The Essentiality of Money in Environments with Centralized Trade

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Abstract

Lagos and Wright (2005) introduced an influential model of monetary exchange in which trade alternates between centralized and decentralized markets and money is essential. A limitation of their model and of the literature that follows is that they do not provide a microfoundation for the process of exchange in the centralized market. In this article, we show that how one models exchange in the centralized market matters for the essentiality of money. More precisely, we describe the centralized market as a market game and find conditions under which money is essential.

Key Words: Money, Centralized Markets, Essentiality.

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1 Introduction

Modern monetary theory is based on the notion that one must be explicit about the frictions in the exchange process that generate an essential role for money. The current benchmark market in this literature is the model introduced in Lagos and Wright (2005) (henceforth LW). The main contribution of LW is constructing an environment in which, unlike in the models of money in the tradition of Kiyotaki and Wright (1993), substantive issues can be analyzed in a tractable manner. The key element of LW is that trade alternates between centralized and decentralized markets.

An important feature of LW and the literature that follows is that they treat trade in the centralized and decentralized markets asymmetrically. While the literature is very explicit about the process of exchange in the decentralized market, it remains silent about how trade takes place in the centralized market, modeling it as a Walrasian market where agents take prices as given and trade against their budget constraints. This asymmetry, however, is at odds with the emphasis that the literature places on taking seriously the process of exchange, and may have important implications for monetary theory in general. In this paper, we model explicitly the process of exchange in the centralized market and ask whether this has implications for the essentiality of money.

The starting point of our analysis is the framework of LW modified in two ways. First, we model the process of exchange in the centralized market as a non–cooperative game, more precisely, as a strategic market game along the lines of Shapley and Shubik (1977).1 There are two reasons for this. First, strategic market games are a natural departure from Walrasian markets. In fact, it is well known that in static settings, equilibrium outcomes of strategic market games converge to Walrasian outcomes as the number of agents increases to infinity (Mas–Colell (1982)). Second, strategic market games allow for a mapping between the formation of prices and the trading decisions of agents, while retaining the idea of centralized markets as large anonymous markets in which agents only observe prices.2

1The classic reference on strategic market games is Shubik (1973). See also Dubey and Shubik (1978) and Postlewaite and Schmeidler (1978). Green and Zhou (2005), Hayashi and Matsui (1996), and Howitt (2005) are examples of applications of market games in the context of monetary theory.

2A third reason for the non–cooperative approach is that if one wants to assess the conditions under which money is essential, one must consider whether agents have the incentive to follow alternative credit–like
Second, while LW assumes a continuum of agents, we consider large but finite populations. In any model, the assumption of a continuum of agents is made for tractability and is only justifiable if it has no substantive economic implications. To put it differently: “The rationale for using the continuum–of–agents model is that it is a useful idealization of a situation with a large finite number of agents, but if equilibria in the continuum model are radically different from equilibria in the model with a finite number of agents, then this idealization makes little sense” (Levine and Pesendorfer (1995), p. 1160). We want to ensure that the essentiality of money is not driven by the continuum population assumption.

Our main message is that providing microfoundations for the process of exchange in the centralized market has important implications for the essentiality of money. Indeed, when one takes into account the impact of individual actions in the centralized market on prices, prices take an informational role. This role of prices can be used to implement desirable non–monetary allocations, rendering money inessential. Hence, modeling centralized trade as a Walrasian market, in which the market power of agents is exogenously set to zero, misses an important aspect of dynamic trading environments. Namely, that even if an agent’s behavior has a negligible impact on current aggregate outcomes, he can still have market power as long as he is informationally relevant, that is, as long as his actions have a measurable impact on prices.\(^3\) In order to make this point clear, we first consider a setting in which the mapping from actions in the centralized market into prices is deterministic, so that individual behavior has a clear impact on prices. In this case, we show that if agents are patient enough, then there exists a non–monetary equilibrium that implements the first–best regardless of the population size.

The assumption of a perfect correlation between individual behavior and prices is not plausible in large populations, though. Hence, any non–essentiality result that depends on this feature is arguably not satisfactory. For this reason, we introduce noise in the map that takes individual actions into prices, thus reducing the ability of prices to convey information about individual behavior when there are many agents. We show that whether agents become informationally negligible or not in large economies critically depends on the ratio between arrangements. The standard Walrasian market does not specify how payoffs are defined off the equilibrium path and thus it is ill–suited to check the feasibility of competing mechanisms of exchange.

\(^3\)Green (1980) makes a similar point in the context of repeated Cournot competition.
the number of agents who participate in trade and the number of goods that are traded in the centralized market, that is, on how “thick” the centralized market is. Thus, our results provide conditions under which specifying the centralized market as a Walrasian market is justified in monetary models.

This article is not the first to address the question of whether the presence of centralized trading matters for the essentiality of money. Aliprantis, Camera and Puzzello (2007) (henceforth ACP) show that money can fail to be essential if individual actions are observable in the centralized market. There are two criticisms of ACP. First, as Lagos and Wright (2008) point out, in LW agents only observe prices in the centralized market, and not individual actions. Second, as acknowledged in ACP, their assumption that individual actions in the centralized market are observed without noise regardless of the population size is important for their result, which subjects their analysis to the criticism raised in the previous paragraph. Unlike ACP, we assume that agents only observe prices in the centralized market and allow for noisy price formation.

The paper is organized as follows. In the next section, we introduce our framework. In Section 3, we analyze the benchmark case in which the map taking actions in the centralized market into prices is deterministic. In Section 4, we consider our main case, the setting in which prices are noisy. Section 5 concludes. All proofs are in the Appendix.

2 The Framework

We first describe the environment and preferences. Then we describe the economy as an infinitely repeated game.

2.1 Environment and Preferences

Time is discrete and indexed by $t \geq 1$. There are two stages at each date and preferences are additively separable across dates and stages. The population consists of a finite number $N$ of infinitely lived agents. Agents do not discount payoffs between stages in a period and have a common discount factor $\delta \in (0, 1)$ across periods. The two stages of a period differ
in terms of the matching process, preferences, and technology. In the first stage, agents are randomly and anonymously matched in pairs. In the second stage, trade takes place in a centralized market.

Agents trade a divisible special good in the decentralized market and a divisible general good in the centralized market. Both the special good and the general good are perishable across stages and dates. There are $S \geq 3$ types of the special good and $G \geq 2$ types of the general good. Each agent is characterized by a pair $(s,g) \in \{1,\ldots,S\} \times \{1,\ldots,G\}$. An agent of type $(s,g)$ can only produce a special good of type $s$ and a general good of type $g$, and only likes to consume a special good of type $s + 1 \pmod{S}$ and a general good of type $g + 1 \pmod{G}$. There is an equal number of agents of each type. In particular, $N \geq SG$ and the probability that an agent is a consumer in a meeting in the decentralized market, which equals the probability that he is a producer, is $N/S(N-1)$. For convenience, in the remainder of the paper we use $g + 1$ as a shorthand for $g + 1 \pmod{G}$ and say that an agent is of type $g$ if he is of type $(s,g)$ for some $s \in \{1,\ldots,S\}$.

**Decentralized Market** An agent who consumes $q \geq 0$ units of the special good he likes enjoys utility $u(q)$, while an agent who produces $q$ units of the special good pays a cost $c(q)$. The functions $u$ and $c$ are strictly increasing and differentiable, with $u$ strictly concave and $c$ convex. Moreover, $u(0) = c(0) = 0$, $u'(0) > c'(0)$, and $u(\bar{q}) = c(\bar{q})$ for some $\bar{q} > 0$.

Trade in the decentralized market takes place as follows. In every single-coincidence meeting the agents in the match simultaneously and independently choose from $\{\text{yes, no}\}$ after learning whether they are consumers or producers. If either agent in the match says no, then the match is dissolved with no trade occurring. If both agents in the match say yes, that is, if both agents agree to trade, then the producer transfers $q^*$ units of the good to the consumer, after which the match is dissolved.\(^4\)

Let $q^* > 0$ be the unique solution to $u'(q) = c'(q)$. Welfare in a single-coincidence meeting is maximized when the producer transfers $q^*$ units of the special good to the consumer.

