Markets and relationships in a learning economy

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ABSTRACT

It is generally agreed that within long-term relationships agents learn the characteristics of their market partners better than through spot transactions. In contrast, little is known on how relationship-based and transaction-based markets compare when agents learn about the aggregate economy from market exchanges. In this paper, we study the market structure that arises in an economy where agents learn the aggregate productivity from market exchanges. The model allows to relate the market structure to macroeconomic fundamentals such as the persistence and volatility of the aggregate productivity and the cross-sectional volatility of productivity.

1. Introduction

In credit, goods and labor markets exchanges occur in two different ways. In some circumstances, agents establish long-term relationships during which they repeatedly exchange with each other. In other circumstances, agents engage in spot transactions, that is, they frequently break matches with their current partners and form matches with new ones. It is generally agreed that one of the reasons agents establish long-term relationships is their desire to acquire information on their market partners. In fact, being repeatedly matched with the same partners allows to learn their characteristics better than through spot transactions.1 Yet, agents also learn about the aggregate economy from market exchanges and it is widely believed that this dimension of learning shapes their behavior. For example, when choosing whether to expand its loan portfolio and gauging the investment opportunities available in the economy, a bank can exploit the information obtained in the exchanges with its current depositors and borrowers. Analogously, when deciding whether to expand its production capacity and estimating the productivity of the workers active in the economy, a firm can use the information obtained in the exchanges with its current workers. Alternatively, think of an entrepreneur who is commercially penetrating a foreign market and is selecting the production capacity to install in this market. The entrepreneur will develop a conjecture about the opportunities available in the new market through her exchanges with local businesses and clients. The knowledge

1 However, it is well established that not necessarily this information on partners’ characteristics is welfare-enhancing. For example, problems of informational monopoly and hold-up can arise (see, e.g., Rajan, 1992).
about the state of the economy or of a sector accumulated through market exchanges can also exert a role in the process of creation of new businesses. When a manager or a worker of a company ponder whether to start their own business, they will likely form an expectation about the investment opportunities in the sector from their exchanges with clients of the company they work for. Indeed, all these mechanisms are deemed to be so important that a growing literature (e.g., Veldkamp and Nieuwerburgh, 2006; Veldkamp, 2005, and Lang and Nakamura, 1990) explains the dynamics of the aggregate economy, such as properties of the business cycle, in models where agents learn the state of the economy (e.g., the aggregate productivity) from market exchanges.

In spite of these considerations, the extant literature focuses almost exclusively on the benefits of long-term relationships in easing the process of learning about agents’ characteristics. In contrast, little is known on how relationship-based and transaction-based markets compare when agents learn about the aggregate economy from market exchanges. Important questions thus remain open: how does the market structure affect the dynamics of the aggregate economy when agents form expectations about changes in the state of the economy from their market experiences? And, in turn, how do the characteristics of the aggregate economy affect the endogenous development of the market structure? For example, can macroeconomic fundamentals help rationalize the cross-country and intertemporal variation in the structure of credit, goods or labor markets? In this paper, we take a step towards answering these questions. We build a decentralized environment where agents learn the aggregate productivity from their private histories in the market. The environment is deliberately parsimonious. In particular, we abstract from widely studied market frictions (moral hazard, adverse selection, externalities) and concentrate on the lack of perfect information on the aggregate productivity. The paper investigates the impact of the market structure on agents’ learning and solves for the market structure that arises in equilibrium as a function of the macroeconomic fundamentals, meant as the persistence and volatility of the aggregate productivity and the cross-sectional volatility of productivity.

A suitable setting for our investigation is an overlapping generation economy. In our economy, young agents learn from their market exchanges with old agents and later on, when they become old, they base their exchanges with the young agents of the following generation upon the beliefs they formed when young. Since they will use the beliefs in the exchanges with the following generation, young agents are not concerned about learning the characteristics of their market partners but only about learning the aggregate state (productivity) of the economy. Put differently, learning has an “aggregate dimension”. An overlapping generation economy is also suitable for its tractability: since agents have a finite life span, the set of beliefs they form does not become too cumbersome during their lives. In our economy, young agents operate investment projects to produce an intermediate good. The output of a project depends jointly on the aggregate productivity and on the productivity of an old agent who participates in the project — for example, because the old agent is a financier who monitors the project or a manager who helps select the investment strategy. Besides contributing an essential input to investment projects, old agents can transform the intermediate good delivered by projects into a final good. The amount of final good they obtain depends on the precision of their beliefs about the aggregate productivity, which in turn stem from their experiences when young. In fact, a young agent learns the aggregate productivity from her history of project outcomes. Yet, her learning is noisy because the productivity of old agents is unobservable and when a young agent observes the output of a project she cannot perfectly disentangle the aggregate productivity from the productivity of the old agent she is matched with.

We let young agents choose between two trading regimes. In one regime, a young agent remains matched with the same old agent over time; in the other regime, young agents continuously break matches and form new ones. For what said above, what drives agents’ choice of trading regime and, hence, the market structure is their desire to maximize the precision of the beliefs they form through market exchanges. The analysis yields rich implications regarding the impact of macroeconomic fundamentals on the market structure. In particular, we obtain that when the persistence of the aggregate productivity is low the economy features a relationship-based market. If instead the persistence of the aggregate productivity is high, the economy has a transaction-based market as long as the cross-sectional volatility of productivity is sufficiently small or the volatility of the aggregate productivity is sufficiently large. We extensively comment these findings throughout the paper but to grasp the kind of forces at work it is useful to elaborate on one of the results more in detail. Consider the effect of the persistence of the aggregate productivity. The benefit of long-term relationships in the learning process is that, since they remain matched with the same partners over time, agents can better control for variations in the productivity of their partners. The cost of long-term relationships is instead that agents confuse persistent changes in the aggregate productivity with the persistence in the productivity of their partners. For example, they tend to misattribute repeated high project outputs to having stayed matched with a high productivity old agent rather than to a persistently high aggregate productivity. The less persistent the aggregate productivity, the better a young agent in a long-term relationship can extract information on the productivity of her partner from project outcomes and, hence, the better she can control for variations in her partner’s productivity. Moreover, the less persistent the aggregate productivity, the less an agent in a relationship confuses the persistence in the aggregate productivity with the persistence in the productivity of her partner. Both these effects render long-term relationships appealing when the aggregate productivity has low persistence.

The plan of the paper is as follows. In the next section, we relate the paper to the literature. Section 3 lays out the setup of the model and Section 4 solves for the equilibrium. Section 5 considers robustness issues. Section 6 concludes. All proofs are relegated to Appendices A and B.
2. Prior literature

There is a growing literature that analyzes learning from market exchanges in macroeconomic environments (see, for example, Amador and Weill, 2009, and references therein). In particular, several studies have recently rationalized key properties of the aggregate economy, such as its response to shocks and its cyclical behavior, as the outcome of a process of learning from exchanges. Veldkamp (2005) and Veldkamp and Nieuwerburgh (2006), for instance, explain business cycle asymmetries — meant as the fact that booms are gradual while crashes are short and sharp — in a model of the credit market with learning. Lang and Nakamura (1990) develop a framework where shocks to the returns of risky projects are magnified and prolonged by a learning process. This strand of literature does not focus on the structure of markets. Our paper contributes to it by uncovering the interplay between the market structure and the aggregate economy via agents’ learning. In this literature, the paper also shares some features with the studies that compare the ability of different financial systems to aggregate information. Boot and Thakor (1997), for example, stress that in market-centered financial systems prices can transmit information better than in bank-centered ones. Our paper can contribute to these studies by helping understand how within a financial system where prices reveal little information different configurations of the financial system affect information aggregation (for instance, offering insights into the emergence of relationship-based or transaction-based banking systems).

