Patent Pools, Litigation and Innovation

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Abstract
This paper analyzes patent pools and their effects on litigation incentives, overall royalty rates, and social welfare when patent rights are probabilistic and can be invalidated in court. With probabilistic patents, the license fees reflect the strength of the patents. We show that patent pools of complementary patents can be used to discourage litigation by depriving potential licensees of the ability to selectively challenge patents and making them committed to a proposition of all-or-nothing in patent litigation. If patents are sufficiently weak, patent pools with complementary patents reduce social welfare as they charge higher licensing fees and chill subsequent innovation incentives.

Keywords: Patent Pools, Probabilistic Patent Rights, Patent Litigation, Complementary Patents

JEL: O3, L1, L4, D8, K4

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1 Introduction

This paper analyzes patent pools and their effects on litigation incentives, overall royalty rates, and social welfare when patent rights are probabilistic. The existing literature on patent pools shows that the procompetitive effects of patent pools crucially depend on the relationship between the constituent patents. If the patents are complementary in nature, patent pools can reduce overall licensing royalties by internalizing pricing externalities and are thus procompetitive. However, if the patents are substitutes, then patent pools can be used as a collusive mechanism that eliminates price competition, and are thus anticompetitive (Shapiro, 2001; Lerner and Tirole, 2004).

We frame our model to consider the dynamic effects of patent pools by investigating the effects of patent pools for subsequent innovations that build on patents in the pools. The procompetitive effects of patent pools for complementary patents naturally apply to dynamic innovation incentives. As patent pools can mitigate the “patent thicket” problem for current users, they reduce royalty rates for subsequent innovations as well. As a result, follow-on innovators are less burdened by the royalty rates and innovation is promoted. However, this simple conclusion may not hold if we entertain the possibility that patents are probabilistic and can be invalidated in court. In such cases, royalty rates will reflect the strength of patents. If patents are weak, then overall royalty rates can be low with independent licensing. Patent pools of complementary patents can be used as a mechanism to discourage patent litigation by depriving potential licensees of the ability to selectively challenge patents. This imposes a proposition of all-or-nothing in patent litigation and enables patent holders to charge higher royalty rates. Patent pools can thus be used to shield weak patents from potential litigation.

Our paper is motivated by recent trends in high-tech industries. As products become more complex and sophisticated, they tend to encompass numerous complementary technologies. In addition, the innovation process is typically cumulative with new technologies building upon previous innovations (Scotchmer, 1991). To reflect such an environment, we consider a setup in which the development of a new technology requires licensing of multiple complementary patents owned by different firms. With complementary patents, patent pools are considered to be an effective way to mitigate the problem of patent thickets and reduce transaction costs. For instance, the Antitrust Guidelines for the Licensing and
Acquisition of Intellectual Property (1995), jointly published by the U.S. Department of Justice and the Federal Trade Commission, recognizes that inclusion of complementary or essential patents in a patent pool is procompetitive. We point out that such a sanguine view about patent pools with complementary patents may not be justified if we consider probabilistic patent rights.

To illustrate this, we develop the notion of the “litigation margin” that relates the patent holders’ ability to set license fees to litigation incentives by potential licensees. When the patent holders set their license fees, they need to consider the effects of a price increase on demand and litigation incentives by potential licensees. Since the incentives to litigate and invalidate patents decrease with the strength of the patents, the litigation margin is the binding constraint for patent holders when patents are weak. We show that patent pools provide a channel to relax the litigation margin, which leads to elevated license fees. Thus, the welfare effects of patent pools with complementary patents depend on whether the demand margin or the litigation margin is binding. When the demand margin is binding, the conventional result holds and patent pools are welfare-enhancing because they eliminate the pricing externality among patent holders. However, if patents are weak and the litigation margin is binding, patent pools can be welfare-reducing. Our paper thus formalizes the idea expressed in the Duplan case in which the court concluded that “[t]he ... patents in suit were known ... to be weak and, ..., they [the parties] were confident that these patents could be invalidated.” The main purpose of the patent pool in the case was “to protect the parties from challenges to the validity of their patents” in order to gain “the power to fix and maintain prices in the form of royalties which they... exercised thereafter.”

Package licensing by a patent owner is akin to patent pools and raises similar issues in combining multiple intellectual property rights. Package licensing has long been recognized as potentially anti-competitive, as a form of tying or bundling by competition authorities and the courts; especially, when a patent owner refuses to grant individual licenses (or alternatively, by charging a license fee that is invariant with respect to the number of patents). In particular, there have been concerns that package licensing can be used as a leverage mechanism to extend market power from legitimate patent claims to illegitimate

ones. For instance, in American Security Co. v. Shatterproof Glass Corp., 268 F.2d 769 (3rd Circuit, 1959), the court condemned package licensing that required the licensee to pay the same price regardless of the number of patents the licensee implemented. The point that the bundling of intellectual property rights might decrease “private incentives to challenge the IP” has also been raised in the Department of Justice and the Federal Trade Commission Report (2007) on IP. However, the analysis of package licensing with probabilistic patents has not been carefully explored in the theoretical literature. We formalize this idea and show that the assessment of the likely welfare effects of patent pools depends on whether the demand margin or the competitive margin is binding with probabilistic patents even when the constituent patents are complementary. This implies that the assessment is more nuanced and fact-intensive than recognized before.

Gilbert and Katz (2006) provide an analysis of package licensing and compare the welfare properties of package licensing to those of component licensing under which each patent is licensed separately without any quantity discount. They focus on the effects of package licensing on the licensee’s incentives to invent around patents and invest in complementary assets. Package licensing in their model plays the role of raising licensee fees for the remaining technologies when the licensee succeeds in inventing around only part of the patent portfolio included in the package. As a result, package licensing can attenuate the incentives to invent around patents in comparison with component licensing. The reduced incentives can be welfare-enhancing because inventing-around activities are privately beneficial, but socially wasteful. As in Gilbert and Katz, the combination of separately owned patents under the common administration of a patent pool plays a similar role of raising licensee fees for the surviving patents when the licensee fails to invalidate them all in court. However, in our model, the reduced incentives to litigate for the licensee may induce a higher overall licensing fee in the presence of patent pools even for complementary patents, thus retarding future innovations.

Our results seem to be consistent with recent empirical findings. Lampe and Moser (2010, 2013, 2014) and Joshi and Nerkar (2011) provide the first empirical tests of the effects of patent pools on innovation incentives. More specifically, Lampe and Moser (2010, 2013) study the Sewing Machine Combination (1856-1877), the first patent pool in U.S. history, whereas Joshi and Nerkar (2011) study the effects of patent pools in the recent global

\[3\]See Rubinfeld and Maness (2005) for more discussion on this.
optical disc industry. In both cases, they find that patent pools inhibit, rather than enhance, innovation by insiders (pool members) and outsiders (licensees). In particular, Lampe and Moser (2010, 2013) show that the Sewing Machine Combination patent pool discouraged patenting and innovation. They attribute the negative incentive effects of the patent pool to the fact that patent pools create more formidable entities in court and thus increase the threat of litigation for outside firms, as their data show that outside firms were at a greater risk of being sued while the pool was active, lowering expected profits and discouraging innovation by outsiders. Lower rates of innovation by outsiders in turn reduced incentives for pool members to innovate. Lampe and Moser (2014) further extend their empirical analysis to examine patent pools in 20 industries in the 1930s. They find a substantial decline in patenting after the formation of a pool and come to the same conclusions as with the sewing machine industry. We develop a dynamic model of innovation in the presence of uncertain patent validity and litigation that is consistent with this empirical evidence on patent pools. In particular, our analysis shows that patent strength is an important consideration in the evaluation of patent pools as it affects the terms of licensing when the litigation margin binds.

