Social norms and money

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Abstract

In an economy where there is no double coincidence of wants and without public record-keeping of past transactions, money is usually seen as the only mechanism that can support exchange. In this paper we show that, as long as the population is finite and agents are sufficiently patient, a social norm establishing gift-exchange can substitute for money. However, for a given discount factor, population growth eventually leads to the breakdown of the social norm. Additionally, increases in the degree of specialization in the economy can also undermine the social norm. By contrast, monetary equilibrium exists independent of the population size. We conclude that money is essential as a medium of exchange when the population is large.

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1. Introduction

The reasons for the existence of money and how it emerged in the economy are fundamental questions in monetary theory but only recently have they been addressed in a systematic way. In Kiyotaki and Wright (KW) (1989) and the

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literature that followed, money exists to play the role of a medium of exchange. However, there are other mechanisms (or institutions) in society that can play the same role. Kocherlakota (1998a, b) shows that every allocation that can be attained with money can be attained with memory as long as there is perfect public record-keeping of past transactions. We will show that a social norm establishing gift-exchange can also be a substitute for money in some situations.¹

Memory is excluded, by construction, from the environment considered by KW. Despite departing from the Walrasian economy in the sense that they make the exchange process non-trivial, they still preserve anonymous market as the locus of trade. Hence, since agents are indistinguishable from one another at all times, there is no scope for exploiting the history of past transactions. This is why there is an interesting role for money. Our objective is not to reject the idea that money actually plays the role of a medium of exchange. This is an obvious feature of modern societies and we believe that the KW environment captures in a natural and simple way the fact that money helps to overcome frictions in the exchange process. Our concern is the study of the conditions under which alternative ways of addressing the exchange problem may or may not be viable. By considering this issue, we gain insight into the reasons money emerged as the dominant mechanism. The main result in our paper establishes that, while exchange can be supported by a social norm in small societies, money has an essential role when the population is large.²

The outline of the paper is as follows. In the next section, we will first discuss the conditions under which memory can be used in the KW environment when there is perfect and public record-keeping. However, our focus is the analysis of exchange patterns without record-keeping. The main result of Section 2 establishes that in small populations there exists an equilibrium involving gift-exchange, and money is not essential. However, as population grows the social norm equilibrium eventually breaks down. We also show that an increment in the degree of specialization has a similar effect. In Section 3 we show the existence of a monetary equilibrium in the KW environment with a finite number of agents (previous analysis of the model always assume a continuum of agents). We prove that money can be valued as medium of exchange independent of the size of the population. In Section 4 we conclude.

2. The model

Consider an economy with a finite number \(N\) of infinitely-lived agents (for simplicity, assume \(N\) is even). Following KW (1993), assume that there are a large


²Following the literature, we say that money is essential if there exists desirable allocations that can be achieved with money and not without money.
number of indivisible consumption goods, which comes in units of size one. Each agent in the economy derives utility $u$ from a fraction $x$ of these goods, although which goods vary from agent to agent. At every date, agents enter the exchange sector and are pairwise matched under a uniform random matching technology. As for the production technology we assume that at every meeting each agent produces one unit of a commodity, drawn randomly from the set of all commodities. There is no production cost. However, in order for exchange to take place, there is a transaction cost $c$ that must be paid by the agent producing the good, with $u > c$. An agent does not consume his own good. Finally, agents discount the future with a discount factor $\beta$.

In our environment, double-coincidence of wants meetings happen with probability $\delta = x^2$ and single-coincidence meetings happen with probability $\sigma = x(1 - x)$. These parameters capture the extent to which commodities and tastes are differentiated in the economy. Although we are mainly interested in environments without memory, we begin with the case of perfect memory as a benchmark for what follows.

### 2.1. Memory

Suppose the economy has a record-keeping device that keeps track of all its transactions, with this device being common knowledge across agents. In other words, every agent in the economy can observe the history of transactions of every other agent. Under these assumptions consider an exchange rule stating the following (denote it rule 1): every time an agent meets someone who likes his good, he gives the good to her, as long as she has done so for others in the past. If any agent deviates, he/she does not receive or give goods anymore. Note that autarky is always an equilibrium in this economy. But we can show that, for some parameter values, the above rule is an equilibrium that makes every agent better off than in autarky. Clearly, this rule is efficient.

**Lemma 1.** If $\beta \geq c/(c + (\sigma + \delta)(u - c))$, the strategy profile associated with rule 1 constitutes a subgame perfect Nash equilibrium.