The paper also relates to the literature on the costs and benefits of long-term relationships (see, e.g., Dewatripont and Maskin, 1995, and Williamson and Aiyagari, 2000). In this literature, an important stream of research examines the role of long-term relationships in information acquisition about agents’ characteristics. More in general, a consensus has formed that relationship-based markets ease agents’ acquisition of information on their partners because, being repeatedly matched with the same partners, agents have more opportunities to learn about their characteristics (see, for example, Rajan, 1992, and Dell’Ariccia and Marquez, 2004, for theoretical models and Boot, 2000, for a review). This strand of literature essentially leaves the aggregate economy in the shade. In fact, it mostly neglects the fact that agents learn about the aggregate economy besides the characteristics of their partners and that the market structure can shape this process. Investigating this issue is important because it is often argued that the variation of the market structure contributes to the heterogeneous performance of different economies (see, e.g., Allen and Gale, 2000, and references therein for the case of the credit market).

3. The model

In this section, we lay out the setup. We keep the setup general so that it can be applied to a variety of scenarios in credit, labor and product markets. At the end of this section, we discuss some applications.

Environment Consider an infinite horizon economy. Time is divided into discrete periods and each period has two sub-periods, “morning” and “afternoon”. The economy is populated by a sequence of overlapping generations of two-period lived agents, with a unit continuum of agents per generation. Agents consume a final good in their second period of life, when they are old, deriving utility \( u = c \) from consumption.

In every period, each young agent can carry out two investment projects to produce an intermediate good, one project in the morning (m) and one in the afternoon (a). A project entails no effort but requires an input (e.g., advice, monitoring, management skills) of an old agent. At the end of the morning (afternoon), the project yields an amount \( e^{\text{Yat}}_t \) (\( e^{\text{Yat}}_t \)) of intermediate good, where the project log productivity \( Y_{mt} \) \( (Y_{at}) \) equals the sum of the log aggregate productivity \( \pi_{mt} \) \( (\pi_{at}) \) and the log productivity \( \varepsilon_{mt} \) \( (\varepsilon_{at}) \) of the old agent. Throughout, we assume that \( \pi_{mt} = \pi_{a1} \) and \( \pi_{at} = \lambda \pi_{mt} + u_t \), where \( \lambda \in [0, 1] \) and \( u_t \sim N(0, \kappa^2) \). We also posit that the log productivity of an agent is constant over time and satisfies \( \varepsilon_{mt} = \varepsilon_{at} \sim N(0, \sigma^2) \).

Each young agent can also set up a technology that she will operate in the second period of her life. By operating this technology, an agent can produce final good from the intermediate good delivered by the projects she joins when old. The output \( Y_t \) of this technology depends on the precision of the old agent’s belief about the log aggregate productivity in the morning. Precisely, \( Y_t = Y(e^{\text{Ymt}}, e^{\text{Yat}}) + \gamma P^f_t \), where \( Y(.) \) is an increasing function of the intermediate good delivered by the projects, \( \gamma \) is a positive constant and \( P^f_t \) is the precision of the belief. For example, this specification can capture the idea that the cost an agent has to sustain to set up and operate the technology is larger the poorer is her understanding of the state of the economy.

Our economy features decentralized interactions and imperfect information. In every sub-period young and old agents are randomly and pairwise matched. In a match, the productivity of the old agent is her private information. Moreover, no agent, young or old, observes the aggregate productivity and all agents form beliefs about it from the histories of their
projects when young. We define the precision \( P_j \) of an agent’s belief as (minus) the expected squared difference between the log aggregate productivity \( \pi_{mt} \) and the average belief \( \pi_{mt}^e \) the agent forms after a history of project productivities when young (i.e., \( P_j = -E((\pi_{mt} - \pi_{mt}^e)^2) \)).

**Market structure** At the beginning of every morning, each young agent chooses between a long-term relationship \((j = R)\) and spot transactions \((j = T)\). In a long-term relationship, the young agent is matched with the same old agent in the morning and in the afternoon. With spot transactions, at the beginning of the afternoon the young agent breaks the match formed in the morning and the agents are re-matched with new partners. To guarantee that in the afternoon a young agent who chose spot transactions can always find a new partner, we assume that with a small probability \( \alpha > 0 \) between the morning and the afternoon a match exogenously breaks down. For tractability, we also assume that in a match the old agent has full bargaining power so that an old agent appropriates the whole output \( Y_t \).

**Summary** We summarize the timing of events by following an agent over her lifetime.

Period \( t \). In the morning of every period \( t \), each young agent chooses between a long-term relationship and spot transactions. Thereafter, she enters a match with one old agent, carries out an investment project and observes the amount of intermediate good delivered by the project. In the afternoon, depending on the occurrence of an exogenous breakdown of the match and on her choice of trading regime, the agent remains matched with the same old agent or enters a match with another old agent. The agent carries out a second project and observes the amount of intermediate good delivered by the project.

Period \( t + 1 \). In the morning and in the afternoon, the old agent enters matches with young agents and provides an input that is necessary for their investment projects. The old agent also operates a technology that transforms the intermediate good delivered by these projects into final good. The output of this technology depends on the precision of her belief about the aggregate productivity in the morning.

**Discussion** The setup we have proposed can represent various scenarios in different markets. It could describe a credit market where new firms obtain (trade) credit from older, established firms, learn from their exchanges with these firms, and then start extending credit to the following generation of firms. The model could also represent a credit market where young agents invest their savings and work in (carry out) projects set up by entrepreneurs and, when they become old, use their savings to set up their own firms. In some overlapping generation models of the credit market, young agents acquire information on (monitor) entrepreneurs. In our framework, young agents acquire information on the aggregate state of the economy. In a related vein, the model could describe a labor or a managerial market where workers or managers learn from their transactions with the clients of the companies they work for. The managers or workers will then start their companies exploiting the knowledge of the sector they have accumulated in these transactions. The model could also be the description of a product market where new or foreign companies learn the state of the sector they are penetrating commercially from their exchanges with local businesses. The entrants will develop a conjecture about the production opportunities available in the new market through this learning process. Clearly, these are just few examples and the reader can think of several other applications of our setup.

4. Learning, decisions, and equilibrium

In this section, we first derive the belief about the aggregate productivity an agent forms when young, conditional on her choice between a long-term relationship and spot transactions. Next, we define the equilibrium concept and characterize the macroeconomic conditions under which young agents choose relationships or spot transactions.

4.1. Learning

All the results of this section are proved in Appendix A. Fix some period \( t \in \mathbb{N} \) and consider the learning process of a generic young agent. We begin with the case in which the agent has chosen to engage in spot transactions when young. We obtain that, conditional on her private history \((Y_{mt}, Y_{at})\) of project log productivities when young, the belief about the log aggregate productivity in the morning of period \( t + 1 \) satisfies

\[
\pi_{mt+1}^T \sim N \left( \frac{\kappa^2 + \sigma^2 \gamma}{(\kappa^2 + \sigma^2)^2 - \lambda^2 \sigma^4} Y_{mt} + \frac{\kappa^2}{(\kappa^2 + \sigma^2)^2 - \lambda^2 \sigma^4} Y_{at}, \frac{\kappa^2 \sigma^2 (\kappa^2 + \sigma^2)}{(\kappa^2 + \sigma^2)^2 - \lambda^2 \sigma^4} \right).
\]

(1)

On average, an agent who has observed a positive (negative) realization of the project log productivity both in the morning and in the afternoon of her first period of life will expect that the log aggregate productivity is positive (negative) in the morning of her second period of life (look at the mean of the distribution in (1)). In contrast, if the agent has observed

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5 To ensure that the output \( Y_t \) is always positive one can appropriately specify \( Y(.) \) and \( y \).

6 Without loss of generality, to simplify notation, we think of \( \alpha \) as arbitrarily small and omit it from the computations.
a mixed history of one project with positive log productivity and one project with negative log productivity, whether she expects a positive or a negative log aggregate productivity will depend on the weights she assigns to these outcomes in forming her belief. The way an agent combines the outcomes of her projects and, hence, the belief induced by a history depend on the three macroeconomic fundamentals, the persistence $\lambda$ of the aggregate productivity, the cross-sectional volatility of productivity $\sigma^2$, and the volatility of the aggregate productivity $\kappa^2$. For example, inspection of the mean of the distribution in (1) reveals that the smaller $\kappa^2$ and the larger $\sigma^2$, the smaller the weights that the agent will attribute to the observed project log productivities $y_{mt}$ and $y_{at}$ when updating her prior belief. Intuitively, the agent will treat a large positive output realization with skepticism and attribute it to a high productivity of her partner if the probability of large realizations of the aggregate productivity is small ($\kappa^2$ is small) or the probability of a high productivity of her partner is large ($\kappa^2$ is large).

