer borrowers face a lower *implied* interest rate under joint liability – just as they would under full information. In this way, group lending harnesses social information to vary the interest rate implicitly by riskiness, thus improving risk-pricing.

This is true even under an unsophisticated pooling strategy, where the lender simply offers all comers a standard joint liability contract (r, c). Whether the lender knows it or not, if matching is homogeneous, the contract embeds discounts for safe borrowers and can draw more of them into the market. Unsophisticated group lending can be responsible for reviving a lending market, underpinning a substantial increase in intermediation.

3.2 Variations on the baseline model

Consider a finite population of borrowers rather than a continuum. Matching into perfectly homogeneous groups is generally impossible, but in any equilibrium all groups will be rank-ordered by riskiness, i.e. maximally homogeneous. That is, for any group size $k \geq 2$, the k riskiest borrowers match together, the next k riskiest borrowers match together, and so on (Ahlin, 2017). Given rank-ordered matching, group lending has qualitatively similar risk-pricing advantages over individual lending: safer borrowers have generally safer partners, so they face lower implied interest rates. Thus, the theory does not critically rely on unrealistically large numbers of borrowers or perfectly homogeneous groups.

Consider instead removing the assumption that borrowers know each other's riskiness. If riskiness is uncorrelated with any characteristics that do drive matching, then matching would be *random* with respect to riskiness, instead of homogeneous. In a large borrowing pool, all borrowers would then face the same implied interest rate, in expectation, equivalent to matching with a borrower of average riskiness in the pool. With no variation in ex ante implied interest rate across borrowers, group lending would lose its risk-pricing advantage over individual lending in this context and could not draw additional borrowers into the market.[20] But, if borrowers matched into "homogeneous" groups based on non-risk charac-

[20] See Ahlin and Townsend (2002, section 5.4.7) and Ahlin (2015, Lemma 3).

teristics that are predictive of riskiness – e.g. due to proximity or friendship – one would still observe some degree of group homogeneity in riskiness. Interestingly, group lending would still embed an implicit discount for safe borrowers, for the reasons discussed. To the extent that groups formed somewhat homogeneously in riskiness, group lending could still be a force for expanding the lending market.

Next, consider group size. For simplicity, the theory in this paper is for groups of fixed size two. In a model with fixed group size larger than two, the same forces are at work: standard joint liability contracts induce homogeneous matching, which gives safe borrowers discounts in their implicit interest rates, and can improve efficiency by drawing them into the market (Ahlin, 2015). That is, with larger groups, it remains homogeneous matching that is critical for the market-reviving effects of group lending.[21]

Summarizing so far, within-group homogeneity in riskiness obtains in a number of settings, allowing group lending to improve risk-pricing and facilitate more efficient lending.

However, joint liability per se does not necessarily lead to homogeneous matching – the contract details matter. Sadoulet (1999) and Guttman (2008) consider *dynamic* contracts where liability for one's partner carries the threat of being denied future loans if both borrowers fail. In this context, the group payoff function can exhibit substitutability rather than complementarity, leading safer borrowers to match with riskier partners.[22]

However, these non-homogeneous matching results hold under contract forms that are not claimed to be optimal. To our knowledge the literature does not establish any efficiency properties of joint liability contracts that give rise to substitutability of types and non-

In a somewhat different context involving costly auditing, Armendariz and Gollier (2000) show how joint liability can raise efficiency even with random matching. The idea is that risky borrowers pay more under joint liability if audited when successful, since their projects have higher returns when successful.

[21]Even if group size is endogenous, if types are complements and the payoff function is sum-based, any two equilibrium groups must be rank-ordered regardless of their equilibrium sizes. Otherwise, one could rearrange the borrowers within the two groups, holding group sizes fixed, and raise the payoffs of at least one group of borrowers – contradicting equilibrium (Ahlin, 2017).

[22]Intuitively, having a more reliable partner is worth more to riskier borrowers, since they more often need their partner to be successful in order to continue receiving loans.

In a *static* model, Ahlin (2015) provides an example of a joint liability contract (for groups of three or more borrowers) that makes riskiness types substitutes rather than complements in the payoff function and leads to non-homogeneous group formation.

homogeneous matching. Efficiency of some such contract may yet be shown; further, lenders may blunder in model selection or contract design, or operate under distorting political constraints – especially when heavily subsidized through the government budget, as our lender is. For these reasons, the empirical tests will consider the prediction of non-homogeneous matching based on type substitutability, but focus primarily on the model's main prediction of homogeneous matching based on type complementarity.

3.3 Matching over degree and type of risk

This section adds a second dimension of heterogeneity and points out a potential pitfall of relying on voluntary matching. We add to the baseline model the possibility for correlated risk. Given the agricultural setting of many micro-lenders, including the one in our data, this is a potentially important extension. However, it is rarely modeled in the group lending literature, and to our knowledge not at all in the context of endogenous group formation.

Given two borrowers i and j with unconditional probabilities of success p_i and p_j, respectively, the joint output distribution can be written uniquely as:

	j Succeeds (p_j)	j Fails $(1-p_j)$
i Succeeds (p_i)	$p_i\, p_j + \epsilon_{ij}$	$p_i(1-p_j) - \epsilon_{ij}$
i Fails $(1-p_i)$	$(1-p_i)p_j - \epsilon_{ij}$	$(1-p_i)(1-p_j) + \epsilon_{ij}$

(5)

The case of $\epsilon_{ij} \equiv 0$ is the case of independent returns considered by Ghatak. A positive (negative) ϵ_{ij} gives positive (negative) correlation between borrower returns.

Correlation parameter ϵ_{ij} may differ across pairs of borrowers $\{i, j\}$. We proceed by placing a simple structure on correlations which ensures that $\epsilon_{ij} = \epsilon > 0$ for any two borrowers facing the same types of risk, and $\epsilon_{ij} = 0$ for all other pairings.

Assume there are two i.i.d. broadly shared sources of uncertainty, or "shocks", A and B. Each equals 1 or -1 with equal probability. Every agent is assumed to be exposed to risk from either shock A or shock B, or neither (N). Let $s_i \in \mathcal{S} \equiv \{A, B, N\}$ denote agent i's shock exposure-type. Shock exposure-type is known by all agents but not the lender.[23]

The probability of success of an agent with $s_i = A$ and project risk parameter p_i equals $p_i + \gamma A$, for some $\gamma > 0$. That is, if there is a good shock ($A = 1$), the agent's success probability is $p_i + \gamma$; a bad shock ($A = -1$) lowers the agent's success probability to $p_i - \gamma$. This agent's project outcome is independent of shock B. The success probability of an agent with $s_i = B$ and project risk parameter p_i is exactly analogous: $p_i + \gamma B$, independent of A. The remaining agents, with $s_i = N$, succeed or fail independently from A and B.

With these assumptions, the ϵ_{ij} of expression 5 varies across borrowers i and j in a straightforward way. Let $\epsilon \equiv \gamma^2$ and $\kappa_{i,j} = 1\{s_i = s_j = A \,||\, s_i = s_j = B\}$. Then

$$\epsilon_{ij} = \kappa_{i,j}\, \epsilon \ .$$

That is, returns are positively correlated for borrowers exposed to the same type of risk ($\kappa_{i,j} = 1$), because probabilities of success are pushed in the same direction by the shock.[24] For borrowers not exposed to the same risk ($\kappa_{i,j} = 0$), $\epsilon_{ij} = 0$, because the shocks each borrower is exposed to are independent.

In summary, the correlation structure boils down to $\epsilon_{ij} = \epsilon$ ($\epsilon_{ij} = 0$) for pairs exposed (not exposed) to the same shock. The payoff of borrower i when matched with borrower j is now

$$\pi_{ij} = \overline{R} - rp_i - c[p_i(1 - p_j) - \epsilon\kappa_{i,j}] = \overline{R} - rp_i - cp_i(1 - p_j) + c\epsilon\kappa_{i,j} \ . \tag{6}$$

The last term ($c\epsilon$) represents a payoff boost from matching with a partner exposed to the same risk. Payoffs are boosted because positive correlation of returns in the group lowers chances of having to bail out one's partner.

In this context, the following can be shown:

[23]In reality, the lender may have some clues, e.g. borrower occupation. One can interpret this assumption as applying to the unobserved aspects of risk exposure.

[24]With probability 1/2, the shock to which both are exposed is good and the probability of both succeeding is $(p_i + \gamma)(p_j + \gamma)$; similarly, with probability 1/2 the probability of both succeeding is $(p_i - \gamma)(p_j - \gamma)$. The unconditional probability of both succeeding is thus $p_ip_j + \gamma^2$.

Proposition 1. *Assume a continuum of borrowers. In equilibrium, almost every group is perfectly homogeneous in both riskiness ($p \in [\underline{p}, \overline{p}]$) and shock exposure-type ($s \in \{A, B, N\}$).*

Thus, groups match homogeneously in riskiness (p_i) and shock exposure-type (s_i); each group contains either all A-risk, all B-risk, or all N-risk borrowers. The intuition for shock exposure homogeneity is simple: borrowers choose to anti-diversify their groups so as to lower their chances of facing liability for their partners.

However, homogeneous matching in exposure-type, i.e. anti-diversification, works against efficient lending. By raising correlated risk within borrowing groups, it lowers the *effective* rate of joint liability. In the extreme case of perfectly correlated risk, for example, the effective rate of joint liability is 0 regardless of how the bank sets c: when one borrower fails, they both do, so joint liability payments are never made. In general, the greater the correlation, the smaller the parameter space over which group lending can achieve fully efficient lending (Debrah, 2017). Here is a dimension of voluntary matching that does not work in favor of efficiency.