**Centralized Market** Production in the centralized market requires effort. We assume there exists an upper bound $\pi > 0$ on the amount of effort an agent can exert in a period.\(^4\)

\(^4\) The same results obtain if the producer can choose the quantity $q$ he transfers to the consumer. The approach we follow simplifies the description of strategies considerably.
An agent who consumes $x \geq 0$ units of the general good he likes obtains utility $U(x)$, while an agent who exerts effort $x$ incurs disutility $x$. The function $U$ is differentiable, strictly increasing and strictly concave, with $U(0) = 0$, $U'(0) > 1$, and $\lim_{x \to \infty} U'(x) = 0$.

Trade in the centralized market takes place as follows. There are $G$ trading posts, one for each type of general good. In every period $t$ each agent $j \in \{1, \ldots, N\}$ simultaneously and independently chooses: (i) the effort $y^j_t$ he directs to the post that trades the good he can produce; (ii) the vector $b^j_t = (b^{j_1}_t, \ldots, b^{j_G}_t)$ of bids he submits to the trading posts. We assume that $\sum_{g=1}^G b^{j_g}_t \leq y^j_t$, that is, the sum of an agent’s bids cannot exceed the total amount of effort he contributes to the trading post. This assumption, which is similar to the assumption in Shapley–Shubik (1977) that agents cannot bid more than their endowments, does not matter for our results and is made for expositional simplicity.\footnote{Drawing a parallel with Shapley and Shubik (1977), one could think that effort has a tangible and perishable counterpart that can be used to make bids. It is possible to show that the results in our paper are unchanged if we assume that there exists an exogenous upper bound on how much an agent can bid in every period.}

Let $\mathcal{N}_p(g)$ be the set of agents who can produce the general good (of type) $g$. The total amount of effort $Y^g_t = \sum_{j \in \mathcal{N}_p(g)} y^j_t$ directed to production in the trading post $g$ yields $\theta^g_t Y^g_t$ units of the general good $g$, where $\theta^g_t$ is a stochastic shock to production in the post $g$. The price of the general good $g$ in period $t$ is

$$p^g_t = \frac{\sum_{j \in \mathcal{N}} b^{j,g}_t}{\theta^g_t \sum_{j \in \mathcal{N}_p(g)} y^j_t},$$

where we adopt the convention that $0/0 = 0$. Agents observe the prices but not the realization of the production shocks. The quantity of the general good $g$ that agent $j$ obtains in period $t$ is then given by $x^{j,g}_t = b^{j,g}_t / p^g_t$. Note that for all $g \in \{1, \ldots, G\}$,

$$\theta^g_t \sum_{j \in \mathcal{N}_p(g)} y^j_t = \frac{1}{p^g_t} \sum_{j=1}^N b^{j,g}_t = \sum_{j=1}^N \frac{b^{j,g}_t}{p^g_t} = \sum_{j=1}^N x^{j,g}_t,$$

so that aggregate supply is always equal to aggregate demand in each trading post.

The introduction of aggregate shocks to production is a natural way of making the map from individual actions into prices non-deterministic. In our analysis, we take the shocks $\theta^g_t$ to be independently and identically distributed over time and across trading posts according
to a differentiable cdf $\Omega$ with $\mathbb{E}[\Omega] = 1$. We assume that $\Omega$ has support in some interval $[\theta_{\text{min}}, \theta_{\text{max}}]$, where $0 < \theta_{\text{min}} < \theta_{\text{max}} < \infty$. Note that the map between actions and prices is deterministic when $\theta_{\text{min}} = \theta_{\text{max}} = 1$. The assumption that $\theta_{\text{min}} > 0$ and $\theta_{\text{max}} < \infty$ is natural. If either $\theta_{\text{min}} = 0$ or $\theta_{\text{max}} = \infty$, the shocks to production can be so extreme that irrespective of the population size, the output in a trading post where only one agent exerts effort can be greater than the output in another trading post where all agents exert maximum effort.

Let $U : \mathbb{R}_+ \to \mathbb{R}_+$ be given by

$$U(x) = \int U(\theta x) d\Omega(\theta).$$

It is immediate to see that $U_\Omega$ is strictly concave. Thus, the problem $\max_{x \geq 0} U_\Omega(x) - x$ has a unique solution, that we denote by $x^*$. Note that $U'(0) = U'(0)E[\Omega] > 1$, and so $x^* > 0$. We assume that $x^* < \overline{x}$. Since the disutility from effort is linear in the amount of effort, welfare in the centralized market is maximized when for each realization $(\theta^1, \ldots, \theta^G)$ of the shocks to production, the agents of type $g$ consume $\theta^g x^*$ units of the general good they like. This requires that total effort for the production of each type of general good is $(N/G)x^*$.

Note that there are gains from trade in the centralized market. This is in contrast to LW, where all agents produce and consume the same good in the centralized market, and so only the exchange of goods for money is beneficial. Since our focus is on non monetary equilibria, it is natural to assume gains from trade in the centralized market. Lagos and Wright (2003) point out that their assumption of no gains from trade in the centralized market is only meant to simplify the analysis.

### 2.2 The Game

The economy is an infinitely repeated game in which the stage game consists of one round of trade in the decentralized market followed by one round of trade in the centralized market. We describe strategies in the repeated game by means of automata. Let $A_1 = \{\text{yes, no}\}$ be the action set of an agent in a single–coincidence meeting in the decentralized market and

$$A_2 = \left\{ a_2 = (y, (b^1, \ldots, b^G)) : y \leq \overline{x} \text{ and } \sum_{g=1}^{G} b^g \leq y \right\}$$
be the action set of an agent in the centralized market. An automaton for an agent of type 
\((s, g)\) is a list \(\{W, w^0, (f_1, f_2), (\tau_1, \tau_2)\}\) where: (i) \(W\) is the set of states; (ii) \(w^0 \in W\) is the initial state; (iii) \(f_1 : W \times \{1, \ldots, S\} \times \{1, \ldots, G\} \rightarrow A_1\) and \(f_2 : W \rightarrow A_2\) are decision rules in the decentralized and centralized markets, respectively; (iv) \(\tau_1 : W \times \{1, \ldots, S\} \times \{1, \ldots, G\} \times A_1^2 \rightarrow W\) and \(\tau_2 : W \times A_2 \times \mathbb{R}_+^G \rightarrow W\) are transition rules in the decentralized and centralized markets, respectively.

If the decision rules for an agent of type \((s, g)\) are given by \((f_1, f_2)\), then the agent’s behavior in state \(w\) is as follows: (i) he chooses \(f_1(w, s', g')\) in a single–coincidence meeting in the decentralized market if his partner’s type is \((s', g')\); (ii) he chooses \(f_2(w)\) in the centralized market. If the transition rules for an agent of type \((s, g)\) are given by \((\tau_1, \tau_2)\), then: (i) \(\tau_1(w, s', g', a_1, a'_1)\) is the agent’s new state when he enters the decentralized market in state \(w\) if he chooses \(a_1\), and his partner is of type \((s', g')\) and chooses \(a'_1\); (ii) \(\tau_2(w, a_2, p)\) is the agent’s new state when he enters the centralized market in state \(w\), chooses \(a_2\), and observes the vector of prices \(p\).\(^6\) We restrict attention to strategy profiles in which the set of states is the same for all agents and agents of the same type follow the same automaton.

Given a strategy profile \(\sigma\), a profile of states for an agent is a map \(\pi : W \times \{1, \ldots, S\} \times \{1, \ldots, G\} \rightarrow \{1, \ldots, N - 1\}\) such that \(\pi(w, s, g)\) is the number of agents of type \((s, g)\) who are in state \(w\), excluding the agent himself if he is of type \((s, g)\) Denote the set of all state profiles by \(\Pi\). A belief for an agent is a map \(p : \Pi \rightarrow [0, 1]\) such that \(\sum_{\pi \in \Pi} p(\pi) = 1\), where \(p(\pi)\) is the probability the agent assigns to the event that the profile of states is \(\pi\). Let \(\Delta\) be the set of all possible beliefs. A belief system for an agent is a map \(\mu : W \rightarrow \Delta\). In an abuse of notation, we use \(\mu\) to denote the profile of belief systems where all agents have the same belief system \(\mu\).