Consider now the case in which the agent has established a long-term relationship when young. We obtain that, conditional on her private history $(y_{mt}, y_{at})$ of project log productivities, the belief about the log aggregate productivity in the morning of period $t+1$ satisfies

$$
\pi_{mt+1}^R \sim N \left( \frac{\sigma^2 \lambda^2 - (1-\lambda^2) (\kappa^2 + \sigma^2)}{\kappa^2 + 2\sigma^2} y_{mt} + (\kappa^2 + \sigma^2) y_{at}, \frac{\kappa^2 \sigma^2 (\kappa^2 + \sigma^2)}{(\kappa^2 + 2\sigma^2)(\kappa^2 + \sigma^2(1-\lambda^2))} \right),
$$

(2)

Inspection of (1) and (2) reveals that, for a given history $(y_{mt}, y_{at})$, the belief after a relationship coincides with the belief after spot transactions when the cross-sectional volatility of productivity $\sigma^2$ equals zero.

Comparison of (1) and (2) yields key insights. The aggregate productivity expected by an agent who has experienced two positive project log productivities $y_{mt}$ and $y_{at}$ is higher after spot transactions than after a long-term relationship. For example, letting $\pi_{mt+1}^R$ and $\pi_{mt+1}^S$ denote the means of the distributions in (1) and (2), we prove that, for all $\sigma^2 > 0$ and $\kappa^2 > 0$, $\pi_{mt+1}^R > \pi_{mt+1}^S > 0$ when $y_{mt} = y_{at} > 0$ (and symmetrically $\pi_{mt+1}^R < \pi_{mt+1}^S < 0$ when $y_{mt} = y_{at} < 0$). Intuitively, after a long-term relationship, if the agent has experienced an “extreme” history with both $y_{mt}$ and $y_{at}$ greater than zero when young, she will be more cautious in attributing this history to a persistently high aggregate productivity because she knows that her reiterated positive log productivities could stem from having remained matched with a highly productive old agent when young. A similar reasoning applies if the agent has experienced an extreme history of two negative project log productivities when young. When a “mixed” history occurs the argument gets reversed. In fact, we prove that, for a given history $(y_{mt}, y_{at})$, the belief after a relationship coincides with the belief after spot transactions when the cross-sectional volatility of productivity $\sigma^2$ equals zero.

Having characterized the belief of the agent after her history when young, we now turn to the probability with which histories occur. Just like the beliefs induced by histories, the probabilities of histories depend on the three macroeconomic fundamentals. For instance, consider the effect of $\lambda$. It is straightforward that, when $\lambda$ is higher, given a positive realization of the project log productivity in the morning, the probability that the project log productivity is positive in the afternoon is larger. This hints at a critical property of the distribution of beliefs. Suppose that the aggregate productivity exhibits a high persistence and, for example, its log is positive both in the morning and in the afternoon. Extreme histories of two high (low) outputs will have high (low) probabilities, whereas mixed histories will have intermediate probabilities. Remember also that, after experiencing an extreme history of two high outputs when young, on average an agent’s belief about the aggregate productivity is higher after spot transactions than after a long-term relationship. Jointly these facts imply that, averaging across histories of project outputs, after long-term relationships old agents are too cautious when the output was repeatedly high in the previous period. Suppose next that the aggregate productivity exhibits a low persistence and its log turns out to be negative in the morning and positive in the afternoon. In this case, mixed histories of a low output followed by a high output will have high probabilities, whereas extreme histories will have intermediate probabilities. In the analysis above, we have shown that, after experiencing a history of a low output followed by a high output when young, on average an agent’s belief about the aggregate productivity is higher after a long-term relationship than after spot transactions. Thus, when the aggregate productivity has low persistence this effect moderates the bias that after a long-term relationship arises following extreme histories.

4.2. Equilibrium

We have now all the elements to solve for the equilibrium. However, before doing so two observations are in order. First, in every period $t \in \mathbb{N}$ all newly born agents start with the same prior belief about the aggregate productivity and expect to face the same decision problem. Hence, in what follows we can drop the subscript $t$ and restrict our analysis to the choices of the members of a generation born in a generic period $t$. Second, given also the assumption that a measure $\alpha > 0$ of
matches exogenously break down between the morning and the afternoon, the choice of a young agent between a long-term relationship and spot transactions does not depend on the choices of other agents. This implies that we can further restrict our analysis to the behavior of a generic agent in a given generation. Let then \( H = \{(y_m, y_a) \mid (y_m, y_a) \in \mathbb{R}^2\} \) denote the set of histories of project log productivity in the morning and in the afternoon that the agent can face when young and let \( j \in \{R, T\} \) be the choice of trading regime that the agent makes at the beginning of her first period of life. Finally, let \( \pi^j_m : H \rightarrow [0, 1] \) be the agent’s belief function about the log aggregate productivity in the morning of her second period of life.

**Definition 1.** An equilibrium is a pair \((j, \pi^j_m)\) such that (i) the choice of trading regime \( j \) maximizes the expected utility of a young agent, (ii) the belief \( \pi^j_m \) is computed by Bayes rule.

When choosing the trading regime the agent compares her expected utility from spot transactions with her expected utility from a long-term relationship. This is tantamount to saying that the agent compares the precision of her belief after spot transactions \( P^T = -E[(\pi_m - \pi^T_m)^2] \) with the precision of her belief after a long-term relationship \( P^R = -E[(\pi_m - \pi^R_m)^2] \) and chooses spot transactions if and only if \( \Delta P = P^T - P^R > 0 \).

Proposition 1 characterizes the equilibrium.

**Proposition 1.** There exists a unique equilibrium. In this equilibrium, all young agents choose spot transactions (a long-term relationship) if and only if

\[
\frac{2\lambda}{\kappa^2 + 2\sigma^2} > \frac{(1 - \lambda^2)(k^2 + \sigma^2)}{(k^2 + \sigma^2)^2 - \lambda^2\sigma^4}.
\]

To grasp the intuition behind Proposition 1, it is useful to decompose the precision \( P = -E[(\pi_m - \pi^j_m)^2] \) of an agent’s belief, expressed in absolute value, into two parts. The first component is simply the variance of the agent’s belief \( V_j \). The second component \( (B_j) \) can be interpreted as the bias of the agent’s belief and measures how much on average the agent’s beliefs is distorted away from the actual aggregate productivity.\(^7\) We obtain that if the agent has chosen a long-term relationship

\[
E[(\pi_m - \pi^R_m)^2] = \frac{2\lambda\kappa^2\sigma^2(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2(k^2 + \sigma^2(1 - \lambda^2))} + \frac{\kappa^2\sigma^2(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2(k^2 + \sigma^2(1 - \lambda^2))},
\]

while if she has chosen spot transactions

\[
E[(\pi_m - \pi^T_m)^2] = \frac{\kappa^2\sigma^2(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2 - \lambda^2\sigma^4}.
\]

We can then decompose the precision gap \( \Delta P \) between spot transactions and a long-term relationship into the sum of the gap between the biases of the beliefs and the gap between the variances of the beliefs. Consider first the difference between the biases of the beliefs. We obtain

\[
\Delta B = B_R - B_T = \frac{2\lambda\kappa^2\sigma^2(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2(k^2 + \sigma^2(1 - \lambda^2))}.
\]

The expression in (4) is positive, which reflects the fact that remaining matched with the same partner over time tends to systematically “contaminate” an agent’s belief about the aggregate productivity with the productivity of her partner. Precisely, as discussed in Section 4.1, an agent tends to misunderstand the persistence of the aggregate productivity for the persistence in the productivity of her partner. Next, consider the difference between the variances of the beliefs. We have

\[
\Delta V = V_R - V_T = \frac{\kappa^2\sigma^2(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2(k^2 + \sigma^2(1 - \lambda^2))} - \frac{\kappa^2\sigma^2(k^2 + \sigma^2)}{(k^2 + \sigma^2)^2 - \lambda^2\sigma^4}.
\]

The expression in (5) is negative, meaning that when an agent engages in spot transactions the variance of her belief is larger than when she engages in a long-term relationship. As discussed in Section 4.1, this occurs because remaining matched with the same partner over time allows an agent to dampen the idiosyncratic volatility of her matches.