Proposition 1 holds with a continuum of borrowers. In a finite population, maximal anti-diversification may be incompatible with maximal riskiness homogeneity, in which case tradeoffs between the two dimensions of matching arise. Homogeneity in one dimension may be (partially) sacrificed to achieve it in the dimension with greater payoff salience. Nonetheless, payoff function complementarities will push toward homogeneity in both dimensions.[25]

A point made in Section 3.2 holds here as well: other ways of implementing joint liability could lead to different matching patterns. For example, dynamic joint liability contracts involving the denial of future loans could lead to formation of diversified groups; diversification would raise chances of partner bailouts that could extend the borrowing relationship. So, the empirical work will consider both diversification and anti-diversification hypotheses, the latter being the focal hypothesis.

[25] As Fox (2010, forthcoming) shows, under a plausible assumption and with a sufficient amount of data on matches and borrower characteristics, complementarities in both dimensions can be identified and estimated, along with the relative strength of the two complementarities. We follow this approach in Section 6.

4 Data and Variable Descriptions

The empirical goal of the paper is to characterize borrower matching with respect to riskiness and types of risk exposure.

4.1 Data description and environment

A subset of the Townsend Thai data are used. In May 1997, a cross section of 192 villages was surveyed, covering four provinces from two contrasting regions of Thailand, both with large agricultural sectors. In each village as many borrowing groups of the Bank for Agriculture and Agricultural Cooperatives (BAAC) as possible were interviewed, up to two. This baseline survey contains data on 262 groups, 200 of which are one of two groups representing their village. Unfortunately for the purposes of this study, the borrower-level data provided in this survey are minimal – they do not include risk variables – and they are all provided by the group's official leader, not the individual borrowers.[26]

Hence, we turn to a resurvey, conducted in April and May 2000. The resurvey data were collected from a random subset of the same villages, stratified at the sub-district (tambon) level. Included are data on 87 groups, 14 of which are the only groups in their village, 70 of which are one of two groups interviewed from the same village, and 3 of which are one of three groups interviewed from the same village.[27] Though observations are fewer, the resurvey data are preferable because individual group members respond to questions on their own behalf, up to five per group and on average 4.5; and because several resurvey questions were designed to measure income risk and correlatedness, the key variables in the theory. In total, we have 36 villages with multiple groups.

The BAAC is a government-operated development bank in Thailand, established in 1966 and the primary formal financial institution serving rural households. It has estimated that

[26] The concern is that when one person responds for all group members, measurement error can be highly correlated within the group, causing homogeneity of matching to be overestimated.

[27] This was apparently a mistake in implementation of the data collection methodology, which capped responses to two groups per village; we use the three-group village anyway.

it serves 4.88 million farm families, in a country that had just over sixty million inhabitants, about two thirds of which lived in rural areas. In the Townsend Thai baseline household survey covering the same villages, BAAC loans constituted 34.3% of the total number of loans, as compared with 3.4% for commercial banks, 12.8% for village-level financial institutions, and 39.4% for informal loans and reciprocal gifts (Kaboski and Townsend, 1998).

The BAAC allowed smaller loans to be backed only with group-based joint liability.[28] This kind of borrowing was widespread: of the nearly 3000 households in the baseline household survey, just over 20% had a group-guaranteed loan from the BAAC outstanding in the previous year. Borrowing in this way required membership in an official BAAC borrowing group and choosing the group-guarantee option on the loan application. The group then faced explicit liability for the loan; that is, the BAAC could opt to follow up with a delinquent borrower *or* other group members in search of repayment. There could also be dynamic repercussions: some group members reported delays or greater difficulties in getting future loans when another group member defaulted. Given lender discretion, it is impossible to specify the exact contract structure, but both static and dynamic elements of joint liability seem operative.

Groups in the data usually have between five and fifteen members; about 15% are larger. Typically, groups were born when borrowers proposed a list of members to the BAAC, and the BAAC then approved some or all members. The BAAC seemed to use its veto power sparingly: only about 12% of groups in the baseline survey reported that the BAAC struck members from the list.[29] We know of no case where the BAAC added members to a list or formed a group unilaterally. Thus, while the BAAC had some say in group formation, it appears that group formation was primarily at the discretion of the borrowers themselves.

[28]The cap on group loans at the time of the baseline survey was 50,000 Thai baht, about $2000. The median group loan was closer to $1000.

[29]This is in response to a free-form question about how the group's original members were determined.

Table 1 — Summary Statistics

Variable	Mean	Median	Std Dev	Min	Max	Obs
Riskiness						
Probability of Success (Future Income)	0.426	0.400	0.253	0.000	1.000	338
Coefficient of Variation (Future Income)	0.449	0.400	0.287	0.000	1.388	313
Type of Risk Exposure						
Expected Earnings from ________ as Pct of Total Earnings:						
Agriculture	50.5%	46.5%	39.1%	0.0%	100.0%	386
Aquaculture	2.8%	0.0%	14.5%	0.0%	100.0%	386
Business	6.0%	0.0%	17.8%	0.0%	100.0%	386
Wages, Salaries	40.7%	32.7%	40.0%	0.0%	100.0%	386
Worst Year for Income	[65.6% last yr, 16.9% yr before, 17.4% same]					390

4.2 Variable descriptions

To characterize matching along the two modeled dimensions, measures that reflect borrower riskiness and within-group correlatedness are necessary. These are summarized in Table 1.

Our main measure of riskiness takes the theory (section 3.1) literally. Group members were asked what their income would be in the coming year if it were a good year (R_{Hi}), what their income would be if it were a bad year (R_{Lo}), and what they expected their income to be ($\overline{R}$). Assuming that income can take only one of two values, R_{Hi} and R_{Lo}, and that $\overline{R}$ represents the mean, the **probability of success**, or p, works out to be

$$p = \frac{\overline{R} - R_{Lo}}{R_{Hi} - R_{Lo}},$$

using the fact that $p\, R_{Hi} + (1-p)\, R_{Lo} = \overline{R}$.[30] Another measure of risk, less directly related to the model, is the **coefficient of variation** of income, C.[31] Based on the same assumed income distribution, this works out to be

$$C \equiv \frac{\sigma_R}{\overline{R}} = \sqrt{\frac{R_{Hi}}{\overline{R}} - 1}\sqrt{1 - \frac{R_{Lo}}{\overline{R}}},$$

which is simply the percentage deviation from expected income, averaged (geometrically) over good and bad outcomes.

Correlatedness is proxied in two ways. First, we create a measure of **occupation**. Borrowers list the most recent year's revenue in more than thirty categories, and expenses in three aggregated categories: agriculture (rice or other crop farming, livestock), aquaculture (raising shrimp or fish), and business (e.g. restaurant, mechanic shop, trading). To transform these revenue and expense data into an individual's occupation, we proxy for the share of income coming from each of four categories: agriculture, aquaculture, business, and wage labor. A simple way of calculating a borrower's income would be the borrower's revenues minus expenses (in each category with expense data). This leads to a practical and a related conceptual problem. Practically, in each category there would be a number of borrowers with negative incomes; conceptually, given risk, one year's net income in a given category may be a quite noisy proxy for usual or expected income in that category. Since revenues seem likely to vary more widely than expenses from year to year, we proxy for income using expense data (in the three categories with expense data). A borrower's agriculture expenses are translated into that borrower's expected agriculture earnings using a sample-wide profit rate (revenues/expenses), calculated as the sum of all agricultural revenues in the sample divided by the sum of all agricultural expenses in the sample; aquaculture and business expected earnings are found analogously. Thus, for each borrower we have a proxy for expected or usual earnings in agriculture, aquaculture, and business, based on that borrower's expenses

[30] The measure described here is used by Ahlin and Townsend (2007b) in their finding of evidence for adverse selection in this credit market.

[31] The coefficient of variation equals the standard deviation divided by the mean.

in each of these categories multiplied by a category-specific, sample-wide factor translating expenses to earnings. Earnings in the fourth category are simply taken as revenues from wages or salaries.[32]

Given occupational vectors for borrowers i and j, $(\omega_{i1}, \omega_{i2}, \omega_{i3}, \omega_{i4})$ and $(\omega_{j1}, \omega_{j2}, \omega_{j3}, \omega_{j4})$, each entry of which is the fraction of total earnings from one occupation, we proxy the degree of correlatedness between borrowers i and j as the negative rectilinear distance between their occupational vectors, $-\sum_{k=1}^{4} |\omega_{ik} - \omega_{jk}|$. This correlatedness measure is maximal for two borrowers with identical occupational vectors, and minimal for two borrowers that drew revenue from no common categories.[33]

Second, we use timing of bad income years, **worst_year**. Specifically, borrowers were asked which year of the past two was worse for household income: "one year ago", "two years ago", or "neither". If borrowers are exposed to correlated shocks, bad income years are more likely to coincide; thus coincidence of bad years can proxy borrowers' correlatedness.

One could certainly envision more informative measures of riskiness and correlatedness than the ones available from this dataset. However, a primary effect from noisiness in these measures is likely to be making random matching harder to reject, and in the limit of pure white noise, impossible. Our conjecture is that mismeasurement is mainly causing an underestimation of systematic matching patterns.

