Tying and Innovation:
A Dynamic Analysis of Tying Arrangements*

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Abstract

This paper analyzes the effects of tying arrangements on R&D incentives. It shows that tying is a means through which a firm can commit to more aggressive R&D investment in the tied goods market. Tying also has the strategic effect of reducing rivals’ incentives to invest in R&D. The strategy of tying is a profitable one if the gains, via an increased share of dynamic rents in the tied goods market, exceed the losses that result from intensified price competition in the market. The welfare implications of tying, and consequently the appropriate antitrust policy, are discussed.

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I. Introduction

This paper investigates the effects of tying arrangements on incentives to innovate. The analysis is particularly relevant in light of the recent antitrust case involving Microsoft. Specifically, it has been alleged that Microsoft’s decision to integrate its Internet browser program, Internet Explorer, with its operating system software, Windows 95, allows Microsoft to leverage its monopoly power in the operating system market into the Internet browser market. The European Commission is also concerned with the way Microsoft produces and sells its Windows operating system and Media Player software. Microsoft’s rivals, led by AOL Time Warner, have alleged that incorporating Media Player as a standard feature of Windows gave the software an unfair advantage over rival programs such as Real Networks’ Real Player (Financial Times, May 10, 2002).

What distinguishes the Microsoft case from previous antitrust cases concerning tying arrangements is that its focus is predominantly on the effects of tying on innovation. Microsoft’s competitors, particularly Netscape, argue that Microsoft’s tying practices lock customers into a single monolithic program and stifle innovation in the industry. Microsoft, in response, contends that the real threat to innovation would come from Government intervention to stop it from bundling products, like a browser, into its Windows operating system. Joel Klein, the former Assistant Attorney General in charge of the Justice Department’s antitrust division at the time, states that mission of the antitrust authorities in the Microsoft case is to “create circumstances in which the right innovation signals are given” (Labaton, 1997). Though the parties involved do not agree on much, all acknowledge that innovation is a key issue in this debate.

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1 The U.S. Department of Justice and Microsoft reached a settlement with Microsoft in November 2001. The settlement agreed with the US Justice Department contains no provision for forced unbundling. Nine states (California, Connecticut, Florida, Iowa, Kansas, Massachusetts, Minnesota, Utah, West Virginia, and the District of Columbia), however, rejected the Justice Department deal and have asked a federal judge in Washington to force Microsoft to offer a completely stripped-down version of Windows with no bundling. As of this writing, the case is in its second month of trial proceedings.
In light of this, it is unfortunate that most existing studies on tying arrangements mainly focus on static price competition and thus misses an important channel through which tying can affect competition. Perhaps more importantly, it is unclear whether the nation's antitrust policy—which was developed in the late nineteenth and early twentieth centuries to address concerns about price practices in the railroad and smokestack industries—is appropriate in the context of the rapidly evolving technology based industries like computing and the Internet (Lohr, 1998). Traditionally, the focus of the nation’s antitrust policy has been to look for the evidence of raising prices by a dominant company. In the information industries, however, there is scant evidence of monopolistic abuse in price in that prices in the information industries have been falling by any means even without adjusting for performance improvements. Economides (2001), for instance, states in his analysis of the Microsoft case that:

If one assumes (as DOJ does in the tying part of the case) that the two markets (operating systems and browsers) are separate and therefore tying occurred, the crucial question is whether tying was anticompetitive. In particular, was the incremental increase in the price of Windows when IE was bundled with it larger than the price increase justified by the value of functionality that IE added? This is the test that the court should have conducted, and it failed to do so. My view is that it is very hard to prove that a quality-adjusted new version of Windows without IE would not have had a higher hedonic price than old Windows. That is, a modest increase in the market price of Windows was likely to be justified by the enhancement of features of Windows even without the inclusion of IE. Thus, if the Court had performed this test, I believe it is likely that it would have found that adding IE functionality to new versions of Windows and distributing IE free of charge for older versions of Windows and for other operating systems did not harm consumers (p. 28).

Considering the central role innovation plays in these industries, I argue that the focus of antitrust enforcement should be rather in ensuring that there is a competitive market for innovation. Basing the nation’s antitrust policy narrowly on the price practices can result in misguided antitrust enforcement.² In this paper, I attempt to fill this gap by providing a framework through which the effects of tying on R&D incentives, and welfare implications thereof, can be analyzed. Specifically, I extend Whinston's (1991)

² Baumol and Ordover (1992) suggest that “the main source of the problem is the fact that the design of defensible antitrust policy for dynamic industries, meaning industries in which product and process innovation constitute key market strategies, raises significant methodological difficulties. These difficulties arise precisely because, when narrowly perceived, antitrust policies seem too much preoccupied with static market power and competition at the expense of dynamic considerations.”
theory of tying and foreclosure to allow for the possibility of R&D investments that precede the price game.

The paper proceeds as follows. In Section II, I briefly review the leverage theory of tying. To highlight the importance of innovation to my story, in Section III, I develop a simple model of tying without R&D competition. I illustrate that tying arrangements intensify price competition and can only reduce the profit of the tying firm; even though bundling allows the tying firm to "steal business" from the rival firm, the cost of it does not justify the practice.³ This confirms the Chicago school’s central contention that tying cannot be used for the purpose of leveraging monopoly power (see Bowman 1957, Posner 1976, and Bork 1978) and highlights the importance of considering the innovation game in the analysis of tying. Section IV extends the basic model by allowing for R&D competition. It shows that tying can serve as a mechanism for the tying firm to commit itself to more aggressive R&D in the tied good market. This commitment also has the strategic effect of dulling the R&D incentives of rival firms. In Section V, I analyze an explicit example of R&D competition to derive closed form solutions. I show that the dynamic gains from bundling can outweigh the static losses; that is, when R&D competition is introduced, bundling can be profitable. I also consider the welfare impact of tying and antitrust implications. Concluding remarks follow in Section VI.

II. The Leverage Theory of Tying

According to the "leverage theory" of tying, a multiproduct firm with monopoly power in one market can monopolize a second market using the leverage provided by its monopoly power in the first market. The leverage theory has been the key intellectual rationale for the historically harsh treatment of tying arrangements by the courts.⁴ The

³ See, however, Carbajo et al. (1990) and Chen (1997) for models where tying is used as a device to segment the market and relax price competition.
leverage theory, however, had come under attack and become largely discredited due to criticisms originating in the Chicago School (see e.g. Bowman 1957, Posner 1976, Bork 1978). As a result, price discrimination, as opposed to leverage, has come to be seen as the main motivation for tying (Stigler 1968, Adams and Yellen 1976, McAfee, McMillan, and Whinston 1989).