\(^7\) We break ties in favor of long-term relationships.

\(^8\) This can be thought as a weighted sum of the biases of the beliefs formed in the different states, with weights given by the probabilities of the states.
In choosing her trading regime a young agent weighed the higher variance associated with spot transactions against the bias associated with a long-term relationship. This trade-off leads to the choice criterion in Proposition 1. Inspection of (3) makes it clear that the choice between spot transactions and long-term relationships, and hence the market structure, depends on the persistence of the aggregate productivity $\lambda$, on its volatility $\kappa^2$, and on the cross-sectional volatility of productivity $\sigma^2$. Proposition 2 relates the market structure to these three macroeconomic fundamentals. The proof is in Appendix B.

**Proposition 2.** Let $\kappa^2 > 0$ and $\sigma^2 > 0$. The market structure depends on the macroeconomic fundamentals as follows:

(i) Persistence of the aggregate productivity ($\lambda$). There exists a unique $\bar{\lambda} \in (0, 1)$ such that all young agents choose spot transactions if and only if $\lambda > \bar{\lambda}$.

(ii) Cross-sectional volatility of productivity ($\sigma^2$).
   
   (a) If $\sqrt{2} - 1 < \lambda < 1$, there exists a unique $\bar{\sigma} > 0$ such that all young agents choose spot transactions if and only if $\sigma^2 < \bar{\sigma}$;
   
   (b) If $\lambda \leq \sqrt{2} - 1$, all young agents choose long-term relationships.

(iii) Volatility of the aggregate productivity ($\kappa^2$).
   
   (a) If $\sqrt{2} - 1 < \lambda < 1$, there exists a unique $\bar{\kappa} > 0$ such that all young agents choose spot transactions if and only if $\kappa^2 > \bar{\kappa}$;
   
   (b) If $\lambda \leq \sqrt{2} - 1$, all young agents choose long-term relationships.

Figs. 1–3 display a graphical illustration of the results. Fig. 1 plots the effect of $\lambda$ on the precision gap $\Delta P$ between spot transactions and long-term relationships that obtains when $\kappa^2 = \sigma^2 = 1$. The effect of the persistence of the aggregate productivity is monotonic: a higher $\lambda$ unambiguously favors spot transactions. Fig. 2 plots the effect of $\sigma^2$ on the precision gap when $\kappa^2 = 1$ and $\lambda = 0.3$ (solid line) or $\lambda = 0.6$ (dashed line). The cross-sectional volatility of productivity has a monotonic effect on the precision gap when $\lambda$ is low (a higher $\sigma^2$ favors long-term relationships) while it has a hump-shaped impact when $\lambda$ is high. Finally, Fig. 3 displays the effect of $\kappa^2$ that obtains when $\sigma^2 = 1$ and $\lambda = 0.3$ (solid line) or $\lambda = 0.6$ (dashed line). A higher volatility of the aggregate productivity tends to favor long-term relationships when $\lambda$ is low, while it has a U-shaped effect when $\lambda$ is sufficiently high.
4.3. Discussion

This section provides further interpretation on the comparative statics results of Proposition 2, discussing how the macroeconomic fundamentals impact the relative bias $\Delta B$ of long-term relationships and the variance gap $\Delta V$ between long-term relationships and spot transactions. This section integrates the discussion in Section 4.1 and most of its arguments derive immediately from the proof of Proposition 2.\(^9\) Let us first consider the role of $\lambda$. We obtain that, for all $\sigma^2$ and $\kappa^2$, a higher $\lambda$ lowers (in absolute value) the variance gap $\Delta V$ between long-term relationships and spot transactions and raises the bias $\Delta B$ of long-term relationships relative to spot transactions. In turn, both these effects drive up the relative precision $\Delta P$ of spot transactions. To understand the impact of $\lambda$ on the variance gap $\Delta V$, consider that when the aggregate productivity is more persistent an agent in a long-term relationship learns less about the idiosyncratic productivity of her partner from the realizations of the project productivities. This implies that the benefit of a long-term relationship in helping the agent control for the idiosyncratic productivity of her partner is smaller. To understand the impact of $\lambda$ on the relative bias $\Delta B$, it is useful to remember the discussion in Section 4.1. In that section, we showed that after experiencing an extreme history of two high outputs when young, an agent’s belief that the aggregate productivity is high is weaker after a long-term relationship than after spot transactions. Put differently, after experiencing such a history an agent is too cautious if she engaged in a long-term relationship because she tends to attribute her repeated high project productivities to a high productivity of her partner. Consider now that the higher $\lambda$, the higher is the probability that the project productivity is persistently high or low (refer to the discussion in Section 4.1). Therefore, a higher $\lambda$ puts more weight on extreme histories. Since extreme histories induce more biased (too cautious) beliefs after long-term relationships, this increases the overall relative bias of long-term relationships relative to spot transactions.

Next, let us turn to discuss the effects of the volatility $\sigma^2$ of the idiosyncratic productivity and the volatility $\kappa^2$ of the aggregate productivity. We find that a larger value of $\sigma^2$ increases the relative bias $\Delta B$ of long-term relationships but reduces the variance gap $\Delta V$ (i.e., renders the belief formed after spot transactions relatively more volatile). In light of what discussed in Section 4.1, an interpretation for the former effect is that when the variance of the productivity of her partner is large an agent who has engaged in a long-term relationship tends to be very cautious after observing repeated high realizations of the project productivity and to attribute such high realizations to large realizations of the productivity of her partner. This exacerbates the agent’s caution in attributing such a history to a persistently high aggregate productivity, thus increasing the relative bias $\Delta B$ of a relationship relative to spot transactions. The reason $\sigma^2$ negatively affects the variance gap $\Delta V$ is also quite intuitive. As discussed in Section 4.1, the benefit of a long-term relationship is that it allows an agent to control for variations in the idiosyncratic productivity of her partner. If the productivity of her partner has high volatility this benefit will be large. This implies that a higher $\sigma^2$ reduces the variance gap $\Delta V$. Clearly, the effects of $\sigma^2$ on $\Delta B$ and on $\Delta V$ we have just discussed work in opposite directions in determining the precision gap $\Delta P$. As Fig. 2 shows, while for low values of $\lambda$ the effect on $\Delta V$ dominates (and a higher $\sigma^2$ always favors long-term relationships) for high values of $\lambda$ the two effects combined generate an hump-shaped impact of $\sigma^2$ on the precision gap. Finally, let us consider the impact of $\kappa^2$. As anticipated in Section 4.1, the effect of $\kappa^2$ generally goes in a direction opposite to that of $\sigma^2$. Indeed, we find that in general a larger variance $\kappa^2$ of the aggregate productivity implies a smaller relative bias $\Delta B$ of long-term relationships and a larger variance gap $\Delta V$. For example, the larger the volatility of the aggregate productivity relative to the cross-sectional volatility of productivity, the smaller the benefit that a long-term relationship conveys to an agent in helping her control for changes in the idiosyncratic productivity of her partner. Thus, for a given $\sigma^2$, a higher value of $\kappa^2$ increases $\Delta V$ (that is, it reduces the variance of the belief formed after spot transactions relative to the variance of the belief formed after a long-term relationship).