5 Univariate Methodology and Results

In this section, we examine matching patterns one characteristic at a time. Section 6 extends the analysis to consider matching along multiple characteristics.

[32]Translating revenues to earnings (rather than expenses to earnings) using a similar strategy produces very similar results, as do a number of variations on this approach.

[33]Euclidean distance seems less appropriate here. It says that a dedicated farmer $(1, 0, 0, 0)$ is closer in occupation to someone who is half in business and half in wage labor $(0, 0, 1/2, 1/2)$ than to someone who is all in business $(0, 0, 1, 0)$ or all in wage labor $(0, 0, 0, 1)$; there is no difference under the rectilinear distance.

5.1 Univariate Methodology

Overview. Consider matching on riskiness alone. Given the assumed contract structure, borrower riskiness types are complements in the group payoff function (see Section 3.1). Type complementarity guarantees that in any equilibrium, every two groups are rank-ordered by riskiness, i.e. maximally homogeneous (see Section 3.2).

We follow two approaches to testing this theory: testing the key functional form *assumption* that the group payoff function exhibits riskiness-type complementarity; and testing the matching-pattern *prediction* that groups are homogeneous. Since the first approach focuses on matching payoffs and the second on matching patterns, we denote them the "payoff" approach and "pattern" approach, respectively.

The *pattern approach* focuses on the key prediction of the model, since it is by making each borrower liable for a self-similar group of borrowers (via homogeneous matching) that the group contract is able to improve risk-pricing without explicitly conditioning on borrower risk. However, this prediction is stark, and easy to reject. This is because any village with two groups that are not perfectly rank-ordered, i.e. maximally homogeneous, is inconsistent with the theory. However, this seems to set the bar too high; given measurement error, matching on other dimensions, or matching constraints, the theory may fail to hold precisely even though it does hold to a degree.[34] That is, even though safe borrowers may not be receiving the maximum possible implicit discount, which would come from *maximally* homogeneous matching, they may be receiving a substantial discount from *moderately* homogeneous matching. Indeed, it is random matching that leads to a zero discount for safe

[34] A typical approach to this issue is to assume that matching is occurring on unobservables as well (see Chiappori and Salanie, 2016, and its references); this would allow matches that were observationally only moderately homogeneous to be both observable in equilibrium and compatible with maximal homogeneity as the unique outcome if matching were based only on observables. Two ways of implementing this approach empirically are to assume some structure on the unobservables (e.g. Choo and Siow, 2006), or to directly assume that matchings that produce higher observable payoffs are more likely to be observed when matching is based both on observables and unobservables (Fox, 2010, forthcoming). The "payoff approach" described next follows the Fox approach in assuming that matchings that produce higher observable payoffs are more likely to be observed. The "pattern approach" differs by assuming that matchings that produce higher observable *homogeneity*, rather than higher observable *payoffs*, are more likely to be observed.

borrowers (in a continuum, Ahlin, 2015); there is much room between random matching and maximally homogeneous matching where safe borrower discounts can be substantially positive, if not maximal. Hence, rather than a null of maximal homogeneity, we test a null of random matching. If random matching can be rejected against the alternative hypothesis of homogeneous matching, we consider the theory to be supported, given there is direct evidence that safer borrowers are bearing liability for a safer set of borrowers, at least moderately so, and thus are receiving lower implicit rates through the group contract.[35]

The *payoff approach* focuses on the underlying forces driving matching by identifying the nature of the group payoff function, in particular whether it features type complementarity or substitutability. An advantage of this approach is that it sidesteps potential pitfalls in trying to identify the nature of matching from observed matching patterns.[36] The group payoff function is taken directly from the theory, in which complementarity vs substitutability is determined by the sign on the riskiness-type interaction term. The assumed contract has a positive sign, i.e. complementarity, but other ways of operationalizing joint liability could give rise to a negative sign, i.e. substitutability (see Section 3.2). Thus, we aim to identify the sign on the type-interaction term. However, the theory predicts quite starkly that every pair of groups maximizes the sum of the two groups' payoffs, whether types are complements or substitutes. This typically rules out all but one borrower grouping in each village. Again, as this seems too stringent a test, we test a null hypothesis of random matching: that the payoffs of observed borrower groupings are not different from what random matching would deliver, assuming the payoff function features complementarity. Rejecting this null in favor of the alternative hypothesis that observed groupings produce higher-than-random complementarity-based payoffs will be taken as evidence in favor of the theory. On the other hand, rejecting the null in favor of the alternative hypothesis that observed groupings

[35] The rationale for this null hypothesis is similar for the ubiquitous t-statistic in a linear regression; in both cases, the null hypothesis is that the variable has no explanatory power, and theory is supported by rejecting the null in the predicted direction.

[36] Pitfalls can arise because, while complementarity makes clear predictions about matching patterns, substitutability's predictions are much weaker and context-sensitive. As a result, substitutability can lead to matching patterns that are observationally similar to complementarity's in certain respects (Ahlin, 2017).

produce lower-than-random complementarity-based payoffs is evidence for substitutability-based matching. This is because the relevant complementarity-based payoffs are equal and opposite to substitutability-based payoffs, differing only in sign; thus, lower-than-random complementarity-based payoffs are higher-than-random substitutability-based payoffs.

In short, the null hypothesis in both approaches is random matching. Support for the theory comes if, compared to random matching, groups look homogeneous (pattern approach) or produce high payoffs under a complementarity-based payoff function (payoff approach).

Consider next matching on type of risk exposure alone. Here borrower types differ horizontally rather than vertically, i.e. they are non-ordered rather than ordered, so that complementarity is not well-defined. Still, a similar approach is feasible. The assumed contract structure implies that the group payoff is diversification-averse, i.e. it is higher the more similar are risk exposure-types within groups; this guarantees that groups are maximally homogeneous in equilibrium (see Section 3.3). In this context, the pattern approach involves comparing observed group homogeneity in risk exposure-type to the degree of homogeneity associated with random matching. The payoff approach involves comparing payoffs of observed groups to the payoffs that random matching would produce, using a diversification-averse payoff function.

We discuss how both group payoffs and homogeneity are measured in Section 5.1.1, and how the random matching hypothesis is tested in Section 5.1.2.

5.1.1 Measuring matching homogeneity and group payoffs

Here we discuss metrics used to measure match homogeneity and group payoffs. Since the ultimate goal is a rank-based comparison of the metrics from observed groupings with the metrics from unobserved groupings, only the metrics' orderings matter, not their scales.

The **pattern approach** relies on comparing homogeneity of observed matches to those of random matches. We use several "homogeneity metrics" to measure homogeneity of a match, or "grouping", which is an assignment of borrowers within a matching market (here, village) into borrowing groups.

The model's main measure for riskiness type is the probability of success, p. Consider data from two groups L and M in village v, of respective sample sizes l and m: $L = (p_1, ..., p_l)$ and $M = (p_{l+1}, ..., p_{l+m})$.

An off-the-shelf way to measure homogeneity of this grouping is via *variance decomposition* of $P = (p_1, ..., p_{l+m})$ into between-group and within-group components. The between-group variance component is maximized in a rank-ordered grouping, so a larger between-group component can be taken as stronger evidence for homogeneous matching. To illustrate, consider a village with 2 groups of size 4, with success probabilities $P = (1, 2, 4, 5, 6, 7, 8, 9)$ (in tenths). Compare the borrower grouping $L = (2, 5, 6, 8)$ and $M = (1, 4, 7, 9)$ with the grouping $L' = (1, 2, 5, 6)$ and $M' = (4, 7, 8, 9)$. The first grouping has a between-group variance component of 0%, while the second grouping has a between-group component of 44%. The higher value reflects the more homogeneous matching of the second grouping – "close" to rank-ordering – while the lower value reflects the more mixed first grouping – equal means, and thus "far" from rank-ordering.

In the theory, group lending works to the extent that safer borrowers are liable for safer partners. A second homogeneity metric hones in on this link: *borrower-partner covariance*, the covariance within a village of a borrower's riskiness with the borrower's observed partners' average riskiness levels. For example, in grouping $L = (2, 5, 6, 8)$ and $M = (1, 4, 7, 9)$, the riskiest borrower ($p = 1$) has average partner riskiness of $20/3 = (4 + 7 + 9)/3$; the borrower-partner covariance is -2.31, the covariance between $(1, 2, 4, 5, 6, 7, 8, 9)$ and $(20/3, 19/3, ..., 4)$. The more homogeneous grouping $L' = (1, 2, 5, 6)$ and $M' = (4, 7, 8, 9)$ has a higher borrower-partner covariance, 1.77. This metric too is maximized in a rank-ordered grouping.