Our paper closely relates to Shapiro (2003) and Choi (2010), who also recognize that IPR associated with patents are inherently uncertain or imperfect, at least until they have successfully survived a challenge in court. Choi’s (2010) analysis focuses on incentives to litigate against each other’s patents between potential pool members (i.e., insiders) and considers patent pools as an attempt to settle disputes on conflicting claims in the litigation process or in expectation of impending litigation. In contrast, this paper investigates incentives to litigate against outsiders with subsequent innovations that build upon existing patents. Shapiro (2003) proposes a general rule for evaluating proposed patent settlements, which is to require that “the proposed settlement generate at least as much surplus for consumers as they would have enjoyed had the settlement not been reached and the dispute instead been resolved through litigation.” Finally, Gilbert (2002) provides a brief history of patent pools and points out that patent pools can be used to protect dubious patents from challenges. This paper provides a theoretical foundation of a mechanism through which

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4 In a related empirical research, Baron and Delcamp (2010) explores the impact of patent pools on firm patenting strategies and show that firms that are already members of a pool are able to include narrower, more incremental and less significant patents than outsiders.
dubious patents can be shielded from challenges to the validity of the patents.

The remainder of the paper is organized as follows. In section 2, as a benchmark case, we analyze the case of ironclad patents and show that patent pools with complementary patents promote subsequent innovations, echoing the basic presumption in the literature enunciated in the Antitrust Guidelines for the Licensing and Acquisition of Intellectual Property (1995). In section 3, we extend the analysis to consider probabilistic patents and explicitly consider strategic incentives to litigate. As a first step, we consider a situation in which only the litigation margin binds by abstracting away from the pricing externalities issue associated with the demand margin. This is to isolate the mechanism through which patent pools deter litigation and elevate royalty rates vis-à-vis independent licensing. In section 4, we analyze the full model that takes into account both the litigation and demand margins. Section 5 considers a public policy that mandates patent pools to engage in individual licensing and its welfare effects. Section 6 expands on the basic model and considers extensions of the model to check the robustness of the main results. The last section concludes. All proofs are relegated to the Appendix.

2 Complementary Patent Pools and Technology Adoption

We consider a situation of multiple patents with dispersed ownership. For example, assume that there are two complementary patents, A and B, which are owned by two separate firms.\footnote{We discuss the general case with \( n \geq 2 \) patents in Section 6.} As emphasized by Scotchmer (1991), innovations are cumulative and the patents are deemed essential, as the commercialization of a new technology or product requires the practice of both patents.\footnote{For instance, the intellectual property to be licensed could be complementary research tools (Schankerman and Scotchmer, 2001) and any user needs to get licenses for both technologies.}

As a benchmark, we look at the case where both patents are ironclad and cannot be challenged in court. We analyze the patent holders’ incentives to form a pool and show how the formation of patent pools can affect future incentives to develop new innovations. Consider the following multi-stage game. In the first stage, the two firms decide whether or not to form a patent pool. In the second stage, they set license fees that allow other firms to use their technologies without infringing them. If they do not form a patent pool, they set the license fees independently. If they do form a patent pool, they can offer a package...
In the third stage, a downstream firm C comes up with a potential innovation of value \( v \), which cannot be practiced without consent of the holders of the essential patents. The cost to implement the innovation is \( c \geq 0 \), where \( c \) is randomly distributed with a cumulative distribution function \( G(.) \) and corresponding density function \( g(.) \). Assume that the reversed hazard rate of \( G(.) \), defined by \( r(.) = g(.)/G(.) \) is monotonically decreasing in its argument.

Two brief comments on this set-up are in order. First, throughout this paper, we allow the possibility of ex ante licensing and analyze the patent holders’ incentives to offer a license for their technology at a fixed price before the cost of subsequent innovation or adoption is sunk. Such ex ante licensing can serve as a commitment mechanism not to hold up against future downstream use of the technologies. Secondly, our analysis also applies to situations where the adoption of the patented technology is by final users rather than intermediate firms who are themselves innovators. In this case \( c \) can be interpreted as the cost of adopting the patented technologies. Or, alternatively, we can think of firm C as a downstream firm that commercializes the patented technologies to the market, in which case \( c \) can be considered as a development cost.

Suppose that both firms offer ex ante license contracts independently. Let \( f_A \) and \( f_B \) be the fixed license fees charged by firm A and B, respectively. Then, firm C develops the innovation only when its development cost is less than \((v - f_A - f_B)\) which occurs with probability \( G(v - f_A - f_B) \). Then, for a fixed royalty rate \( f_i \), firm \( i \) maximizes \( f_i G(v - f_A - f_B) \) with respect to \( f_i \) which yields the first order condition

\[
    f_i = \frac{G(v - f_A - f_B)}{g(v - f_A - f_B)}.
\]

Equation (1) implicitly defines firm \( i \)'s reaction function \( f_i = \Theta(f_j) \). The Nash equilibrium license fees \( f_A^* \) and \( f_B^* \) are at the intersection of the two reaction functions. The monotone reversed hazard rate assumption guarantees the stability and uniqueness of the Nash equilibrium in license fees. With perfect complementarity and ironclad patents, both firms are in a symmetric position and charge \( f_A^* = f_B^* = f^* \). The total royalty rate in the absence of a patent pool is given by \( F^* = f_A^* + f_B^* \).

In contrast, if firms A and B form a patent pool and practice package licensing, the optimal royalty rate maximizes \( FG(v - F) \) with respect to \( F \). Let \( F^{**} \) be the optimal ex
ante fixed licensing fee for the pool. Then, $F^{**}$ satisfies the following first order condition:

$$F^{**} = \frac{G(v - F^{**})}{g(v - F^{**})}. \quad (2)$$

Proposition 1 shows that aggregate license fees are lower when firms form a patent pool. Thus, pools promote subsequent innovation incentives in the presence of ironclad patents.

**Proposition 1** Consider pool formation and licensing with ironclad, complementary patents. When firms form a patent pool, total licensing fees are lower, that is, $F^* = f^*_A + f^*_B > F^{**}$, and there are more downstream innovations. Social and private incentives to form a patent pool are perfectly aligned.

This is a variation of the well-known result that dates back to Cournot’s (1927) analysis of the complementary monopoly problem. Without coordination in licensing fees, each patentee does not internalize the increase in the other patentee’s profits when the demand for the package is increased by a reduction in its price. Thus, a patent pool can decrease the overall royalty rates for the package and simultaneously increase both patentees’ profits and induce more adoption of the technologies. Consequently, social welfare also increases and an argument can be made for a lenient treatment of patent pools in the presence of complementary blocking patents.

### 3 Probabilistic Patent Rights and Litigation with Patent Pools

In the previous section, we have seen that patent pools of complementary technologies have salutary effects of promoting subsequent innovations. However, this conclusion hinges crucially on the assumption of ironclad patents. If we recognize that patent rights are probabilistic and can be invalidated in court when challenged, licensing takes place in the shadow of patent litigation and the licensing terms will reflect the strength of the patents. In this section, we show that if patent pools are used as a mechanism to harbor weak patents and deter patent litigation, then they may induce higher royalty rates than would be paid if licenses were sold separately by independent patent holders.

**A Model of Probabilistic Patents.** To analyze the incentives to form patent pools with probabilistic patents, we represent the uncertainty about the validity of the patents by the

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7 Variables associated with patent pools are denoted with double asterisks.
parameters $p_A = \alpha \geq 0$ and $p_B = \beta \leq 1$, which are the probabilities that the court will uphold the validity of patents $A$ and $B$, respectively, if they are challenged. Without any loss of generality, we assume that patent $B$ is at least as strong as patent $A$, that is, $\alpha \leq \beta$. We assume a symmetric information structure in that $\alpha$ and $\beta$ are common knowledge.