\(^9\) Details are available from the authors on request.
5. Robustness

In the model, we have made a number of simplifying assumptions that are worth discussion. First, we have not allowed old agents to communicate their productivity to young agents. Even allowing for this, several reasons could render such a communication not credible. For example, interacting with a low productivity old agent could entail a cost — possibly even an infinitesimal one — so that such an agent could have no incentive to truthfully reveal her type and thereby lose some surplus to the benefit of her mate. A second simplifying assumption we have introduced is that old agents care about the realization of the aggregate productivity in the morning. An alternative specification would allow the output of an old agent to depend both on the precision of her belief about the aggregate productivity in the morning and on the precision of her belief about the aggregate productivity in the afternoon. Such an extension is straightforward, although analytically cumbersome. Indeed, in such an extension one could also consider a scenario in which, when forming her belief about the aggregate productivity in the afternoon, an old agent can also use the outcome of her match with a young agent in the morning. This specification would further expand the information set of old agents and, hence, would complicate the set of posterior beliefs that one needs to keep track of. We believe, nevertheless, that our qualitative results about the choice of trading regime by young agents would be unchanged. The reason is that, for any belief an agent expects to form when young, her posterior after the morning project when old will be independent of the chosen trading regime.

A third assumption we have made is that young agents choose the trading regime at the beginning of the morning and, hence, they do not condition their choice on the outcome of morning projects. This assumption may appear restrictive if one is interested in the impact of short-run factors, such as the outcome of a single project, on the formation of a long-term relationship. However, in this paper we are interested in isolating the impact of structural macroeconomic fundamentals on the market structure. By postulating that the choice of trading regime occurs in the morning, we better isolate the impact of these structural factors.

A final variation of the model worth discussion is one in which a young agent cares about learning the productivity of her mate besides learning the aggregate productivity. We have developed an extension in which both in the morning and in the afternoon after completing her project a young agent chooses the value of an action (say, an effort level) with the optimal value of this action depending on the productivity of the old agent she is matched with. For instance, such a specification could reflect some form of complementarity between the action of the young agent and the type of the old one. In this extension, a young agent is interested in learning the productivity of her partner to appropriately select the value of her action in the morning and in the afternoon. For the sake of conciseness, we do not present the results of the extension here but we find that the comparative statics results with respect to the three macroeconomic fundamentals carry through to this extended environment.

6. Conclusion

In this paper, we have studied an economy where agents learn the aggregate productivity from market exchanges. In such an economy, the structure — relationship-based or transaction-based — of the market shapes the process of decentralized learning and, hence, agents’ production outcomes. We have characterized macroeconomic conditions under which a transaction-based market or a relationship-based one arises. The model predicts that an economy where the persistence of the aggregate productivity is low should feature a relationship-based market. If instead the persistence of the aggregate productivity is high, the economy should feature a transaction-based market as long as the cross-sectional volatility of productivity is sufficiently small or the volatility of the aggregate productivity is sufficiently large. We believe that, in spite of its abstractness, the model can help understand the interaction between the aggregate economy and the endogenous development of the market structure and, for instance, shed some light on the cross-country and intertemporal variation of the structure of credit, labor and product markets. At the same time, we are aware that a number of interesting issues remain open. For example, being focused on the impact of long-run macroeconomic fundamentals, the paper does not explore the impact that short-run macroeconomic conditions, such as booms or recessions, can have on agents’ learning process and, hence, on the market structure. We leave this and other issues for future research.

Appendix A. Learning

This appendix derives the results in Section 4.1.

A.1. Derivation of beliefs

A.1.1. Spot transactions

Consider the learning process of a young agent who has chosen spot transactions. The agent enters the economy with a belief about the log aggregate productivity in the morning of the current period given by \( \pi_{mt}^T \sim N(0, \kappa^2 \lambda^{-2}) \). During the morning, the agent observes \( y_{mt} = \pi_{mt} + \epsilon_{mt} \). Conditional on the log aggregate productivity \( \pi_{mt} \), the underlying distribution of \( y_{mt} \) is

\[
y_{mt} | \pi_{mt} \sim N(\pi_{mt}, \sigma^2).
\]
This implies that at the end of the morning the agent’s belief about the log aggregate productivity in the morning of the current period is
\[
\pi_{mt}^T \mid y_{mt} \sim N \left( \frac{\kappa^2}{k^2 + \sigma^2(1 - \lambda^2)} y_{mt}, \frac{\sigma^2 k^2}{k^2 + \sigma^2(1 - \lambda^2)} \right).
\]

At the beginning of the afternoon, since \(\pi_{at} = \lambda \pi_{mt} + u_t\), the agent’s belief about the log aggregate productivity in the afternoon of the current period is
\[
\pi_{at}^T \mid y_{mt} \sim N \left( \frac{\lambda k^2}{k^2 + \sigma^2(1 - \lambda^2)} y_{mt}, \frac{\kappa^2 (k^2 + 2 \sigma^2 (1 - \lambda^2))}{k^2 + \sigma^2(1 - \lambda^2)} \right).
\]

During the afternoon, the agent observes \(y_{at} = \pi_{at} + \epsilon_{at}\). Conditional on the log aggregate productivity \(\pi_{at}\), the underlying distribution of \(y_{at}\) is
\[
y_{at} \mid \pi_{at} \sim N(\pi_{at}, \sigma^2).
\]

Thus, conditional on the private history \((y_{mt}, y_{at})\) of project log productivities, the agent’s belief about the log aggregate productivity in the afternoon of the current period is
\[
\pi_{at}^T \sim N \left( \frac{\lambda \sigma^2 k^2}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4 y_{mt}} + \sigma^2 \frac{\lambda \sigma^2 k^2}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4 y_{mt}} + \frac{\lambda k^2}{k^2 + \sigma^2(1 - \lambda^2)} y_{mt}, \frac{\kappa^2 (k^2 + 2 \sigma^2 (1 - \lambda^2))}{k^2 + \sigma^2(1 - \lambda^2)} \right),
\]

which can be rewritten as
\[
\pi_{at}^T \sim N \left( \frac{\lambda \sigma^2 k^2}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4 y_{mt}} + \frac{\lambda k^2}{k^2 + \sigma^2(1 - \lambda^2)} y_{mt}, \frac{\kappa^2 (k^2 + 2 \sigma^2 (1 - \lambda^2))}{k^2 + \sigma^2(1 - \lambda^2)} \right).
\]

Finally, since \(\pi_{mt+1} = \pi_{at}\), at the beginning of period \(t + 1\) the agent’s belief \(\pi_{mt+1}^T\) about the log aggregate productivity in the morning of that period has the same distribution as \(\pi_{at}^T\).

We can now compute \(E[\{\pi_{mt+1} - \pi_{mt+1}^T\}^2]\), that is
\[
E \left[ \left( \pi_{mt+1} - \frac{\lambda \sigma^2 k^2}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4 y_{mt}} - \frac{(k^2 + 2 \sigma^2 (1 - \lambda^2))}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4} \right)^2 \right].
\]

Substituting for the values of \(y_{mt}\), \(y_{at}\) and \(\pi_{mt+1}\), we obtain
\[
E \left[ \frac{(\lambda (1 - \lambda^2)) \sigma^4 \pi_{mt} + \sigma^2 (k^2 + (1 - \lambda^2)) \sigma^4 u_t - \lambda \sigma^2 k^2 \epsilon_{mt} - (k^2 + 2 \sigma^2 (1 - \lambda^2)) \epsilon_{at}}{\sigma^2 (k^2 + (1 - \lambda^2))^2 - \lambda^2 \sigma^4} \right].
\]

After some algebraic manipulations, we can rewrite this expression as
\[
\frac{k^2 \sigma^2 (k^2 + 2 \sigma^2 (1 - \lambda^2))}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4},
\]

which is equal to the variance of \(\pi_{mt+1}^T\).