The same homogeneity metrics can be used with the alternative measure of borrower riskiness, the coefficient of variation. However, they cannot be applied to risk exposure-type s, measured by occupation and worst_year, since these are non-ordered, categorical variables. A suitable metric for these variables is the *chi-squared* independence (or homogeneity) *test* *statistic.*[37] This statistic quantifies deviations from the grouping in which each group has the same proportion of borrowers of each type – thus it is minimized (at 0) under an equal distribution of types across groups, and maximized under maximal group homogeneity. For example, letting A and B be two risk exposure-types – in the data, two occupations or worst years – compare the grouping $L = (A, A, B, B)$ and $M = (A, A, B, B)$ with $L' = (A, A, A, B)$ and $M' = (A, B, B, B)$. The chi-squared test statistic for the first grouping is 0 and for the second grouping is 2.[38]

The **payoff approach** relies on comparing group payoffs in observed matches to those of random matches. Measurement of group payoffs comes directly from the theory. Consider first the baseline model with unidimensional types reflecting only riskiness. Let groups $L = \{i, j\}$ and $M = \{i', j'\}$ be observed in a village, and group payoff functions be $\Pi_L = \pi_{ij} + \pi_{ji}$ and $\Pi_M = \pi_{i'j'} + \pi_{j'i'}$. Using equation 1, the total payoffs in this grouping are

$$\Pi_L + \Pi_M = 4\overline{R} - (r + c)(p_i + p_j + p_{i'} + p_{j'}) + c(p_i p_j + p_j p_i + p_{i'} p_{j'} + p_{j'} p_{i'}) \ .$$

Note that only the interaction terms (the last term in parentheses) can differ across groupings of the four borrowers. Hence, given our ultimate purpose of comparing $\Pi_L + \Pi_M$ against alternative groupings of the same set of borrowers, we can ignore all but these terms. Further, since $c > 0$ – this is the key assumption that delivers complementarity of types – c simply scales these type interaction terms and can be ignored in the comparisons. If $c < 0$ were instead true, which would deliver substitutability of types, the scale of payoffs would be identical, but inverted. Letting p_{-k} be the success probability of borrower k's partner, these terms can be written

$$\sum_{k \in L} p_k p_{-k} + \sum_{k \in M} p_k p_{-k} \ . \tag{7}$$

Taking the theory to data is complicated by the fact that the borrowing groups in the data

[37] The formula and discussion can be found in DeGroot (1986, pp. 536-7, 542-3).

[38] The chi-squared statistic easily accomodates fractional types, e.g. a borrower being 30% of occupation A and 70% of occupation B. It is based on summing number of borrowers of each type within group and village, and these sums are well-defined whether summing parts or wholes of borrowers.

are not pairs, but typically involve 5-15 members; further, the sample contains a maximum of five borrowers per group. Our strategy is to proxy for p_{-k} in expression 7 using the average success probability of the other sampled group members. Specifically, let group G be a set of grouped borrowers, $\mathcal{S}^G$ be the sampled subset of group G, and $\overline{p}_{-k}^{\mathcal{S}^G}$ be the average success probability in $\mathcal{S}^G$ excluding borrower k. The following is our sample estimate of the relevant part of the payoffs (expression 7):

$$\sum_{k\in\mathcal{S}^L} p_k\overline{p}_{-k}^{\mathcal{S}^L} + \sum_{k\in\mathcal{S}^M} p_k\overline{p}_{-k}^{\mathcal{S}^M}. \tag{8}$$

This estimate is simply the sum, over all sampled village borrowers, of the borrower's success probability multiplied by the average success probability of other same-group, sampled borrowers.[39] This can be directly calculated from the data using the success probability variable (see Section 4.2).

Consider next the contract when borrowers have two-dimensional types, capturing riskiness and type of risk exposure. Let $\kappa_{k,-k}$ be the indicator for whether borrower k shares the same risk exposure-type as his partner. Using payoff function 6 gives

$$\begin{aligned}\Pi_L + \Pi_M =& 4\overline{R} - (r+c)(p_i + p_j + p_{i'} + p_{j'}) \\ &+ c\left(\sum_{k\in L} p_k p_{-k} + \sum_{k\in M} p_k p_{-k}\right) + c\epsilon\left(\sum_{k\in L}\kappa_{k,-k} + \sum_{k\in M}\kappa_{k,-k}\right).\end{aligned} \tag{9}$$

There are two types of interaction terms in this payoff expression, involving riskiness and risk exposure-type (the first and second parentheticals on the second line, respectively). To test for homogeneity in risk exposure-type using the univariate techniques of this section, only the interactions involving risk exposure-type are used.[40] Following the above techniques and defining $\overline{\kappa}_{k,-k}^{\mathcal{S}^G}$ as the average correlatedness dummy of borrower k in group G with other

[39]To illustrate, sampled grouping $L = (2,5,6,8)$ and $M = (1,4,7,9)$ has sum of group payoffs of 202 – i.e. $2*19/3 + 5*16/3 + ... + 9*4$ – compared to $234\frac{2}{3}$ for grouping $L' = (1,2,5,6)$ and $M' = (4,7,8,9)$.

[40]That is, we examine each separately in this univariate analysis, matching on riskiness (see equation 8) and on risk exposure-type. Testing both together based on the entire payoff function is reserved for Section 6.

sampled group G members, our measure for the part of payoffs due to correlated risk is

$$\sum_{k \in \mathcal{S}^L} \overline{\kappa}^{\mathcal{S}^L}_{k,-k} + \sum_{k \in \mathcal{S}^M} \overline{\kappa}^{\mathcal{S}^M}_{k,-k} \quad . \tag{10}$$

This measure is simply the sum, over all sampled village borrowers, of the fraction of other same-group, sampled borrowers exposed to the same risk.[41]

The remaining question is how to use the data to proxy for $\kappa_{i,j}$, the indicator for being exposed to the same risk. In the case of worst_year, $\kappa_{i,j}$ is simply proxied by $1\{worst\_year^G_i = worst\_year^G_j\}$; that is, two borrowers are considered exposed to the same risk iff they give the same answer in identifying the worst year. In the case of occupation, a vector with the fraction of earnings coming from each of four broad occupations, $\kappa_{i,j}$ is proxied by the negative rectilineal distance between the borrowers' occupational vectors.[42]

There is an interesting **similarity between the pattern and payoff approaches** in a particular case of matching on riskiness: the relative ranking of any two groupings using the homogeneity metric borrower-partner covariance is identical to those using the payoff function 8. That is, one grouping is more homogeneous than another on the borrower-partner covariance scale iff it produces higher payoffs using the model's payoff function. Thus, in the case of riskiness, the pattern approach (with a particular metric) and the payoff approach exactly coincide.[43] Of course similarity between the approaches is expected for reasons discussed, but exact coincidence is more surprising, and encouraging. Given this equivalence, the borrower-partner covariance results and the results of the payoff approach are reported once only, with labels used interchangeably.

[41] For example, in grouping $L = (A, A, B, B)$ and $M = (A, A, B, B)$, since 1/3 of each of the 8 borrower's fellow group members is exposed to the same shock, the correlation-related payoffs sum to $8 \cdot \frac{1}{3} = 2\frac{2}{3}$. In grouping $L' = (A, A, A, B)$ and $M' = (A, B, B, B)$, the correlation-related payoffs sum to $6 \cdot \frac{2}{3} = 4$.

[42] See Section 4.2. This could be scaled to a $[0, 1]$ range by using $1 - d/2$ rather than $-d$, where $d \in [0, 2]$ is distance; but this scaling does not affect the ordering, and thus produces identical results.

[43] The equivalence becomes clear through inspection of payoff function 8, which is proportional to the village expected value of $p_k \cdot \overline{p}^{\mathcal{S}^G}_{-k}$; this expectation is a key component of the borrower-partner covariance, and critically, the only part that varies across groupings of the same borrowers.

5.1.2 Testing for Random Matching

The previous section used specific functional forms to quantify payoff functions and homogeneity of matching. The remaining components of the statistical test are nonparametric.

The null hypothesis is random matching, specifically, that all groupings of observed borrowers within a village, that preserve observed group sizes, are equally likely. Focusing only on matches that preserve observed group sizes allows us to bypass the issue of optimal group size, as does the theory. Focusing only on within-village matches is based on the assumption that the relevant matching market is the village – a reasonable assumption since villages are relatively small and geographically concentrated.

We first translate each homogeneity and payoff metric of each village to a percentile scale, by using the metric to compare the village's observed grouping with the entire set of possible groupings in the village. Specifically, consider observed groups L and M, of respective sizes l and m, in village v. For all possible combinations of the $l + m$ borrowers into two groups of respective sizes l and m, we calculate the same homogeneity metric or payoff function. The observed village grouping can then be assigned a percentile reflecting how high the observed grouping scores, in homogeneity or payoff terms, compared to all possible matching outcomes. We call this a "homogeneity percentile", in the pattern approach, and a "complementarity percentile", in the payoff approach (with slight abuse of terminology in the risk exposure-type case).

Of course, given finite populations and possible ties, the result will always be a percentile *range*, rather than a point. To illustrate, consider the village with two groups of size four, with success probabilities $P = (1, 2, 4, 5, 6, 7, 8, 9)$. There are $\binom{8}{4}/2 = 35$ groupings of these eight borrowers into two groups of size four. Compared to the grouping $L = (2, 5, 6, 8)$ and $M = (1, 4, 7, 9)$, 32 groupings register higher between-group variance while 3 (including the grouping itself) register exactly the same (zero) between-group variance. Thus this grouping is somewhere between the 0th and 8.6th percentiles in terms of group homogeneity, based on variance decomposition; its homogeneity percentile range is $[0, 8.6]$. Compared to the grouping $L' = (1, 2, 5, 6)$ and $M' = (4, 7, 8, 9)$, 31 groupings have lower, 2 have the same, and 2 have higher between-group variance. This grouping's homogeneity percentile range is thus $[88.6, 94.3]$. Applying this permutation scaling to the borrower-partner covariance metric, or equivalently the payoff function metric, happens to give the exact same homogeneity (and complementarity) percentile ranges: $[0, 8.6]$ to the first grouping, and $[88.6, 94.3]$ to the second grouping.