Recently, however, Whinston (1990) has revived the leverage theory of tying. He shows that if the market structure in the tied good market is oligopolistic, and in the presence of scale economies, tying can be an effective and profitable strategy to alter market structure by making continued operation unprofitable for tied good rivals. Previous models of tying missed this "strategic effect" due to their adherence to the assumption of competitive, constant returns-to-scale structure in the tied good market. It is important to keep in mind, however, that in Whinston’s basic model, inducing the exit of the rival firm is essential for the profitability of tying arrangements. Thus, if the competitor has already paid the sunk cost of entry and there is no avoidable fixed cost, tying cannot be a profitable strategy.

By contrast, the model I develop below demonstrates that even in the absence of exit by the rival firm, tying can be a profitable strategy via its long-term effects on competition through innovation. Thus, my analysis lends credence to Kaplow’s (1985) contention that the traditional criticisms of the leverage theory are “wholly beside the point” since they attempt to disprove the existence of the long-term leverage effect by using static analysis.

Carbajo, de Meza, and Seidman (1990) and Chen (1997) provide an alternative theory of strategic bundling in which bundling plays the role of a product-differentiation

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5 Bowman, for instance, claims that "leveraging, in a word, is no more plausible than lifting oneself by ones bootstraps." These arguments, often associated with the University of Chicago oral tradition, are traceable to Aaron Director.

6 Whinston (1990) points out that if the heterogeneity of consumer preferences are allowed for the tying good, tying can also serve as a price discriminating device and exclusion of the rival firm is not necessary for the profitability of tying. See also Carbajo et al. (1990).
device. As in this paper, bundling does not require the exit of the rival firm to be profitable. Bundling, however, is used to segment the market and relax competition. Thus, the mechanism through which bundling affects firms’ profits and welfare is completely different from this paper.

To highlight the importance of considering R&D competition, I first construct a model with only price competition. As shown in Whinston (1990), bundling induces the tying firm to engage in a more aggressive pricing strategy. The reason is that the tying firm can reap the benefits of selling its monopolized product only in conjunction with the sales of competitively supplied product. As a result, the tying firm will have a larger market share and take sales away from its competitors in the tied good market. Nonetheless, as discussed above, bundling is not profitable for the tying firm unless this strategic foreclosure induces exit by the rival firms because the tying firm’s aggressive pricing induces the rival firms to respond by lowering their own prices.7

When the possibility of R&D competition is considered, however, I show that the profitability of tying can be established through its effect on R&D incentives. The increased market share in the tied good market due to bundling is not a profitable strategy in itself. However, bundling also affects R&D competition. The tying firm’s R&D incentives in the tied good market increase since it can spread out the costs of R&D over a larger number of units, whereas the rival firms’ R&D incentives decrease. If this positive effect via R&D competition dominates the negative effect via price competition, tying can be beneficial for the tying firm even in the absence of exit by the rival firms.

In a previous paper (Choi 1996), I extended the leverage theory to consider the impact of tying on the pace of innovation. In particular, I considered the effect of bundling on R&D incentives in “systems” markets where two complementary products are to be used on a one-to-one basis. Using a model of preemptive innovation, I showed

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7 See also Nalebuff (1999) for an analysis of bundling as an entry-deterrent strategy.
that bundling creates interdependence between constituent markets, thereby turning the two separate R&D games over components into a single one over the system. This allows the tying firm to utilize the unused monopoly slack in one R&D market to bolster its strategic position in the other R&D market.

In the present paper, I analyze independent products and consider more general specifications of R&D. More importantly, I focus on the interaction between the price game and the R&D game, rather than the interaction between the R&D games for the two products. Thus, the paper illustrates another innovation-related channel through which a firm can leverage the monopoly power in one market to gain advantage in another. I emphasize that my focus on independent products is purely for expositional simplicity. As pointed out by Carlton and Waldman (1998), if there is a monopolistic producer for one of the products, price competition with complementary products entails multiplicity of equilibria, depending on the extent to which the monopolistic supplier can practice “price squeeze” in the other competitively supplied component [Ordover, Sykes and Willig (1985)]. The assumption of independent products allows me to avoid this multiple equilibria problem. However, once an assumption is made on the degree of “price squeeze,” the analysis and the intuition behind the main results of the paper carry over to the complementary products case [see Choi, Lee, and Stefanadis (2000) for more details].

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8 In a related model, Farrell and Katz (1998) analyze how compatibility standards shape the nature of innovation and price competition in the presence of network effects. To the extent that compatibility can be unilaterally blocked through proprietary interface standards, the compatibility choice has similar effects to the tying decision in preventing other firms from supplying compatible complementary products. As in this paper, the consideration of R&D incentives is central to firms’ compatibility choices. However, the focus of their paper is rather different from this paper. As is well known in the network externality literature (Katz and Shapiro, 1985; Farrell and Saloner, 1986), the existence of positive feedback generates multiple market equilibria and which equilibrium is selected in the market place crucially depends on expectations. Thus, Farrell and Katz’s main emphasis is on how innovation in networks markets depend sensitively on the nature of consumers’ and complementary product suppliers’ expectations and how public policies can be used to influence these expectations to improve market performance.

9 Farrell and Katz (2000), for instance, assume that the independent producer is able to obtain the full reward when it is the lowest cost producer in the competitively supplied component market, whereas
III. Tying in the Absence of R&D Competition

Consider two independent products, A and B. They are unrelated in the sense that they can be consumed independently and their values to consumers are independent of whether they are consumed separately or together. Consumers, whose total measure is normalized to 1, have a unit demand for each product. To focus on the strategic motive for bundling, I assume that there is no cost advantage or disadvantage associated with bundling.