The timing of the game with probabilistic patents follows the set-up in the previous section with two additional litigation stages after the downstream firm’s decision of whether to buy the licenses or not. If the downstream firm purchases both licenses, the game ends. If the downstream firm decides not to buy one (or both) of the licenses, the patent holder(s) can choose whether to sue for infringement. If a patent pool has not formed, firms A and B make their litigation decision simultaneously.\(^8\) Let $L \geq 0$ be the litigation cost for each firm. In case of litigation with weak patents, firm C will contest the validity of the patent in question. In the second additional stage, the court determines the validity of all challenged patents and litigation outcomes are revealed. If a patent has been challenged and upheld, its holder proposes a new license fee and firm C decides whether to purchase the license or not.\(^9\) In contrast, we assume that the fee for a purchased license can not be raised when the other patent has been challenged and invalidated. This essentially captures that when a patent holder sets a license fee, it is a commitment which can only be revoked when the downstream firm refuses to license and instead infringes and challenges the patent. Meanwhile, when both patents have been challenged and validated, the patentees simultaneously choose their license fee. After the court has invalidated a patent, the downstream firm can use the technology at no cost. If firm C does not acquire a license of a validated or unchallenged patent, it is unable to produce and thus receives a profit of zero. To summarize, the game proceeds as follows.

1. Firms $A$ and $B$ decide whether or not to form a patent pool.
2. Firms $A$ and $B$ set license fees. If they form a patent pool, they coordinate their license fees. Otherwise, they set license fees independently.
3. Firm $C$ draws its cost $c$ from distribution $G(.)$ and decides whether to incur the cost and engage in the subsequent innovation. If Firm $C$ does not engage in the innovation,\(^8\) In Section 6 we also consider the possibility of sequential litigation involving one patent at a time.\(^9\) Farrell and Shapiro (2008) make a similar assumption. They assume that if a patent is ruled valid, any licenses already signed remain in force, but that the patent holder negotiates anew with the downstream firm(s) that lack licenses.
the game ends.

(4) Firm $C$ decides for each technology whether to buy the license or not.

(5) The upstream firms (or the pool) decide(s) whether to sue firm $C$ for infringement. If sued for infringement, firm $C$ contests the validity of the patent in court.$^{10}$

(6) Litigation outcomes are revealed. If a patent has been challenged and upheld, its holder proposes a new license fee for firm $C$. If both patents have been challenged and validated, the upstream firms simultaneously choose their license fee.

(7) If firm $C$ has a license for all non-invalidated patents, it receives a payoff of $v$.

In this set-up the patent holders face not only a demand margin for subsequent innovations as in the previous section, but also a litigation margin. An increases in license fees can lead to firms not developing the innovation or it can result in product development followed by patent litigation. As an intermediate step towards deriving the optimal license fee equilibrium with both active demand and litigation margins, we first consider a game that ignores the demand margin and focuses on the litigation margin. In other words, we assume that firm $C$ always develops the subsequent innovation and we analyze how litigation considerations influence the patentees' licensing decisions. That is, for the current section only, we consider a game with the following stage (3L) instead of (3):

(3L) Firm $C$ develops the subsequent innovation.

One way to think about this is to assume that the downstream firm has no development cost ($c = 0$). In this case, as we show below, firm $C$ always introduces the innovation. This approach allows us to abstract away from the pricing externalities issue associated with the demand margin. In section 4, we consider the full game with both the demand and litigation margins using the results of this section and the previous section. We now solve for the subgame perfect equilibrium with an active litigation margin only.

**Litigation Incentives.** Licensing occurs in the shadow of patent litigation in this framework. Throughout the paper, we focus on parameter values such that the threat of litigation

$^{10}$In an earlier version of this paper, we analyzed a game in which litigation is initiated by firm $C$, and derived qualitatively the same results.
(and counter-litigation) is credible and assume

$$\frac{L}{v} \leq \frac{\alpha}{2\alpha + 1}(1 - \alpha)(1 - \beta).$$

(A)

This condition requires that the cost of litigation is sufficiently small relative to the value of the commercialised downstream product. In the Appendix we show the following lemma.

**Lemma 1** If condition (A) holds, then for any license fee offer, patent holders have an incentive to sue for infringement when the downstream firm is using one or both technologies without a license.

Condition (A) is a sufficient condition that ensures the credibility of infringement suits both when patent holders act independently and when they form a patent pool. As a consequence, the downstream firm has a choice between purchasing a license or entering into litigation to challenge the validity of the patent. In what follows, we consider the license fee equilibrium with independent patent holders and with a patent pool, respectively.

**Licensing Equilibrium with Independent Firms.** Suppose firms A and B propose license fees, $f_A$ and $f_B$, respectively. At this point, following Lemma 1, firm C has four strategic options. First, suppose the downstream firm does not buy any license and litigation ensues with both patent holders. If the court declares both patents invalid, firm C can use both technologies at no cost. If exactly one patent is upheld, its owner charges the monopoly price $v$. If both patents are upheld, there exists a Nash equilibrium in which each patent holder charges $v/2$ and firm C makes no profits.\(^{11}\) Hence, the downstream firm’s expected profit with litigation against patents A and B is

$$V_{AB} = (1 - \alpha)(1 - \beta)v - 2L.$$

Under assumption (A), it holds that $V_{AB} \geq 0$, that is, challenging both patents always dominates remaining inactive. Now suppose firm C does not purchase a license for technology A but acquires a license from B. If, in the resulting litigation, the validity of patent A is upheld, then firm A charges $v - f_B$ and firm C receives no profits. If the patent is

\(^{11}\)There are no equilibria in which the patent holders extract less than the entire surplus $v$. While there exist other equilibria in which the patent holders extract the total value $v$, the symmetric equilibrium seems natural as both technologies are essential and both patents have been validated.
invalidated, the downstream firm can use technology $A$ at no cost. Hence, the expected payoff, when litigation arises with firm A only, is

$$V_A = (1 - \alpha)(v - f_B) - L.$$  

Similarly, the expected profits of challenging patent $B$ and purchasing the license for $A$ are

$$V_B = (1 - \beta)(v - f_A) - L.$$  

Note that the payoff with exactly one litigation decreases in the license fee paid for the other technology. Finally, if firm $C$ accepts both license offers, it receives

$$V_0 = v - f_A - f_B.$$  

What is the optimal licensing (and litigation) strategy for firm $C$? As convention, assume that if the downstream firm is indifferent between two options, it chooses the one that involves less litigation. If the downstream firm is indifferent between purchasing only $A$ or only $B$, the firm randomizes and acquires each license with probability $1/2$. It can be shown that firm $C$ buys both licenses if $V_0 \geq V_A$, that is, if

$$f_A \leq \alpha(v - f_B) + L \quad (3)$$

and $V_0 \geq V_B$, which requires

$$f_B \leq \beta(v - f_A) + L. \quad (4)$$

We illustrate the optimal licensing behavior of firm $C$ in Figure 1 below. The graph depicts the optimal strategy for any license fee pair set by the patent holders. Region 0 in Figure 1 below contains all license fee pairs that jointly satisfy conditions (3) and (4). Let $(f_A, f_B)$ denote the license fee pair at which both conditions hold with equality. Alternatively, the downstream firm prefers not to purchase licenses and challenge both patents if $V_{AB} \geq V_A$,

$$f_B \geq \beta v + \frac{L}{1 - \alpha}. \quad (5)$$
and $V_{AB} \geq V_B$, 

$$f_A \geq \alpha v + \frac{L}{1 - \beta}. \tag{6}$$

Fee region $AB$ of Figure 1 satisfies both of these two conditions. Finally, it is easy to check that there exist license fees that neither satisfy the conditions of region $0$ nor those of region $AB$. For these license fees, the downstream firm is best off buying a license from one patentee while challenging the other patent. With exactly one litigation target, the downstream firm prefers to purchase $B$ and challenge patent $A$ if $V_A \geq V_B$, or

$$f_B \leq \frac{\beta - \alpha}{1 - \alpha} v + \frac{1 - \beta}{1 - \alpha} f_A. \tag{7}$$

If the license fee for patent $B$ is relatively small compared to $f_A$, then the downstream firm challenges patent $A$ (region $A$). Otherwise, it contests the validity of patent $B$ (region $B$).