The same permutation procedure is applied to risk exposure-type metrics. There are 17 combinations with a larger chi-squared test statistic and 18 combinations tied with grouping $L = (A, A, B, B)$ and $M = (A, A, B, B)$. This grouping's homogeneity percentile range is thus $[0, 51.4]$. Calculation of exposure-type payoffs (equation 10) establishes an identical complementarity percentile range, $[0, 51.4]$. Compared to grouping $L' = (A, A, A, B)$ and $M' = (A, B, B, B)$, 18 combinations have less, 1 combination has greater, and 16 combinations have the same chi-squared test statistic and exposure-type payoffs. Hence, this grouping's homogeneity and complementarity percentile ranges are both $[51.4, 97.1]$.

In sum, this permutation procedure scales each raw homogeneity or payoff measure for each village into a percentile range in $[0, 1]$, with higher ranges representing greater homogeneity or higher complementarity-based payoffs, and lower ranges less homogeneity or lower complementarity-based payoffs. This permutation scaling is applied for each variable, village, and metric. The role of this procedure is to put into context the observed degree of matching homogeneity or complementarity-payoff maximization, relative to all possibilities given the observed distribution of borrower types.

The last step is to combine all villages into a single test (per variable and metric) for random matching. For concreteness, consider the pattern approach. We claim that if matching is random, a village's homogeneity percentile is distributed uniformly on $[0, 1]$. For intuition, consider the case of a large number of borrowers in a village, no two groupings of which result in a tie using the given homogeneity metric. If each of the N, say, possible groupings is equally likely, as it is under random matching, then each $1/N$th homogeneity percentile is equally likely to be realized by a given village. That is, a village's homogeneity percentile is drawn from the uniform distribution – approximately, with the difference getting arbitrarily small as N increases.

With smaller numbers of borrowers and ties, a village is assigned a homogeneity percentile *range*, i.e. a set rather than a point. The result still holds as long as the village's homogeneity percentile (point estimate) is drawn from the village's homogeneity percentile range via the uniform distribution.

In short, let a village's homogeneity percentile *range* be calculated by the permutation method described in the previous section; and let its homogeneity *percentile* (point estimate) be drawn at random from the uniform distribution over its homogeneity percentile range. Then the exact distribution of a village's homogeneity percentile under random matching, regardless of the homogeneity metric, is the uniform distribution on $[0, 1]$. The same logic applies to the complementarity percentiles of the payoff approach.

Proposition 2. *Under random matching, a) for any homogeneity metric, a village's homogeneity percentile is drawn from the uniform distribution on* $[0, 1]$*; and b) a village's complementarity percentile is drawn from the uniform distribution on* $[0, 1]$.

The test then constructs a sample CDF from the *observed* village homogeneity (or complementarity) percentiles, and compares it to the uniform distribution using the Kolmogorov-Smirnov (KS) test. If the sample CDF stochastically dominates the uniform, this means villages' homogeneity (or complementarity) percentiles tend to be higher than random matching would give rise to and provides statistical evidence for homogeneous (or complementarity-based) matching. On the other hand, if the sample CDF is stochastically dominated by the uniform, this means villages' homogeneity (or complementarity) percentiles tend to be lower than what random matching would produce, suggesting non-homogeneous (or substitutability-based) matching. We thus report p-values for these KS one-sided tests of stochastic dominance.

Note that any such p-value involves a set of random choices: the random draws that

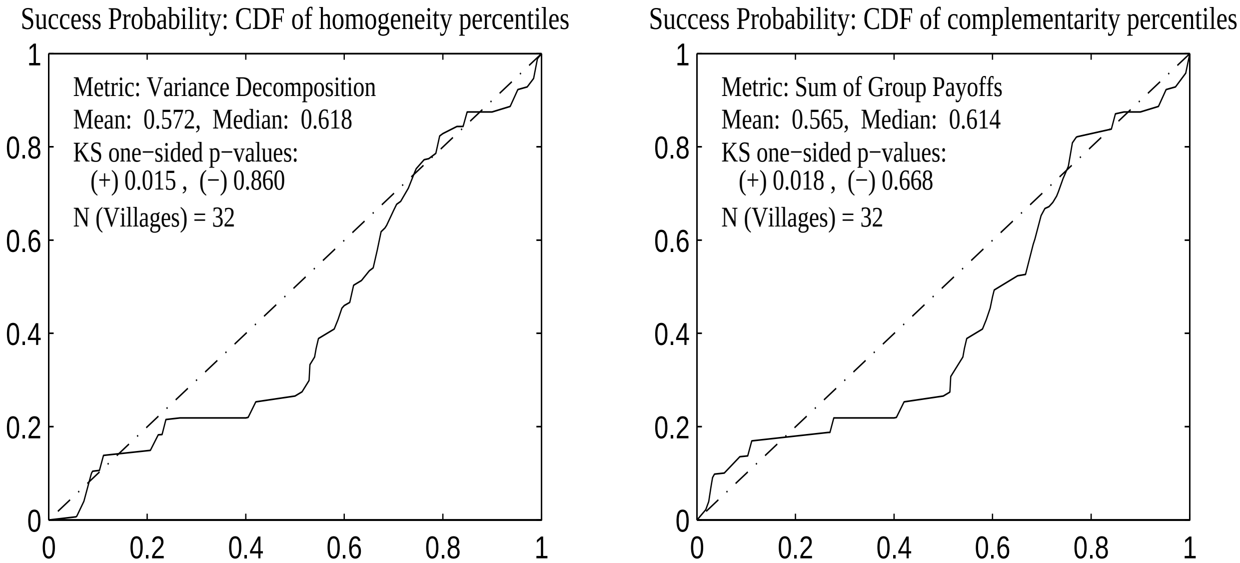

Figure 1: **Probability of success**. Dashed Lines: Uniform CDF. Solid Lines: Sample CDFs of villages' homogeneity percentiles based on variance decomposition (left panel) and of villages' complementarity percentiles based on the group payoffs (right panel).

select villages' homogeneity percentiles from their homogeneity percentile ranges. We report the average p-value across one million tests with independent sets of draws. Since each test by itself produces a valid p-value, i.e. probability under the null of observing a test statistic at least so extreme as the one observed, the average p-value across many draws approximates the expected probability under the null of observing a test statistic at least so extreme across all possible sets of draws, and thus is appropriate for inference.

5.2 Univariate Results

Sorting by riskiness. The first set of results measures riskiness with the **success probability**, or p. Figure 1, left panel, graphs results from the pattern approach, specifically the sample CDF of village homogeneity percentiles based on variance decomposition.[44] According to this metric, the mean (median) village grouping is more homogeneously matched than

[44]For this and all graphs in this Section, the reported p-values are averages over 1 million KS p-values, each based on an independent set of random draws from villages' percentile ranges. Sample CDFs are calculated incorporating the percentile range of each village directly, and means and medians are computed similarly.

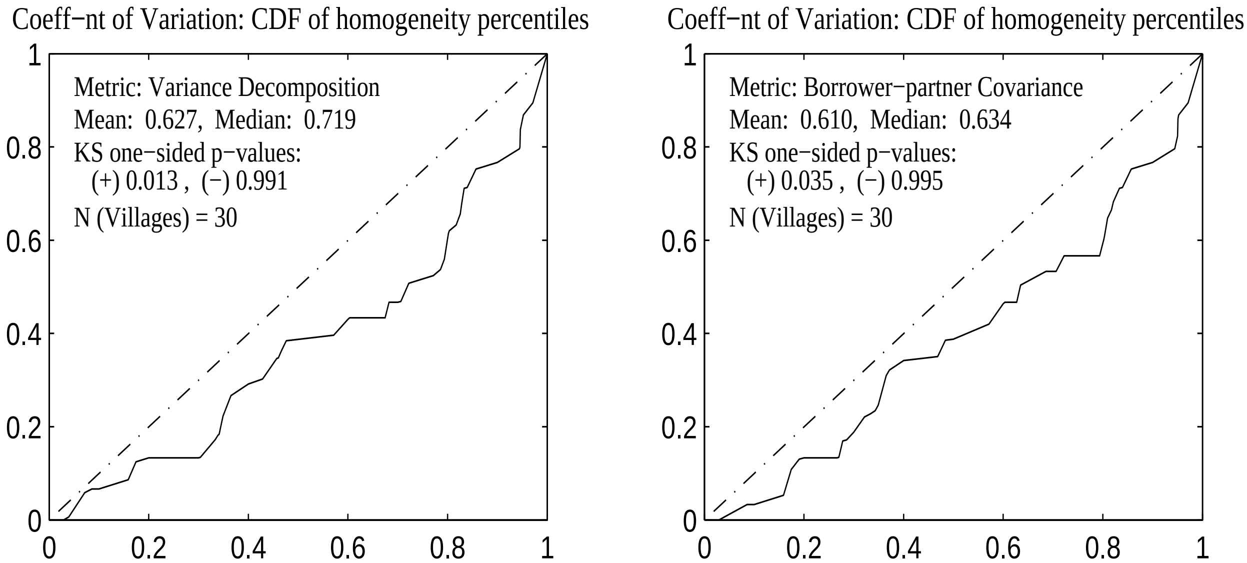

Figure 2: **Coefficient of Variation** for income (standard deviation/mean). Dashed Lines: Uniform CDF. Solid Lines: Sample CDFs of villages' homogeneity percentiles based on variance decomposition (left panel) and based on borrower-partner covariance (right panel).

