Up or Down?
A Male Economist’s Manifesto on the Toilet Seat Etiquette

By

Jay Pil Choi*

Michigan State University

November 2002

Abstract

This paper develops an economic analysis of the toilet seat etiquette, that is, whether the toilet seat should be left up or down. I investigate whether there is any efficiency justification for the presumption that men should leave the toilet seat down after use. I find that the “down rule” is inefficient unless there is a large degree of asymmetry in the inconvenience costs of shifting the position of the toilet seat across genders. I show that the “selfish” or the “status quo” rule that leaves the toilet seat in the position used dominates the down rule in a wide range of parameter spaces including the case where the inconvenience costs are the same. The analysis can be applied to other shared facilities that can be customized to each user’s preference.

JEL Classification: D7, H4.
Keywords: gender, economic efficiency, social norm.

Correspondent:
Jay Pil Choi
Department of Economics, Michigan State University
East Lansing, MI 48824
E-mail: choijay@msu.edu

* I thank Carl Davidson and Roger Lagunoff for helpful discussions and many colleagues for sharing their experiences. I am solely responsible for the views expressed in this paper.
Dear Annie: I read with interest the letters about putting down the toilet seat. I've been browbeaten by various women for the past 60 years about proper seat etiquette, starting with my mother. If I forget to put the seat down even once, my wife reminds me for hours about this life-threatening situation.

I know you said the last column was the final word on the subject, but I hope you'll reopen the issue. I want to ask women: Who gave you exclusive ownership of the bathroom? If men are nice enough to put the lid down, why can't you ladies lift it up when you are done? When I suggested this to my wife, she wanted to have me taken out and shot. It's time to rebel! – Fed Up in Salem, Ore.

Dear Fed Up: What is it about toilet seats that excites people? We received hundreds of letters on this subject and decided the "last word" would have to wait – Kathy Mitchell and Marcy Sugar.

1. Introduction

Should the toilet seat be left up or down after use? This is a question that arises when members of the opposite sex share the same toilet. For some reason, this seemingly trivial question elicits passion from all sorts of people. It has become a topic of national debates in popular syndicated columns by Ann Landers and TV sitcoms such as ABC’s “Home Improvement” and NBC’s “3rd Rock from the Sun.” It is clear that this age-old debate is divided by the gender. Women complain that it should be the man’s responsibility to lower the toilet seat after use. Men seem to question why women should be the free-rider all the time.

Despite high emotions in the debate, scientific inquiries into this issue are sparse. In fact, it is not obvious why there should be a presumption that men are expected to leave the toilet seat down after use. In this paper, I investigate whether there is any

---

1 Annie’s Mailbox by Kathy Mitchell and Marcy Sugar, October 29, 2002. Annie’s Mailbox is written by Kathy Mitchell and Marcy Sugar, long time editors of the Ann Landers syndicated column.
2 To quote Larry James, a personal relationship counsellor, “The most hotly contested battlefield in the gender wars may not be necessarily be in the bedroom. It may be the bathroom. The seat-up vs. seat-down debate rages on …”
3 “Leaving the toilet seat up” is often described as a problem and there is even a toilet seat that goes down automatically after about two minutes, claiming that it has the perfect solution to the problem.
I find that the down rule is inefficient unless there is large asymmetry in the inconvenience costs of shifting the position of the toilet seat across genders. I show that the “selfish” or the “status quo” rule that leaves the toilet seat in the position used dominates the down rule in a wide range of parameter spaces including the case where the inconvenience costs are the same. The intuition for this result is easy to understand. Imagine a situation in which the aggregate frequency of toilet usage is the same across genders, i.e., the probability that any visitor will be male is \( \frac{1}{2} \). With the down rule, each male visit is associated with lifting the toilet seat up \textit{before} use and lowering it down \textit{after} use, with the inconvenience costs being incurred twice. With the selfish rule, in contrast, the inconvenience costs are incurred once and only when the previous visitor is a member of different gender. The worst case under the selfish rule would occur when the sex of the toilet visitor strictly alternates in each usage. Even in this case, the total inconvenience costs would be the same as those under the down rule if the costs are symmetric. If there is any possibility that consecutive users are from the same gender, the selfish rule strictly dominates the down rule since it keeps the option value of not incurring any inconvenience costs in such an event. This logic can be extended to the case of asymmetric aggregate frequency of toilet usage across genders.