We can thus summarize the downstream firm’s optimal litigation and licensing as follows.

**Lemma 2** If $f_A$ and $f_B$ are both sufficiently low, firm $C$ buys both licenses. If $f_A$ and $f_B$ are both sufficiently high, it challenges the validity of both patents. Otherwise, firm $C$ purchases one license and litigates against the other patent holder.
Let us now analyze the license fee equilibrium. In the absence of a patent pool, patentees A and B set their license fees independently and maximize their respective expected profits. As shown in the Appendix, an individual patentee’s best response function is a limit licensing strategy that ensures that the downstream firm purchases the firm’s license rather than challenging the patent. The optimal limit license fee depends on the fee charged by the other patentee. If firm $j$’s license fee is low ($f_j \leq \overline{f}_j$), the limit license fee of patentee $i$ is the highest fee at which the downstream firm prefers to purchase both licenses to challenging firm $i$’s patent. For intermediate fees, the limit licensing occurs just below the fee that would make the downstream firm indifferent between challenging firm A or firm B’s patent, that is, just above or below where (7) holds with equality. For higher values of $f_j$, the downstream firm challenges the other firm’s patent and the limit license fee for firm $i$ satisfies $V_j = V_{AB}$. Hence, the unique intersection of the best response functions is at

$$(\overline{f}_A, \overline{f}_B) = (\overline{f}_A, \overline{f}_B) = \left( \frac{(1 - \alpha)L + \alpha(1 - \beta)v}{1 - \alpha \beta}, \frac{(1 - \beta)L + \beta(1 - \alpha)v}{1 - \alpha \beta} \right)$$

and total license fees with independent patent holders are $F^* = \overline{f}_A + \overline{f}_B$. Intuitively, an increase in a firm’s patent strength raises the limit license fee it can charge. Thus, a patentee’s equilibrium license fee increases in the strength of its own patent and decreases in the other patent’s strength.

**Proposition 2** Consider the license fee equilibrium with independent patent holders when only the litigation margin is binding. Independent patent holders set limit licensing fees that prevent litigation with the downstream firm. A firm’s equilibrium license fee increases in the litigation cost and the strength of its own patent. It decreases in the strength of the other patent.

**Patent Pool with Package License and Comparison.** Suppose the patent holders form a pool and sell a package license to the two patents for a fee $F$. The patent pool maximizes the joint profits of the patent holders. The downstream firm can either buy the package license or enter into litigation and challenge both pool patents or remain inactive. When firm $C$ opts not to buy and to infringe on the patents, it needs to successfully challenge both pool patents in order to earn any profits. By assumption (A), challenging

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12 In Section 5 we consider the case of a patent pool selling individual licenses rather than a package license.
both patents dominates remaining inactive. The downstream firm buys the package license if \( v - F \geq V_{AB} \), or

\[
F \leq F^{**} = [1 - (1 - \alpha)(1 - \beta)]v + 2L, \tag{8}
\]

where \( F^{**} \) is the limit license fee for the patent pool. In order to avoid the cost of litigation, it is always optimal for the pool to set the limit license fee.\(^{13}\)

We are now in a position to compare the aggregate limit license fees charged by a pool and independent patentees when the litigation margin is binding. Figure 2 below depicts the aggregate license fee for each case in the \((f_A, f_B)\) space. Independent patent holders set their equilibrium fees such that the downstream firm is indifferent between buying both licenses or challenging exactly one patent. In contrast, a patent pool sets its package fee \( F^{**} \) such that the downstream firm is indifferent between buying all licenses and challenging all patents. It thus holds that

\[
V_A|_{f_B = \bar{f}_B} = V_B|_{f_A = \bar{f}_A} = V_0|_{F = \bar{F}} > V_{AB} = V_0|_{F = \bar{F}^{**}}
\]

and the next proposition follows.

**Proposition 3** Suppose only the litigation margin is binding. A patent pool with a package license charges a higher aggregate limit license fee compared to independent patent holders, that is, \( \bar{F}^* = \bar{f}_A + \bar{f}_B < F^{**} \).

In the presence of weak patents and litigation, we get the reverse result of Proposition 1. A patent pool issuing a package license is able to charge higher license fees than independent patent holders. Two arguments explain this result. First, the pool’s package license imposes an all-or-nothing litigation proposition on the downstream firm. The only way to reduce its payment to the pool is to successfully challenge all of the patents in the pool. By contrast, when individual patent holders market their license, the downstream firm reduces its royalty payments with any successful litigation challenge. This ability to selectively challenge patents increases the downstream firm’s incentives to litigate with independent

\(^{13}\) Setting a higher fee and inducing litigation is never profitable since the expected profits from litigation are less than the pool’s limit license fee,

\[
[1 - (1 - \alpha)(1 - \beta)]v - 2L < \bar{F}^{**}.
\]
patent holders. In this sense, pools can shield weak patents. Put differently, it is always easier for the patent holders to satisfy the pool’s limit licensing constraint (8) rather than the conditions (3) and (4) jointly when patents are licensed separately. This mechanism allows the pool to charge a higher aggregate license fee.

Second, independent patent holders are unable to sustain higher license fees than \((\bar{f}_A, \bar{f}_B)\) because they are engaged in a Bertrand-type competition with respect to litigation. Suppose both individual patent holders set their fees above their equilibrium fees \((\bar{f}_A, \bar{f}_B)\). In fee regions A and B, the downstream firm challenges exactly one patent and the holders’ license fees determine the litigation target. However, each individual patent holder is better off reducing its license fee to avoid a possible challenge against its own patent. Hence, this litigation externality creates downward pressure on license fees, and individual patent holders compete each other down to the limit licensing levels.\(^{14}\)\(^{15}\)

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\(^{14}\)This fee competition of not being the one litigation target of the downstream firm arises for intermediate license fees of the other patent holder. In those cases, the best response of a patent holder is to limit price and shave the fee that makes the downstream firm indifferent between challenging one or the other patent.

\(^{15}\)In Section 5, we consider a patent pool that issues individual licenses and is able to internalize this litigation externality.

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![Figure 2: Equilibrium licensing fee with and without patent pool](image-url)
Note that the pool’s ability to charge higher aggregate license fees does not disappear when litigation costs are zero as

\[ \bar{F}^*(L = 0) - \bar{F}'(L = 0) = \frac{\alpha \beta (1 - \alpha)(1 - \beta)v}{1 - \alpha \beta} > 0. \]

As the cost \( L \) increases, the pool’s all-or-nothing litigation proposition further increases the difference between the aggregate limit license fees since

\[ \frac{\partial \bar{F}^*}{\partial L} - \frac{\partial \bar{F}^*}{\partial L} = \frac{\alpha + \beta - \alpha \beta}{1 - \alpha - \beta} > 0. \]

We have shown that patent pools can elevate the total licensing fees when they are used to shield weak patents from the threat of litigation. However, the elevated licensing fees have no efficiency consequences in the simple model where only the litigation margin binds. Licensing fees are just a transfer between the patent holders and the downstream firm. The only source of inefficiency is costly litigation, which does not arise in equilibrium. In the next section, we extend our model to allow both the demand and the litigation margin to bind.