57% (62%) of all groupings of the same borrowers that preserve observed group sizes. The random-matching benchmark, the uniform, is graphed as a dashed line. The KS test rejects random matching at the 5% level, against the alternative of homogeneous matching, that is, that the true distribution of village homogeneity percentiles first-order stochastically dominates the uniform. These results point to matching by riskiness that, while not rank-ordered, is statistically distinguishable from random matching in the direction of homogeneity.

Figure 1, right panel, graphs results from the payoff approach, specifically the sample CDF of village complementarity percentiles based on the payoff function. The results are quite similar.[45] The mean (median) village grouping produces higher complementarity-based payoffs than 56% (61%) of possible groupings, and random matching is rejected at the 5% level against the alternative of complementarity-based matching.

A second measure of riskiness is the **coefficient of variation** of projected income. Figure 2 graphs results from the pattern approach, the left panel using the variance decomposi-

[45] Recall that these results are identical to those using the borrower-partner covariance homogeneity metric.

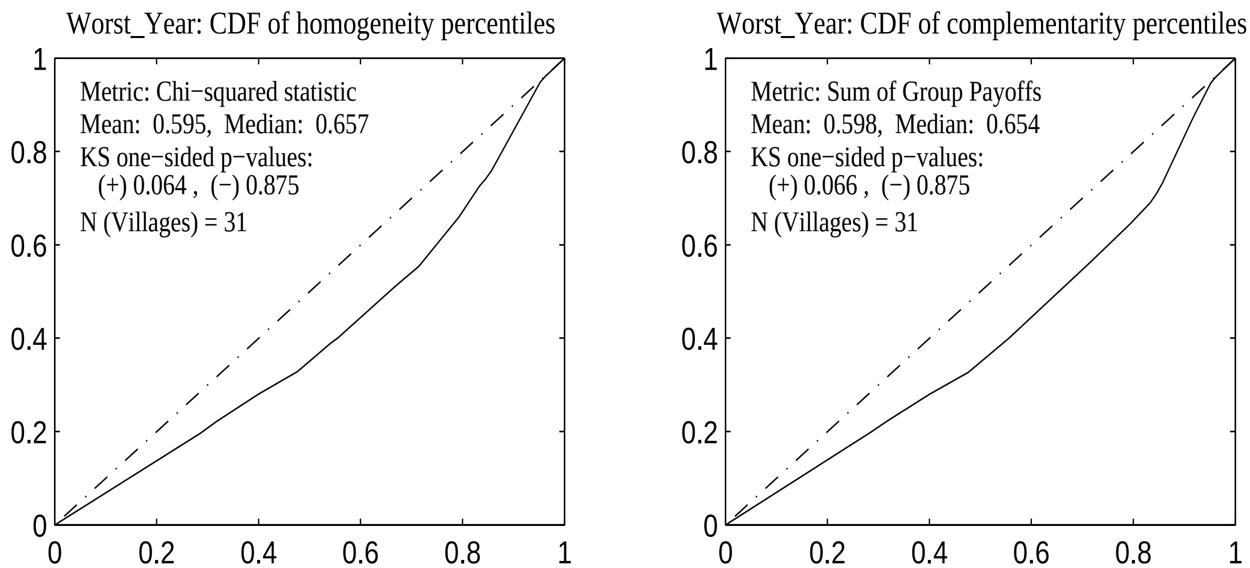

Figure 3: **Worst_Year** for income. Dashed Lines: Uniform CDF. Solid Lines: Sample CDFs of villages' homogeneity percentiles based on the chi-squared statistic (left panel) and of villages' complementarity percentiles based on the group payoff function (right panel).

tion metric, and the right panel using the borrower-partner covariance metric. The variance decomposition results give strong evidence of homogeneous matching: the mean (median) village grouping is more homogeneously matched than 63% (72%) of all possible borrower groupings, and random matching is rejected at the 5% level against the alternative of homogeneous matching. Similar results obtain using the borrower-partner covariance: the mean (median) is 61% (63%), and the KS test rejects random matching at the 5% level.

Overall, the data give solid evidence for a non-negligible degree of homogeneous (and complementarity-based) matching by riskiness; random matching is consistently rejected at the 5% level. From the standpoint of the theory of section 3, this suggests that safe borrowers are indeed receiving lower implicit borrowing rates because they tend to have safer partners.

Sorting by risk exposure-type. We next examine diversification within groups, first using the coincidence of **worst_year** within groups. Figure 3, left panel, reports results from the pattern approach using the chi-squared homogeneity metric. The average (median) village grouping has greater within-group homogeneity of bad income years than 60% (66%) of

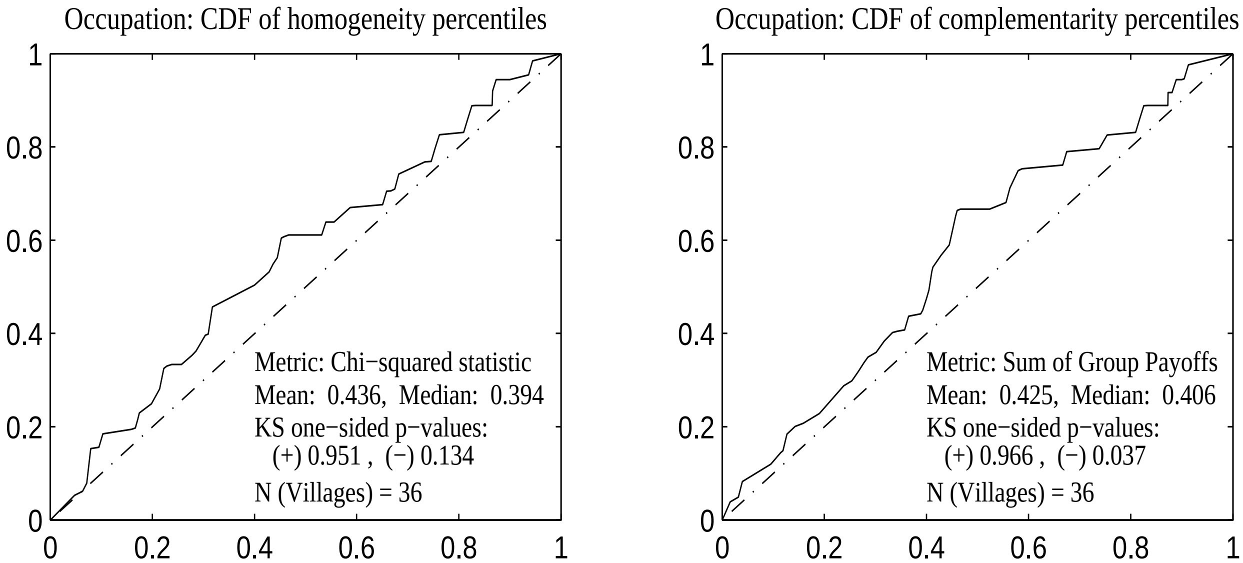

Figure 4: Agricultural **Occupation**. Dashed Lines: Uniform CDF. Solid Lines: Sample CDFs of villages' homogeneity percentiles based on the chi-squared statistic (left panel) and of villages' complementarity percentiles based on the group payoff function (right panel).

possible groupings, and random matching is rejected at the 10% level against the alternative of homogeneous matching (anti-diversification). The payoff method results, right panel, are nearly identical: the average (median) village grouping produces higher diversification-averse payoffs than 60% (65%) of possible groupings, and random matching is rejected at the 10% level.[46]

Occupational diversification results, from the pattern and payoff approaches, are graphed in Figure 4. Results are nearly opposite those from worst_year. Using the chi-squared metric, the average (median) village grouping has greater within-group occupational *heterogeneity* than 56% (61%) of possible groupings.[47] Using the payoff metric, the average (median) village grouping produces higher diversification-*loving* payoffs than 57% (59%) of possible

[46]One might wonder whether correlated risk within groups is not due to matching, but to the joint liability contract itself, which can make one borrower's bad year a bad year for others who are liable. This is unlikely because the survey question used for worst_year is designed to refer to income prior to transfers. Indeed, when asked for the reason for the bad year, about 85% of responses are agricultural shock-related – prices, weather, or pests. The survey follows up by asking how the household responded to the bad income year, and here is where a number of the responses have to do with transfers.

[47]Equivalently, it has higher within-group occupational homogeneity than only 44% (39%) of possible groupings, as the Figure reports.

groupings. Random matching is rejected at the 5% level in the payoff case, and at the 15% level in the pattern case, against the alternative of heterogeneous, diversification-loving matching.

Interestingly, groups are somewhat anti-diversified along income lines (worst_year), but somewhat diversified along occupational lines. One interpretation is that the lender encourages diversification within groups along observable dimensions, including by occupation, but that the borrowers are able to achieve some anti-diversification by exploiting other, unobserved characteristics. This would suggest that anti-diversification is occurring, and partially undoing the risk-pricing improvements, but not to the degree it would be if groups were less occupationally diverse.

5.3 Discussion of Univariate Results

The univariate tests establish that groups are moderately homogeneous in riskiness, as well as in risk exposure-type when measured by income shocks. This evidence is in line with the theory, but not proof of riskiness or exposure-type causing matching behavior. For example, it may be that friends or relatives group together, and that friends or relatives are similar along risk dimensions. Or, perhaps monitoring is easier within a group of similarly-occupied individuals, who by nature of their occupation face similar amounts and types of risk (though the observed occupational diversity casts doubt on this particular story).