The remainder of the paper is organized in the following way. In section II, I compare three plausible rules for the toilet seat position – up, down, and selfish – on an

\[^4\text{Internet search generated the following non-economic/scientific reasons for the down rule. First, there is an argument that being considerate to one’s love partner’s needs supports things going well in and out of the bedroom. To quote a phrase in the internet (available at http://www.celebratelove.com/littlethings.htm), “Foreplay begins with putting the toilet seat down without being asked!” Second, it is not good Feng-Shi to leave the toilet seats up. Third, a toilet is not the most attractive household appliance. Closing the lid improves its appearance and prevents things from falling into the bowl. The last argument, however, proposes not only the seat down but also the lid down.}\]
efficiency criterion. I show that the selfish rule always dominates the other two if the inconvenience costs of changing the toilet seat position are the same across genders. In section III, I characterize the optimal rule for the toilet seat position. It turns out that the selfish rule is the most efficient rule in a wide range of parameter spaces. I also derive the condition that the down rule can be the most efficient one when the inconvenience costs are asymmetric. Section IV extends the analysis to the case where the inconvenience costs are heterogeneous even within the same gender. Section V contains concluding remarks.

II. The Basic Model

I consider the usage of a toilet that is shared by members of the opposite sex. Assume that the proportion of male to all users of a certain toilet is given by $\alpha$. Let me assume the frequency of using a toilet by male and female is the same without loss of generality. If one gender uses the toilet more often, this asymmetry can be reflected in $\alpha$. Thus, the parameter $\alpha$ represents the relative aggregate frequency of male using the toilet.\[5\]

I analyze an infinite horizon discrete time framework where the toilet is used once in each period. The discount factor is given by $\delta$. With the assumption about the relative frequency of the toilet usage by each gender, the probability that the user is male in each period is given by $\alpha$.\[6\] The inconvenience cost of lowering the toilet seat for women is given by $c_w$. The corresponding cost of lifting the toilet seat for men is given by $c_m$. The relative frequency of men going “number 1” vs. “number 2” can be also incorporated in the parameter $\alpha$.\[5\] Equivalently, I could envision a continuous time model in which the arrival rate is given by a Possion process with the arrival rate being a function of the number of total users. The probability that a particular arrival is male is given by $\alpha$. I derive essentially the same results with this continuous model.\[6\]

---

5 The relative frequency of men going “number 1” vs. “number 2” can be also incorporated in the parameter $\alpha$.
6 Equivalently, I could envision a continuous time model in which the arrival rate is given by a Possion process with the arrival rate being a function of the number of total users. The probability that a particular arrival is male is given by $\alpha$. I derive essentially the same results with this continuous model.
by $c_m$. My goal in this section is to compare the expected aggregate inconvenience costs of three rules – down, up, and selfish – concerning the position of the toilet seat. In this comparative analysis, I abstract from other considerations such as being considerate to members of the opposite sex, aesthetic aspects, the wear costs of the seat hinge, etc.

**The Down (Female-Friendly) Rule**

This is a rule that leaves the position of the seat down after one is done with the bathroom task. In particular, this rule implies that each visit by a male member will be associated with the inconvenience costs of $2c_m$ whereas female members will incur no costs.

Let $V_{m}^{DOWN}$ and $V_{f}^{DOWN}$ denote the value functions with the down rule when the particular user in the current period is male and female, respectively. Then, these value functions satisfy the following recursive relationships.

\[
V_{m}^{DOWN} = -2c_m + \delta(\alpha V_{m}^{DOWN} + (1-\alpha)V_{f}^{DOWN}) 
\]

\[
V_{f}^{DOWN} = \delta(\alpha V_{m}^{DOWN} + (1-\alpha)V_{f}^{DOWN})
\]

By solving these two equations, we can get

\[
V_{m}^{DOWN} = -\left(1 - \frac{\delta \alpha}{1-\delta}\right)(2c_m) 
\]

\[
V_{f}^{DOWN} = -\frac{\delta \alpha}{1-\delta}(2c_m)
\]

