1. Fixed-\textit{b} Inference for Testing Structural Change in a Time Series Regression (with Timothy J. Vogelsang)

Many existing research such as Kiefer and Vogelsang (2005) document the finite-sample size distortion of heteroskedasticity and autocorrelation (HAC) robust tests. The size distortions can be more severe with smaller sample size and this is definitely a problem for tests of structural change since researchers often split the sample into two regimes (pre- and post-break) to conduct inference. Splitting the sample will make the HAC estimators for each regime subject to more finite-sample bias and randomness and this in turn will cause the test to suffer from finite-sample issue such as over-rejection problems. The fixed-\textit{b} asymptotic theory originally developed for non-structural change setting is known to have better approximation to the finite-sample distribution of the test statistics. This advantage of the fixed-\textit{b} inference is expected to be even more pronouncing for tests of structural change but a fixed-\textit{b} theory for the structural change setting needs to be newly developed.

Chapter 1 develops an asymptotic theory for testing the presence of structural change in a weakly dependent time series regression model. The cases of full structural change and partial structural change are considered. A HAC estimator is involved in the construction of the test statistics. Depending on how the long run variance for pre- and post-break regimes is estimated, two types of heteroskedasticity and autocorrelation (HAC) robust Wald statistics, denoted by $Wald^{(F)}$ and $Wald^{(S)}$ are analyzed. The fixed-\textit{b} asymptotics established by Kiefer and Vogelsang (2005) is applied to derive the limits of the statistics with the break date being treated as a priori. The fixed-\textit{b} limits turn out to depend on the location of break fraction and the bandwidth ratio as well as on the kernel being used. For both Wald statistics the limits capture the finite-sample randomness existing in the HAC estimators for the pre- and post-break regimes. The limit of $Wald^{(F)}$ further captures the finite-sample covariance between the pre-break regression slope estimator and the post-break slope estimator. The fixed-\textit{b} limit stays the same and is pivotal for $Wald^{(F)}$ irregardless of whether some of the regressors are not subject to structural change. Critical values for the tests are obtained by simulation methods. Monte Carlo simulations compare the finite sample size properties of the two Wald statistics and a local power analysis is conducted to provide guidance on the power properties of the tests. This Chapter extends its analysis to cover the case of the break date being unknown. Supremum, mean and exponential Wald statistics are considered and finite sample size distortions are examined via simulations with newly tabulated fixed-\textit{b} critical values for these statistics.
2. A Test of the Null of Integer Integration against the Alternative of Fractional Integration (with Christine Amsler and Peter Schmidt)

The research in Chapter 2 is motivated by the fact that sometimes both the KPSS test of the hypothesis of short memory and a unit root test reject their respective null hypotheses. This could be interpreted as a contradiction, but another explanation is that the series is fractionally integrated, with properties between those of a short memory series and a unit root series.

Chapter 2 proposes a test of the null hypothesis of integer integration against the alternative of fractional integration. The null of integer integration is satisfied if the series is either $I(0)$ or $I(1)$, while the alternative is that it is $I(d)$ with $0 < d < 1$. The test is based on two statistics. We reject the null if the KPSS test rejects $I(0)$ and a unit root test rejects $I(1)$. We propose a new unit root test to use in this testing procedure, which is a lower-tailed KPSS test based on the data in differences, and we call our test of the null of integer integration the "Double KPSS" test.

We show that the test has asymptotically correct size under the null that the series is either $I(0)$ or $I(1)$, and we also show that the test is consistent against $I(d)$ alternatives for all $d$ between zero and one. These statements are true under the assumption that the number of lags used in long-run variance estimation goes to infinity with sample size, but more slowly than sample size. We refer to this as "standard asymptotics." This requires some original asymptotic theory for our new unit root test, and also for the KPSS short memory test for the case that $d = 1/2$. We also consider "fixed-$b$ asymptotics" as in Kiefer and Vogelsang (2005).

The paper investigates the finite-sample size and power of the Double KPSS test, using both the critical values based on standard asymptotics and the critical values based on fixed-$b$ asymptotics. The test is more accurate when it uses the fixed-$b$ critical values. We conclude that we can distinguish integer integration from fractional integration, but it takes a rather large sample size to do so reliably.

3. Extensions of Fixed-$b$ Structural Change Test

In Chapter 3, non-trivial extensions of Chapter 1 are addressed. First, as a supplementary result to Chapter 1, this Chapter provides details on fixed-$b$ inference for null hypotheses in which coefficients for only one of the two regimes are involved. It also contains results on fixed-$b$ inference on the coefficients which are not subject to a structural break in the partial structural change regression model. The second part of this chapter relaxes the assumption that there is no structural break in the distribution of the regressor and error term. A leading case with this assumption relaxed is the one in which either the long run variance or the limit of sample second moment of the regressors is allowed to have a structural break as well. In this situation the Wald statistics introduced in Chapter 1 do not have pivotal limiting distributions under the fixed-$b$ approach. This result alone implies that the traditional asymptotics might be giving even poorer approximations when allowing for heterogenous long run variances across regimes although the heterogeneity should not change the asymptotic distribution under the traditional approach. Chapter 3 seeks for an alternative test statistic whose fixed-$b$ limit is pivotal.