A.1.2. Long-term relationships

Consider the learning process of a young agent who has chosen a long-term relationship. Conditional on the realization of \(y_{mt}\), the agent’s belief about the log aggregate productivity in the afternoon of the current period is the same as the belief formed after spot transactions, that is
\[
\pi_{at}^R \mid y_{mt} \sim N \left( \frac{\lambda k^2}{k^2 + \sigma^2(1 - \lambda^2)} y_{mt}, \frac{\kappa^2 (k^2 + 2 \sigma^2 (1 - \lambda^2))}{k^2 + \sigma^2(1 - \lambda^2)} \right).
\]

During the afternoon, the agent observes \(y_{at} = \pi_{at} + \epsilon_{at}\). Note that the realization of the idiosyncratic shock is the same as in the morning. This implies that an agent is able to update her belief about the idiosyncratic shock after observing \(y_{mt}\) in the morning. Precisely, conditional on the idiosyncratic shock \(\epsilon_{mt}\), the underlying distribution of the observation \(y_{mt}\) is
\[
y_{mt} \mid \epsilon_{mt} \sim N \left( \epsilon_{mt}, \frac{\sigma^2 (1 - \lambda^2)}{k^2 + \sigma^2(1 - \lambda^2)} \right).
\]

In turn, the agent’s belief about the idiosyncratic shock after observing \(y_{mt}\) is
\[
\epsilon_{at}^R \mid y_{mt} \sim N \left( \frac{\sigma^2 (1 - \lambda^2)}{k^2 + \sigma^2(1 - \lambda^2)} y_{mt}, \frac{\sigma^2 k^2}{k^2 + \sigma^2(1 - \lambda^2)} \right).
\]
As a result, conditional on the log aggregate productivity in the afternoon, the underlying distribution of the observation \( y_{at} \) is

\[
y_{at} \mid \pi_{at}, y_{mt} \sim N\left( \pi_{at} + \frac{\sigma^2 (1 - \lambda^2)}{\kappa^2 + \sigma^2 (1 - \lambda^2)} y_{mt}, \frac{\sigma^2 \lambda^2}{\kappa^2 + \sigma^2 (1 - \lambda^2)} \right).
\]

The expected value of \( y_{at} \) conditional on \( \pi_{at} \) and \( y_{mt} \) differs from \( \pi_{at} \) by \( \frac{\sigma^2 (1 - \lambda^2)}{\kappa^2 + \sigma^2 (1 - \lambda^2)} y_{mt} \). Since the agent wants to form a belief about the aggregate productivity \( \pi_{at} \) in the afternoon, she adjusts the observation \( y_{at} \) by subtracting \( \frac{\sigma^2 (1 - \lambda^2)}{\kappa^2 + \sigma^2 (1 - \lambda^2)} y_{mt} \) from its realization. This implies that

\[
\begin{align*}
\left[ y_{at} - \frac{\sigma^2 (1 - \lambda^2)}{\kappa^2 + \sigma^2 (1 - \lambda^2)} y_{mt} \right] \mid \pi_{at}, y_{mt} \sim & N\left( \pi_{at}, \frac{\sigma^2 \lambda^2}{\kappa^2 + \sigma^2 (1 - \lambda^2)} \right) \quad \text{and variance}
\end{align*}
\]

After some algebraic manipulations, we obtain

\[
\pi^R_{mt+1} \sim N\left( \frac{\sigma^2 \lambda^2 - (1 - \lambda^2) (\kappa^2 + \sigma^2)}{\kappa^2 + \sigma^2 (1 - \lambda^2)} y_{mt} + \frac{(k^2 + \sigma^2) y_{at}}{k^2 + 2 \sigma^2} \right),
\]

Because \( \pi_{mt+1} = \pi_{at} \), at the beginning of period \( t + 1 \) the agent’s belief \( \pi^R_{mt+1} \) about the log aggregate productivity in the morning of that period has the same distribution as \( \pi^R_{at} \).

Finally, we need to compute \( E[\pi_{mt+1} - \pi^R_{mt+1}]^2 \), that is

\[
E \left[ \left( \pi_{mt+1} - \frac{\sigma^2 \lambda^2 - (1 - \lambda^2) (k^2 + \sigma^2)}{k^2 + \sigma^2 (1 - \lambda^2)} y_{mt} + \frac{(k^2 + \sigma^2) y_{at}}{k^2 + 2 \sigma^2} \right)^2 \right].
\]

Substituting for the values of \( y_{mt}, y_{at} \), and \( \pi_{mt+1} \), we obtain

\[
E \left[ \frac{(\sigma^2 (1 - \lambda^2) (k^2 + \sigma^2) \pi_{mt} + \sigma^2 (1 - \lambda^2) \lambda u_t - \kappa^2 (k^2 + \sigma^2) \lambda e_{mt})^2}{(k^2 + 2 \sigma^2)^2 [(k^2 + \sigma^2 (1 - \lambda^2))]^2} \right].
\]

After some algebraic manipulations, we can rewrite this expression as

\[
\frac{\kappa^2 \sigma^2 (k^2 + \sigma^2)}{(k^2 + 2 \sigma^2)^2 [(k^2 + \sigma^2 (1 - \lambda^2))]^2} + \frac{2 \lambda k^2 \sigma^4 (k^2 + \sigma^2)}{(k^2 + 2 \sigma^2)^2 [(k^2 + \sigma^2 (1 - \lambda^2))]^2}.
\]

The first term is equal to the variance of \( \pi^R_{mt+1} \). This implies that the second term captures the bias of the posterior.

A.2. Comparison of beliefs

In what follows, we compare the beliefs formed under the two trading regimes.

(i) We prove the claim that \( \pi^R_{mt+1} > \pi^R_{mt+1} > 0 \) when \( y_{mt} = y_{at} > 0 \) and \( \pi^R_{mt+1} < \pi^R_{mt+1} < 0 \) when \( y_{mt} = y_{at} < 0 \). Using the previous results in this appendix to substitute for \( \pi^R_{mt+1} \), we obtain that for \( y_{mt} = y_{at} \) the inequality \( \pi^R_{mt+1} > \pi^R_{mt+1} \) becomes

\[
\frac{(k^2 + \sigma^2) k^2 + \lambda \sigma^2 k^2}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4} > \frac{\sigma^2 (k^2 - (1 - \lambda^2) (k^2 + \sigma^2))}{k^2 + 2 \sigma^2} + \frac{(k^2 + \sigma^2)}{k^2 + 2 \sigma^2},
\]

which can be rewritten as

\[
\frac{1}{(k^2 + \sigma^2)^2 - \lambda^2 \sigma^4} > \frac{1}{(k^2 + \sigma^2 (1 - \lambda^2))(k^2 + 2 \sigma^2)}.
\]
After some manipulations, we obtain that this inequality holds if and only if \((1 - \lambda^2)(\kappa^2 + \gamma^2) > 0\), which is always satisfied.

(ii) We prove the claim that \(\pi_{mt+1}^T > \pi_{mt+1}^R > 0\) when \(-y_{mt} = y_{at} > 0\) and \(\pi_{mt+1}^T < \pi_{mt+1}^R < 0\) when \(-y_{mt} = y_{at} < 0\). We obtain

\[
\frac{(k^2 + \sigma^2) - \sigma^2 \beta^2 (1 - \lambda^2) (k^2 + \gamma^2)}{k^2 + 2\sigma^2} > \frac{(k^2 + \sigma^2)k^2 - \lambda \kappa^2 \beta^2}{(k^2 + \sigma^2)^2 - \lambda \kappa^2 \beta^2},
\]

which can be rewritten as

\[
\frac{k^2 (k^2 + \sigma^2) - \lambda \kappa^2 \beta^2 + 2\sigma^2 (1 - \lambda^2) (k^2 + \gamma^2)}{(k^2 + 2\sigma^2)^2 (1 - \lambda^2)} > \frac{k^2 (k^2 + \sigma^2) - \lambda \kappa^2 \beta^2}{(k^2 + \sigma^2)^2 - \lambda \kappa^2 \beta^2}.
\]

After some computations, this becomes

\[
k^4 + (3 + \lambda) \sigma^2 \kappa^2 + 2(1 - \lambda^2) \sigma^4 > 0,
\]

which is always satisfied.