---

\[7\] Even though I use the term inconvenience costs, $c_f$ and $c_m$ can encompass other types of costs such as “unwittingly placing one’s bottom directly on the porcelain” and risk of falling in by sitting down without looking when the seat is up or “leaving sprinkles on the seat” when it is down, respectively.
Since the probability of a particular arrival being male is $\alpha$, the value function associated with the down rule is:

$$V^{DOWN} = \alpha V^{DOWN}_m + (1-\alpha) V^{DOWN}_f = -\frac{\alpha}{1-\delta} (2c_m) \quad (5)$$

**The Up (Male-Friendly) Rule**

This is a rule that leaves the position of the seat up after one is done with the bathroom task. In this case, all the inconvenience costs are incurred by females. The case is a mirror image of the down rule and the value function of this rule can be derived in an analogous way.

Let $V^{UP}_m$ and $V^{UP}_f$ denote the value functions when the particular user is male and female, respectively. Then, these value functions satisfy the following relationships.

$$V^{UP}_m = \delta[\alpha V^{UP}_m + (1-\alpha) V^{UP}_f] \quad (6)$$

$$V^{UP}_f = -2c_f + \delta[\alpha V^{DOWN}_m + (1-\alpha) V^{UP}_f] \quad (7)$$

By solving these two equations, I can derive

$$V^{UP}_m = -\frac{\delta(1-\alpha)}{1-\delta} (2c_f) \quad (8)$$

$$V^{UP}_f = -\left(1 - \frac{\delta(1-\alpha)}{1-\delta}\right) (2c_f) \quad (9)$$

Since the probability that a particular arrival is male is $\alpha$, the value function associated with the down rule is:

$$V^{UP} = \alpha V^{UP}_m + (1-\alpha) V^{UP}_f = -\frac{(1-\alpha)}{1-\delta} (2c_f) \quad (10)$$

A comparison of equations (5) and (10) yields the following proposition.
Proposition 1. The down rule is more efficient than the up rule if and only if
\[
\frac{c_f}{c_m} > \frac{\alpha}{1-\alpha}.
\]

The Selfish (Status Quo) Rule

This is a rule that leaves the position of the seat as it was used.

Let \( V_{m}^{SQ} \) and \( V_{f}^{SQ} \) denote the value functions when the particular user is male and female, respectively under the selfish rule. Then, these value functions satisfy the following relationships.

\[
V_{m}^{SQ} = -(1-\alpha) \, c_m + \delta[\alpha V_{m}^{SQ} + (1-\alpha) V_{f}^{SQ}]
\]  
(11)

\[
V_{f}^{UP} = -\alpha \, c_f + \delta[\alpha V_{m}^{SQ} + (1-\alpha) V_{f}^{SQ}]
\]  
(12)

By solving these two equations, I get

\[
V_{m}^{SQ} = \left[ \frac{(1-\alpha)(1-\delta(1-\alpha))}{1-\delta} c_m + \frac{\delta\alpha(1-\alpha)}{1-\delta} c_f \right]
\]  
(13)

\[
V_{f}^{SQ} = \left[ \frac{\delta\alpha(1-\alpha)}{1-\delta} c_m + \frac{\alpha(1-\delta\alpha)}{1-\delta} c_f \right]
\]  
(14)

Since the probability that a particular arrival is male is \( \alpha \), the value function associated with the down rule is:

\[
V_{m}^{SQ} = \alpha V_{m}^{SQ} + (1-\alpha) V_{f}^{SQ} = -\frac{\alpha(1-\alpha)}{1-\delta}(c_m + c_f)
\]  
(15)

Comparisons of equations (5), (10), and (15) give me the following result. See also figure 1.
Proposition 2. If the inconvenience costs are the same across genders ($c_m = c_f$), the selfish rule dominates both the up and down rules.

Figure 1. Comparisons of the Up, Down, and Selfish Rule for the Symmetric Inconvenience Costs ($c_m = c_f = c$)
The intuition for Proposition 2 is easy to understand. With either up or down rule, each member of one gender group has to incur the inconvenience costs two times with each usage. This practice can be obviously inefficient in the event that consecutive users are from the same gender to which the inconvenience costs are attributed. This inefficiency can be avoided by using the selfish rule since the inconvenience costs are incurred only when the consecutive users are from different genders. Even in such a event, the aggregate costs would be the same as those under the up or down rule if the inconvenience costs are the same across genders.