However, if the goal is to assess whether the Ghatak theory is an empirically plausible explanation of group lending's popularity and ability to revive credit markets, these simple univariate results are in some ways preferable to alternatives. The reason is that the safe-borrower discount embedded in lending to homogeneous groups exists *regardless of how* groups end up homogeneous by riskiness. Whether borrowers consciously considered the risk of their partners in forming groups, or simply formed groups with friends or relatives who happened to have similar risk characteristics, the tests show that safe borrowers end up with safer partners. Given this homogeneous matching by riskiness, the joint liability stipulation is less onerous for safe borrowers, and they get an implicit discount in their borrowing rate. This is exactly the discount that in theory allows group lending to draw more borrowers into the market. The point is that, in this framework, matching that is homogeneous by riskiness – by whatever mechanism – is all that is needed for group lending to offer an improvement in contracting.

Thus, testing directly the degree of risk homogeneity is arguably the most informative approach to testing this main point of the Ghatak theory. Conversely, rejecting the Ghatak theory based on causally identifying, e.g., kinship and not riskiness as the key matching determinant could be misguided, if the evidence pointed to risk-homogeneous groups. Such a result would cast doubt on the nature of the matching process in the Ghatak theory, but could still be fully compatible with the theory's basic explanation for group lending's success, if riskiness generally correlates among kin.

A similar argument can be made about the extended model that incorporates correlated risk. If there is unconditional evidence for anti-diversification of risk, then that is enough to raise the concern that some of the contractually stipulated joint liability is being undone – whether or not the anti-diversification is a conscious choice on the part of borrowers.

Thus, we believe the results of this section are directly informative about the ability and limitations of the theory to explain the rise of group lending and microcredit.

One shortcoming of these results is that they cannot differentiate between homogeneous matching and group conformity, in which groups gravitate toward similar risk choices *because* they are grouped together. Ideal to distinguish these two stories would be risk data that pre-dates group formation, which we unfortunately lack and must leave for future work.[48]

6 Multivariate Methodology and Results

The univariate results are consistent with both dimensions of risk – riskiness and type of risk exposure – being important for matching. Of course, a matching pattern along one

[48]Barr et al. (2015) analyze matching into community-based organizations (CBOs) using pre-match data.

dimension could be due to matching occurring on another dimension. While the univariate results are directly informative, as argued in the previous Section, it is helpful to understand whether both dimensions are salient in determining matching. Here we use a multivariate approach that allows both dimensions of risk simultaneously to affect payoffs and matching, and is able to identify complementarity in both dimensions and tradeoffs between the two.

Specifically, we use the matching maximum score estimator of Fox (forthcoming) to estimate parameters of the model's group payoff function that determine whether it displays complementarity or substitutability, along both dimensions.[49] The estimator works by choosing parameters that most frequently give observed agent groupings higher payoffs than feasible, unobserved agent groupings. It has been shown consistent in an environment with many matching markets (as in our setting), assuming that groupings that give higher observable surplus are more likely to be observed.

Consider observed groups L and M in village v. Let $\tilde{L}$ and $\tilde{M}$ denote an alternative arrangement of the borrowers from L and M into two groups of the original sizes. As in section 5, we assume that borrowers can match with any others in their village; thus $\tilde{L}$ and $\tilde{M}$ represent a feasible, unobserved grouping. If $\Pi_G(\phi)$ gives the sum of payoffs of any group G as a function of parameters ϕ, theory predicts

$$\Pi_L(\phi) + \Pi_M(\phi) \geq \Pi_{\tilde{L}}(\phi) + \Pi_{\tilde{M}}(\phi). \tag{11}$$

The matching maximum score estimator chooses parameters ϕ that maximize the score, i.e. the number of inequalities of the form 11 that are true, where each inequality corresponds to a different unobserved grouping $\tilde{L}, \tilde{M}$.

Our set of unobserved groupings, and thus inequalities, comes from all k-for-k borrower swaps across two groups in the same village.[50] For example, if we have data on five borrowers

[49] A reduced-form estimation that included more controls than we use could also be interesting. However, this is not as attractive in part because our dataset lacks data on social networks and many SES indicators.

[50] If the larger group in a village has sample size m and the smaller group has sample size n, k is capped at $\min\{n, m-1\}$.

in each of two groups in the same village, there are $5 \times 5 = 25$ one-for-one swaps, $10 \times 10 = 100$ two-for-two swaps, and so on.

Consider the model's expression for group payoffs $\Pi_L + \Pi_M$ from section 3.3, reproduced from equation 9 here:

$$\begin{aligned}\Pi_L + \Pi_M =& 4\overline{R} - (r+c)(p_i + p_j + p_{i'} + p_{j'}) \\ &+ c\left(\sum_{k \in L} p_k p_{-k} + \sum_{k \in M} p_k p_{-k}\right) + c\epsilon\left(\sum_{k \in L} \kappa_{k,-k} + \sum_{k \in M} \kappa_{k,-k}\right).\end{aligned}$$

Note that all terms in the group payoff function that do not involve interactions between borrower characteristics drop out of inequality 11, since they appear identically on both sides; hence, we can ignore the non-interaction terms, i.e. all but the second line.

We proceed as in section 5.1.1. Since groups contain more than 2 members and since our data contain a subset of each group (up to 5 members), we use a sample analog expression for the payoff function. Again, let G be defined as a set of grouped borrowers, $\mathcal{S}^G$ as the sampled subset of group G, k as a sampled group-G borrower, $\overline{p}_{-k}^{\mathcal{S}^G}$ as the average success probability in the sampled subset of group G excluding borrower k, and $\overline{\kappa}_{k,-k}^{\mathcal{S}^G}$ as the average correlatedness dummy of borrower k with other sampled group-G borrowers. Then, the sample analog to the (relevant part of the) payoff function is[51]

$$\Pi_L + \Pi_M = \beta_1\left(\sum_{k \in \mathcal{S}^L} p_k \overline{p}_{-k}^{\mathcal{S}^L} + \sum_{k \in \mathcal{S}^M} p_k \overline{p}_{-k}^{\mathcal{S}^M}\right) + \beta_2\left(\sum_{k \in \mathcal{S}^L} \overline{\kappa}_{k,-k}^{\mathcal{S}^L} + \sum_{k \in \mathcal{S}^M} \overline{\kappa}_{k,-k}^{\mathcal{S}^M}\right), \tag{12}$$

where $\beta_1 = c$ and $\beta_2 = c\epsilon$. Given measures of borrower riskiness (p_i^G's) and correlatedness ($\kappa_{i,j}$'s), the β's can be estimated, but only up to scale, since multiplication by any positive scalar would preserve the inequality. Note that ϵ would be identified as β_2/β_1. However, this would require measures that capture the existence of correlation ($\kappa_{i,j}$) as distinct from the intensiveness (ϵ) of correlation; our measures of correlatedness are proxies and cannot

[51] Use of samples of groups rather than entire groups represents a departure from Fox's analysis; we conjecture that his arguments extend straightforwardly to this case as well.

be used to push the estimation this far.

Hence, we focus on identifying whether the payoff function exhibits riskiness-type complementarity, and whether it is diversification-averse. That is, we test whether both β's are positive. A positive $\beta_1(= c)$ guarantees complementarity of types in the payoff function, which drives homogeneous matching; a negative β_1 drives matching based on substitutability of types. A positive β_2 guarantees diversification-averse payoffs, i.e. payoffs that rise with greater similarity in risk-exposure within groups, as predicted by the theory; a negative β_2 guarantees diversification-loving payoffs.

Probabilities of success p_k are measured as discussed in Section 4.2; correlatedness is proxied using similarity in worst_year or occupation, as described in Sections 4.2 and 5.1.1. If there are V villages indexed by v, and each village v has two (sampled) groups, L_v and M_v, the estimator comes from

$$max_{\beta_1 \in \{-1,1\}, \beta_2} \sum_{v=1}^{V} \sum_{\tilde{L}_v, \tilde{M}_v} 1\{\Pi_{L_v} + \Pi_{M_v} \geq \Pi_{\tilde{L}_v} + \Pi_{\tilde{M}_v}\},$$

where the alternate groupings $\tilde{L}_v$ and $\tilde{M}_v$ come from all k-for-k borrower swaps, as discussed above, and β_1 is normalized to $+1$ or -1 in estimation given identification is only up to scale.

We also estimate based on a slightly different objective function, where the score is the sum of all villages' *fractions* of correct inequalities rather than *numbers* of correct inequalities. This weights each village equally in its contribution to the estimation and provides a more similar basis of comparison with the univariate KS results, where each village counts as a single draw from a distribution.