**Proof.** Let $V_c$ be the expected payoff of an agent on the equilibrium path and let $V_a = 0$ be the value function after he deviates. We need to determine if an agent will produce whenever he is called upon to do so. Since the environment is stationary, we only need to check a one-shot deviation. If he produces the good he receives $-c + \beta V_c$, and if he does not produce he receives $V_a = 0$. Thus to follow rule 1 is optimal

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3The assumptions on the technology process are basically the same as in KW (1993). The difference is that we collapse the production activity into the exchange process, i.e., agents produce while matched to each other. This modification allows us to treat agents as ex-ante (before the match) homogeneous even though they are heterogeneous regarding their preferences. This approach will be useful later and it does not affect the main insights from the original environment.
whenever \(-c + \beta V_c \geq 0\). One can calculate:

\[
V_c = \sigma(u + \beta V_c) + \sigma(-c + \beta V_c) + \delta(u - c + \beta V_c) + (1 - 2\sigma - \delta)\beta V_c
\]

\[
\Leftrightarrow V_c = \frac{(\sigma + \delta)(u - c)}{(1 - \beta)}.
\]

Hence, rule 1 is a Nash equilibrium if and only if

\[
\beta \geq \frac{c}{c + (\sigma + \delta)(u - c)}.
\]

For any history off the equilibrium path, if an agent is still cooperating he is only going to cooperate with the remaining cooperators, otherwise he is going to be punished with autarky. If he already deviated, there is no gain to cooperate since the corresponding payoff is \(-c\). Hence, there are no deviations off the equilibrium path, and rule 1 is a subgame perfect Nash equilibrium. □

This result is basically the same as the one obtained by Kocherlakota (1998a). The problem with this approach is that the gift-exchange equilibrium depends upon perfect and public record-keeping of past transactions, which is a very strong assumption. Next we will show that record-keeping is not needed, as long as agents are sufficiently patient.

2.2. Social norms

In what follows we are going to use the notion of a contagion equilibrium developed by Kandori (1992). Under a uniform random matching technology, Kandori shows that cooperation can be obtained in a one-shot prisoner’s dilemma with a social norm establishing that: “....a single defection by a member means the end of the whole community trust, and a player who sees dishonest behavior starts cheating all of his opponents. As a result, defection spreads like an epidemic and cooperation in the whole community breaks down.” (1992, p. 69). The contagious defection differs from the rules of punishment previously considered. First, without public record-keeping of transactions, agents only realize that a deviation happened when they meet a deviant agent. Moreover, punishment cannot be personalized. Upon seeing a defection, an agent punishes the whole community, and this is the reason that such a pattern can be well represented by the idea of a social norm.

Consider the same environment as before but now there is no public record-keeping of past transactions in the economy, each agent can only observe his own private history. Assume that all agents in this economy behave in accordance to the following social norm:

“Every time an agent meets another, in a single-coincidence meeting where the latter likes his good, he gives the good as long as everyone has done so in the past for him. If in a meeting an agent fails to give a good that the other agent likes, neither agent will ever produce again in single-coincidence meetings. In double-coincidence of wants meetings, agents exchange goods simultaneously, irrespective of their previous private history. In any other situation, there is no exchange at all”.
Under the above norm, an agent’s expected payoff can be calculated as follows. Let \( V_n \) be the expected payoff when all agents cooperate and let \( V_d \) be the expected payoff of an agent who triggers the contagious punishment. Note that, on the equilibrium path, the social norm specifies the same behavior as rule 1 (from Lemma 1). Therefore, \( V_n = V_c \), i.e.

\[
V_n = \frac{(\sigma + \delta)(u - c)}{1 - \beta}.
\]

(3)

Now, if an agent deviates from the equilibrium path, he obtains (the calculation of \( V_d \) is done in Appendix A):\(^4\)

\[
V_d = \frac{\beta \delta (u - c)}{1 - \beta} + \sum_{t=1}^{\infty} \beta^t e_1 A' \pi \sigma u,
\]

(4)

where

\[
e_1 = (1, 0, 0, ..., 0), \ N\text{-dimensional},
\]

\[
A = \{(a_{ij})\}_{N \times N}, \text{ where } (a_{ij}) = \Pr(D_{t+1} = j \mid D_t = i),
\]

\[
\pi = (\pi_i), \ N\text{-dimensional, where } \pi_i = \Pr(\text{defector meets a cooperator} \mid D_t = i),
\]

\[
D_t = \text{number of defectors at time } t.
\]

**Proposition 1.** Let \( N > 2 \). There exists a discount factor \( \beta^* \) such that, for every \( \beta \geq \beta^* \), the strategy profile corresponding to the social norm constitutes a sequential equilibrium.

Before giving a formal proof let us outline here the main steps we are going to follow. First, we show that in an economy where agents are patient, the social norm is a Nash equilibrium. The intuition here is simple: under the threat of a contagious punishment, an agent knows that an initial defection is eventually going to spread out over the entire economy. Therefore, if he is sufficiently patient, he will not deviate.

Second, we prove that, for a further restriction on the set of discount factors, the equilibrium is also sequential. This means that an agent, after seeing a defection, is going to follow the social norm and defect himself from that period on. As Kandori (1992) points out, this is the hard part of the proof. The intuition for this difficulty runs as follows. When an agent sees a defection he may want to cooperate in the next period in an attempt to slow down the contagious process. In order to deal with this issue, we will follow the proof in Ellison (1994). In the context of the prisoner’s dilemma, Ellison shows that for sufficiently patient agents, the incentives to slow down the contagion and cooperate are smaller than the benefits of defecting in a world that is already under a contagious punishment. The idea is that, since

\(^4\)To calculate \( V_d \), we calculate first a closed form expression for the contagious process in this environment. It turns out that the process analyzed by Kandori (1992) can be seen as a particular case of the one analyzed here.
cooperation is breaking down anyway, one extra deviation will have a limited impact. The crucial element in Ellison’s proof is to show that the marginal benefit of not spreading a defection is decreasing in the number of agents already defecting.