I cannot rule out the optimality of, say, the down rule if the inconvenience costs are asymmetric across genders. My analysis, however, suggests that to justify the down rule on efficiency grounds, the inconvenience costs for female should be very high relative to those for male. More precisely, the condition for the down rule to dominate the selfish rule is \( \gamma = \frac{c_f}{c_m} > \frac{1+\alpha}{1-\alpha} \). For instance, if male and female users visit the toilet with the same frequency (\( \alpha = 1/2 \)), the inconvenience costs for female should be three times higher than the corresponding costs for male to justify the down rule.

Up to now, I have considered only three potential mechanisms. These three rules, however, are not the only rules we can entertain. For instance, I can imagine a rule such that the position of the seat should be restored to the prior position before use. Alternatively, I can also consider a mutually considerate rule in which male users leave the seat down whereas female users leave the seat up after use. In the next section, however, I show that all these rules are dominated by one of the three rules I have considered. Thus, restricting my attention to the three rules does not entail any loss of generality in the analysis.
III. Characterization of the Optimal Rule: A Mechanism Design Approach

In the previous section, we compared three simple rules that can be used for the toilet seat position. The task of this section is to derive the most efficient rule among all possible mechanisms. I show that one of the three rules discussed in the previous section is always optimal. Thus, restricting my attention to the three rules does not entail any loss of generality if the only concern is to minimize the aggregate inconvenience costs of toilet users.

The general rule can be considered a collection of four numbers (\(\sigma_{um}, \sigma_{dm}, \sigma_{uf}, \sigma_{df}\)) where \(\sigma_{ij}\) denotes the probability that the seat be down after use when the position of the seat before use is \(i\) and the visitor is \(j\), where \(i = u, d\) and \(j = m, f\). The first subscripts \(u\) and \(d\) denote up and down, respectively and the second subscripts \(m\) and \(f\) denote male and female, respectively. The objective is to search for the best mechanism that minimizes the aggregate inconvenience costs.

In the Appendix, I prove that the position of the seat before one’s use should not count in the optimal rule.

**Lemma.** The optimal rule should depend only on the gender of the user, not the position of the seat before one arrives.

With the help of lemma, I can restrict my search for the optimal mechanism to a class of rules that can be written as \((\sigma_m, \sigma_f)\), where \(\sigma_m\) and \(\sigma_f\) are the probabilities that the toilet seat should be in the down position after usage by a male and a female, respectively.
Let $V_m(\sigma_m, \sigma_f)$ and $V_f(\sigma_m, \sigma_f)$ be the corresponding present discounted value when a particular user in the current period is male and female, respectively.

\[
V_m(\sigma_m, \sigma_f) = -[\alpha \sigma_m + (1-\alpha) \sigma_f] c_m - \sigma_m c_m + \delta[\alpha V_m + (1-\alpha) V_f]
\]

\[
V_f(\sigma_m, \sigma_f) = -[\alpha (1-\sigma_m) + (1-\alpha)(1-\sigma_f)] c_f - (1-\sigma_f) c_f + \delta[\alpha V_m + (1-\alpha) V_f]
\]

Then, the corresponding value function for the rule $(\sigma_m, \sigma_f)$ can be written as

\[
V(\sigma_m, \sigma_f) = \alpha V_m(\sigma_m, \sigma_f) + (1-\alpha) V_f(\sigma_m, \sigma_f) = -\frac{\alpha M + (1-\alpha)F}{1-\delta},
\]

where $M = [\alpha \sigma_m + (1-\alpha) \sigma_f] c_m + \sigma_m c_m$ and $F = [\alpha (1-\sigma_m) + (1-\alpha)(1-\sigma_f)] c_f + (1-\sigma_f) c_f$

The search for the optimal mechanism is equivalent to solving

\[
\min_{\sigma_m, \sigma_f} \alpha M + (1-\alpha)F
\]

**Proposition 3.** The optimal toilet etiquette is given by the following:

Let $\gamma = \frac{c_f}{c_m}$ be the relative cost of changing the toilet seat position for male and female.