We consider sequential equilibria of the repeated game. The first–best is achieved if in each period trade takes place in all single–coincidence meetings in the decentralized market and welfare is maximized in the centralized market.

\(^6\)For simplicity, we only define transition rules for agents in the decentralized market if they participate in a single–coincidence meeting. In what follows, we always assume that an agent’s state does not change if he participates in a non–coincidence meeting.
3 Deterministic Prices

In order to make clear that being explicit about the exchange process in the centralized market has important implications for the essentiality of money, in this section we study the benchmark case in which the map that takes individual actions in the centralized market into prices is deterministic. We show that in this case there exists an equilibrium that sustains the first-best if agents are patient enough. In what follows, we let $e_g$, with $g \in \{1, \ldots, G\}$, be the vector with all entries equal to zero except the $g$th entry, which is equal to one, and $e$ be the vector with all entries equal to one.

Define $\sigma^*$ to be the strategy profile in which an agent of type $g$ behaves according to the following automaton. The set of states is $W = \{C, D, A\}$ and the initial state is $C$. The decision rules are given by

$$f_1(w, s', g') = \begin{cases} 
\text{yes} & \text{if } w \in \{C, D\} \\
\text{no} & \text{if } w = A
\end{cases}$$

and

$$f_2(w) = \begin{cases} 
(x^*, x^*e_{g+1}) & \text{if } w = C \\
(\overline{x}, 0) & \text{if } w = D \\
(0, 0) & \text{if } w = A
\end{cases}$$

For instance, an agent in state $C$ behaves as follows. In the decentralized market, he agrees to trade regardless of his partner’s type. In the centralized market he contributes effort $x^*$ to the trading post $g$ and bids $x^*$ at the trading post $g + 1$. The transition rules are given by

$$\tau_1(w, s', g', a_1, a'_1) = \begin{cases} 
C & \text{if } w = C \text{ and } (a_1, a'_1) = (\text{yes}, \text{yes}) \\
D & \text{if } w = C \text{ and } (a_1, a'_1) \neq (\text{yes}, \text{yes}) \\
w & \text{if } w \in \{D, A\}
\end{cases}$$

and

$$\tau_2(w, a_2, p) = \begin{cases} 
C & \text{if } w \in \{C, D\} \text{ and } p \in \{e\} \cup \mathcal{P} \\
A & \text{if } w \in \{C, D\} \text{ and } p \notin \{e\} \cup \mathcal{P} \text{ or } w = A
\end{cases},$$

where $\mathcal{P}$ is the set of possible price vectors in the centralized market when $N - 2$ agents are in state $C$ and the two remaining agents are in state $D$. For instance, an agent in state $C$ in a single-coincidence meeting in the decentralized market remains in $C$ only if trade takes place in his match, otherwise he moves to state $D$. Likewise, an agent in state $C$ in the centralized market stays in $C$ if the price he observes belongs to $\{e\} \cup \mathcal{P}$, otherwise he moves
to state $A$. Note that a necessary condition for a price vector to be an element of $\mathcal{P}$ is that total effort by the agents exceeds the sum of their bids by $2\varpi$. This fact is useful later on.

Now let $\mu^*$ be the belief system such that: (i) an agent in state $C$ believes that all other agents are in state $C$; (ii) an agent in state $A$ believes that all other agents are in state $A$; (iii) an agent in state $D$ believes that there exists one other agent in state $D$ and the remaining agents are in state $C$.\footnote{Since under $\sigma^*$ all agents in state $D$ behave in the same way, the continuation payoff of an agent in state $D$ does not depend on the type of the other agent who is in the same state. Thus, there is no loss of generality in describing the belief of an agent in state $D$ as we do above.} Clearly, $(\sigma^*, \mu^*)$ is a consistent assessment and $\sigma^*$ implements the first–best. We have the following result.

**Proposition 1.** Suppose that $\varpi + U(x^*) - x^* \geq c(q^*)$. There exists $\delta' \in (0,1)$ independent of $N$ and $G$ such that $(\sigma^*, \mu^*)$ is a sequential equilibrium for all $\delta \geq \delta'$.

A sketch of the proof of Proposition 1 is as follows. Consider first an agent in state $C$ in a single–coincidence meeting in the decentralized market. If he is a producer, his flow payoff gain from a one–shot deviation is $c(q^*)$. However, in the centralized market meeting that immediately follows, he exerts effort $\varpi$ without receiving anything in return, incurring a payoff loss of $\varpi + U(x^*) - x^*$. Since $\varpi + U(x^*) - x^* \geq c(q^*)$ by assumption, the one–shot deviation is not profitable.

Consider now an agent in state $C$ in the centralized market. First note that any one–shot deviation by the agent leads to a price vector $p \notin \mathcal{P}$. Indeed, total effort by the other agents is equal to the sum of their bids. Given that the agent can at most exert effort $\varpi$, it is not possible to have total effort exceeding the sum of bids by $2\varpi$. Moreover, it turns out that no one–shot deviation with $p = e$ leads to a flow payoff gain. Since any one–shot deviation with $p \notin \{e\} \cup \mathcal{P}$ triggers permanent autarky, we can then conclude that no one–shot deviation is profitable if the agent is patient enough.

To finish, we need to show that behavior off the path of play is credible. It is immediate to see that no agent in state $A$ has an incentive to deviate. It is also immediate to see that no agent is ever in state $D$ in the decentralized market. Consider then an agent in state $D$ in the centralized market. For the same reason as in the previous paragraph, we are done if we show that any one–shot deviation by the agent leads to a price vector $p \notin \{e\} \cup \mathcal{P}$. Since one
other agent is in state $D$, the remaining $N - 2$ agents are in state $C$, and the agent cannot
bid more than his effort, any one-shot deviation by the agent implies that total production
differs from the sum of bids by an amount $\eta \in [\bar{x}, 2\bar{x})$, which leads to the desired result.

Note that our strategy of proof is quite different from the strategy of proof in ACP. Their
environment is very much like a repeated prisoner’s dilemma in the sense that communicating
a defection to the population in the centralized market involves taking an action that is
myopically optimal. In our setting, communicating a defection is costly in terms of flow
payoffs. What sustains the threat of punishment is that if an agent deviates off the path of
play, this leads to an even greater punishment.

Finally, note that Proposition 1 holds without the condition that $c(q^*) \leq \bar{x} + U(x^*) - x^*$. In our
candidate equilibrium, since cooperation is restored after agents observe a price in $\mathcal{P}$,
the only punishment for an agent who defects in the decentralized market is his payoff loss
in the subsequent round of trading in the centralized market. In order for such a punishment
to be effective, it must be that $c(q^*)$, the cost of cooperating in the decentralized market, is
small enough. The restriction $c(q^*) \leq \bar{x} + U(x^*) - x^*$ can be dropped if a defection in the
decentralized market were to lead to a greater punishment, as it would be the case if a price
in $\mathcal{P}$ led to a number of periods of no trade in both markets.

4 Noisy Prices

The analysis with deterministic prices shows that providing microfoundations for the process
of exchange in the centralized market matters for the essentiality of money. However, the
assumption that prices are perfectly correlated with actions, which implies that agents are
always informationally relevant, and thus money is not essential, is not plausible in large
economies. In this section we analyze the main case in which the price formation process
is not deterministic. Our main result is that whether agents are informationally relevant in
large economies depends on the structure of the centralized market.

The presence of noise in prices implies that the probability that an agent’s actions can
affect prices in a noticeable way is small if in each trading post total effort and bids by the
other agents are large. Then, a natural conjecture is that agents are informationally relevant
if total activity in each trading post is not too large. A sufficient condition for this is that the number of trading posts is not small relative to the population size, that is, the centralized market is not thick. In what follows we show that market thickness, as measured by the ratio $N/G$, is indeed a key determinant of the agents' informational relevance.

Our first result is that if $G$ is fixed, so that market thickness increases with $N$, then the first–best is not an equilibrium outcome when the population is large enough no matter how patient agents are.