**Proof.** First, we need to check if an agent will offer a commodity when he meets another that likes his good in a single-coincidence meeting. Note that, on the equilibrium path, these are the only meetings where an agent may want to deviate. In double-coincidence of wants meetings, since agents exchange goods simultaneously, the best choice is always to offer the good. Therefore, we only need to consider the inequality

\[-c + \beta V_n \geq V_d,
\]

where

\[V_d = \frac{\beta \delta (u - c)}{(1 - \beta)} + \sum_{t=0}^{\infty} \beta^t e_1 A' \sigma u - \sigma u.\]

If we replace the last column of $A$ by zeros (this replacement is justified because $D_t = N$ is the absorbing state and the $N$th element of $\pi$ is zero) the following equality holds (where $A'$ is the matrix obtained from $A$ after the replacement)

\[(I - \beta A)^{-1} \pi = \sum_{t=0}^{\infty} \beta^t A' \pi = \sum_{t=0}^{\infty} \beta^t A'' \pi = (I - \beta A')^{-1} \pi.\]

Now, since the number of defectors never declines $A'$ is upper-triangular and so is $(I - A')$. The determinant of an upper-triangular matrix is the product of its diagonal elements, which are all strictly positive for $(I - A')$. Hence, $(I - \beta A')^{-1} \pi < \infty \iff (I - \beta A)^{-1} \pi < \infty$. We can rewrite $V_d$ as

\[V_d = \frac{\beta \delta (u - c)}{(1 - \beta)} + e_1 (I - \beta A)^{-1} \pi \sigma u - \sigma u.\]

Substituting this expression in the inequality $-c + \beta V_n \geq V_d$, we obtain

\[-c + \frac{\beta \delta (u - c)}{(1 - \beta)} \geq e_1 (I - \beta A)^{-1} \pi \sigma u - \sigma u.\]

When $\beta \to 1$, the expression on the left-hand side goes to infinity while the expression on the right-hand side is finite. Hence, there exists $\beta$ such that, for all $\beta \geq \beta$, there will be no deviations from the equilibrium path. This implies that our strategy profile is a Nash equilibrium. It remains to prove that, for sufficiently patient agents, the equilibrium is also sequential. Since this part of the proof is long, we leave it to Appendix B.

We reiterate that the history of economy-wide transactions is not used to implement the social norm, only the personal history. However, since it takes more time to punish a deviant in this environment, as compared to one with perfect public record-keeping, the discount factor needed to support the equilibrium is higher.
Proposition 1 establishes conditions under which a social norm can implement gift-exchange. However, it can also determine when this norm cannot be implemented. For a fixed $\beta$, the value of $N$ is the key factor. When the population increases, defection requires a longer time to contaminate all agents. If the discount factor does not change, we will reach a point where agents will no longer have an incentive to follow the social norm and the equilibrium will break down. The following proposition can be proved:

**Proposition 2.** For a fixed discount factor $\beta$, there exists a value of $N$, say $N'$, such that, for all $N > N'$ the social norm cannot be sustained as a Nash Equilibrium.

The proof is contained in the Appendix C and it is similar to a result in Kandori (1992, p. 78). Proposition 2 holds for any value of $\delta$ and $\sigma$. In what follows we are going to change our focus and see what happens when $\sigma$ changes, for a given value of $N$. We will consider a particular case where $\delta = 0$ and reinterpret $\sigma$ as measuring the degree of specialization in the economy. If the economy is very specialized, $\sigma$ is low, i.e., there are many goods but agents only like a small fraction of them. We can also motivate the use of $\sigma$ in the following way. Suppose agents specialize more in production, for example, agents that used to produce 10 distinct goods now produce only one good. This implies that the probability of meetings where an agent likes the goods produced by the other goes down, and this is exactly what happens when $\sigma$ decreases.

In order to see the effects of a change in the degree of specialization over the social norm equilibrium, we need to calculate $\frac{\partial}{\partial \sigma} \left( \frac{\beta Vc}{C0} Vd \right)$. If it is positive, increasing the degree of specialization in the economy weakens the equilibrium we have been considering so far. We do not have the analytical solution for this derivative but we analyzed the value of sign $\left( \frac{\partial}{\partial \sigma} \left( \frac{\beta Vc}{C0} Vd \right) \right)$ under distinct population sizes and for various values of $\beta$ and $\sigma$. We considered $N = 6, 14$; $\beta = 0.1, 0.3, 0.5, 0.7, 0.9$ and $\sigma = 0.1, 0.3, 0.5$. In all cases, $\frac{\partial}{\partial \sigma} \left( \frac{\beta Vc}{C0} Vd \right) > 0$, supporting the idea that specialization of the economy reduces the incentives to follow the equilibrium. The intuition for this result runs as follows. First, the reduction of single-coincidence of wants meetings reduces the expected gains from trade over time, which creates more incentive for an agent to deviate. Second, since interactions which involve exchange become less frequent when $\sigma$ decreases, the contagion process tends to be slower.