**Appendix B. Proofs of propositions**

This appendix contains the proofs of Propositions 1 and 2.

**B.1. Proof of Proposition 1**

In every period, a young agent decides between spot transactions and a long-term relationship in order to maximize her expected consumption of final good when old. The total amount of final good an old agent consumes is given by \(Y = Y(e^{\kappa^T}e^{\gamma^T} + \gamma P^T)\), where \(j \in \{R, T\}\). \(y_{mt}\) and \(y_{at}\) correspond to the realized log productivities of the projects the agent joins in the morning and in the afternoon of her second period of life, while \(P^T\) is the precision of the agent’s belief. First, note that \(y_{mt}(y_{at})\) only depends on the aggregate productivity in the morning (afternoon) and on the agent’s own productivity. Thus, it cannot be affected by the agent’s choice of trading regime. This implies that a young agent will choose spot transactions (a relationship) if and only if

\[
P^R = -E[(\pi_{mt}^m - \pi_{mt}^E)^2] < (>) - E[(\pi_{mt}^m - \pi_{mt}^T)^2] = P^T.
\]

In Appendix A, we have obtained that

\[
E[(\pi_{mt} - \pi_{mt}^E)^2] = \frac{k^2 \sigma^2 (k^2 + \sigma^2) (k^2 + 2\sigma^2) + 2\lambda \kappa^2 \sigma^4 (k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2 (k^2 + \sigma^2 (1 - \lambda^2))}
\]

and

\[
E[(\pi_{mt} - \pi_{mt}^T)^2] = \frac{k^2 \sigma^2 (k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2 - \lambda \kappa^2 \beta^2}.
\]

Thus, a young agent will choose spot transactions (a long-term relationship) if and only if

\[
\frac{k^2 \sigma^2 (k^2 + \sigma^2) (k^2 + 2\sigma^2) + 2\lambda \kappa^2 \sigma^4 (k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2 (k^2 + \sigma^2 (1 - \lambda^2))} > (\leq) \frac{k^2 \sigma^2 (k^2 + \sigma^2)}{(k^2 + \sigma^2)^2 - \lambda \kappa^2 \beta^2}.
\]

After some computations, we can rewrite this inequality as (3).

**B.2. Proof of Proposition 2**

(i) Persistence of the aggregate productivity (\(\lambda\)).

Let \(F(\lambda) = E[(\pi_{mt+1}^m - \pi_{mt+1}^E)^2] - E[(\pi_{mt+1}^T - \pi_{mt+1}^T)^2]\). We have

\[
F(\lambda) = \frac{k^2 \sigma^4 (k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2 (k^2 + \sigma^2 (1 - \lambda^2))} \left\{ \frac{2\lambda \kappa^2 (k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2 - \lambda \kappa^2 \beta^2} - \frac{(1 - \lambda^2) (k^2 + \sigma^2)}{(k^2 + \sigma^2)^2 - \lambda \kappa^2 \beta^2} \right\}.
\]

The agent will choose spot transactions if and only if \(F(\lambda) > 0\). For all \(k^2 > 0\) and \(\sigma^2 > 0\),

\[
F(0) = - \frac{k^2 \sigma^4}{(k^2 + 2\sigma^2) (k^2 + \sigma^2)} < 0
\]

and
\[
\lim_{\lambda \to 1} F(\lambda) = \frac{2\sigma^4(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)^2} > 0.
\]

Since \( F(\lambda) \) is continuous in \( \lambda \), for all \( k^2 > 0 \) and \( \sigma^2 > 0 \) there must exist at least one value of \( \lambda \in (0, 1) \) such that \( F(\lambda) = 0 \). Moreover,

\[
\frac{\partial F(\lambda)}{\partial \lambda} = \frac{2\kappa^2\sigma^4(k^2 + \sigma^2)[1 + \frac{1}{k^2 + 2\sigma^2} + \frac{(k^2 + \sigma^2)\kappa^2(k^2 + 2\sigma^2)}{(k^2 + 2\sigma^2)[k^2 + \sigma^2(1 - \lambda^2)]}]}{(k^2 + 2\sigma^2)}
\]

and the derivative of \( F(\lambda) \) evaluated at \( \lambda = 0 \) is

\[
\frac{\partial F(\lambda)}{\partial \lambda} [F(\lambda) = 0] = \frac{2\kappa^2\sigma^4(k^2 + \sigma^2)}{(k^2 + 2\sigma^2)}\left\{\frac{1}{k^2 + 2\sigma^2} + \frac{(k^2 + \sigma^2)\kappa^2(k^2 + 2\sigma^2)}{([k^2 + \sigma^2]^2 - \lambda^2\sigma^4)^2}\right\}
\]

which is positive for all \( k^2 > 0 \) and \( \sigma^2 > 0 \). This implies that the value of \( \lambda \) such that \( F(\lambda) = 0 \) is unique and that, calling this value \( \lambda^* \), \( F(\lambda) > 0 \) if and only if \( \lambda > \lambda^* \). Thus, all young agents will choose spot transactions if and only if \( \lambda > \lambda^* \).

(ii) Cross-sectional volatility of productivity \( (\sigma^2) \).

For notational convenience, denote \( k^2 \equiv \kappa_p \) and \( \sigma^2 \equiv \sigma_p \). Let \( G(\sigma_p) = E[(\pi_{mt+1} - \pi_{mt+1}^{eR})^2] - E[(\pi_{mt+1} - \pi_{mt+1}^{eT})^2] \). We have

\[
G(\sigma_p) = \frac{\kappa_p\sigma_p^2(\kappa_p + \sigma_p)}{(\kappa_p + 2\sigma_p)[\kappa_p + \sigma_p(1 - \lambda^2)]}\left\{\frac{2\lambda}{\kappa_p + 2\sigma_p} - \frac{(1 - \lambda^2)(\kappa_p + \sigma_p)}{(\kappa_p + \sigma_p)^2 - \lambda^2\sigma_p^2}\right\}.
\]

The agent will choose spot transactions as long as

\[
\tilde{G}(\sigma_p) = \frac{2\lambda}{\kappa_p + 2\sigma_p} - \frac{(1 - \lambda^2)(\kappa_p + \sigma_p)}{(\kappa_p + \sigma_p)^2 - \lambda^2\sigma_p^2} > 0.
\]

Fix \( \kappa_p > 0 \), \( \sigma_p > 0 \) and consider \( \tilde{G}(\kappa_p) = 0 \). After some computations, this equation becomes

\[
-2(1 - \lambda)(1 - \lambda^2)\sigma_p^2 + (4\lambda - 3 + 3\lambda^2)\kappa_p\sigma_p + (2\lambda - 1 + \lambda^2)\kappa_p^2 = 0,
\]

which has the solutions

\[
\sigma_p^- = \frac{4\lambda - 3(1 - \lambda^2) - \sqrt{(4\lambda - 3(1 - \lambda^2))^2 + 8(1 - \lambda)(1 - \lambda^2)(2\lambda - 1 + \lambda^2)}}{4(1 - \lambda)(1 - \lambda^2)}
\]

and

\[
\sigma_p^+ = \frac{4\lambda - 3(1 - \lambda^2) + \sqrt{(4\lambda - 3(1 - \lambda^2))^2 + 8(1 - \lambda)(1 - \lambda^2)(2\lambda - 1 + \lambda^2)}}{4(1 - \lambda)(1 - \lambda^2)}.
\]

Note that \( \sigma_p^- < \sigma_p^+ \). We first show that \( \sigma_p^- \leq 0 \), that is

\[
4\lambda - 3(1 - \lambda^2) \leq \sqrt{(4\lambda - 3(1 - \lambda^2))^2 + 8(1 - \lambda)(1 - \lambda^2)(2\lambda - 1 + \lambda^2)}.
\]