The market for product A is monopolized by firm 1 with unit production cost of \( c_A \). All consumers have valuation of \( v_A (> c_A) \) for product A. It is assumed that entry to market A is not feasible. The market for product B, however, is served by two firms, firm 1 and firm 2, who engage in price competition. Their unit production costs for product B are the same and given by \( c_B \). As in Whinston (1990), I assume that product B is differentiated. For analytical simplicity, I consider a Hotelling-type price competition for product B in which two firms are located at the end points of the unit interval. Consumers are uniformly distributed along the interval. They demand only one unit of the good B with reservation values of \( v_B \), which is assumed to be sufficiently large to ensure that the market is covered. We identify a consumer with the point in the interval that represents her ideal variety of a product B. A consumer buying a product B located at the distance of \( x \) away from her ideal variety will incur utility loss of \( tx \), in addition to the price of the good, where \( t \) is a “transport” cost parameter.

Carlton and Waldman (2002) assume that the monopolistic supplier can extract half of the surplus created by the independent producer.

10 In the Microsoft case, the relationship between the operating system and Internet browser program is unclear. At the first blush, they seem to constitute complementary products that form a system. As a technical matter, however, an Internet browser program might some day serve as a substitute operating system – a platform on which other applications program run – and could eliminate the importance of Windows. This future threat is believed to be a main reason why Microsoft is so keen on the Internet browser market. For an analysis of tying in systems markets, see Choi and Stefanadis (2001).

11 Firm 1 may have a patent or have an installed base that makes entry unprofitable in the presence of network externalities (Farrell and Saloner, 1986).
Both firms are already in the market and have paid sunk cost of entry, if there is any. Thus, in contrast to Whinston (1990), entry and exit are not issues in this model. In such a case of no exclusion of the rival firm, tying is always a weakly dominated strategy if the production cost of each firm is given and cannot be altered. To see this, I consider the following two-stage game.

In the first stage, firm 1 (the monopolistic supplier of product A) decides whether or not to bundle the two products. As in Whinston (1990) and Carbajo, de Meza, and Sediman (1990), I assume that this precommitment is made possible through costly investments in product design and the production process.

A price game ensues in the second stage with the bundling decision in the previous stage as given. The timing assumption reflects the fact that the bundling decision through product design is a longer term decision that cannot be modified easily compared to the price decision. The outcomes are described below and depend on firm 1’s bundling decision in the first stage.

1. No Bundling

If the two products are not bundled, they can be analyzed independently. With the assumption of identical consumers and rectangular demand, firm 1 can extract the whole consumer surplus in market A and have profits of \((v_A - c_A)\). In market B, we have the

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12 If the two products are complementary components used in fixed proportions to comprise a system, firm 1 is trivially able to exclude firm 2 by bundling. However, Whinston (1990) shows that firm 1 never finds it worthwhile to bundle in order to monopolize market B.


14 The precommitment to tying is plausible in the context of the Microsoft case if we believe Microsoft's claim that its operating system and web browser program are so integrated that the browser, Internet Explorer, cannot be removed without disabling the operating system, Windows. The reason is that when Microsoft upgraded Internet Explorer to version 3.0, it placed some of the program’s improved code into files that also contained instructions for operating system functions. The sharing of these files, called dynamic linked libraries, in the design of software makes these two programs difficult to separate without jeopardizing the stability of each program. For example, in a recent court filing, Microsoft argued that in the newly released Windows 98 operating system, Internet Explorer is so tightly integrated that it would “take many months (if not years) to develop and test” the operating system without the browser and until then the product “would be of no commercial value” (The NY Times, May 22, 1998).
standard Hotelling type competition. We can derive the demand function for each firm as (see Tirole, 1988, Chapter 7 for details):

\[ q_i(p_{B1}, p_{B2}) = \frac{1}{2} + \frac{p_{Bj} - p_{Bi}}{2t}, \ i=1,2 \text{ and } i \neq j. \]

Each firm chooses \( p_{Bi} \) to maximize its own profit given the other firm’s price \( p_{Bj} \) and its marginal cost \( c_B \).

\[ \text{Max } (p_{Bi} - c_B)(\frac{1}{2} + \frac{p_{Bj} - p_{Bi}}{2t}) \]

The first order condition for firm \( i \) is given by \( p_{Bj} + c_B + t - 2p_{Bi} = 0 \), or

\[ p_{Bi} = R_{Bi}(p_{Bj}, c_B) = \frac{p_{Bj} + c_B + t}{2}, \ i=1,2 \text{ and } i \neq j. \]

The Nash equilibrium in prices when both firms’ marginal costs are \( c_B \) is given by

\[ p_{Bi}^*(c_b, c_b) = c_b + t \]

Each firm’s profit in market B is given by

\[ \pi_{Bi} = \frac{t}{2} \]

Thus, the overall profit for the monopolist is given by

\[ \Pi_1 = (v_A - c_A) + \frac{t}{2} \]

2. Bundling

Suppose that the monopolist bundles product A and B and charges the price of \( \tilde{P} \) for the bundled product. In this case, consumers have two choices. The first option is to buy the bundled product from the monopolist at the price of \( \tilde{P} \) and the second one is to buy only product B from firm 2. For the first option to be chosen by the consumer located at \( x \), \( \tilde{P} \) should satisfy the following condition:

\[ v_A + v_B - \tilde{P} - tx \geq v_B - \tilde{p}_{B2} - t(1-x) \]

We can derive the demand function for each firm as (8):

\[ \tilde{q}_1(\tilde{P}, \tilde{p}_{B2}) = \frac{1}{2} + \frac{\tilde{p}_{B2} - \tilde{P} + v_A}{2t}, \quad \tilde{q}_2(\tilde{P}, \tilde{p}_{B2}) = \frac{1}{2} + \frac{\tilde{P} - \tilde{p}_{B2} - v_A}{2t} \]

\[ \text{Variables corresponding to bundling are denoted with a tilde.} \]
Substituting these demand functions into each firm’s profit functions and maximizing with respect to \( \tilde{P} \) and \( \tilde{p}_{B2} \), respectively, yield the following reaction functions:

\[
(9) \quad \tilde{P} = \tilde{R}_1(\tilde{p}_{B2}; c_B) = \frac{(\tilde{p}_{B2} + c_B + t) + (c_A + v_A)}{2}
\]

\[
(10) \quad \tilde{p}_{B2} = \tilde{R}_{B2}(\tilde{P}; c_B) = \frac{(\tilde{P} - v_A) + c_B + t}{2}
\]

To interpret the result above, it is useful to consider a fictitious price \( \tilde{p}_{B1} = \tilde{P} - v_A \) as an implicit price charged for product B in the bundle. Then, we can rewrite Eq. (9) as:

\[
(9') \quad \tilde{p}_{B1} = \tilde{R}_1(\tilde{p}_{B2}; c_B) - v_A = \frac{\tilde{p}_{B2} + [c_B - (v_A - c_A)] + t}{2} = R_{B1}(p_{B2}; c_B - s_A),
\]

where \( s_A = v_A - c_A (> 0) \) denote the monopoly surplus in market A.