We conclude that, in societies where the economy is either too large or too specialized, it is unlikely to observe exchange based on a social norm.

3. **Money**

In this section we introduce money in the economy and study the conditions under which monetary equilibrium exists. We consider the same environment as before. We assume no record keeping and we do not take into account the possibility that agents
use a social norm. This study is in the same spirit of KW (1993) but with a finite population. Previous analysis of this type of model always use a continuum.

Following the literature, money is indivisible and agents can store at most one unit. This implies that the only potential monetary exchange involves one unit of money for one unit of a good. Money enters the economy in the following way. In period one, $M$ units are randomly distributed among the agents. Agents are aware of the quantity of money in the economy, therefore they can calculate the expected payoff under a monetary equilibrium.5

In equilibrium an agent’s expected value from a match depends on whether or not he is holding money. When an agent (for example, agent $i$) does not have money there is a probability $s$ that he is matched with another with a preference for his good. In this case, with probability $M/(N-1)$ the other agent is holding money and so in a monetary equilibrium agent $i$ gives the good (at a cost $c$) and obtains a continuation payoff of $\beta V_1$. With probability $(N-1-M)/(N-1)$ the other party has no money and there is no trade. There is a probability $s$ that $i$ meets an agent which has the good he likes and probability $(1-2\sigma-\delta)$ that he faces a no-coincidence of wants meeting. In both cases, there is no trade and the continuation payoff is $\beta V_0$. Finally, there is a probability $\delta$ of a double-coincidence of wants meeting, in which case, the pair simply exchange goods.6 This reasoning provides the expected value of not holding money. By similar reasoning, we can obtain the expected value of holding money.

Let $V_0$ denote the expected payoff of an agent without money and $V_1$ the expected payoff of an agent with money. Then the above reasoning yields:

\[
V_0 = \sigma \left[ \frac{M}{N-1} (-c + \beta V_1) + \frac{(N-1-M)}{N-1} \beta V_0 \right] + \sigma \beta V_0
\]

\[
+ \delta \left[ \frac{M}{N-1} (u - c + \beta V_0) + \frac{(N-1-M)}{N-1} (u - c + \beta V_0) \right]
\]

\[
+ (1-2\sigma-\delta) \beta V_0,
\]

(10)

\[
V_1 = \sigma \left[ \frac{(M-1)}{N-1} \beta V_1 + \frac{(N-M)}{N-1} (u + \beta V_0) \right] + \sigma \beta V_1
\]

\[
+ \delta \left[ \frac{(M-1)}{N-1} (u - c + \beta V_1) + \frac{(N-M)}{N-1} (u - c + \beta V_1) \right]
\]

\[
+ (1-2\sigma-\delta) \beta V_1.
\]

(11)

5 The problem of calculating the expected payoff in a monetary equilibrium is harder when agents do not know $M$. See Araujo and Camargo (2003).

6 Note that we are assuming here a version of the KW environment where agents with money are able to produce. In this situation, when an agent with money meets another without money in a double-coincidence of wants, we cannot simply assume that agents exchange goods. Another possibility is that they exchange one unit of good for one unit of money. However, in a related environment, Rupert et al. (2001) proved that the unique subgame perfect equilibrium in pure strategies involves a barter exchange, and the same argument applies here.
Notice that since we assumed random production types, one does not need to know who (which types) have money, which makes it easier to calculate the expected payoff of an agent.

A monetary equilibrium exists if and only if \(-c + \beta V_1 \geq \beta V_0\). Letting \(\gamma = M/N\), and substituting into the equations for \(V_0\) and \(V_1\), we can, after some algebraic manipulation, provide the necessary and sufficient conditions for the equilibrium in terms of the primitives of the model. Proposition 3 describes these conditions. Corollary 1 establishes that for any population size there exists a discount factor \(\beta^* < 1\) such that, for all \(\beta \geq \beta^*\) a monetary equilibrium exists.

**Proposition 3.** A monetary equilibrium exists if and only if

\[
\beta \geq \frac{c}{c + (1 - \gamma)\sigma(u - c)N/(N - 1)}.
\]

**Proof.** For a monetary equilibrium, we must have \(-c + \beta V_1 \geq \beta V_0\). From the expressions for \(V_0\) and \(V_1\), this condition is equivalent to the condition in (12).

**Corollary 1.** For any \(N\), a sufficient condition for a monetary equilibrium is \(\beta \geq c/(c + (1 - \gamma)\sigma(u - c))\).