**Proposition 2.** Fix $G > 1$. For every strategy profile $\sigma$ that implements the first–best and for all $\delta \in (0, 1)$, there exists $N' \geq 1$ such that $\sigma$ is not a Nash equilibrium if $N \geq N'$.

The idea of the proof is as follows. Consider a strategy profile $\sigma$ that implements the first–best. There are two channels through which an agent can be punished for not producing in the decentralized market. The first is through a standard contagion process that is limited to the agents who participate in single–coincidence meetings without production. The second is through changes of prices in the centralized market. It is well–known that when there are many agents, the first channel is not effective in disciplining behavior, see Kandori (1992) and Araujo (2004). Now observe that if $\sigma$ is to implement the first–best, then in every period total effort in each trading post must be $(N/G)x^*$. As it turns out, when the population is large, this is only possible in equilibrium if all agents always bid $x^*$ for the general good they like. However, when total effort and bids in each trading post are $(N/G)x^*$, an agent’s impact on the distribution of prices is negligible when there are many agents. This implies that deviations in the decentralized market from the behavior prescribed by $\sigma$ take a long time to influence prices. Thus, when the population is large, the second channel is also not effective in disciplining behavior. To put it differently, in large populations, efficiency in the centralized market is incompatible with efficiency in the decentralized market.

A natural question to ask is whether some trade in the decentralized market can be sustained as an equilibrium outcome in large populations when the number of trading posts is fixed. As the discussion in the previous paragraph suggests, this may be possible if one sacrifices efficiency in the centralized market by keeping the volume of trade in some trading posts small enough to make agents informationally relevant. We conjecture that there exists
$\delta' \in (0, 1)$ such that efficient trade in the decentralized market is an equilibrium outcome for all $\delta \geq \delta'$ regardless of the population size.

A key element in the proof of Proposition 2 is that when all agents exert effort $x^*$ and bid $x^*$ for the good they like, the set of possible price vectors one can observe in the centralized market has a nonempty interior. This is the case if in each period the shocks to production are independent across posts, as we have assumed. However, this is also the case if the shocks to production in each trading post are the sum of a common component and an idiosyncratic component. Thus, Proposition 2 is valid under a more general specification of the shocks to production than we assumed.

The preceding discussion suggests that if $N/G$ is bounded above, which would be the case, for instance, if each trading post has a finite capacity, then as long as agents are patient enough, the first–best is an equilibrium outcome regardless of the population size. Proposition 3 below shows that this is indeed the case. Thus, how exchange in the centralized market is organized matters for the essentiality of money when prices are noisy.

**Proposition 3.** Suppose that \( \lim_{N \to \infty} N/G < \infty \). If there exists $\Lambda > 0$ such that $\Omega'(\theta) \geq \Lambda$ for all $\theta \in [\theta_{\min}, \theta_{\max}]$, then there exists $\delta'' \in (0, 1)$ independent of $N$ such that the first–best is an equilibrium outcome for all $\delta \geq \delta''$.

It turns out that the above result holds under the weaker assumption that $\Omega'(\theta_{\min}) > 0$ and $\Omega'(\theta_{\max}) > 0$. We discuss this at the end of the proof of Proposition 3 in the Appendix. Moreover, when either $\Omega'(\theta_{\min}) = 0$ or $\Omega'(\theta_{\max}) = 0$, it is possible to show that for all $\varepsilon > 0$, there exists $\delta'' \in (0, 1)$ independent of $N$ such that if $\delta \geq \delta''$, then there exists an equilibrium in which all agents obtain a payoff within distance $\varepsilon$ of the first–best.

## 5 Concluding Remarks

In this paper we investigate the relationship between the essentiality of money and centralized trading. A key feature of our analysis is that we explicitly model the process of exchange in the centralized market. Doing so means that one has to introduce a map between individual actions in the centralized market and prices. Even though we restrict attention to a particular
map, the one derived from a strategic market game, our message is quite general. Namely, that the essentiality of money is tied to the informational relevance of agents, which depends on the market structure. Thus, modeling the centralized market as a Walrasian market, where agents are informationally irrelevant by assumption, needs to be justified. Our analysis shows that if we take the centralized market to be organized as a market game, then money is essential as long the ratio between the number of agents who participate in trade and the number of goods that are traded in the centralized market is sufficiently large.

6 Appendix

6.1 Proof of Proposition 1

Let $V_{DM}^C$ and $V_{CM}^C$ be the (discounted) lifetime payoffs to an agent in state $C$ before he enters the decentralized and centralized markets, respectively. Then,

$$V_{DM}^C = \frac{1}{1-\delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U(x^*) - x^* \right\} \quad \text{and} \quad V_{CM}^C = U(x^*) - x^* + \delta V_{DM}^C.$$ 

Now let $V_A$ be the lifetime payoff to an agent in state $A$. It is easy to see that $V_A = 0$. Finally, let $V_D$ be the lifetime payoff to an agent in state $D$ before he enters the centralized market. Since an agent in state $D$ in the centralized market believes that there are $N-2$ agents in state $C$ and one other agent in state $D$, he believes that the price vector in the centralized market will lie in the set $\mathcal{P}$. Thus,

$$V_D = -\bar{x} + \delta V_{CM}^C.$$ 

We start with incentives in state $C$. An agent in state $C$ in the decentralized market has no profitable one-shot deviation if

$$-c(q^*) + V_{CM}^C = -c(q^*) + U(x^*) - x^* + \delta V_{DM}^C \geq V_D = -\bar{x} + \delta V_{CM}^C,$$

which is satisfied since $\bar{x} + U(x^*) - x^* \geq c(q^*)$. Consider then an agent in state $C$ in the centralized market. Without loss of generality, we can assume that the agent’s type is $g = 1$.

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8 Another trading mechanism would be a double auction. Large double auctions have also been used to provide non-cooperative foundations for competitive markets. See Rustichini et al. (1994) and Cripps and Swinkels (2006).
Let $a_2 = (y, (b^1, \ldots, b^G)) \neq (x^*, x^* e_2)$ be the agent’s action and denote the corresponding vector of prices by $p = (p^1, \ldots, p^G)$. We first show that there exists no profitable one-shot deviation by the agent when $a_2$ is such that $p = e$. It is immediate to see that $p^g = 1$ for $g > 1$ if, and only if, $b^2 = x^*$ and $b^g = 0$ for $g > 2$. Moreover, $p^1 = 1$ if, and only if, $b^1 = y - x^*$. Thus, when $a_2 \neq (x^*, x^* e_2)$ and $p = e$, the agent’s flow payoff is $U(x^*) - y$ with $y > x^*$, which is smaller than $U(x^*) - x^*$. The desired result follows from the fact $V_{DM}^C$ is the highest continuation payoff possible for the agent.

Now we show that if the agent is patient enough, then he has no profitable one-shot deviation when $a_2$ is such that $p \neq e$. From the main text, we know that any one-shot deviation by the agent necessarily leads to a price vector $p \notin P$. Hence, a one-shot deviation with $p \neq e$ leads to state $A$, which implies a loss of continuation payoffs equal to $V_{DM}^C$. Next, note that the agent’s flow payoff from $a_2$ is $U(b^2/p^2) - y$, where

$$p^2 = \frac{(\frac{N}{G} - 1) x^* + b^2}{\frac{N}{G} x^*}.$$  

It is easy to see that $b^2/p^2$ is maximized when $b^2 = y - b^1$. Thus, the highest flow payoff gain possible for the agent given the choice of $y$ in $a_2$ is

$$\Delta(y) = U\left(y - \frac{\frac{N}{G} x^*}{\frac{N}{G} - 1} x^* + y\right) - y - [U(x^*) - x^*].$$

Since $U$ is strictly concave and $U^{\prime\prime} = 1$, we then have that

$$\Delta(y) \leq U^{\prime\prime}\left\{y - \frac{\frac{N}{G} x^*}{\frac{N}{G} - 1} x^* + y - x^*\right\} - (y - x^*) = y - \frac{x^* - y}{\frac{N}{G} - 1} x^* + y \leq \frac{x^*}{2},$$

where we use the fact that $N \geq SG$ and $S \geq 3$. From this, one can see that there exists $\delta^1 \in (0, 1)$ independent of $N$ and $G$ such that no one-shot deviation is profitable if $\delta \geq \delta^1$.