This implies that after bundling firm 1 behaves as if its cost of B were \( c_B - s_A \); bundling makes firm 1’s reaction function in market B shift inwards. The reason is that after bundling firm 1 can realize the monopoly surplus of \( s_A \) only in conjunction with the sale of product B. Thus, the firm is willing to sell product B up to the loss of \( s_A \). As a result, firm 1 will price more aggressively after bundling and captures a larger market share in market B.\(^{16}\)

Bundling, however, is not a profitable strategy. With competition in strategic complement, firm 1’s aggressive pricing in turn invites lower prices by firm 2. As a result, both firms’ profits are reduced. Unless this reduced profit induces firm 2 to exit from the market, bundling cannot be an optimal strategy. To see this, the Nash equilibrium prices with bundling can be derived by solving (9) and (10) simultaneously:

\[
(11) \quad \tilde{P}^* = c_B + t + \frac{2c_A + v_A}{3} = v_A + \left[ c_B + t - \frac{2s_A}{3} \right]
\]

\(^{16}\) In the terminology of Fudenberg and Tirole (1984), bundling is a "top dog" strategy, while non-bundling softens price competition and is a "puppy dog" strategy. See Bulow et al. (1985) and Tirole (1988) for a discussion of the taxonomy of business strategies.
\[ \tilde{p}_{B2}^* = c_B + t - \frac{s_A}{3} \]

The market shares for each firm in market B are given by
(12) \[ \bar{q}_1(\tilde{p}^*, \tilde{p}_{B2}^*) = \frac{1}{2} + \frac{s_A}{6t}, \quad \bar{q}_2(\tilde{p}^*, \tilde{p}_{B2}^*) = \frac{1}{2} - \frac{s_A}{6t} \]

We assume that \( s_A < 3t \). This assumption is made to ensure that firm 2 does not exit from the market after bundling. The resulting equilibrium profits are given by:
(13) \[ \tilde{\Pi}_1 = \frac{v_A - c_A}{3} + \frac{(v_A - c_A)^2}{18t} + \frac{t}{2} = \frac{s_A}{3} + \frac{s_A^2}{18t} + \frac{t}{2} \]

It can be easily shown that \( \tilde{\Pi}_1 < \Pi_1 = (v_i - c_i) + \frac{t}{2} = s_A + \frac{t}{2} \) under the assumption that \( s_A < 3t \). Therefore, bundling is not a profitable strategy.

**IV. Tying and the Incentive to Innovate**

In this section, I extend the basic model of Section III by introducing the possibility of R&D, thereby endogenizing the final production cost of each firm. Specifically, I analyze a three-stage game identical to that above, except that firms engage in R&D competition before the pricing game. That is, in the first stage, firm 1 decides whether or not to bundle. In the second stage, the two firms engage in cost reducing R&D activities. A price game ensues in the third and final stage, with the cost structure inherited from the realizations of R&D. As usual, I solve the game via backwards induction.

The analysis of the third stage is the same as above. To focus on the impact of tying arrangements on R&D competition in the tied good market, I ignore the possibility of R&D in market A and focus on the incentives for R&D in market B.\[^{17}\] Let us assume that each firm can reduce the unit production cost of B by \( \Delta \) with the investment costs of

\[^{17}\]In Choi (1996), in contrast, the leverage of monopoly power occurs as a result of creating an interdependence of R&D competition between the two product markets. Thus, the model in the present paper abstracts from this mechanism by assuming the R&D possibility in only one market.
Φ(Δ), where Φ′(.)>0 and Φ′′(.)>0.

1. No Bundling

In this case, consumers’ purchase decisions for each product are independent of each other, which implies that each market can be analyzed separately.

Let Δ1 and Δ2 be the levels of cost-reduction by firm 1 and firm 2, respectively, in the R&D stage. Then, post R&D unit production costs become (cB−Δi) and (cB−Δ2) for firm 1 and firm 2, respectively. It can be easily verified that each firm’s reaction function in price competition can be written as:

\[ p_{Bi} = R_{Bi}(p_{Bj}; \Delta_i) = \frac{p_{Bj} + (c_B - \Delta_i) + t}{2}, \quad i=1,2 \text{ and } i \neq j. \]

The Nash equilibrium in prices is given by

\[ p_{Bi}^*(\Delta_1, \Delta_2) = t + c_B - \frac{2\Delta_i + \Delta_j}{3} \]

Each firm’s demand is given by

\[ \bar{q}_{i}^*(\Delta_1, \Delta_2) = \frac{1}{2} + \frac{\Delta_i - \Delta_j}{6t}, \]

Each firm’s post R&D profit in market B is given by

\[ \pi_{Bi}^*(\Delta_1, \Delta_2) = \frac{t}{2} + \frac{\Delta_i - \Delta_j}{3} + \frac{(\Delta_i - \Delta_j)^2}{18t} \]

Therefore, firm i chooses Δi to solve

\[ \text{Max } \pi_{Bi}^*(\Delta_1, \Delta_2) - \Phi(\Delta_i) = \frac{t}{2} + \frac{\Delta_i - \Delta_j}{3} + \frac{(\Delta_i - \Delta_j)^2}{18t} - \Phi(\Delta_i) \]

The first order condition for the optimal level of cost reduction for firm i is given by:

\[ \frac{1}{3} + \frac{\Delta_i - \Delta_j}{9t} = \Phi'(\Delta_i), \]

which implicitly defines the reaction function in the R&D game stage Δi = Θi(Δj), where i=1,2 and i ≠ j. We assume that the second order conditions for the firms’ maximization problems are satisfied, that is, \( \Phi''(\Delta_i) > 1/9t \). By totally differentiating (18), it can be

\[ 18 \text{ In the previous version, I consider a more general R&D specification where the nature of R&D outcome is stochastic and derive essentially the same results.} \]
easily verified that the reaction functions in the R&D stage game are negatively sloped with the property of strategic substitutes.

\[(19) \quad \Theta_i'(\Delta_j) = \frac{1}{9t} \left[ \text{second order condition} \right] < 0 \]

The symmetric equilibrium in the R&D stage is given by \( \Delta_1^* = \Delta_2^* = \Delta^* \), where \( \Delta^* \) is uniquely defined by \( \Phi'(\Delta^*) = 1/3 \). I further assume that \(|\Theta_i'(\Delta_j)| < 1\), which is to say that \( \Phi''(\Delta_i) > 2/9t \). This condition ensures the stability of the Nash equilibrium.