Proposition 3 shows that when agents are sufficiently patient there exists a monetary equilibrium, regardless of the size of population. This is not the case for a social norm equilibrium. If the population is large agents have incentive to deviate from the social norm because it takes a long time until they are reached by the contagion process. However, in a monetary equilibrium agents must obtain money if they want to consume in a single-coincidence of wants meeting. Hence, as long as the probability of a single-coincidence of wants is not too small, a monetary equilibrium always exists. Finally, since there is no public record of past transactions, we cannot use memory in order to implement exchange. This reasoning leads to the main result of the paper, which can be stated as follows: *in an economy with a large population there are allocations that only money can achieve, hence money is essential.*

Finally, notice that the social norm equilibrium (when it exists) is more efficient than a monetary equilibrium. The reason is very simple: in a social norm equilibrium, there exists no single or double coincidence of wants meeting in which exchange does not occur. In other words, the social norm can implement the best possible allocation. In a monetary equilibrium this allocation is not attainable, since a necessary condition for exchange to take place in single-coincidence meetings is that the agent demanding goods must have money.\(^7\)

\(^7\)The inefficiency of the monetary equilibrium reflects our assumption that agents are randomly pairwise matched. Corbae et al. (2003) show that money is efficient as a medium of exchange in an environment with endogenous non-random matching.
4. Conclusion

The objective of this paper was to show that social norms can support exchange in the Kiyotaki–Wright environment if population is not too large and agents are sufficiently patient. The importance of this result is that it gives a possible explanation for the dominant role of money as compared to social norms in modern economies. As long as agents believe that money is valuable as a medium of exchange, we can support a monetary equilibrium, independent of the size of the population. Hence, even though social norms played a role in the past sustaining the exchange process, for example in village economies,8 they cannot support exchange in a modern economy.

In general, the basic results obtained in this paper are robust to several modifications of the environment. That is, as long as the population is finite and agents are sufficiently patient, gift-exchange can be supported in the KW environment as a sequential equilibrium. Moreover, population growth eventually leads to the breakdown of the norm, while a monetary equilibrium exists independent of the population size. However, the fact that we could obtain a simple expression for the contagion process depends on the assumption that agents can be treated as ex-ante homogeneous.

In our economy, there is no public record of past transactions, so, memory cannot be used. This leads to the conclusion that money is essential as a medium of exchange in a highly populated economy. Specialization, by reducing the value of future benefits and the speed of the contagion process, also increases the incentives for agents to deviate. Hence, even if population size is fixed, the social norm equilibrium can collapse if the degree of specialization in the economy increases.

The implications here are distinct from Kocherlakota (1998a, b). The conclusion from Kocherlakota’s work is that if the cost of keeping track of the past is too high, memory cannot substitute for money, and money becomes essential. Our conclusion is that money is essential not due to the complexity of the past but to the complexity of the present, to the fact that is hard to punish defectors in a world that is either too large in terms of population or too specialized.

Appendix A. Calculation of $V_d$

Let $D_t$ be equal to the number of defectors at time $t$. We want to calculate the matrix $A = (a_{ij})$, where $a_{ij} = P(D_{t+1} = j \mid D_t = i)$. First, note that since the number of defectors is non-decreasing over time, $a_{ij} = 0$ whenever $j < i$. Moreover, if $j \geq i$ and $(j-i) > \min(D_t, N-D_t)$, $a_{ij} = 0$ since in order to increase the number of defectors by $(j-i)$ it is necessary to have at least $(j-i)$ defectors and cooperators. Another situation where it is easy to identify the value of $a_{ij}$ is when $i = 1$. In this case, after a first deviation of a player, in the next period there will be exactly 2 defectors in the

8A good reference in this direction is Landa (1994), which gives a historical analysis of gift-exchange.
economy. That is, \( a_{ij} = 0 \) if \( j \neq 2 \) and \( a_{ij} = 1 \) if \( j = 2 \). The non-trivial case is when \( i \neq 1, j \geq i \) and \( (j - i) \leq \min(D_i, N - D_i) \). In this case, we have the following lemma:

**Lemma A.1.** If \( i \neq 1, j \geq i \) and \( (j - i) \leq \min(D_i, N - D_i) \), the probability of moving from \( i \) to \( j \) defectors \( (a_{ij}) \) is equal to

\[
a_{ij} = \max(\tau | m - 2\tau \geq j - i) \sum_{\tau=0}^{\max(\tau | m - 2\tau \geq j - i)} \binom{M}{m-2\tau} \binom{m}{m-2\tau} (m-2\tau)! \binom{m-2\tau}{j-i} \times \sigma^{j-i}(1-\sigma)^{(m-2\tau)-(j-i)}S(2\tau)S(M - (m - 2\tau)) \frac{S(N)}{S(N)}
\]

where

- \( m = \min(D_i, N - D_i) \),
- \( M = \max(D_i, N - D_i) \),
- \( S(x) = \text{number of different ways to make} \frac{x}{2} \text{ pairs out of} x \text{ agents} \),
- \( S(0) = 1 \).

**Proof.** In order to generate \( (j - i) \) defectors, we need to have at least \( (j - i) \) meetings between defectors and cooperators. We have to consider situations where the number of meetings between cooperators and defectors is above \( (j - i) \) since a cooperator does not always turn into a defector upon meeting a defector. More specifically we need to consider up to \( m \) meetings between a cooperator and a defector, since this is the maximum possible number of meetings of this kind.