This is clearly true for all \( \lambda \leq \lambda^* \), where \( \lambda^* \) is the solution to \( 4\lambda - 3(1 - \lambda^2) = 0 \). Consider next the case in which \( \lambda > \lambda^* \). After some computations, the above inequality becomes \( 2\lambda - 1 + \lambda^2 \geq 0 \), that is \( \lambda > \sqrt{2} - 1 \). Since \( \lambda = \frac{\sqrt{13} - 2}{3} > \sqrt{2} - 1 \) this is always true. We now describe the region where \( \sigma_p^+ > 0 \), that is

\[
\sqrt{(4\lambda - 3(1 - \lambda^2))^2 + 8(1 - \lambda)(1 - \lambda^2)(2\lambda - 1 + \lambda^2)} > 3(1 - \lambda^2) - 4\lambda.
\]

This is clearly true for all \( \lambda > \lambda^* \). If \( \lambda < \lambda^* \), after some algebraic manipulations the above inequality becomes \( 2\lambda - 1 + \lambda^2 > 0 \), that is \( \lambda > \sqrt{2} - 1 \). In this range, and together with the fact that \(-2(1 - \lambda)(1 - \lambda^2) < 0 \), we obtain that there exists a unique \( \sigma > 0 \) such that \( \tilde{G}(\sigma) > 0 \) whenever \( \sigma_p < \sigma \) (the agent chooses spot transactions) and \( \tilde{G}(\sigma) > 0 \) whenever \( \sigma_p > \sigma \) (the agent chooses a long-term relationship). In turn, if \( \lambda \leq \sqrt{2} - 1 \) we have that \( \sigma_p^+ \leq 0 \), which implies that \( \tilde{G}(\sigma_p) \leq 0 \) (the agent will choose a long-term relationship).

(iii) Volatility of the aggregate productivity \( (k^2) \).

For notational convenience, let \( k^2 \equiv \kappa_p \), \( \sigma^2 \equiv \sigma_p \) and \( H(\kappa_p) = E[(\pi_{mt+1} - \pi_{mt+1}^{eR})^2] - E[(\pi_{mt+1} - \pi_{mt+1}^{eT})^2] \). We have

\[
H(\kappa_p) = \frac{\kappa_p\sigma_p^2(k_p + \sigma_p)}{(\kappa_p + 2\sigma_p)[\kappa_p + \sigma_p(1 - \lambda^2)]}\left\{\frac{2\lambda}{\kappa_p + 2\sigma_p} - \frac{(1 - \lambda^2)(\kappa_p + \sigma_p)}{(\kappa_p + \sigma_p)^2 - \lambda^2\sigma_p^2}\right\}.
\]

The agent will choose spot transactions as long as
\[ H(\kappa_p) = \frac{2\lambda}{\kappa_p + 2\sigma_p} - \frac{(1 - \lambda^2)(\kappa_p + \sigma_p)}{(\kappa_p + \sigma_p)^2 - \lambda^2\sigma_p^2} > 0. \]

Fix \( \kappa_p > 0 \), \( \sigma_p > 0 \) and consider \( H(\kappa_p) = 0 \). After some computations, this equation becomes

\[
(2\lambda - 1 + \lambda^2)\kappa_p^2 + (4\lambda - 3 + 3\lambda^2)\sigma_p\kappa_p - 2(1 - \lambda)(1 - \lambda^2)\sigma_p^2 = 0,
\]

which has the solutions

\[
\kappa_p^- = \frac{3(1 - \lambda^2) - 4\lambda - [16\lambda^2 - 8(3 - 2(1 - \lambda))\lambda(1 - \lambda^2) + [9 - 8(1 - \lambda)](1 - \lambda^2)^2]^{\frac{1}{2}}}{4\lambda - 2(1 - \lambda^2)}
\]

and

\[
\kappa_p^+ = \frac{3(1 - \lambda^2) - 4\lambda + [16\lambda^2 - 8(3 - 2(1 - \lambda))\lambda(1 - \lambda^2) + [9 - 8(1 - \lambda)](1 - \lambda^2)^2]^{\frac{1}{2}}}{4\lambda - 2(1 - \lambda^2)}
\]

Consider first the case in which \( 2\lambda - 1 + \lambda^2 > 0 \), that is \( \lambda > \sqrt{2} - 1 \). In this region, since the denominator of \( \kappa_p^- \) and \( \kappa_p^+ \) is positive, we have \( \kappa_p^- < \kappa_p^+ \). We claim that \( \kappa_p^+ > 0 \) for all \( \lambda < 1 \). The proof is by contradiction, that is assume that \( \kappa_p^+ \leq 0 \). This implies

\[
\{16\lambda^2 - 8[3 - 2(1 - \lambda)]\lambda(1 - \lambda^2) + [9 - 8(1 - \lambda)](1 - \lambda^2)^2\}^{\frac{1}{2}} \leq 4\lambda - 3(1 - \lambda^2).
\]

Clearly, if \( 4\lambda - 3(1 - \lambda^2) \leq 0 \) this inequality is never satisfied. Assume then that \( 4\lambda - 3(1 - \lambda^2) > 0 \). In this case, after some computations the above inequality becomes \( 2\lambda - 1 + \lambda^2 \leq 0 \), which contradicts our assumption that \( 2\lambda - 1 + \lambda^2 > 0 \). Thus, \( \kappa_p^+ > 0 \) for all \( \lambda < 1 \). Now, note that \( \kappa_p^- \leq 0 \) as long as

\[
\{16\lambda^2 - 8[3 - 2(1 - \lambda)]\lambda(1 - \lambda^2) + [9 - 8(1 - \lambda)](1 - \lambda^2)^2\}^{\frac{1}{2}} \geq 3(1 - \lambda^2) - 4\lambda.
\]

Let \( \lambda \) be the solution to \( 3(1 - \lambda^2) - 4\lambda = 0 \) and observe that \( \lambda > \sqrt{2} - 1 \). If \( \lambda \geq \lambda \), \( 3(1 - \lambda^2) - 4\lambda \leq 0 \) and the above inequality is always true. If, instead, \( \lambda \in (\sqrt{2} - 1, \lambda) \) then \( 3(1 - \lambda^2) - 4\lambda > 0 \) and the above inequality is equivalent to \( 2\lambda - 1 + \lambda^2 > 0 \), which is always true in the case under consideration. This implies \( \kappa_p^- \leq 0 \). Since we have already shown \( \kappa_p^+ > 0 \), we can conclude that, for all \( 1 > \lambda > \sqrt{2} - 1 \), there exists \( \kappa > 0 \) such that \( H(\kappa_p) \leq 0 \) when \( \kappa_p \leq \kappa \) (the agent chooses a long-term relationship) and \( H(\kappa_p) > 0 \) when \( \kappa_p > \kappa \) (the agent chooses spot transactions).

Consider next the case in which \( 2\lambda - 1 + \lambda^2 < 0 \), that is \( \lambda < \sqrt{2} - 1 \). This implies \( \kappa_p^- < \kappa_p^+ \). In what follows, we prove that \( \kappa_p^- < 0 \) in this range. As seen above, we can write \( \kappa_p^- < 0 \) as

\[
\{16\lambda^2 - 8[3 - 2(1 - \lambda)]\lambda(1 - \lambda^2) + [9 - 8(1 - \lambda)](1 - \lambda^2)^2\}^{\frac{1}{2}} < 3(1 - \lambda^2) - 4\lambda.
\]

Since \( \lambda > \sqrt{2} - 1 \), we know that \( 3(1 - \lambda^2) - 4\lambda > 0 \). We can thus rewrite the above inequality as \( 2\lambda - 1 + \lambda^2 < 0 \), which is always true in the case under consideration. As a result, we have that, for all \( \lambda < \sqrt{2} - 1 \), \( H(\kappa_p) < 0 \) for any \( \kappa_p > 0 \) (the agent always chooses a long-term relationship).

Finally, if \( \lambda = \sqrt{2} - 1 \), \( H(\kappa_p) = 2\sigma_p(1 - \sqrt{2})[\kappa_p + 2(2 - \sqrt{2})\sigma_p] < 0 \) and the agent chooses a long-term relationship for all \( \kappa_p > 0 \).

References