2. Bundling

Let \( \tilde{\Delta}_1 \) and \( \tilde{\Delta}_2 \) be the levels of cost-reduction by firm 1 and firm 2, respectively, in the R&D stage with bundling. By proceeding in the same way as in section III, I can derive the equilibrium bundle price by firm 1 and firm 2’s component price for product B as follows:

\[(20) \quad \tilde{P}^* = v_A + [c_B + t - 2\tilde{\Delta}_1 + \frac{3}{2} \tilde{\Delta}_2 - \frac{2s_A}{3}] \]

\[\tilde{p}_{B2}^* = c_b + t - 2\tilde{\Delta}_2 + \frac{3}{2} \tilde{\Delta}_1 - \frac{s_A}{6t} \]

\[\tilde{q}_1^*(\tilde{\Delta}_1, \tilde{\Delta}_2) = \frac{1}{2} + \frac{\tilde{\Delta}_1 - \tilde{\Delta}_2 + s_A}{6t} \]

\[\tilde{q}_2^*(\tilde{\Delta}_1, \tilde{\Delta}_2) = \frac{1}{2} + \frac{\tilde{\Delta}_2 - \tilde{\Delta}_1 - s_A}{6t} \]

Given \( \tilde{\Delta}_1 \) and \( \tilde{\Delta}_2 \), each firm’s post R&D profits are given by

\[(21) \quad \tilde{\pi}_1^*(\tilde{\Delta}_1, \tilde{\Delta}_2) = \frac{1}{2} + \frac{s_A + \tilde{\Delta}_1 - \tilde{\Delta}_2 + (s_A + \tilde{\Delta}_1 - \tilde{\Delta}_2)^2}{3} \quad \text{and} \quad \frac{18t}{18t} \]

\[\tilde{\pi}_{B2}^*(\tilde{\Delta}_1, \tilde{\Delta}_2) = \frac{1}{2} + \frac{-\tilde{\Delta}_2 - \tilde{\Delta}_1 - s_A + (\tilde{\Delta}_2 - \tilde{\Delta}_1 - s_A)^2}{3} \quad \frac{18t}{18t} \]

Therefore, firm 1 chooses \( \tilde{\Delta}_1 \) to solve

\[\text{Max} \quad \tilde{\pi}_1^*(\tilde{\Delta}_1, \tilde{\Delta}_2) - \Phi(\tilde{\Delta}_1) = \frac{1}{2} + \frac{s_A + \tilde{\Delta}_1 - \tilde{\Delta}_2 + (s_A + \tilde{\Delta}_1 - \tilde{\Delta}_2)^2}{3} \quad \frac{18t}{18t} - \Phi(\tilde{\Delta}_1) \]

The first order condition for the optimal level of cost reduction for firm 1 is given by:

\[(22) \quad \frac{1}{3} + \frac{s_A + \tilde{\Delta}_1 - \tilde{\Delta}_2}{9t} = \Phi'(\tilde{\Delta}_1) \]
Equation (22) implicitly defines firm 1’s reaction function under bundling, \( \tilde{\Delta}_1 = \tilde{\Theta}_1 (\tilde{\Delta}_2) \).
The comparison of (18) and (22) immediately gives the result that for any given level of
cost reduction by firm 2, \( \Delta_2, \tilde{\Delta}_1 = \tilde{\Theta}_1 (\Delta_2) > \Delta_1 = \Theta_1 (\Delta_2): \) firm 1 has greater incentives for
R&D after bundling. The reason for this result is that a firm’s incentives for R&D are
directly proportional to its market share. With tying arrangements, as shown above, the
market share of firm 1 for product B increases due to its incentives to price more
aggressively. As a result firm 1’s R&D incentives increase.

In contrast, firm 2’s market share for product B decrease with firm 1’s tying
arrangements. Thus, firm 2’s R&D incentives are reduced as a result of bundling. To
verify this, note that firm 2’s maximization problem in the R&D stage is given by:
\[
\max \tilde{\pi}_{B_2}^* (\tilde{\Delta}_1, \tilde{\Delta}_2) - \Phi(\tilde{\Delta}_2) = t_2 + \frac{1}{\frac{3}{2}} + \frac{\tilde{\Delta}_2 - \tilde{\Delta}_1 - s_4}{9t} + \frac{(\tilde{\Delta}_2 - \tilde{\Delta}_1 - s_4)^2}{18t} - \Phi(\tilde{\Delta}_2)
\]
The first order condition for firm 2’s optimal cost reduction is given by:
\[
(23) \quad \frac{1}{3} + \frac{\tilde{\Delta}_2 - \tilde{\Delta}_1 - s_4}{9t} = \Phi'(\tilde{\Delta}_2)
\]
Equation (23) defines firm 2’s reaction function under bundling, \( \tilde{\Delta}_2 = \tilde{\Theta}_2 (\tilde{\Delta}_1) \). The
comparison of (18) and (23) gives the desired result, that is, for any given level of \( \Delta_1, \tilde{\Delta}_2 = \tilde{\Theta}_2 (\Delta_1) < \Delta_2 = \Theta_2 (\Delta_1) \). Let \( \tilde{\Delta}_1^* \) and \( \tilde{\Delta}_2^* \) denote the Nash equilibrium R&D investment
levels for the firm 1 (the tying firm) and firm 2 (the rival firm), respectively, under
bundling. Then, \( (\tilde{\Delta}_1^*, \tilde{\Delta}_2^*) \) can be derived by at the intersection of the two reaction
functions \( \tilde{\Theta}_1 \) and \( \tilde{\Theta}_2 \).

**Proposition 1.** With bundling, the tying firm’s R&D investment level increases \( (\tilde{\Delta}_1^* > \Delta_1^*) \), and the rival firm’s R&D investment level decreases \( (\tilde{\Delta}_2^* < \Delta_2^*) \). The magnitudes
of changes in each direction increase with \( s_4 \).