Fix a number of meetings \( m - 2\tau \), for some \( \tau \geq 0, \tau \leq \max(\tau | m - 2\tau \geq j - i) \). The number of possible distinct partitions in groups of two (where one agent is a cooperator and another is a defector) with size \( m - 2\tau \) is equal to \( \binom{M}{m-2\tau} \binom{m}{m-2\tau} (m-2\tau)! \). Now, for each of these partitions, the chance of the number of defectors increase to \( (j - i) \) follows a binomial distribution with parameter \( \sigma \), and is equal to \( \binom{m-2\tau}{j-i}(1-\sigma)^{(m-2\tau)-(j-i)} \). Now, since we are fixing attention on only \( m - 2\tau \) meetings between a defector and a cooperator, the number of possible ways of choosing pairs with the rest of the population where cooperator meets cooperator and defector meets defector is equal to \( S(2\tau)S(M - (m - 2\tau)) \). By varying \( \tau \) for all its possible values, we obtain the number of possible partitions of the population in pairs such that the number of defectors increase to \( (j - i) \), and dividing this number by \( S(N) \) gives the probability we are looking for. \( \square \)

Notice that the contagion process described by Kandori (1989,1992) can be seen as a particular case of the one analyzed here, which happens when a cooperator always starts to defect after meeting a defector, i.e., when \( \sigma = 1 \). Now, we can calculate the payoff obtained by an agent who deviates from the proposed social norm equilibrium. First, an agent only has incentive to deviate when he meets another agent that has a preference for his good in a single-coincidence meeting. Hence, the
overall payoff after a deviation is equal to:

\[ V_d = \frac{\beta \delta (u - c)}{(1 - \beta)} + \sum_{i=1}^{\infty} \beta^i e_1 A \pi \sigma u. \]  

(A.1)

Appendix B. Proof of Proposition 1—continuation

This part of the proof follows closely Ellison (1994). Let \( k \) correspond to the number of agents who are not cooperating at the start of period \( t \). Let \( f(k, \beta) \) be \( i \)'s per period continuation payoff from period \( t \) on when \( k \) agents (\( i \) included) do not plan to cooperate while all the others intend to do so. Note that in order for an agent to defect he must meet another that likes his good in a single-coincidence of wants meeting. Our objective is to show that, for sufficiently high values of \( \beta \), every agent will be willing to spread the contagious punishment and start defecting after seeing a defector. Formally, we have to show that

\[ \frac{\beta}{(1 - \beta)} [f(j, \beta) - f(j + 1, \beta)] \leq c \quad \text{for all } j \geq 2. \]  

(B.1)

The left-hand side indicates the agent's gain if he slows down the contagion and the right-hand side indicates his cost. First, we show that \( f \) is convex in \( k \), for all \( k \geq 1 \). Note that

\[ f(k, \beta) = E_{(W, X)} g(k, \beta, w, x) \]  

(B.2)

where \((W, X)\) is a random vector whose realization is a pairing of all agents in each period, and the corresponding goods produced in every match. From now on, without loss of generality, we focus our attention on the payoffs received by player 1. Therefore, function \( g \) reflects 1’s continuation payoff (given \( w \) and \( x \)) when he and players 2 to \( k \) are going to defect if they have the opportunity to do so. For convenience, let \( h(k, \beta, w, x) \) be player 1’s continuation payoff when he, players 2 to \( k \) and player \( N \) are not cooperating. Obviously

\[ E_{(W, X)} g(k + 1, \beta, w, x) = E_{(W, X)} h(k, \beta, w, x). \]  

(B.3)

We will prove that, for all realizations \((w, x)\) and all \( k \geq 1 \)

\[ g(k, \beta, w, x) - h(k, \beta, w, x) \geq g(k + s, \beta, w, x) - h(k + s, \beta, w, x). \]  

(B.4)

This implies that

\[ f(k, \beta) - f(k + 1, \beta) \geq f(k + s, \beta) - f(k + s + 1, \beta) \]  

(B.5)

so that \( f \) is convex in \( k \). First, we establish some notation that will help us in what follows.

- Let \( P(t, w, x) \) be the set of producers in single-coincidence meetings at date \( t \), given \((w, x)\), and let \( P(t, w, x) \) be the complement of \( P(t, w, x) \).
- Let \( C(t, k, w, x) \) indicates the set of agents who are still cooperating in period \( t \) when players 1, 2, 3, \ldots, \( k \) intend to start defecting in period 1. Moreover, define
\( \overline{C(t,k,w,x)} \) as the complement of \( C(t,k,w,x) \). We can obtain \( C(t,k,w,x) \) recursively in the following way, where \( o_i(t,w) \) is \( i \)'s match at date \( t \), given \( w \):

\[
\begin{align*}
C(1,k,w,x) &= \{k+1, \ldots, N\}, \\
C(t+1,k,w,x) &= \{i \in C(t,k,w,x) \mid o_i(t,w) \in C(t,k,w,x) \cup \overline{C(t,k,w,x)} \cap \overline{P(t,w,x)}\}.
\end{align*}
\]  

(\text{B.6})