Consider incentives in state $D$. Since no agent is ever in state $D$ in the decentralized market, we only need to consider an agent in state $D$ in the centralized market. Suppose, once again without loss of generality, that the agent’s type is $g = 1$. Let $a_2 = (y, (b^1, \ldots, b^G)) \neq (x^*, 0)$ be his action and denote the corresponding vector of prices by $p = (p^1, \ldots, p^G)$. In order to find an upper bound for the agent’s flow payoff gain, suppose he can place a bid $b^2 = \pi$ in the post $g = 2$ without having to exert effort. Since total production of the good...
\( g = 2 \) is at most \((N/G - 1)x^* + \bar{x}\) (when the other agent in state \( D \) is of type \( g = 2 \)), total bids for this good by the other agents are at least \((N/G - 2)x^*\). Given that \(N/G \geq S \geq 3\), an upper bound for the flow payoff the agent can obtain is then given by

\[
U \left( \bar{x} \frac{N - 1}{N - 2} x^* + \bar{x} \right) = \frac{N}{G} \left( 1 + \frac{x^*}{N - 2} \right) \leq U(2\bar{x}),
\]

which is independent of \( N \) and \( G \). Now recall from the main text that any one-shot deviation by the agent necessarily leads to a price vector \( p \notin \{e\} \cup \mathcal{P} \). The same argument as in the previous paragraph then shows that there exists \( \delta^2 \in (0, 1) \) independent of \( N \) and \( G \) such that no one-shot deviation is profitable if \( \delta \geq \delta^2 \).

To finish, since state \( A \) is absorbing and involves no trade in both markets, it is immediate to see that no one-shot deviation is profitable in this state. We can then conclude that \((\sigma^*, \mu^*)\) is an equilibrium as long as \( \delta \geq \max\{\delta^1, \delta^2\} \).

### 6.2 Proof of Proposition 2

A necessary condition for the ex-ante welfare to be maximized in the centralized market is that for each \( g \in \{1, \ldots, G\} \), all agents of type \( g - 1 \) submit the same bid \( b > 0 \) and total effort for the production of good \( g \) is \((N/G)x^*\). Consider a strategy profile \( \sigma \) that implements the first-best and let \( b_t^g > 0 \) be the (on the path of play) bid that the agents of type \( g - 1 \) submit to the trading post \( g \) in period \( t \). Since agents cannot bid more than their effort, \( b_t^g > x^* \) implies that total effort for the production of good \( g - 1 \) in period \( t \) is greater than \((N/G)x^*\), a contradiction. Hence, \( b_t^g \leq x^* \) for all \( t \geq 1 \) and \( g \in \{1, \ldots, G\} \). We claim that if \( b_t^g < x^* \) for some \( t \geq 1 \) and \( g \in \{1, \ldots, G\} \), then for each \( \delta \in (0, 1) \), there exists \( N' \geq 1 \) such that \( \sigma \) is not a Nash equilibrium if \( N \geq N' \).

Suppose that \( b_t^g < x^* \). Since total effort for the production of good \( g \) in period \( t \) is \((N/G)x^*\), at least one agent of type \( g - 1 \) exerts effort \( x^* \) or more in period \( t \). Consider one such agent and suppose he deviates by increasing his bid from \( b_t^g \) to \( x^* \). The agent’s flow payoff gain from this deviation is

\[
\Delta = U_\Omega \left( x^* \frac{N}{G} x^* \frac{N - 1}{N} b_t^g + x^* \right) - U_\Omega(x^*) \geq U_\Omega \left( x^* \left\{ 1 + \frac{(S - 1)(x^* - b_t^g)}{(S - 1)b_t^g + x^*} \right\} \right) - U_\Omega(x^*),
\]
since \( N/G \geq S \). In particular, \( \Delta \) is positive regardless of the population size (but it does depend on \( b_t^g \)).

Now observe that if the realized value of the period–t shock to production in the post \( g \) is \( \theta^g \), then the (on the path of play) price of good \( g \) is \( b_t^g / \theta^g x^* \). Hence, the deviation under consideration leads to a punishment only if

\[
\left( \frac{N}{G} - 1 \right) b_t^g + x^* > \frac{b_t^g}{\theta^g \frac{N}{G} x^*} \Leftrightarrow \theta^g < \theta_{\min} \left( 1 + \frac{x^* - b_t^g}{\frac{N}{G} b_t^g} \right).
\]

Given that the greatest punishment possible for an agent is permanent autarky, that is, no trade in both markets in all subsequent periods, an upper bound for the agent’s payoff loss after his deviation is

\[
\begin{align}
\lambda & \frac{\delta}{1 - \delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U_\Omega(x^*) - x^* \right\},
\end{align}
\]

where

\[
\lambda = \Pr \left\{ \theta^g \in \left[ \theta_{\min}, \theta_{\min} \left( 1 + \frac{x^* - b_t^g}{\frac{N}{G} b_t^g} \right) \right] \right\}.
\]

Since \( \lim_{N \to \infty} \lambda = 0 \), we can then conclude that there exists \( N' \geq 1 \) such that \( \Delta \) is greater than (1) for all \( N \geq N' \). This establishes the desired result.

Let now \( \sigma \) be a strategy profile implementing the first–best such that \( b_t^g = x^* \) for all \( t \geq 1 \) and \( g \in \{1, \ldots, G\} \). Note that in order for \( \sigma \) to implement the first–best, it must be that on the path of play all agents always exert effort \( x^* \) in the centralized market. We claim that for each \( \delta \in (0, 1) \), there exists \( N' \geq 1 \) such that \( \sigma \) is not a Nash equilibrium if \( N \geq N' \). We divide the argument in two parts.

**Part 1** Suppose that \( M < N/G \) agents in the centralized market deviate from the behavior that \( \sigma \) prescribes on the path of play, that is, they either do not exert effort \( x^* \) or submit a bid for the good they like that is different from \( x^* \). By relabeling the agents if necessary, we can assume that the agents under consideration are the agents \( 1 \) to \( M \). Let \( b_{t,j}^g \) be the bid of agent \( j \in \{1, \ldots, M\} \) in the post \( g \) and \( y_{t,j}^g \) be the effort that \( j \) exerts for the production of good \( g \). Note that \( y_{t,j}^g = 0 \) if \( j \) is not of type \( g \). Moreover, let \( m_g \) be the number of agents in \( \{1, \ldots, M\} \) who are of type \( g \). Given a realization \( \theta = (\theta^1, \ldots, \theta^G) \) of the shocks to production, the vector of prices in the centralized market is then given by
\[ \hat{p}(\theta) = \left( (1/\theta_1)\hat{p}_1, \ldots, (1/\theta_G)\hat{p}_G \right), \] where
\[ \hat{p}_g = \frac{(N/G - m_{g-1})x^* + \sum_{j=1}^M b_j^g - 1}{(N/G - m_g)x^* + \sum_{j=1}^M y_j^g}. \]

Since \[ |(m_g - m_{g-1})x^* + \sum_{j=1}^M b_j^g - 1 - \sum_{j=1}^M y_j^g| \leq M(x^* + \pi), \]

it is easy to see that for all \( g \in \{1, \ldots, G\} \) such that
\[ |\hat{p}_g - 1| \leq \kappa_N(M) = \frac{M(x^* + \pi)}{(N/G - M)x^*} \]

for all \( g \in \{1, \ldots, G\} \). Note that \( \kappa_N(M) \) is increasing in \( M \) and \( \lim_{N \to \infty} \kappa_N(M) = 0 \). In particular, for each \( M \geq 1 \), \( \hat{p}_g \) converges uniformly to \( 1 \) as \( N \) increases to infinity.