**Sketch of the Proof.** With bundling by firm 1, both firms’ reaction curves shift. We can
consider the change in equilibrium as a result of sequential shifts of the two reaction
curves. Let \((\hat{\Delta}_1^*, \hat{\Delta}_2^*)\) be the intersection point of \(\tilde{\Theta}_1(\Delta_2)\) and \(\Theta_2(\Delta_1)\). Since \(\tilde{\Theta}_1(\Delta_2)\) is an outward shift of \(\Theta_2(\Delta_1)\), we have \(\hat{\Delta}_1^* > \Delta_1^*\) and \(\hat{\Delta}_2^* < \Delta_2^*\) with the stability of Nash equilibrium. \(\tilde{\Theta}_2(\Delta_1)\) is an inward shift of \(\Theta_2(\Delta_1)\), which implies that \(\tilde{\Delta}_1^* > \hat{\Delta}_1^*\) and \(\tilde{\Delta}_2^* < \hat{\Delta}_2^*\). Thus, we have \(\tilde{\Delta}_1^* > \Delta_1^*\) and \(\tilde{\Delta}_2^* < \Delta_2^*\). See Figure 1. A rigorous proof is provided in the Appendix.

Figure 1. Equilibrium in R&D Investment under Bundling and Nonbundling

Proposition 1 tells us that the tying firm’s R&D incentives increase at the expense of the rival firm’s. The intuition for this result is simple. It is just a manifestation of the

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19 See the appendix for the details of the proof.
“appropriability of innovation benefit hypothesis” which asserts that the incentives to innovate are directly proportional to the appropriability of the innovation benefits. As seen in the previous section, tying allows the tying firm to capture a larger market share in the tied good market that translates into a larger profit from an innovation. Thus, the tying decision can be considered as a commitment device to more aggressive R&D investment. It also renders some credibility to the argument that tying by a dominant firm can stifle innovation incentives by competitors in the tied good market.

In my model, market foreclosure does not necessarily lead to exclusion of the rival firm. Rather, market foreclosure in the product market translates into foreclosure in R&D markets. In the static model of price competition where the industry rent is fixed, bundling reduces the tying firm’s overall profits since it intensifies the effective price competition in the tied good market. However, in the presence of dynamic rents that can be created through R&D, bundling may be a profitable strategy. The change in R&D incentives through bundling enables the tying firm to capture a larger share of dynamic rents. If this effect outweighs the negative effect of more aggressive price competition, bundling will be a privately optimal strategy even in the absence of exit by the rival firm.

This is an important point. One of the most startling aspects of the recent Microsoft case is that Robert Bork, a prominent member of the Chicago school who questioned the validity of the leverage theory, was retained by Netscape, asserting that Microsoft violated the law by using its dominant position in computer operating systems to promote its own browser over that of its rival. To the critics accusing him of "selling his soul", Robert Bork responds by referring to the case of Lorain Journal Company v. United States in 1951. The Lorain Journal, a daily newspaper in Lorain, Ohio, had a virtual monopoly of the mass dissemination of news and advertising in the town. When

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20 The result is consistent with the empirical evidence in Vanderwerf (1990) who finds that most innovations in electronic wire preparation equipment have come from firms who also produced parts effectively tied to the equipment. He explains his finding by positing that when parts and machines can be tied, firms that sell both can appropriate greater economic benefit from equipment innovation than can firms that sell equipment only.
there was a threat to the monopoly with the establishment of radio station WEOL in a nearby town, the newspaper refused to accept local advertising from any advertiser that used WEOL. The Lorain Journal’s practice was deemed predatory by the Supreme Court, in violation of Section 2 of the Sherman Act. Bork (1998) argues that there is an exact parallel between the Lorain Journal case and the Microsoft case. Then, he reminds his critics that 20 years ago he wrote that the Lorain Journal case had been correctly decided. His critics, however, argue that the analogy is not exact because the Lorain case was concerned with the impending extinction of a radio broadcast station while Netscape is not facing such a fate. My model suggests that the exclusion of the rival firm is not necessary for tying arrangements to be privately optimal and/or to have potentially anti-competitive effects in cases where the effects arise mainly from distorted R&D incentives.

V. Private Incentives for Bundling and Welfare Analysis

In this section, I analyze private incentives to engage in bundling and social welfare implications of bundling. To this end, I further assume that the cost of R&D is given by \( \Phi(\Delta) = \frac{k}{2} \Delta^2 \), where \( k \) represents the R&D cost parameter. With this assumption, we can derive closed-form solutions for the optimal levels of R&D and equilibrium profits under bundling and no bundling. Without bundling, the equilibrium cost reduction is given by

\[
\Delta_1 = \Delta_2 = \Delta^* = \frac{1}{3k}
\]

The equilibrium profits are

\[
\Pi_1 = s + \frac{t}{2} - \frac{1}{18k}
\]
\[
\Pi_2 = \frac{t}{2} - \frac{1}{18k}
\]

With bundling, the equilibrium R&D levels are given by:
\begin{equation}
\begin{align*}
\Delta_1^* &= \frac{1}{3k} + \frac{s_A}{9kt - 2} \\
\Delta_2^* &= \frac{1}{3k} - \frac{s_A}{9kt - 2}
\end{align*}
\end{equation}

Notice that with a Hotelling-type competition in product market B and quadratic R&D cost function, the aggregate equilibrium cost reduction is the same regardless of firm 1’s bundling decision, that is, \( \Delta_1^* + \Delta_2^* = \Delta_1^* + \Delta_2^* = \frac{2}{3k} \). This feature will be useful in our welfare analysis later.

The equilibrium profits under bundling are given by:
\begin{equation}
\Pi_1 = \frac{t}{2} + \frac{1}{3} \left( s_A + \frac{2s_A}{9kt - 2} \right) + \frac{1}{18t} \left( s_A + \frac{2s_A}{9kt - 2} \right)^2 - \frac{k}{2} \left( \frac{1}{3k} + \frac{s_A}{9kt - 2} \right)^2
\end{equation}

Needless to say, firm 1 will engage in bundling if \( \Pi_1^* > \Pi_1 \).