4 The Interplay of the Demand Margin and Litigation Incentives

We have considered two extreme cases where either only the demand margin or only the litigation margin binds. Now we analyze the full game as described at the beginning of section 3, in which both margins figure into the patentees’ licensing decisions. With ironclad patents, the patent holders’ licensing decisions are driven solely by the demand margin, captured by the innovation cost distribution of the downstream firm, which yields a license demand function \( G(v - f_A - f_B) \). With probabilistic (weak) patents, patentees also need to pay attention to the litigation incentives, as setting too high a license fee may trigger a patent challenge by the downstream firm. As will be shown below, the optimal license fees will depend on whether the demand or the litigation margin is binding.

**Equilibrium Licensing Fees with Independent Licensing.** Let us first consider the licensing equilibrium when both firms set fees independently without forming a patent pool.
When both the litigation and demand margins potentially bind, the best response function of each patentee depends on the relative position of the reaction function \( f_i = \Theta(f_j) \) from section 2 and the downstream firm’s litigation conditions from section 3. In particular, three possibilities may arise, depending on which margin binds for each firm.

**Case 1: Litigation margins not binding.** When litigation costs are high and patents are strong, the license fee equilibrium with the demand margin is not constrained by litigation incentives. This holds when the equilibrium fee from section 2 is less than the lowest equilibrium fee with a binding litigation margin, that is,

\[
f^* < \overline{f}_A \leq \overline{f}_B.
\]  

(9)

In this case, the downstream firm has no incentive to litigate when firms set their equilibrium licensing fees derived in the analysis of section 2, and patentees behave as if their patent were ironclad. The equilibrium license fees are thus given by \( f_A^* = f_B^* = f^* \).

**Case 2: Both litigation margins binding.** When litigation costs are sufficiently small, each firm’s limit litigation fee from section 3 is less than its best response to the rival’s limit litigation fee, that is, \( \overline{f}_i \leq \Theta(\overline{f}_j) \). For \( \alpha \leq \beta \), both conditions are satisfied if

\[
\overline{f}_B \leq \Theta(\overline{f}_A).
\]  

(10)

In this case, the litigation margin binds for both firms. When firm \( j \) sets \( \overline{f}_j \), firm \( i \) has no incentive to increase its fee as it would trigger a challenge against its own patent. In addition, condition (10) ensures that both patent holders have no incentive to decrease their license fee either. Thus, in a subgame perfect equilibrium, each patentee sets its licensing fee at the level that deters litigation and we get the same equilibrium fees as in section 3, that is \( (\overline{f}_A, \overline{f}_B) \).

When both patents are of equal strength \( (\alpha = \beta) \), conditions (9) and (10) cannot be violated at the same time. This means that in a subgame perfect equilibrium of the full game, firms are either constrained by the demand margin and price like in section 2, or they are constrained by the litigation margin and set the equilibrium fees of section 3.

**Case 3: Litigation margin only binds for firm A.** If patents are of asymmetric strength, a third case can arise in which conditions (9) and (10) are both violated. In this case, holder A with the weaker patent is constrained by the litigation limit whereas holder B operates
on the demand margin. They delegate the formal proof of this discussion to the Appendix and state the main result for the licensing game with independent patent holders.

**Lemma 3** When litigation costs and patent strengths are sufficiently low, there exists a unique subgame perfect equilibrium in which the patent holders set their limit litigation license fees at $f_A$ and $f_B$, respectively.

**Equilibrium License Fees with Patent Pool.** Now suppose that firms $A$ and $B$ form a patent pool. Again, the optimal package license fee depends on whether the demand or the litigation margin binds. Note that the pool’s optimal fee from section 2, $F^{**}$, is completely determined by the demand conditions (that is, the cost distribution function $G$), while the limit license fee $F^{**}$ is determined by the strength of the patents and litigation costs. When the pool patents are sufficiently strong and litigation is costly, it holds that $F^{**} < F^{***}$ and the pool can set its package license fee as if its patents are ironclad, as there is no threat of litigation by the downstream firm. Otherwise, the litigation margin binds and the patent pool sets its limit license fee at $F^{***}$ to deter litigation. Hence, the patent pool’s optimal licensing fee is given by $\min[F^{**}, F^{***}]$.

**Comparison of Aggregate License Fees.** The welfare effects of patent pools depend on whether patent pools elevate or reduce the overall licensing fees paid by the downstream firm. From the above analysis it is clear that in Case 1 where independent firms are not constrained by the litigation margin, the standard result with ironclad patents obtains and a pool charges lower aggregate fees as it avoids royalty stacking.

However, when the litigation margin binds for independent patent holders, that is, condition (10) is satisfied, we get the same result as in section 3 and patent pools are able to extract a higher total license fee. Since $F^{***} < F^{**}$, a sufficient condition for this to hold is that the unconstrained license fee of the patent pool $F^{**}$ is larger than the aggregate limit license fee $F^*$ when the litigation margin binds for independent patent holders. Since

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16 We analyze this case more closely in our working paper, Choi and Gerlach (2013), and show that in such situations, a pure-strategy equilibrium in license fees might fail to exist.

17 If condition (9) holds, patent pools charge lower fees since

$$\min\{F^{**}, F^{***}\} \leq F^{**} < F^*.$$
\( F^{**} = \Theta(0) \) we get that \( \overline{F} < F^{**} \) if and only if

\[
\overline{F}_B \leq \Theta(0) - \overline{F}_A. \tag{11}
\]

A necessary condition for (11) to hold is that the litigation margins bind with independent patent holders, that is, \( \overline{F}_B \leq \Theta(\overline{F}_A) \). Furthermore, we can explicitly solve this condition and show that it is satisfied if and only if

\[
L \leq L' = \frac{1 - \alpha\beta}{2 - \alpha - \beta}\Theta(0) - \frac{\alpha + \beta - 2\alpha\beta}{2 - \alpha - \beta} v.
\]

Upon inspection, we find that if the patent strength parameters are sufficiently small, then there always exists a threshold value \( L' (> 0) \) such that for any litigation cost \( L < L' \), the total license fee is higher with a patent pool. We thus get the following result.

**Proposition 4** Consider the full game with both the demand and litigation margins. If patents are sufficiently weak and litigation cost is low relative to the value of the innovation, then patent pools hinder subsequent innovations and reduce welfare.

The condition in Proposition 4 arises when the threat of the downstream firm litigating against weak patents is sufficiently strong. In such a case, patent pools can be used for safe-harboring weak patents from litigation in order to elevate the overall licensing fees.

**An Example.** To illustrate these results further, consider an example with symmetric patent strengths \( \alpha = \beta \) and let the development cost \( c \) be distributed uniformly on \( [0, 1] \), with the value of innovation normalized at \( v = 1 \). In this case, it is always optimal to develop the subsequent innovation. However, with patent rights and licensing, the downstream firm develops the new product only when \( c + f_A + f_B < 1 \). Thus, the demand function for the joint licenses is given by \( (1 - f_A - f_B) \), which is the probability that the development cost satisfies the condition \( c + f_A + f_B < 1 \) given the uniform distribution of \( c \). It is then easily verified that the optimal license fees with demand margins are given by \( F^* = 1/3, \overline{F}^* = 2/3 > F^{**} = 1/2 \). When the litigation margins bind, the license fees are determined by the strength of the patents and the cost of litigation. With symmetric patents, we get \( \overline{F}_A = \overline{F}_B = (L + \alpha)/(1 + \alpha) \) and \( \overline{F}^{**} = \alpha(2 - \alpha) + 2L > \overline{F}^* \). In the game with potentially binding demand and litigation margins, independent patent holders charge the limit license
fee when (10) is satisfied, or \( L \leq (1 - 2\alpha)/3 \). Otherwise, the demand margin binds and aggregate fees are \( F^* \). We furthermore check that condition (11) holds, that is, \( \overline{F} = F^* \) if and only if \( L \leq L' = (1 - 3\alpha)/4 \). Moreover, since the pool charges the limit litigation fee if \( \overline{F} = F^* \), or \( L \leq (1 - 2\alpha(2 - \alpha))/4 < L' \), we get the following result.