Maximization is carried out using the genetic algorithm routine in Matlab. Results from four estimations that alternate between the two objective functions and the two proxies for correlated risk are reported in Table 2. The point estimates are based on the 32 villages with sufficient data, and the corresponding 3767 total inequalities. Inference is carried out

Table 2 — Matching Maximum Score Estimation

Variable		Number	Share	Number	Share
Success Probability	Est.	+1	+1	+1	+1
	p-val.		Super-consistent		
Worst_Year	Est.	0.404	0.325^{***}		
	p-val.	0.350	0.045		
Occupation	Est.			-0.378^{*}	-0.302^{**}
	p-val.			0.110	0.065
Number of Inequalities		3767		3767	
Number of Villages			32		32
Maximized Objective Fn.		2346	19.72	2517	19.41
Percent Correct		62.3%	61.6%	66.8%	60.7%

Note: Each column corresponds to a different estimation; differences arise from the objective function used (noted atop each column) and the proxies for correlated risk. P-values are from one-sided tests for a negative (positive) true parameter if the point estimate is positive (negative). They are constructed using subsampling methods on 200 subsamples, each containing 24 distinct villages. Significance at the 5%, 10%, and 15% level denoted by ***, **, and *, respectively.

by subsampling, as suggested by Fox (forthcoming).[52]

We find that the estimated coefficient on probabilities of success is consistently positive.[53] Thus, even when controlling for correlated risk measures, including occupational similarity, riskiness has explanatory power for group formation consistent with complementarity. This supports the model, since complementarity is the basis for homogeneous matching and hence group lending's improved risk-pricing.

The correlated risk results are also similar to the univariate results. The estimates for worst_year are consistently positive, and significant in one of two cases, suggesting a diversification-averse payoff function. The estimates for occupation are consistently negative, and mildly significant in both cases, suggesting a diversification-loving payoff function. This is the same pattern observed in the univariate results, and may be explained by

[52] For each estimation, we create 200 subsamples containing 24 villages' data, by randomly sampling without replacement from the 32 villages. Estimation is carried out for each subsample. Operating under the assumption of $\sqrt[3]{n}$-convergence, appropriate for multiple-market estimation (Fox, forthcoming), one can apply the distribution of $\left(\frac{24}{32}\right)^{1/3}(\hat{\beta}_{24,i} - \hat{\beta}_{32})$ to $(\hat{\beta}_{32} - \beta_0)$ to construct confidence intervals, where $i \in \{1, ..., 200\}$ corresponds to the subsamples, $\hat{\beta}_{24,i}$ are the subsample estimates, $\hat{\beta}_{32}$ is the full-sample estimate, and β_0 is the true parameter. See Politis et al. (1999, 2.2).

[53] These estimates are denoted super-consistent because they converge at a rate faster than the typical root-n (see also Fox, forthcoming). Practically speaking, we see more than 99% of subsamples produce estimates equal to +1 in three specifications, and 95% in the fourth.

lender-encouraged occupational diversification combined with anti-diversification on other income-relevant dimensions.

Overall, we interpret these results as supportive of the univariate results, and thus of both aspects of the theory, with the exception noted.

7 Conclusion

In the context of joint liability lending and unobserved risk, theory suggests that borrowers will match homogeneously by riskiness; this embeds an implicit discount for safe borrowers and can draw them into the market, increasing intermediation and improving efficiency. We develop tests of this hypothesis about matching behavior, and find supportive evidence from Thai microcredit groups: groups are more homogeneous in riskiness than random matching would predict. Thus, safe borrowers are effectively paying lower interest rates, since they bear liability for safer partners. This simple feature of group lending is one plausible mechanism by which credit markets were revived among poor households around the world.

However, theory also suggests that borrowers may match to anti-diversify risk, in order to minimize potential liability for fellow group members. While the first kind of matching works in favor of efficiency, the second works against it by limiting the lender's ability to use group lending effectively. The data here suggest that some anti-diversification is indeed occurring, judging by income though not by occupation.

From a policy standpoint these results show that voluntary matching by borrowers may also have its downside. Within-group correlated risk works against the lender's interests and, in equilibrium, the borrowers'. The results suggest that lenders may want to intervene to promote risk diversification within groups – for example, requiring occupational diversity – but only if this intervention does not substantially undermine homogeneous matching by riskiness. It may also be optimal to separate borrowers with high vs. low correlated risk into different borrowing pools (Debrah, 2017), e.g. by having separate contracts, or even separate institutions, for agricultural vs. non-agricultural clientele.

The paper leaves some open questions for future work to address. First, the risk and correlation measures used here could be improved upon. Future work with income histories and/or more detailed elicitations of future income distributions could perhaps push the analysis further, including in a more quantitative direction. Second, it would be ideal for matching tests to use measures of risk that pre-date group formation, to distinguish matching behavior from within-group conformity that occurs after group formation. Third, richer datasets that include data on social networks, physical distances, etc., could potentially be used to identify whether risk-homogeneity and anti-diversification are purposeful or are by-products of other matching considerations. They could also help quantify and pinpoint matching frictions in these environments. Finally, more research on exactly how microcredit has been able to open new markets is needed to unravel this mystery, and to shed light on what elements were and are critical to its success.

Appendix

Proof of Proposition 1. Consider an equilibrium assignment. There are six sets into which all equilibrium groups can be partitioned: AA, BB, NN, AB, AN, BN, where the set names denote the pair of risk exposure-types of all groups within the partition.

The cross-partial of group payoff functions with respect to p_i and p_j is still given by equation 3. Thus the baseline result of homogeneous matching holds in any set of groups within which correlatedness is fixed for all possible pairings of borrowers within the set – AA, BB, and NN.

It remains to show that the sets AB, AN, and BN have zero measure in equilibrium. Consider AB, for example. Riskiness complementarity implies rank-ordering within risk exposure-type. That is, if (i, j) and (i', j') are equilibrium groups and borrowers i, i' (j, j') are A-risk (B-risk), then one of the following pairs of statements must hold: $p_i \geq p_{i'}$ and $p_j \geq p_{j'}$, or $p_{i'} \geq p_i$ and $p_{j'} \geq p_j$. Otherwise, the grouping (i, j') and (i', j) would raise surplus by increasing payoffs from riskiness complementarity without altering the nature of the exposure-type matching.

Given this fact and if set AB has positive measure, then for any $\delta > 0$, there must exist two groups (i, j) and (i', j') with $|p_i - p_{i'}| < \delta$ and $|p_j - p_{j'}| < \delta$. Fix $\delta = \sqrt{\epsilon/4}$ and two such groups. We will show that with riskiness levels so close, the gains from anti-diversification (matching A with A, B with B) outweigh any losses from decreased similarity in riskiness.

Without loss of generality, let (i,j) be the safer group, i.e. $p_i \geq p_{i'}$ and $p_j \geq p_{j'}$. Using equation 6, the sum of both groups' payoffs can be written

$$4\overline{R} - (r+c)(p_i + p_j + p_{i'} + p_{j'}) + 2c(p_i p_j + p_{i'} p_{j'}) \ ,$$

since no borrowers are exposed to the same shocks. An (i,i') and (j,j') grouping would instead pay

$$4\overline{R} - (r+c)(p_i + p_j + p_{i'} + p_{j'}) + 2c(p_i p_{i'} + p_j p_{j'}) + 4c\epsilon \ ;$$

the last term capturing the gains from anti-diversification. Now if $p_{j'} \geq p_i$ or $p_{i'} \geq p_j$, then the new grouping is rank-ordered by riskiness, so $p_i p_{i'} + p_j p_{j'} \geq p_i p_j + p_{i'} p_{j'}$ and surplus has increased. If instead $p_{j'} < p_i$ and $p_{i'} < p_j$, then all four riskiness levels $(p_i, p_j, p_{i'}, p_{j'})$ are within 2δ of each other, which caps the difference between $p_i p_{i'} + p_j p_{j'}$ and $p_i p_{i'} + p_j p_{j'}$ at $4\delta^2 = \epsilon$.[54] In this case too, surplus has increased. Since this alternate grouping raises surplus, the matching must not be an equilibrium; we thus contradict the hypothesis that AB has positive measure. By a similar argument, AN and BN cannot have positive measure.

Proof of Proposition 2. Let there be N possible borrower groupings in a given market (here, village), and $K \leq N$ unique values that arise when the given metric (homogeneity metric or payoff function) is applied to the N groupings, with values $v_1 < v_2 < ... < v_K$. (Ties involve $K < N$.) Let n_i be the number of groupings that give rise to value v_i and N_i be the number of groupings that give rise to any value $v \leq v_i$, with $N_0 \equiv 0$; then $N_i = \sum_{k=1}^{i} n_k$ and $N_K = N$. A village whose grouping gives rise to value v_i has matching percentile range (i.e. homogeneity or complementarity percentile range) of $[\frac{N_{i-1}}{N}, \frac{N_i}{N}]$. The village's matching percentile is then drawn uniformly from its matching percentile range.

We show next that a village's matching percentile z, constructed in this way, is distributed uniformly under random matching, i.e. $F(z) = z$. Fix $z \in [0,1]$. There exists some $i \in \{1, 2, \ldots, K\}$ such that $z \in [\frac{N_{i-1}}{N}, \frac{N_i}{N}]$. Then the probability that a village's matching percentile is less than z, i.e. $F(z)$, is the probability that its grouping leads to any value strictly less than v_i plus the probability that its grouping leads to value v_i *and* its percentile picked from the uniform on $[\frac{N_{i-1}}{N}, \frac{N_i}{N}]$ is below z. Given that if matching is random, a village's

[54]For more detailed derivation, see Ahlin (2009).

grouping will result in value v_i with probability $\pi_i \equiv n_i/N$, this equals:

$$F(z) = \sum_{k=1}^{i-1} \pi_k + \pi_i \int_{\frac{N_{i-1}}{N}}^{z} \frac{1}{\frac{N_i - N_{i-1}}{N}} \, dz = \sum_{k=1}^{i-1} \frac{n_k}{N} + \frac{n_i}{N}\frac{N}{n_i}(z - \frac{N_{i-1}}{N}) = z \quad .$$

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