We normalize the transportation cost parameter \( t = 1 \) to further analyze the incentives to bundle. With this normalization, the stability condition \( k > 2/9 \) and no exit condition in the absence of R&D competition \( s_A < 3 \) are given by \( k > 2/9 \) and \( s_A < 3 \), respectively. It is noteworthy to observe that firm 2 is foreclosed with R&D competition even if it is not in the absence of R&D competition if \( 3 - \frac{2}{3k} < s_A < 3 \). In that case which is represented by the dotted area in Figure 2, tying is always optimal. If \( s_A < 3 - \frac{2}{3k} \), firm 2 will engage in R&D and has a positive market share. In such a case, the shaded area in Figure 2 shows the parameter region in which bundling is profitable for firm 1, where

\begin{align*}
A &= \tilde{\Pi}_1 - \Pi_1 = \frac{s_A}{3} + \frac{2s_A}{3(9k - 2)} + \frac{(s_A + \frac{2s_A}{9k - 2})^2}{18} - \frac{k}{2} \left( \frac{1}{3k} + \frac{s_A}{9k - 2} \right)^2 + \frac{1}{18k} - s_A
\end{align*}

represents the benefit from tying for firm 1.
In our model, we can show that bundling is always social welfare reducing even if it can be privately optimal. There are several sources of inefficiency in our model due to bundling. First, we know that the aggregate cost reduction is the same across the regimes. With $\Phi''(.)>0$ the most efficient way to achieve a given level of aggregate cost reduction is to have the symmetric outcome, which is the case without bundling. With bundling, there is asymmetry in the incentives for R&D. As a result, the marginal cost of $\Delta i$ is not equalized with firm 1 investing too much and firm 2 investing too little compared to the no bundling case. The asymmetry in cost reduction efforts translates into asymmetry in post R&D production cost, which in turn introduces asymmetric market shares. In a
Hotelling model, the market division that minimizes the total transportation costs is equal market division. Finally, there will be some consumers who will forego the consumption of product A due to bundling. For these consumers who opt to buy only product B, the benefit of $s_A$ is lost. All three effects of bundling move in the same direction to reduce welfare in our model. I formalize the discussion above in Proposition 2 and prove it in the Appendix.

**Proposition 2.** Social welfare decreases with bundling.

*Proof.* See the Appendix.

It should be kept in mind that the strong result we have in Proposition 2 is due to the deterministic nature of R&D specification and horizontal product differentiation assumed in the model. When the R&D specification is amended to account for uncertainty and products are less differentiated, there are in general two aspects of R&D to consider in evaluating the efficiency of R&D competition. R&D competition promotes a *diversity* of research lines and thus increases the aggregate probability of success (the level of cost reduction) if the outcome of research project is uncertain. On the other hand, R&D competition can also result in the *duplication* of research efforts to the extent that their outcomes are correlated (Dasgupta and Maskin, 1987). The desirability of unfettered R&D competition hinges on the trade-off between diversity and duplication. Thus, we cannot rule out the possibility that tying arrangements enhance welfare by serving as a mechanism to coordinate and eliminate duplicative R&D activities if R&D outcomes are uncertain.\(^{21}\) In such a case, intimate knowledge of the industry and the nature of R&D competition would be required to assess the effects of tying on welfare.

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\(^{21}\) For such a possibility, see the working paper version of this paper, Choi (1998), in which I consider an R&D specification with stochastic outcomes and homogeneous (or vertically differentiated) products. Gilbert and Riordan (2002) also draw an ambiguous welfare results in which bundling serves as a coordinating mechanism in the R&D game as in Choi (1998).
VI. Concluding Remarks

The paper points out the possibility that bundling can tilt the playing field in favor of the tying firm in the R&D market. Tying arrangements prevent competitors from having a fair chance to reach consumers. This market foreclosure translates into reduced R&D incentives for the rival firms. Thus, it is possible that tying arrangements drive better products and services out of the market.

In this paper, we constructed a deterministic R&D model with horizontal product differentiation in which the effects of tying arrangements on social welfare were unambiguously negative. However, it is also possible that tying arrangements enhance welfare by serving as a mechanism to coordinate and eliminate duplicative R&D activities if R&D outcomes are stochastic and products are less differentiated (Choi, 1998). Presumably, this uncertainty concerning the welfare effects of tying suggests the difficulty of a simple legal standard to apply in antitrust cases. Another important extension would be to consider an asymmetric situation before R&D (see, Choi et al. (2000), and Gilbert and Riordan (2002) for such an analysis]. Nonetheless, the present paper formalizes the mechanism through which tying results in foreclosure in the innovation market. It also points out what are the crucial elements to consider in evaluating the welfare effects of tying. In particular, the nature of R&D competition that will take place without bundling and the degree of horizontal product differentiation should be an important criterion.

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22 The ambiguity concerning the welfare effects of bundling arises due to the nature of equilibrium under no bundling. Due to the existence of multiple equilibria (two pure strategy equilibria and one mixed strategy equilibrium), the welfare effects hinge on which one is the focal point in the selection of equilibrium.

23 In practice, other elements ignored in the simple model should be also considered. For instance, if the product is differentiated and/or if the demand curve is downward-sloping in the tied good market, the duplication of R&D activities would be less important. In this case, tying arrangements would have tendency to be more anti-competitive. However, as Microsoft argues, there may be also offsetting effects of tying such as enhanced performance due to a seamless integration of products if the tying good and the tied good are often used together.
References


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Appendix

Proof of Proposition 1:

Under bundling, the first order conditions for the optimal level of cost reduction for firms 1 and 2 are given by (22) and (23), respectively:

\[(A.1) \quad \frac{1}{3} + \frac{s_A + \Delta_1 - \Delta_2}{9t} = \Phi'(\Delta_1)\]
\[(A.2) \quad \frac{1}{3} + \frac{\Delta_2 - \Delta_1 - s_A}{9t} = \Phi'(\Delta_2)\]

I perform a comparative static analysis by totally differentiating (22) and (23). In matrix form,

\[
\begin{bmatrix}
\frac{1}{9t} - \Phi''(\Delta_1) & -\frac{1}{9t} \\
-\frac{1}{9t} & \frac{1}{9t} - \Phi''(\Delta_2)
\end{bmatrix}
\begin{bmatrix}
\frac{d\Delta_1}{ds_A} \\
\frac{d\Delta_2}{ds_A}
\end{bmatrix}
= \begin{bmatrix}
-\frac{1}{9t} \\
\frac{1}{9t}
\end{bmatrix}
\]