By construction, \( \hat{p}(\theta) \) does not belong to the set \( P_{\text{path}} \) of price vectors one can observe on the path of play if, and only if, there exists \( g \in \{1, \ldots, G\} \) such that
\[ (1/\theta_g)\hat{p}_g \notin [1/\theta_{\text{max}}, 1/\theta_{\text{min}}]. \]

If \( \hat{p}_g < 1 \), the probability that (3) does not happen is the probability that \( \theta_g \leq \hat{p}_g \theta_{\text{max}} \), which is \( \Omega(\hat{p}_g \theta_{\text{max}}) \). If \( \hat{p}_g > 1 \), the probability that (3) does not hold is the probability that \( \theta_g \geq \hat{p}_g \theta_{\text{min}} \), which is \( 1 - \Omega(\hat{p}_g \theta_{\text{min}}) \). Hence, the probability that \( \hat{p}(\theta) \notin P_{\text{path}} \) is
\[ 1 - \prod_{g=1}^G \left[ \Omega(\hat{p}_g \theta_{\text{max}}) \mathbb{I}\{\hat{p}_g < 1\} + \Omega(\hat{p}_g \theta_{\text{min}}) \mathbb{I}\{\hat{p}_g \geq 1\} \right] \]
\[ \leq 1 - \prod_{g=1}^G \min \{ \Omega([1 - \kappa_N(M)]\theta_{\text{max}}), 1 - \Omega([1 + \kappa_N(M)]\theta_{\text{min}}) \} = \pi_N(M), \]

where \( \mathbb{I} \) is the indicator function and the inequality follows from (2). Notice that \( \pi_N(M) \) is increasing \( M \) and such that \( \lim_{N \to \infty} \pi_N(M) = 0 \).

To finish, observe that
\[ U_\Omega(x^*) = \int U(\theta x^*)d\Omega(\theta) \leq \int U(\theta x^*/\hat{p}_g)d\Omega(\theta) + \frac{x^*(1 - \hat{p}_g)}{\hat{p}_g} \int \theta U'_{x^*}(\hat{p}_g)d\Omega(\theta) \]
\[ \leq U_\Omega(x^*/\hat{p}_g) + \frac{x^*|1 - \hat{p}_g|}{\hat{p}_g} \int \theta U'_{x^*}(\hat{p}_g)d\Omega(\theta), \]

where the first inequality follows from the strict concavity of \( U \). From (2) and the fact that \( \int \theta U'(\theta x^*)d\Omega(\theta) = 1 \), it is easy to see that for each \( M \geq 1 \), there exists \( N_1 \geq 1 \) such that
\[ U_\Omega(x^*) \leq U_\Omega(x^*/\hat{p}_g) + \frac{2\kappa_N(M)}{1 - \kappa_N(M)} \]

(5)
for all \( N \geq N_1 \). Note that \( U_\Omega(x^*/\bar{p}_g) - x^* \) is the payoff in the centralized market to an agent of type \( g - 1 \) who exerts effort \( x^* \) and bids \( x^* \) for the good he likes when \( M \) other agents deviate from the behavior that \( \sigma \) prescribes on the path of play.

**Part II)** Consider the following deviation: (\( i \)) in the decentralized market, never agree to trade if a producer and always agree to trade if a consumer; (\( ii \)) in the centralized market, always exert effort \( x^* \) and always bid \( x^* \) for the good one likes. In what follows we show that there exists \( N' \geq 1 \) such that this deviation is profitable if \( N \geq N' \), which establishes the desired result.

First, let \( T \) be such that
\[
\frac{(1-\varepsilon)(1-\delta^T)}{1-\delta} \left\{ \left(1-\varepsilon\right) \frac{N}{S(N-1)} u(q^*) + U_\Omega(x^*) - \varepsilon - x^* \right\} > \frac{1}{1-\delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U_\Omega(x^*) - x^* \right\}.
\]
Consider now an agent who follows the deviation described above and let \( O_t \) be the event that up to (but not including) period \( t \) the price vectors in the centralized market are all in \( P_{\text{path}} \), in which case no more than \( 2^t - 1 \) agents in \( t \) deviate from the behavior prescribed by \( \sigma \) on the path of play. Then, conditional on \( O_t \), we have that: (\( i \)) the probability that the agent’s partner in a single–coincidence meeting in period \( t \) does not agree to trade is bounded above by \( (2^t - 1)/N \); (\( ii \)) the probability that the price vector in the centralized market in period \( t \) does not belong to \( P_{\text{path}} \) is bounded above by \( \pi_N(2^t - 1) \). Note that in (\( ii \)) we used the fact that \( \pi_N(M) \) is increasing in \( M \). Therefore, since the right side of (5) is also increasing in \( M \), a lower bound for the agent’s payoff is
\[
\frac{1-\delta^T}{1-\delta} \left[ 1 - \pi_N(2^T - 1) \right] \left\{ \frac{N}{S(N-1)} \left( 1 - \frac{2^T - 1}{N - 1} \right) u(q^*) + U_\Omega(x^*) - \frac{2x^*\kappa_N(2^T - 1)}{1 - \kappa_N(2^T - 1)} - x^* \right\}
\]
as long as \( N \geq N_1 \). Given that \( \lim_{N\to\infty} \kappa_N(2^T - 1) = \lim_{N\to\infty} \pi_N(2^T - 1) = 0 \), it is easy to see that there exists \( N' \geq \max\{N_1, N_2\} \) such that the deviation is profitable if \( N \geq N' \).

### 6.3 Proof of Proposition 3

As in the proof of Proposition 1, for simplicity we consider a strategy profile in which the only punishment for an agent who defects in the decentralized market is his payoff loss in
the subsequent round of trading in the centralized market. Thus, as before, in order for such a punishment to be effective, it must be that \( c(q^*) \) is small enough. More precisely, in what follows, we assume that there exists \( 0 < \kappa < \min\{x^*, \bar{x} - x^*\} \) such that

\[
-c(q^*) + U\Omega(x^*) - x^* \geq U\Omega(x^* - \kappa) - (x^* - \kappa).
\]

This assumption can be dropped if a defection in the decentralized market were to lead to a greater expected punishment.

Define \( \sigma^* \) to be the strategy profile in which an agent of type \( g \) behaves according to the following automaton. The set of states is \( W^g = \{C, D_{-1}^g, D_0^g, \{D_0^g\}_{g' \neq g}, A\} \) and the initial state is \( C \). The decision rules are

\[
f_1(w, s', g') = \begin{cases} 
\text{yes} & \text{if } w \neq A \\
\text{no} & \text{if } w = A
\end{cases}
\quad \text{and} \quad
f_2(w) = \begin{cases} 
(x^*, x^* e_{g+1}) & \text{if } w = C \\
(x^* - \kappa, (x^* - \kappa) e_{g+1}) & \text{if } w = D_{-1}^g \\
(x^* + \kappa, (x^* + \kappa) e_{g+1}) & \text{if } w = D_0^g \\
(x^* e_{g+1} + (x^* - \varepsilon) e_{g+1}) & \text{if } w = D_g^g \\
(0, 0) & \text{if } w = A
\end{cases}
\]

where \( \varepsilon > 0 \) is small enough that \( U\Omega(\varepsilon) - x^* < 0 \). The transition rules are

\[
\tau_1(w, s', g', a_1, a'_1) = \begin{cases} 
C & \text{if } w = C \text{ and } (a_1, a'_1) \in \{(yes, yes), (no, no)\} \\
D_g^g & \text{if } w = C, (a_1, a'_1) \in \{(yes, no), (no, yes)\}, \text{ and } g' \neq g \\
D_{-1}^g & \text{if } w = C, (a_1, a'_1) = (no, yes), \text{ and } g' = g \\
D_0^g & \text{if } w = C, (a_1, a'_1) = (yes, no), \text{ and } g' = g \\
w & \text{if } w \neq C
\end{cases}
\quad \text{and} \quad
\tau_2(w, a_2, p) = \begin{cases} 
C & \text{if } w \neq A \text{ and } p \in \tilde{P} \\
A & \text{if } w \neq A \text{ and } p \notin \tilde{P} \text{ or } w = A
\end{cases}
\]

where \( \tilde{P} = \{p \in \mathbb{R}^G_+ : p = ((1/\theta^1), \ldots, (1/\theta^G)) \text{ with } (\theta^1, \ldots, \theta^G) \in [\theta_{\text{min}}, \theta_{\text{max}}]^G\} \). By construction, the profile \( \sigma^* \) implements the first–best.