**Corollary 1** Suppose \( c \) is uniformly distributed on \([0,1]\), patents are of equal strength and \( v \) is normalized to 1. In this case, condition (11) is necessary and sufficient for patent pools to charge higher aggregate license fees than individual license holders.

Our analysis so far has taken an *ex post* view by considering a situation in which upstream firms already hold patents, and has not considered the effects of pools on *ex ante* incentives to innovate for upstream firms. Since firms join a pool only if this increases their profits, the prospect of forming a patent pool encourages innovation. However, in our framework with probabilistic patents, we also need to consider the effects of patent pools on the quality of innovations. The ability to shield weak patents with higher license fees provides more incentives to produce weak patents rather than strong patents; if stronger patents are associated with higher quality innovations, the prospect of patent pools may lead to more weak patents of suspect quality.

## 5 Patent Pool with Individual Licenses

In the previous section, we showed that patent pools can be anticompetitive even with complementary patents, once we account for the probabilistic nature of patent rights. By offering package licensing, patent pools deprive the downstream firm of the ability to selectively challenge patents. This allows a patent pool to charge higher licensing fees relative to independent licensing. By contrast, in this section, we discuss the case where the pool offers individual licenses for each patented technology and coordinates pricing. We first characterize the optimal individual license fees for the pool, discuss the pool’s optimal form of licensing, and derive conditions under which mandatory individual licensing increases total welfare.

**Profit-Maximizing Individual License Fees.** Suppose the patent pool issues individual licenses for each patent, charging \( f_A \) and \( f_B \), respectively. In this case, the downstream firm’s litigation behavior is the same as in the analysis with independent patent holders.
However, the patent pool maximizes the joint profits from both patents. In what follows, we focus on the case where only the litigation margin binds. The case where both margins are operating can be found in the proof of the next lemma. With individual licenses, the pool has three strategic options: limit licensing both patents, exactly one patent or inducing challenges against both patents. First consider the strategy of limit pricing exactly one patent and inducing litigation against the other patent. The highest possible limit license fee for patent $i$ while the downstream firm challenges patent $j$ is $f_i = p_i v + L/(1 - p_j)$. At this license fee, the patent pool makes an expected profit of

$$p_j v + (1 - p_j) [p_i v + \frac{L}{1 - p_j}] - L = [1 - (1 - \alpha)(1 - \beta)] v.$$  

Limit licensing exactly one patent yields the same expected payoff independent of which patent is selected as the litigation target.\(^{18}\) Furthermore, the pool always prefers limit licensing one patent to inducing litigation against both patents (in which case it would get the same profits minus the cost of litigation of $2L$). Finally, limit licensing exactly one patent dominates limit licensing both at $(f_A, f_B)$ if the above expected profits exceed $F^* = f_A + f_B$ or

$$\frac{L}{v} \leq \frac{\alpha\beta(1 - \alpha)(1 - \beta)}{2 - \alpha - \beta}.$$  

Thus, when litigation costs are sufficiently small, a patent pool with individual licenses is best off selling one license and inducing a legal challenge against the other patent. As we show in the Appendix to the next lemma, this result readily generalizes to the case where both demand and litigation margins are operating.\(^{19}\)

**Lemma 4** Consider a patent pool issuing individual licenses. There exists a threshold value $L''$, with $0 < L'' < L'$, such that for $L \leq L''$, the patent pool’s optimal license fees induce the downstream firm to buy the license for one patent and challenge the other patent. For higher litigation costs, $L > L''$, the patent pool charges aggregate limit license fees of $\min \{F^*, F^{**}\}$ and no litigation occurs.

\(^{18}\)This implies that setting fees such that the downstream firm is indifferent between litigating $A$ or $B$ yields the same payoff as fees at which it strictly prefers challenging one patent.

\(^{19}\)The only difference is that the patent pool’s local maximizer in regions $A$, $B$ and $0$ can be interior. Hence, candidate maximizers of the pool’s fee setting problem are the interior solution or the limit licensing fees $(f_A, f_B)$ of region $0$, the interior solution to regions $A/B$ or, as above, the corner solution at $f_i = p_i v + L/(1 - p_j)$.  

21
This result is somewhat surprising. If the litigation cost is sufficiently small \((L \leq L'')\), litigation arises, although the joint profits of the upstream and downstream firms are lower compared to licensing arrangements that avoid litigation. The reason for this is that the pool’s limit license fee for avoiding an additional litigation increases exponentially in the number of patents that are to be challenged. Thus, when faced with one patent litigation, the downstream firm is willing to pay a higher fee for the other patent to avoid a situation where it has to challenge both patents successfully to get any returns. From the pool’s perspective, the gain from this fee extraction with one sold license has to be weighed against the cost of litigation against the other patent. Hence, selling one license only is optimal when litigation costs are low.

Let us also briefly relate this result to the limit license equilibrium with independent patent holders. The lemma shows that the upstream profits may be maximized with (proposed) fees above the equilibrium limit license fees for independent patent holders from sections 3 and 4. As mentioned above, such license fees are not sustainable with independent patent holders as there exists a unilateral incentive to reduce the license fee to avoid a challenge against the holder’s own patent. A patent pool with individual licenses can internalize this externality and sustain the upstream profit-maximizing license fee levels.

**Profits, Welfare and Patent Pool Policy.** We now consider whether and how the pool’s private incentives to use package versus individual licenses align with social incentives. First compare the patent pool’s profits and welfare when licenses are sold in a package or individually.

**Proposition 5** Package licensing yields (weakly) higher profits for the patent pool compared to selling individual licenses. If \(L \leq L''\), package licensing is welfare superior to individual licenses. For higher values of \(L\), individual licensing yields weakly higher welfare.

Patent pools prefer package licensing as it reduces the strategic litigation options for the downstream firm and thus increases the aggregate limit license fee. By contrast, there is a welfare trade-off between package and individual licenses. Package licensing leads to higher aggregate limit licensing fees while individual licenses may induce litigation against one patent and high fees for the other patent. Thus, package licensing is socially efficient when litigation cost is relatively low as it prevents litigation and excessive fees for one patent. For higher litigation costs, individual licenses are efficient in that they prevent the pool from
shielding weak patents with high limit license fees.

From our analysis in section 4, we know that for \( L \leq L' \), a pool charges higher package license fees resulting in lower welfare compared to independent patent holders. One policy option is thus blocking patent pool formation in situations where the threat of litigation is large, that is, when litigation costs are small relative to the value of the innovation. Short of prohibiting patent pools, Proposition 5 suggests that mandatory individual licensing can increase total welfare with patent pools.

**Corollary 2** For intermediate litigation costs \( L'' < L \leq L' \), mandatory individual licensing for patent pools increases welfare and implements the same outcome as with independent patent holders.

For intermediate values of the litigation cost, a patent pool with individual licenses charges the same limit licensing fees as individual patent holders. Hence, in those situations, a policy that mandates patent pools to offer individual licenses increases welfare and implements the same efficiency outcome as with individual patent holders. In the next section we discuss another policy option, which is to allow the pool to sell a package license under the requirement that each patent holder also markets an individual license to its patent.