By using Cramer’s rule, I can derive

\[(A.3) \quad \frac{d\Delta_1}{ds_A} = \frac{\begin{vmatrix}
\frac{1}{9t} - \Phi''(\Delta_1) & -\frac{1}{9t} \\
-\frac{1}{9t} & \frac{1}{9t} - \Phi''(\Delta_2)
\end{vmatrix}}{\frac{1}{9t} - \Phi''(\Delta_1) - \frac{1}{9t} - \Phi''(\Delta_2)} > 0\]

since the denominator is positive with the stability condition we imposed \((\Phi''(\Delta_i) > 2/9t)\). Similarly,

\[(A.4) \quad \frac{d\Delta_2}{ds_A} = \frac{\begin{vmatrix}
\frac{1}{9t} - \Phi''(\Delta_1) & -\frac{1}{9t} \\
-\frac{1}{9t} & \frac{1}{9t} - \Phi''(\Delta_2)
\end{vmatrix}}{\frac{1}{9t} - \Phi''(\Delta_1) - \frac{1}{9t} - \Phi''(\Delta_2)} < 0\]

Since the equilibrium cost reduction levels without bundling, \(\Delta_1^*\) and \(\Delta_2^*\) correspond to the case where \(s_A = 0\). Thus, I have \(\Delta_1^* > \Delta_1^*\) and \(\Delta_2^* < \Delta_2^*\).
Proof of Proposition 2:

Social welfare without bundling can be written as

(A.5) \[ W = s_A + (v_B - c_B) + \left\{ \left[ \Delta_1* q_1* (\Delta_1*, \Delta_2*) + \Delta_2* q_2* (\Delta_1*, \Delta_2*) \right] - \frac{k}{2} \left[ (\Delta_1*)^2 + (\Delta_2*)^2 \right] \right\} - \left[ \int_0^{\gamma_{\Delta_1*}(\Delta_2*)} t \; dx + \int_{\gamma_{\Delta_1*}(\Delta_2*)}^\gamma t \; dx \right] = s_A + (v_B - c_B) + [ DR - TC ], \]

where \( DR = \left\{ \left[ \Delta_1* q_1* (\Delta_1*, \Delta_2*) + \Delta_2* q_2* (\Delta_1*, \Delta_2*) \right] - \frac{k}{2} \left[ (\Delta_1*)^2 + (\Delta_2*)^2 \right] \right\} \) represents the dynamic rents from the R&D stage and \( TC = \left[ \int_0^{\gamma_{\Delta_1*}(\Delta_2*)} t \; dx + \int_{\gamma_{\Delta_1*}(\Delta_2*)}^\gamma t \; dx \right] \) represents the total “transportation costs.”

In contrast, social welfare with bundling can be written as

(A.6) \[ \tilde{W} = s_A\tilde{q}_1* (\tilde{\Delta}_1, \tilde{\Delta}_2) + (v_B - c_B) + \left\{ \left[ \tilde{\Delta}_1* \tilde{q}_1* (\tilde{\Delta}_1*, \tilde{\Delta}_2*) + \tilde{\Delta}_2* \tilde{q}_2* (\tilde{\Delta}_1*, \tilde{\Delta}_2*) \right] - \frac{k}{2} \left[ (\tilde{\Delta}_1*)^2 + (\tilde{\Delta}_2*)^2 \right] \right\} - \left[ \int_0^{\gamma_{\tilde{\Delta}_1*}(\tilde{\Delta}_2*)} t \; dx + \int_{\gamma_{\tilde{\Delta}_1*}(\tilde{\Delta}_2*)}^\gamma t \; dx \right] = s_A\tilde{q}_1* (\tilde{\Delta}_1, \tilde{\Delta}_2) + (v_B - c_B) + [ \tilde{D}R - \tilde{TC} ], \]

where \( \tilde{D}R = \left\{ \left[ \tilde{\Delta}_1* \tilde{q}_1* (\tilde{\Delta}_1*, \tilde{\Delta}_2*) + \tilde{\Delta}_2* \tilde{q}_2* (\tilde{\Delta}_1*, \tilde{\Delta}_2*) \right] - \frac{k}{2} \left[ (\tilde{\Delta}_1*)^2 + (\tilde{\Delta}_2*)^2 \right] \right\} \) and \( \tilde{TC} = \left[ \int_0^{\gamma_{\tilde{\Delta}_1*}(\tilde{\Delta}_2*)} t \; dx + \int_{\gamma_{\tilde{\Delta}_1*}(\tilde{\Delta}_2*)}^\gamma t \; dx \right] \).

Now, I show that \( [ DR - TC ] > [ \tilde{D}R - \tilde{TC} ] \). To this end, I consider the following problem:

Max \( \Theta(\alpha, \Delta_1, \Delta_2) = \Delta_1 \left( \frac{1}{2} + \alpha \right) + \Delta_2 \left( \frac{1}{2} - \alpha \right) - \frac{k}{2} \left[ (\Delta_1)^2 + (\Delta_2)^2 \right] \)

subject to

\( \Delta_1 + \Delta_2 = \frac{2}{3k} \)

Then, it can be easily shown that \( \Theta(\alpha, \Delta_1, \Delta_2) \) is maximized at \( \alpha = 0, \Delta_1 = \Delta_2 = \frac{1}{3k} \).

Both \( [ DR - TC ] \) and \( [ \tilde{D}R - \tilde{TC} ] \) can be represented as values of the function \( \Theta(\alpha, \Delta_1, \Delta_2) \) by choosing the values of \( \alpha, \Delta_1, \) and \( \Delta_2 \) appropriately. However, \( [ DR - TC ] = \Theta(0, \)
\[
\frac{1}{3k} , \frac{1}{3k} \]
and is the maximum value of \(\Theta(\alpha, \Delta_1, \Delta_2)\). Therefore, \([DR - TC] > [\tilde{D}\tilde{R} - \tilde{T}\tilde{C}]\). In addition, \(s_A \geq s_A q^* (\tilde{\Delta}_1, \tilde{\Delta}_2)\) since \(q^* (\tilde{\Delta}_1, \tilde{\Delta}_2) \leq 1\). Taken together, we have \(W > \tilde{W} \).