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\(^9\)The definition of automata presented in Section 2 assumes that the set of states is the same regardless of the agent’s type in the centralized market. We can extend the definition of \( \sigma^* \) to accommodate this requirement as follows. The set of states is \( W = \{A, C\} \cup_{g \in [1, \ldots, G]} \{D_{-1}^g, D_0^g, \{D_0^g\}_{g' \neq g}\} \). The decision rules \( f_1 \) and \( f_2 \) for an agent of type \( g \) are such that \( f_1(w, s', g') = \text{yes} \) and \( f_2(w) = (x^*, x^* e_{g+1}) \) if \( w \notin W^g \). The transition rules \( \tau_1 \) and \( \tau_2 \) for an agent of type \( g \) are such that \( \tau_1(w, s', g', a_1, a'_1) = \tau_2(w, a_2, p) = w \) if \( w \notin W^g \). Since an agent of type \( g \) is never on a state \( w \notin W^g \) there is no need to check for one–shot deviations in such states.
Now let $\mu^*$ be the belief system where: (i) an agent in state $C$ believes that all other agents are in state $C$; (ii) an agent in state $A$ believes that all other agents are in state $A$; (iii) an agent of type $g$ in state $D^g_1$ believes that there is one agent of type $g$ in state $D^g_0$ and the remaining agents are in state $C$; (iv) an agent of type $g$ in state $D^g_0$ believes that there is one agent of type $g$ in state $D^g_1$ and the remaining agents are in state $C$; (v) an agent of type $g$ in state $D^g_{g'}$ with $g' \neq g$, believes that there is one agent of type $g'$ in state $D^g_{g'}$ and the remaining agents are in state $C$. Clearly, $(\sigma^*, \mu^*)$ is a consistent assessment. In what follows we show that there exists $\delta' \in (0,1)$ independent of $N$ such that $(\sigma^*, \mu^*)$ is a sequential equilibrium when $\delta \geq \delta'$.

Let $V^C_{DM}$ and $V^C_{CM}$ be the lifetime payoffs to an agent in state $C$ before he enters the decentralized market and the centralized market, respectively. Then,

$$V^C_{DM} = \frac{1}{1-\delta} \left\{ \frac{N}{S(N-1)} [u(q^*) - c(q^*)] + U(x^*) - x^* \right\}$$

and

$$V^C_{CM} = U(x^*) - x^* + \delta V^C_{DM}.$$ 

Now let $V^g_D$ be the lifetime payoff to an agent of type $g$ in state $D \in \{D^g_1, D^g_0, \{D^g_{g'} \} \}_{g' \neq g}$ before he enters the centralized market. Since such an agent believes that the vector of prices will lie in the set $\tilde{P}$, we have that

$$V^g_D = \begin{cases} 
U(x^* - \kappa) - (x^* - \kappa) + \delta V^C_{DM} & \text{if } D = D^g_1 \\
U(x^* + \kappa) - (x^* + \kappa) + \delta V^C_{DM} & \text{if } D = D^g_0 \\
U(\varepsilon) - x^* + \delta V^C_{DM} & \text{if } D = D^g_{g'}
\end{cases}$$

Note that $U(\varepsilon) - x^* < U(x^* - \kappa) - (x^* - \kappa)$ by construction. Finally, observe that the lifetime payoff to an agent in state $A$ is $V_A = 0$.

It is immediate to see that no one–shot deviation is profitable in state $A$. Let us start with incentives in state $C$ then. An agent in the decentralized market has no profitable one–shot deviation if

$$-c(q^*) + U(x^*) - x^* + \delta V^C_{DM} \geq U(x^* - \kappa) - (x^* - \kappa) + \delta V^C_{DM},$$

which is satisfied by construction. Consider now an agent in the centralized market and assume, without loss of generality, that his type is $g = 1$. Let $a_2 = (y, (b^1, \ldots, b^G)) \neq \ldots \neq (y, (b^1, \ldots, b^G))$
\((x^*, x^* e_2)\) be the agent’s action. There are two possible types of one–shot deviations. One that leads to a price vector in \(\tilde{P}\) with probability one and one that does not. The first type of one–shot deviation involves \(b^2 = x^*\), \(b^1 = y - x^*\), and \(y > x^*\). It is easy to see that this reduces the agent’s flow payoff, and so is not profitable (given that \(V_C^{\text{DM}}\) is the highest continuation payoff possible for the agent).

Consider then a one–shot deviation that leads to state \(A\) with positive probability. Since setting \(b^g > 0\) for some \(g \geq 3\) reduces flow payoffs and does not increase continuation payoffs, we can assume that \(b^g = 0\) for \(g \geq 3\). Now observe that the agent’s flow payoff from \(a_2\) is \(U_\Omega(b^2/\tilde{p}^2) - y\), where

\[
\tilde{p}^2 = \frac{(\frac{N}{G} - 1)x^* + b^2}{\frac{N}{G}x^*}.
\]

It is easy to see that \(b^2/\tilde{p}^2\) is maximized when \(b^2 = y - b^1\). Thus, the highest flow payoff gain possible for the agent given the choice of \(y\) in \(a_2\) is

\[
\Delta(y) = U_\Omega \left( y \frac{\frac{N}{G}x^*}{(\frac{N}{G} - 1)x^* + y} \right) - y - [U_\Omega(x^*) - x^*].
\]

Since \(U_\Omega\) is strictly concave and \(U_\Omega(x^*) = 1\), we have that

\[
\Delta(y) \leq y \frac{x^* - y}{(\frac{N}{G} - 1)x^* + y}.
\]

Note that \(\Delta(y) > 0\) only if \(y < x^*\). Suppose then that \(y < x^*\). This implies that the one–shot deviation leads to state \(A\) if the realized value \(\theta^2\) of the shock to production in the post \(g = 2\) is such that

\[
\tilde{p}^2 < \frac{1}{\theta_{\text{max}}} \iff \theta^2 > \theta_{\text{max}} \left( 1 - \frac{x^* - y}{\frac{N}{G}x^*} \right).
\]

Since \(b^2 \leq y\) and \(\Omega'(\theta)\) is bounded below by \(\Lambda > 0\), a lower bound on the expected continuation payoff loss from the one–shot deviation is

\[
\Lambda \theta_{\text{max}} \frac{x^* - y}{\frac{N}{G}x^*} \delta V_C^{\text{DM}}.
\]

Given that

\[
\frac{N}{G}x^* \Delta(y) \leq y(x^* - y) \frac{\frac{N}{G}x^*}{(\frac{N}{G} - 1)x^* + y} \leq \frac{3}{2} y(x^* - y),
\]

we can then conclude that the one–shot deviation is not profitable if

\[
\Lambda \theta_{\text{max}} \delta V_C^{\text{DM}} \geq \Lambda \theta_{\text{max}} \frac{\delta}{1 - \delta} \left\{ \frac{1}{S} [u(q^*) - c(q^*)] + U_\Omega(x^*) - x^* \right\} > \frac{3}{2} x^*.
\]
It is easy to see from the last condition that there exists \( \delta^1 \in (0,1) \) independent of \( N \) such that no one–shot deviation in state \( C \) in the centralized market is profitable if \( \delta \geq \delta^1 \).