Then, the optimal rule is characterized by two critical values of $\gamma$ ($\gamma$ and $\gamma'$) such that:

1. The toilet should be down if $\gamma > \gamma = \frac{1+\alpha}{1-\alpha}$

2. The toilet should be left as it was used if $\frac{\alpha}{2-\alpha} = \gamma < \gamma' = \frac{1+\alpha}{1-\alpha}$

3. The toilet should be up if $\gamma < \gamma' = \frac{\alpha}{2-\alpha}$
Proof. Since the objective function $\alpha M + (1-\alpha)F$ is a linear function of $\sigma_m$ and $\sigma_f$, I have corner solutions except the knife-edge cases. By differentiating $\alpha M + (1-\alpha)F$ with respect to $\sigma_m$ and $\sigma_f$, the optimal rule is given by:

$$\sigma_m = \begin{cases} 1 & \text{if } \gamma > \frac{1+\alpha}{1-\alpha} \\ \text{any number between 0 and 1} & \text{if } \gamma = \frac{1+\alpha}{1-\alpha} \\ 0 & \text{if } \gamma < \frac{1+\alpha}{1-\alpha} \end{cases}$$

$$\sigma_f = \begin{cases} 1 & \text{if } \gamma > \frac{\alpha}{2-\alpha} \\ \text{any number between 0 and 1} & \text{if } \gamma = \frac{\alpha}{2-\alpha} \\ 0 & \text{if } \gamma < \frac{\alpha}{2-\alpha} \end{cases}$$

Since $\frac{\alpha}{2-\alpha} < \gamma < \frac{1+\alpha}{1-\alpha}$, we have the desired result. Figure 2 summarizes the optimal configuration for different values of $\alpha$ and $\gamma$. Q.E.D.

My analysis can be easily extended to the case of time-varying $\gamma$ and $\alpha$. These values, for instance, can change depending on the time of the day. The mistake costs of "unwittingly placing one's bottom directly on the porcelain" or risk of falling in are presumably higher during the nighttime when the light is turned off. If this is the case, the optimal rule could be time-dependent, with the selfish rule during the daytime and the down rule during the nighttime.\[8\]

---

8 A countervailing argument against the down rule during the nighttime is that nocturia (needing to urinate frequently during the night time) is more common with men due to BPH (benign prostatic hyperplasia). I thank Carl Davidson for this observation.
Figure 2. The Optimal Rules with Asymmetric Inconvenience Costs
IV. Heterogeneous Costs within the Same Gender

In the previous sections, I assumed that the inconvenience costs are the same within the same gender. I extend the analysis to the case where different users have different inconvenience costs even within the same gender. Let me assume that $c_m$ and $c_f$ are distributed according to continuous distribution functions $G(.)$ and $H(.)$, respectively on $[0, \infty)$. Let $\bar{c}_m$ and $\bar{c}_f$ denote the mean values of the inconvenience costs for male and female, respectively:

$$\bar{c}_m = \int_{0}^{\infty} c_m dG = E(c_m), \quad \bar{c}_f = \int_{0}^{\infty} c_f dH = E(c_f)$$

It is clear that with heterogeneous inconvenience costs, the optimal rule should be characterized with two critical values $c_m^*$ and $c_f^*$ such that a male visitor should put the toilet seat down if and only if his $c_m \leq c_m^*$ and a female visitor should put the toilet seat up if and only if her $c_f \leq c_f^*$. Then, $\sigma_m = G(c_m^*)$ and $\sigma_f = 1 - H(c_f^*)$ using our previous notation. The value function with the critical values of $c_m^*$ and $c_f^*$ can be written as

$$V(c_m^*, c_f^*) = -\frac{\alpha M + (1 - \alpha) F}{1 - \delta},$$

where $M = [\alpha G(c_m^*) + (1 - \alpha)(1 - H(c_f^*))] \bar{c}_m + G(c_m^*) E(c_m \mid c_m \leq c_m^*)$ and $F = [\alpha(1 - G(c_m^*)) + (1 - \alpha) H(c_f^*)] \bar{c}_f + H(c_f^*) E(c_f \mid c_f \leq c_f^*)$.

With this observation, the optimal critical values for $c_m^*$ and $c_f^*$ can be derived by solving

$$\min_{c_m^*, c_f^*} \alpha M + (1 - \alpha) F$$

The first order condition with respects to $c_m^*$ and $c_f^*$ can be written as:

$$c_m^* = -\alpha \bar{c}_m + (1 - \alpha) \bar{c}_f$$
Since inconvenience costs are nonnegative, the two first order conditions cannot be satisfied as interior solutions simultaneously. Taking the boundary conditions into consideration, I can state the following result.

