GEO 428: Slope
October 2, 2012

Introduction
Slope: measure of steepness of a surface at a location
Can be measured at many locations and mapped
Important Environmental Variable
  Flow of water across DEM
  Habitat indicator
  Site suitability / selection
  Cross-country movement modeling

Elements of Slope
Defined by a plane tangent to surface
  Gradient: maximum rate of change
  Aspect: Direction of Gradient
Direction important!
  Sign is positive or negative

High School Math
'Slope = Rise / Run'
More precisely:
  Percent Slope = 100 * Rise/Run
  Degrees Slope = angle opposite the rise
tan (alpha) = rise / run
  alpha = arctan (rise / run)

Examples

Properties
Degrees range from (+/-) 0 – 90
Percent ranges from (+/-) 0 – infinity

Calculus
For functions of form $z = f(x)$
First derivative of the function, solved for location, is the slope
Precisely, it is equal to rise/run for a line tangent to the function at location
Why slope is called a terrain derivative!

Calculus Example
Consider $f(x) = 2x^3 - 10x^2 + 6x + 20$
What is the slope at x=1? X=3?

Calculating 1st Derivative
$f(x) = 2x^3 - 10x^2 + 6x + 20$
$f'(x) = 6x^2 - 20x + 6 + C$
For $x = 1$
For $x = 3$
Extending to 2-D
A point on a surface has slope:
\[ S = \arctan \sqrt{p^2 + q^2} \]
where \( p \) is the west-east gradient
\[ p = f_x = \frac{\partial f}{\partial x} \]
And \( q \) is the north-south gradient
\[ q = f_y = \frac{\partial f}{\partial y} \]
How to calculate \( p \) and \( q \)?

Problems for Terrain
No continuous function defined
- Limited subset of known elevations
Scale important
- Not well defined at a point
- Slope is a function of 'run'
Direction important
- Steepness is a function of direction

Finding Slope for TINs
Intrinsic property of TIN facets
Identify facet that a location lies within
Choose a direction
- Typically direction of steepest slope
Solve for rise/run for this direction
May be stored as attribute of TIN
- Or calculated on-the-fly

Finding Slope on Raster DEMs
No single correct method!
Typically give similar but not identical answers
Challenges:
- identifying 'run' distance
- identifying 'run' direction

Raster Notation
Let \( E \) be the cell for which to calculate slope
\( Z_a \) is elevation for cell \( A \), etc.

A Common Algorithm
Horn (1981), implemented in A/I Grid
Averages east-west & north-south gradient:
\[ \frac{dz}{dx} = \frac{(Z_a + 2*Z_d + Z_g) - (Z_c + 2*Z_f + Z_i)}{8*\text{cell resolution}} \]
\[ \frac{dz}{dy} = \frac{(Z_a + 2*Z_b + Z_c) - (Z_g + 2*Z_h + Z_i)}{8*\text{cell resolution}} \]
Then calculate percent slope as:
\[ \text{slope}(E) = \sqrt{(\frac{dz}{dx})^2 + (\frac{dz}{dy})^2} \]
Issues with this Algorithm
Elevation at E not used to estimate slope
   Slope is unaffected by altitude of E
x,y units and z units must be the same
   Should be ground units, not degrees!
ArcGIS supports square cells only
   Simplifies algorithm
dz/dx and dz/dy look a bit like rise/run

Other Approaches
Maximum gradient
4 neighbor methods
8/9 neighbor methods
3-cell, etc...

Maximum Gradient
Identify neighbor with greatest difference
   Set this difference = dz
Slope = dz / dx
Value at E matters
Poorer results / Sensitive to error & rough terrain

4 Neighbor Methods
Ignore diagonals, e.g.:
   dz/dx = (Zd – Zf) / 2 * cell resolution
   dz/dy = (Zb – Zh) / 2 * cell resolution
% slope = sqrt (dz/dx^2 + dz/dy^2)
Studies have suggested these methods more accurate than 8 neighbor
   Especially for smooth surfaces
Not often implemented in GIS
A class project?

8/9 Neighbor Methods
Fit a surface to 3x3 point grid
   e.g. a polynomial fit via OLS
   Generate slope from this surface
At least half a dozen plausible methods have been suggested
Generally similar results to Horn

Visualizing Slope
Slope is usually more rugged than elevation
   NEVER values below 0!
Difficult to visualize
Draping over DEM
Shading via standard deviations:
   6 classes, breaks at mean, mean +- 0.6 SD, mean +- 1.2 SD
Harlan, KY
USGS 7.5' DEM
Dissected Plateau
Very steep-sided valleys
Range: 0 – 50 degrees
Mean: 19 degrees
SD: 7.5 degrees

Lake Cachuma, CA
Mountainous
  elev. range 1150 m
Large, flat lake surface
Range: 0 – 47 degrees
Mean: 15 degrees, SD: 9.8 degrees
2 slope maps with 11 classes:
  First using 0.5 SD breaks
  Second using equal interval breaks

Galapagos Volcano: Cerro Azul
SRTM Data in Landserf
  Slope ranges from 0-68 degrees

Elevation Info vs. Slope
NED 30 m. near Park City, Utah

Combining Slope and Elevation

Slope in High-Res DEMs
MSU DEM from 3.2 million lidar returns

Low Slopes: white & yellow
High Slopes: magenta & blue
What shows up?

Summary
Important variable for much work
Usually, gradient is meant
  Slope is both gradient and aspect
No best method for raster DEMs
Alternative methods give different results
Visualization
Next Time: Aspect, Curvature, EXAM discussion! Bring questions!