### 6 Extensions

**Licensing and litigation with more than two patents.** In this extension we show that the qualitative results of our above analysis hold more generally for \( n \geq 2 \) complementary technologies when patent strength is symmetric (and equal to \( \alpha \)). It is again useful to first consider the model when only the litigation margin is binding. Suppose patent holder \( i \) offers a license for patent \( i \in \{1,2..n\} \) at a fee \( f_i \). The optimal licensing and litigation strategy of the downstream firm can be characterized as follows.\(^{20}\)

**Lemma 5** There exists a \( l^* \in \{0..n\} \) such that the downstream firm buys \( l^* \) of the (weakly) cheapest licenses and challenges the remaining \( n-l^* \) patents. The optimal number of patent challenges increases in the overall licensing fee.

\(^{20}\) We give the \( n \)-firm version of condition (A) that ensures upstream litigation incentives are in place in the proof to the next lemma.
For a given number of litigation cases, the downstream firm never challenges a patent with a low license fee whilst buying a more expensive license of another patent. The incentive to litigate depends on the overall licensing fee and its distribution. Challenging the marginal patent implies the risk of losing the case and the net returns from buying the infra-marginal licenses. Hence, the higher the license fees, the lower the potential loss from litigation and the higher the number of patent challenges.

Now consider the best response function of an individual patent holder for a given fee profile of the other $n - 1$ patentees. Suppose patent holder $j$’s license fee is ranked between 1 and $l^* - 1$. In this case, slightly increasing its own license fee is always optimal until the patent at rank $l^*$ is challenged. Next assume that owner $j$’s patent is the marginal patent at rank $l^*$. The downstream firm prefers not to challenge patent $j$ if

$$(1 - \alpha)^{n-l^*} (v - \sum_{r=1}^{l^*-1} f_r - f_j) - (n - l^*)L \geq (1 - \alpha)^{n-l^*+1} (v - \sum_{r=1}^{l^*-1} f_r) - (n - l^* + 1)L$$

which holds if

$$f_j \leq \frac{L}{(1 - \alpha)^{n-l^*}} + \alpha(v - \sum_{r=1}^{l^*-1} f_r).$$

This is the $n$-firm equivalent of condition (3) in section 3. Again we can show that patent holder $j$ has no incentive to violate this condition and prefers to limit license to avoid litigation.\(^{21}\) Hence, in the unique symmetric fee setting equilibrium, firms charge $f_i = \mathcal{F}^*(n)$ such that (12) holds with equality and we get

$$\mathcal{F}^*(n) = \frac{L + \alpha v}{1 + \alpha(n - 1)}.$$ 

The individual equilibrium license fee is decreasing in $n$ as more patents increase the total infra-marginal license fees which raise incentives to challenge patents. Note, however, that the total licensing fee for the downstream firm $n\mathcal{F}^*(n)$ is increasing in the number of patents.

Next consider a patent pool offering a package license for all $n$ patents at a fee $F$. The downstream firm buys the package license if and only if

$$F \leq \mathcal{F}^{**}(n) = (1 - (1 - \alpha)^n)v + nL.$$ 

\(^{21}\)This is demonstrated in the proof of the next proposition.
The probability of invalidating all patents in court is decreasing in the number of patented technologies. Thus, the limit license fee for a patent pool with license packaging increases in $n$.

Let us compare the aggregate license fees with independent patent holders and a pool. Since the pool’s limit license fee increases exponentially and faster than the aggregate fees for the individual patent holders, we can show in the Appendix to the next proposition that

$$F^*(n) > n \tilde{f}^*(n).$$

Hence, when only the litigation margin is operating, patent pools charge more than individual patentees for any $n \geq 2$. Now suppose the demand margin binds. Note that while the aggregate license fee with independent patent holders $n f^*(n)$ increases, the pool’s optimal package fee $F^*$ is invariant in the number of patents $n$. This means that there must exist an upper bound on the number of patents, above which the litigation margin for independent patent holders is unable to restrict the fees below the pool’s optimal package fee $F^*$. In other words, with active demand and litigation margins, pools set higher license fees than independent patentees if and only if the number of patents is not too large. The next proposition makes this statement more precise.

**Proposition 6** If $n \tilde{f}^*(n) \leq F^*$, then patent pools with $n \geq 2$ patents charge higher total license fees and reduce total welfare relative to independent patent holders. This condition is harder to satisfy, the higher the number of complementary patents. There exists a finite upper bound $\overline{n}$ on the number of patents, above which pools increase total welfare.

**Independent Licensing Requirement.** Lerner and Tirole (2004) argue that if policy makers cannot establish whether patents are substitutes or complements, an independent licensing requirement can screen in welfare-enhancing patent pools and prevent the formation of welfare-reducing pools. Independent licensing means that individual patents that are part of the pool are also licensed from the original patent owner. To illustrate the effect of independent licensing in our model, we consider the case of $n$ patents of symmetric strength $\alpha$ when the litigation margin is binding. We consider the same timing as in Lerner and Tirole (2004). First, the patent pool sets a fee $F$ for the package license. Then, the individual patent owners non-cooperatively set their fee $f_i$ for an individual license to their patent.
After the license fees are set, the game continues as before. That is, the downstream firm decides whether to buy the package license, buy individual licenses, or whether to infringe and induce litigation. We assume that pool members share royalties equally.

Our first result establishes whether a patent pool with independent licensing by its members is still able to sustain higher fees relative to a situation where no pool has formed.

**Proposition 7** Consider a patent pool with independent licensing by its members. If \( n > 2 \), then there exist equilibria in which a patent pool sells its package at a total license fee exceeding the total price charged in the absence of a pool.

Independent licensing is not sufficient to prevent the patent pool from setting higher fees when there are more than two patents involved. To see this, let the pool charge a \( F \leq \bar{F}^{**} \) such that the downstream firm prefers buying the package license to not buying any individual license and induce litigation against all patents. Furthermore, suppose the individual license fees are symmetric and set at a level \( f_i = f \) such that the downstream firm prefers litigating against all patents rather than buying individual licenses to any of the patents. With such a fee structure, the downstream firm buys from the pool and an individual patent holder would get its stake holding of \( F/n \) from the pool. Alternatively, the patent holder could deviate to a license fee \( g < f \) in order to induce the downstream firm to buy its license and enter litigation with the other patent holders rather than buying the pool package. The limit license fee \( \bar{g} \) for the deviator satisfies

\[
(1 - \alpha)^{n-1}(v - \bar{g}) - (n - 1)L = v - F.
\]

An individual patent holder has no incentive to deviate if \( F/n \geq \bar{g} \). As the limit license fee increases faster in \( F \) than the individual stake holding, there exists a maximum package license fee, \( F^{ILR} \), a pool can charge without inducing deviation. We show in the Appendix that if \( n > 2 \), it holds that \( F^* < F^{ILR} < \bar{F}^{**} \). Despite the individual licensing requirement, the pool is able to charge a higher price than in the absence of patent pools. When there are exactly two patents, \( F^{ILR} \) is equal to \( F^* \) and, in the terminology of Lerner and Tirole (2004) the pool is strongly unstable.

For more than two patents, the independent licensing requirement is unable to prevent pools from charging higher license fees in the context of weak, complementary patents. In the following we consider the effect of independent licensing with an additional constraint.
Suppose the pool is required to unbundle the patents. That is, the pool is not allowed to give a package discount and has to sell the package license at a fee not exceeding the sum of the individual license fees.

**Proposition 8** Consider independent licensing together with an unbundling requirement. Then, there exists no equilibrium in which the pool is able to sell its package license at a fee exceeding total license fees in the absence of a patent pool.