- Let \( D(t,w,x) \) be the set of agents who want to defect in period \( t \) when player \( N \) intends to start defecting in period 1. \( D(t,w,x) \) can be obtained recursively as follows:

\[
\begin{align*}
D(1,w,x) &= \{N\}, \\
D(t+1,w,x) &= D(t,w,x) \cup \{i \mid o_i(t,w) \in D(t,w,x) \cap P(t,w,x)\}. \quad \text{(B.7)}
\end{align*}
\]

We are now in position to calculate \( g(k,\beta,w,x) - h(k,\beta,w,x) \), i.e., the difference in 1’s continuation payoff between a situation where he and \( k-1 \) others start defecting in period 1 and a situation where he and \( k \) others start defecting in period 1. This difference is equal to

\[
\sum_{t=1}^{\infty} (1 - \beta)\beta^{t-1} u^{\gamma} [o_1(t,w) \in C(t,k,w,x) \cap D(t,w,x) \cap P(t,w,x)], \quad \text{(B.8)}
\]

where \( \gamma(E) \) is a function equal to one or zero depending upon the deterministic condition \( E \) being false or true. Intuitively, in any period \( t \), \( g(k,\beta,w,x) \) and \( h(k,\beta,w,x) \) are different only if player 1’s match at this date is a producer in a single-coincidence of wants meeting (therefore, he belongs to the set \( P(t,w,x) \)) with the following behavior: he plans to cooperate whenever players 1 to \( k \) start defecting in period 1 (therefore, he belongs to the set \( C(t,k,w,x) \)) and plans not to cooperate when players 1 to \( k \) and player \( N \) start defecting in period 1 (therefore, he belongs to the set \( D(t,w,x) \)).

Note that, for any possible realization \( (w,x) \), if the number of defectors at date 1 increases from \( k \) to \( k+s \), the number of defectors at any point in time is also going to increase. Then, from the definition of \( C(t,k,w,x) \), we have

\[
C(t,k+s,w,x) \subset C(t,k,w,x) \quad \text{for all} \quad (w,x)
\]

(B.9)

and

\[
C(t,k+s,w,x) \cap D(t,w,x) \cap P(t,w,x) \subset C(t,k,w,x) \cap D(t,w,x) \cap P(t,w,x),
\]

for all \( (w,x) \).

This result implies

\[
\sum_{t=1}^{\infty} (1 - \beta)\beta^{t-1} u^{\gamma} [o_1(t,w) \in C(t,k,w,x) \cap D(t,w,x) \cap P(t,w,x)]
\]
\[
\geq \sum_{t=1}^{\infty} (1 - \beta)\beta^{t-1} u Y \{ o_1(t, w) \\
\in C(t, k + s, w, x) \cap D(t, w, x) \cap P(t, w, x) \}.
\]  
(B.10)

Therefore, for all \((w, x)\)

\[
g(k, \beta, w, x) - h(k, \beta, w, x) \geq g(k + s, \beta, w, x) - h(k + s, \beta, w, x).
\]  
(B.11)

This gives us the convexity of \(f\), for any \(k \geq 1\). From the convexity of \(f\), we can restrict attention to inequality (B.1) evaluated at \(j = 2\). That is, we only have to show that, for sufficiently high \(\beta\)

\[
\frac{\beta}{1 - \beta} [f(2, \beta) - f(3, \beta)] \leq c.
\]  
(B.12)

We now show that, for all \(\beta \geq \beta_0\), where \(\beta_0\) is as defined in the first part of the proof (see p. 7)

\[
\frac{\beta}{1 - \beta} [f(1, \beta) - f(2, \beta) - f(2, \beta) - f(3, \beta)] \geq \gamma(\beta) > 0.
\]  
(B.13)

This will allow us to find a positive range of discount factors (close enough to \(\beta = \beta_0\)) where, restricted to this set, the social norm is a sequential equilibrium. From expression (B.8) we know that \(f(1, \beta) - f(2, \beta) - f(2, \beta) - f(3, \beta)\) is equal to

\[
E_{(w, x)} \left\{ \sum_{t=1}^{\infty} (1 - \beta)\beta^{t-1} u Y \{ o_1(t, w) \in (C(t, 1, w, x) \\
- C(t, 2, w, x) ) \cap D(t, w, x) \cap P(t, w, x) \} \right\}.
\]  
(B.14)

The second term of this sum \((t = 2)\) is

\[
(1 - \beta)\beta u \Pr \{ o_1(2, w) \in (C(2, 1, w, x) \\
- C(2, 2, w, x) ) \cap D(2, w, x) \cap P(2, w, x) \}.
\]  
(B.15)

The objective is to find a realization \((w, x)\) such that

\[
(C(2, 1, w, x) - C(2, 2, w, x)) \cap D(2, w, x) \cap P(2, w, x)
\]  
(B.16)

is a non-empty set. This implies that the probability stated on (B.15) is positive.