To finish, consider incentives in states \( D \in \{ D^g_0, D^g_0, \{ D^g_g \}_g \neq g \} \). No agent can be in such state in the decentralized market. Consider then an agent in state \( D \in \{ D^g_0, D^g_0, \{ D^g_g \}_g \neq g \} \) in the centralized market. Once again, we assume, without loss of generality, that the agent’s type is \( g = 1 \). We only consider the case in which \( D = D^1_g \) for some \( g' \neq 1 \). The analysis in the other cases is very similar. Let \( a_2 = (y, (b^1, \ldots, b^G)) \neq (x^*, \varepsilon e_2 + (x^* - \varepsilon)e_{g+1}) \) be the agent’s action. Note that \( b^g > 0 \) for \( g \notin \{2, g' + 1\} \) is never optimal for it reduces the agent’s flow payoff. Also note that we can restrict attention to one–shot deviations where \( y = b^2 + b^{g'+1} \). In fact, if \( y > x^* \) and \( y > b^2 + b^{g'+1} \), the agent can increase his flow payoff and (weakly) reduce the probability that the state changes to \( A \) by reducing \( y \) while keeping \( b^2 \) and \( b^{g'+1} \) the same. If \( y \leq x^* \) and \( y > b^2 + b^{g'+1} \), the agent can reduce the probability that the state changes to \( A \) by either increasing \( b^{g'+1} \) or increasing \( b^2 \). Now observe that the agent’s flow payoff from \( a_2 \) is \( U_a(b^2/\hat{p}^2) - y \), where

\[
\hat{p}^2 = \frac{N_G x^* - \varepsilon + b^2}{N_G x^*}.
\]

Thus, the flow payoff gain for the agent given a choice of \( y \) and \( b^2 \) in \( a_2 \) is

\[
\Delta(y, b^2) = U_a \left( b^2 \frac{N_G x^*}{N_G x^* - \varepsilon + b^2} \right) - y - \left[ U_a (\varepsilon) - x^* \right].
\]

There are two types of one–shot deviations that we need to consider: (i) the choice of \( y \) in \( a_2 \) is \( y \geq x^* \); (ii) the choice of \( y \) in \( a_2 \) is \( y < x^* \). Consider case (i) first. In this case, only an increase in \( b^2 \) is profitable. Suppose then that \( b^2 > \varepsilon \). This implies that a one–shot deviation leads to state \( A \) if the realized value \( \theta^2 \) of the shock to production in the trading post \( g = 2 \) is such that

\[
\hat{p}^2 > \frac{1}{\theta_{\min}^2} \Leftrightarrow \theta^2 < \theta_{\min} \left( 1 + \frac{b^2 - \varepsilon}{N_G x^*} \right).
\]

The expected continuation payoff loss from the deviation is then at least

\[
\Delta \theta_{\min} \frac{b^2 - \varepsilon}{N_G x^*} \delta V_C^{\text{DM}}.
\]
Now note that
\[ \Delta(x^*, b^2) = U_\Omega \left( b^2 \frac{N G}{G} x^* - \varepsilon + b^2 \right) - U_\Omega(\varepsilon) \leq U'_\Omega(\varepsilon), \]
and that for any \( b^2 > \varepsilon \), \( \Delta(y, b^2) \leq \Delta(x^*, b^2) \) for all \( y \geq x^* \). Thus, the one–shot deviation is not profitable if
\[ \lambda \theta_{\min} \frac{1}{G} x^* \delta V_{DM} \geq U'_\Omega(\varepsilon). \] (7)
Since \( \lim_{N \to \infty} N/G < \infty \), there exists \( \Gamma \) such that \( N/G \leq \Gamma \) for all \( N \). Hence, there exists \( \delta^2 \in (0, 1) \) independent of \( N \) and \( G \) (but dependent on \( \Gamma \)) such that (7) holds for all \( \delta \geq \delta^2 \).

Consider now case \( (ii) \). In this case, since the agent increases his flow payoff by reducing his disutility of production, a one–shot deviation can involve either an increase or a decrease in \( b^2 - \varepsilon \). If \( b^2 - \varepsilon > 0 \), the one–shot deviation leads to state \( A \) when the realized values \( \theta^1 \) and \( \theta^2 \) of the shocks to production in the trading posts \( g = 1 \) and \( g = 2 \) are such that
\[ \frac{N G}{G} x^* - \varepsilon + b^2 \theta^2 \frac{N G}{G} x^* > \frac{1}{\theta_{\min}} \text{ or } \frac{N G}{G} x^* \left( \frac{N G}{G} - 1 \right) x^* + y > \frac{1}{\theta_{\min}}. \]
A lower bound for the probability of this event is
\[ \lambda \theta_{\min} \frac{b^2 - \varepsilon}{N G x^*} + \lambda \theta_{\min} \frac{x^* - y}{\left( \frac{N G}{G} - 1 \right) x^* + y} - (\lambda \theta_{\min})^2 \frac{N G}{G} x^* \left( \frac{N G}{G} - 1 \right) x^* + y \geq \lambda \theta_{\min} \frac{1}{G} \max \{ b^2 - \varepsilon, x^* - y \}. \]
If \( b^2 - \varepsilon < 0 \), the one–shot deviation leads to state \( A \) when the realized values \( \theta^1 \) and \( \theta^2 \) of the shocks to production in the trading posts \( g = 1 \) and \( g = 2 \) are such that
\[ \frac{N G}{G} x^* - \varepsilon + b^2 \theta^2 \frac{N G}{G} x^* < \frac{1}{\theta_{\max}} \text{ or } \frac{N G}{G} x^* \left( \frac{N G}{G} - 1 \right) x^* + y > \frac{1}{\theta_{\min}}. \]
The probability of this event is bounded below by
\[ \lambda \theta_{\max} \frac{\varepsilon - b^2}{N G x^*} + \lambda \theta_{\min} \frac{x^* - y}{\left( \frac{N G}{G} - 1 \right) x^* + y} - \lambda^2 \theta_{\min} \theta_{\max} \frac{N G}{G} x^* \left( \frac{N G}{G} - 1 \right) x^* + y \geq \lambda \theta_{\min} \frac{1}{G} \max \{ b^2 - \varepsilon, x^* - y \}. \]
Hence, the expected continuation payoff loss from the deviation is at least
\[ \lambda \theta_{\min} \frac{1}{G} \max \{ |b^2 - \varepsilon|, x^* - y \} \delta V_{DM}^C. \]
To finish, observe that
\[
\Delta(y, b^2) = x^* - y + \Delta(x^*, b^2) \leq [1 + U'_\Omega(\varepsilon)] \max \{b^2 - \varepsilon, x^* - y\},
\]
and so the one-shot deviation is not profitable if
\[
\Delta \theta_{\min} \frac{1}{\frac{N}{G}x^*} \max \{|b^2 - \varepsilon|, x^* - y\} \delta V_C^{DM} \geq [1 + U'_\Omega(\varepsilon)] \max \{b^2 - \varepsilon, x^* - y\}. 
\] (8)

Since there exists \( \Gamma > 0 \) such that \( N/G \leq \Gamma \) for all \( N \), it is straightforward to see that there exists \( \delta^3 \in (0, 1) \) independent of \( N \) and \( G \) such that (8) holds for all \( \delta \geq \delta^3 \). This establishes the desired result.

We now argue that the strategy profile in the proof of Proposition 3 is still an equilibrium under the weaker assumption that \( \Omega'(\theta_{\min}) > 0 \) and \( \Omega'(\theta_{\max}) > 0 \) if agents are patient enough. For this, consider an agent in state \( C \) in the centralized market. It is easy to see from the argument leading to (6) that the only one-shot deviations that are possibly profitable are the ones in which \( x^* - y \) is close to zero, for otherwise the probability that such deviations lead to autarky are bounded away from zero. Now observe that when \( x^* - y \) is small, so that \( x^* - b^2 \) is small as well,
\[
\Omega(\theta_{\max}) - \Omega \left( \theta_{\max} \left( 1 - \frac{x^* - b^2}{N \frac{x^*}{G}} \right) \right) \approx \Omega'(\theta_{\max}) \theta_{\max} \frac{x^* - b^2}{N \frac{x^*}{G}}.
\]

Thus, if we let \( \lambda = \Omega'(\theta_{\max}) \), the same argument that follows the derivation of (6) shows that no one-shot deviation is profitable. A similar argument shows that if \( \Omega'(\theta_{\max}) > 0 \) and \( \Omega'(\theta_{\min}) > 0 \), then that there are no profitable one-shot deviations for an agent in state \( D \in \{D^g_{-1}, D^g_0, \{D^g_{g'}\}_{g' \neq g}\} \) in the centralized market. Since there is no other instance in which the properties of \( \Omega \) matter for the proof of Proposition 3, we are done.

References


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