**Proposition 4.** With the heterogeneous inconvenience costs, the optimal toilet rule is given by the following:

(1) If \( \frac{\overline{c}_f}{\overline{c}_m} > \frac{\alpha}{1-\alpha} \), women should leave the seat as it was used and a male visitor should put the toilet seat down after use if \( c_m \leq c_m^* - \alpha \overline{c}_m + (1-\alpha) \overline{c}_f \) and use the selfish rule otherwise.

(2) If \( \frac{\overline{c}_f}{\overline{c}_m} < \frac{\alpha}{1-\alpha} \), men should leave the seat as it was used and a female visitor should put the toilet seat up after use if \( c_f \leq c_f^* = \alpha \overline{c}_m - (1-\alpha) \overline{c}_f \) and use the selfish rule otherwise.

**V. Concluding Remarks**

In this paper, I conducted an economic analysis of the most efficient rule concerning the position of the toilet seat when the objective is solely to minimize the aggregate costs of inconvenience. This research is in line with an argument that social norms are instruments of collective optimization when the price system is in applicable.9

The main result is that unless there is a large degree of asymmetry in the inconvenience

---

9See Arrow (1971).
costs of changing the seat position across users, the selfish rule is the most efficient one. In addition, the selfish rule is incentive-compatible in that it can be self-enforcing without any outside sanctions for violating the rule.

The analysis has applicability to a wide variety of circumstance in which a facility is used by different people and the facility can be customized according to the preference of each user. For instance, a car can be shared by family members who need different configurations of the driver seat. The same logic implies that it would be most efficient to leave the seat position as it was used last if the inconvenience costs of reconfiguring the position is the same across family members. A computer shared by different people is another example.

---

10 This result, however, should not be construed as implying that selfish behavior is optimal in other bathroom etiquettes. My analysis will apply only to the cases where different users have conflicting preferences. For instance, every user prefers clean toilets. In this case, cleaning after use for the next visitor should be the proper etiquette. This is especially so since cleaning one’s own is less unpleasant than the previous user’s.
Appendix: Proof of Lemma

To derive the optimal rule, I allow the possibility that a visitor’s decision can depend on the toilet position prior to one’s use. The general rule can be considered a collection of four numbers \( (\sigma_{um}, \sigma_{dm}, \sigma_{uf}, \sigma_{df}) \) where \( \sigma_{ij} \) denotes the probability that the seat be down after use when the position of the seat before use is \( i \) and the visitor is \( j \), where \( i = u, d \) and \( j = m, f \).

Suppose that the visitor in a particular period is male. At the time of his arrival, the toilet seat could be either up or down. Let \( V_{um} \) and \( V_{dm} \) be the value functions when the position of the toilet is up and down at the time of visit, respectively. I also denote corresponding value functions by \( V_{uf} \) and \( V_{df} \) when the visitor is female. Then, I can write

\[
V_{um} = \max_{\sigma_{um}} \left[ -c_m + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right] + (1-\sigma_{um})\left[ -c_m + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right]
\]

\[
V_{dm} = \max_{\sigma_{dm}} \left[ -c_m + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right] + (1-\sigma_{dm})\left[ -c_m + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right]
\]

Comparison of \( V_{um} \) and \( V_{dm} \) reveals that the optimal values for \( \sigma_{um} \) and \( \sigma_{dm} \) should be the same (\( \sigma_{um} = \sigma_{dm} \)), that is, the optimal rule should be forward-looking and not contingent on the position prior to the visit.

Similarly, when the visitor in a particular visit is female, we can write the value functions as

\[
V_{uf} = \max_{\sigma_{uf}} \left[ -c_f + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right] + (1-\sigma_{uf})\left[ -c_f + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right]
\]

\[
V_{df} = \max_{\sigma_{df}} \left[ -c_f + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right] + (1-\sigma_{df})\left[ -c_f + \delta \{ \alpha V_{dm} + (1-\alpha)V_{df} \} \right]
\]

Once again, comparison of \( V_{uf} \) and \( V_{df} \) reveals that the optimal values for \( \sigma_{uf} \) and \( \sigma_{df} \) should be the same (\( \sigma_{uf} = \sigma_{df} \)).
References