Consider a realization \((w', x')\) such that agent 2 meets agent \(N\) in period 1 in a single-coincidence meeting where \(N\) is the producer. Moreover, in period 2, agent 1 meets agent 2 in a single-coincidence meeting where agent 2 is the producer. From the definition of \(C(t, k, w, x)\) and \(D(t, w, x)\)

\[
2 \in C(2, 1, w', x') \quad \text{but} \quad 2 \notin C(2, 2, w', x'), \quad N \in C(2, 1, w', x'), \quad N \in C(2, 2, w', x') \quad \text{and} \quad D(2, w', x') = \{2, N\}.
\]  
(B.17)

So

\[
[C(2, 1, w', x') - C(2, 2, w', x')] \cap D(2, w', x') \cap P(2, w', x') = \{2\}.
\]  
(B.18)
The probability of the event \((w', x')\) is equal to
\[
\frac{1}{(N-1)^{\sigma}} \frac{1}{(N-1)^{\sigma}} = \left( \frac{\sigma}{(N-1)} \right)^2.
\]
Therefore, for all \(\beta \geq \beta'\)
\[
\frac{\beta}{(1-\beta)} [f(1, \beta) - f(2, \beta) - (f(2, \beta) - f(3, \beta))] \geq \left( \frac{\beta \sigma}{(N-1)} \right)^2 u \geq \left( \frac{\beta \sigma}{(N-1)} \right)^2 u = \gamma(\beta) > 0.
\]
Now, from \(f(k, \beta) = E_{(W,Y)}(k, \beta, w, x)\) and (B.8) we can show that
\[
\frac{\partial}{\partial \beta}([\beta/(1-\beta)](f(1, \beta) - f(2, \beta))] \mid_{\beta=\beta_0}.
\]
This implies that, for some \(\eta < \gamma(\beta)/2\), \(\exists \beta \in (\beta_0, 1)\) such that
\[
\frac{\beta}{(1-\beta)} [f(1, \beta) - f(2, \beta)] = c + \eta.
\]
Moreover, for \(\beta \in (\beta_0, \beta)\)
\[
c \leq \frac{\beta}{(1-\beta)} [f(1, \beta) - f(2, \beta)] \leq c + \eta.
\]
From expression (B.13) we obtain, for all \(\beta \in (\beta_0, \beta_0)\)
\[
\left( \frac{\beta}{(1-\beta)} [f(2, \beta) - f(3, \beta)] \right) \leq \frac{\beta}{(1-\beta)} [f(1, \beta) - f(2, \beta)]
\]
\[
\left( \frac{\beta}{(1-\beta)} [f(1, \beta) - f(2, \beta)] \right) - \gamma(\beta) \leq c - \eta.
\]
Therefore, inequality (B.12) is satisfied. Since we already know that for all \(\beta \geq \beta_0\), the social norm is a Nash equilibrium, we obtain that for all \(\beta \in (\beta_0, \beta)\), this norm is also a sequential equilibrium.

As a last step we just apply Ellison’s Lemma 2 (1994, p. 580). Suppose there is a non-empty interval \( (\delta_0, \delta_1) \) such that \( s(\delta) \) is a sequential equilibrium with outcome \( a \), for all \( \delta \in (\delta_0, \delta_1) \). Ellison shows that there exists \( \delta^* = \delta_0/\delta_1 \) such that, for all \( \delta \geq \delta^* \), we can define a strategy profile \( s^*(\delta) \) which is also a sequential equilibrium with outcome \( a \). In our case, we only need to define \( \beta^* = \beta_0/\beta \) and obtain that, for every \( \beta \geq \beta^* \), the social norm is a sequential equilibrium. This concludes our proof.

**Appendix C. Proof of Proposition 2**

Based on Kandori’s Proposition 3 (1992, p. 78). Suppose that an agent wants to deviate from the social norm equilibrium at a given point in time, say \( T \). Let \( V_c \) and \( V_d \) be the continuation payoffs for this agent when he follows and when he deviates from the equilibrium. Let \( I_t \) be the set of players whose behavior at \( T + t \) is affected by the defection at time \( T \). If player \( i \) is the defector at \( T \), \( I_t \) is at most equal to
\( \{i, \mu(i, T)\} \) and for \( t > 1 \), \( I_t \) is at most equal to \( \{j; j \in I_{t-1} \) or \( j = \mu(k, t-1) \) for some \( k \in I_{t-1} \}\), where \( \mu(i, t) \) indicates the agent who matches with \( i \) at date \( t \).

The number of players in \( I_t \) is maximal when every member of \( I_t \) is matched with a member of \( N - I_t \) and the latter is supposed to receive a good from the former but did not (so that he turns into a defector in the next period). Hence, an upper bound of \( \#I_t \) is \( \min \{2^t, N\} \). So, \( \min \{2^t, N\}/(N - 1) \) is an upper bound for the probability of the agent who deviated at \( T \) to meet a defector at \( T + t \). This reasoning implies the following inequality:

\[
V_c - V_d \leq \sum_{t=1}^{\infty} \beta^t \min \frac{\{2^t, N\}}{N - 1} u \to 0 \text{ when } N \to \infty.
\] (C.1)

Finally, we have

\[
\lim_{N \to \infty} \beta V_c - V_d = 0 < c,
\] (C.2)

which implies that there exists a value of \( N \), say \( N' \), such that, for every \( N > N' \), the social norm cannot be supported as a Nash Equilibrium.

References