In the presence of weak patents, independent licensing together with a strict unbundling requirement can screen out welfare-reducing patent pools. The unbundling requirement equates total license fees from the pool’s package and the independent licensors, that is, \( F = nf \). This means that if the pool is setting a package fee larger than the limit litigation fee \( F^* = n \bar{f}^* \) (charged in the absence of a pool), it is no longer optimal for the downstream firm to buy the pool package, and litigation involving at least one patent will arise. Faced with a positive probability of litigation, an individual patentee makes strictly less than \( f = F/n \) in expected terms. Moreover, deviating to a slightly lower fee makes the deviator’s license the cheapest one which avoids litigation with certainty, and yields license revenues of approximately \( f \). Hence, it can be shown that deviations from any license fee combination \( F = nf > F^* \) are always profitable. The highest pool package license fee that can be sustained is the limit litigation fee \( F^* = nf^* \) which obtains in the absence of a pool.

**Sequential litigation.** In our analysis so far, we assume that once the downstream firm infringes on both patents, litigation challenges arise simultaneously. This is a good description of many situations in which a short lead time to commercialisation is crucial. In some situations, however, the patentees and the downstream firm might be able to use a sequential litigation strategy instead. We briefly discuss how the analysis with a binding litigation margin would change under this assumption.

First consider a patent pool when the downstream firm has not purchased any license. Filing suits sequentially entails the same overall probability of having both patents invalidated during litigation compared to simultaneous litigation. However, it might save the cost of the second litigation if the first challenged patent is upheld. Hence, suing the downstream firm for infringing on the stronger patent \( B \) first is optimal as it allows for the highest prob-
ability of saving litigation cost.\textsuperscript{22} Sequential litigation also saves expected litigation cost for the downstream firm in the case when it infringes on both patents. The downstream firm’s expected profits with sequential litigation starting with patent $B$ is simply $V_{AB}^s = V_{AB} + \beta L$. In turn, a patent pool selling a package license practices limit pricing at $V_{AB}^a = V_0$ with a limit license fee of $F^s = F^{**} - \beta L$. Due to the potential litigation cost savings for the downstream firm, the pool’s limit license fee is lower with sequential litigation.

Now consider independent patentees selling individual licenses and suppose the downstream firm has decided not to buy any license. In a sequential litigation set-up, the patentees have the choice whether to sue straight away or wait. The outcome of such a strategic timing game depends on the additional assumptions with respect to when the patentees set their new fees if successful.\textsuperscript{23} However, independent of whether the downstream firm faces simultaneous or sequential litigation after infringing on both patents, the unique license fee equilibrium with independent patent holders is at $(f_A, f_B)$ as in Section 3. This is obviously the case when, in the equilibrium of the timing game, the patentees sue the downstream firm simultaneously. But it also holds with sequential litigation in equilibrium. The reason is that sequential litigation only occurs when license fees are sufficiently such that the downstream firm prefers not buying any license to exactly one license. In other words, sequential litigation affects the limit license between challenging one or both patents, that is, conditions (5) and (6) in Section 3. It does not affect the intersection of the best response function and the equilibrium license fee with independent patent holders.\textsuperscript{24}

It remains to compare aggregate license fees with and without a patent pool. Aggregate license fees are higher in the presence of a pool if and only if $F^s > f_A + f_B$ or

$$\frac{\alpha(1 - \beta)}{1 - \alpha \beta} [(1 - \beta)L + (1 - \alpha)\beta v] > 0$$

which always holds. The possibility of sequential litigation reduces the limit license fee for the pool but the qualitative nature of the results in section 3 does not change.

\textsuperscript{22} For simplicity, and to make sequential litigation more profitable, assume that there is no discounting of future profits.

\textsuperscript{23} Suppose patentee $j$ waits while $i$ sues and his patent is upheld in court. If patentee $i$ can set a new fee before $j$’s decision to sue, he would set a fee of $v$ and no more litigation would arise. In this case, there is a first mover advantage and both sequential or simultaneous litigation can arise. In contrast, if patentees set their fees after both patentees have decided to sue, the same payoffs as in Section 3 obtain independent of the order of suits.

\textsuperscript{24} We discuss this in more detail in our working paper Choi and Gerlach (2013).
7 Concluding Remarks

This paper analyzes the effects of patent pools with complementary patents on incentives to develop subsequent innovations. We find that the effects of patent pools depend on the strength of patents included in the pool. If patents are relatively strong, then the conventional result holds that pools with complementary patents mitigate the double marginalization problem and reduce overall licensing fees, which promotes subsequent innovations. However, if patents are relatively weak, patent pools can be used as a mechanism to deter litigation that would invalidate the patents in the pool. Package licensing of complementary patents imposes an all-or-nothing proposition in litigation on downstream firms. This allows patent pools to safe-harbour weak patents which would be targeted in litigation if the licenses would be sold independently. Our analysis shows that if patents are sufficiently weak, patent pools reduce social welfare as they raise total licensing fees and hinder subsequent innovations. This conclusion is robust to extensions of our analysis, which allow for more than two patents and sequential litigation strategies. We further explore the policy implications of mandated individual licenses to make the pool patents more vulnerable to litigation and command lower limit license fees. We find that the welfare effects of such policy mandates crucially depend on the size of the litigation cost relative to the value of the innovation. We also show that enforcing an independent licensing requirement for pool patents is not sufficient to prevent the pool from charging higher aggregate license fees. Hence, overall, our analysis suggests that a blanket approval of patent pools based on the complementary nature of the included patents is not warranted, and a more cautious approach that takes into account the strength of the patents and incentives to litigate is called for.
Appendix

Proof of Proposition 1. As we require this analysis with \( n \geq 2 \) patents in section 6, we prove the result for more than two firms at this point. The first-order condition for patent holder \( i \in \{1..n\} \) is

\[
G(v - \sum_{j=1}^{n} f_j) - f_i g(v - \sum_{j=1}^{n} f_j) = 0.
\]

Hence the equilibrium license fees \( (f_1^*..f_n^*) \) satisfy

\[
nG(v - \sum_{j=1}^{n} f_j^*) - \sum_{j=1}^{n} f_j^* g(v - \sum_{j=1}^{n} f_j^*) = 0.
\]

Evaluate the first order condition (2) for the patent pool at \( F^* = \sum_{j=1}^{n} f_j^* \), which yields

\[
G(v - F^*) - F^* g(v - F^*) = -(n - 1)G(v - F^*) < 0.
\]

This implies the desired result that \( F^* > F^{**} \).\end{proof}

Proof of Lemma 1. Suppose no license has been purchased and there are independent patent holders. Given patent holder \( j \) sues, patent holder \( i \) sues if

\[
p_i p_j v/2 + p_i (1 - p_j) v - L \geq 0.
\]

Since LHS increases in \( p_i \), the condition is harder to satisfy for firm \( i = A \), thus the binding constraint for both patent holders to sue is

\[
\alpha \beta v/2 + \alpha(1 - \beta) v - L \geq 0. \tag{App-1}
\]

Consider a patent pool (which offers a package license or independent licenses) when no license has been bought. The pool sues for infringement against both patents rather than for infringement against only the stronger patent if

\[
\beta v + \alpha (1 - \beta) v - 2L \geq \beta v - L \quad \text{or} \quad L \leq \alpha (1 - \beta) v. \tag{App-2}
\]

If this condition holds, condition (App-1) is always satisfied. When (App-1) holds, we have \( L \leq \beta v \) and no litigation at all is not optimal for the pool. Hence, the pool always sues
for infringement against both patents. Now suppose only one license, say for technology 
\( j \), has been purchased. The litigation condition for independent patent holders and a pool 
issuing individual licenses is the same, that is, \( p_i(v - f_j) - L \geq 0 \) or \( f_j \leq v - L/p_i \). Let 
\( V_{j0} \) denote the downstream firm’s payoff when buying with \( f_j > v - L/p_i \) such that there 
is no incentive to sue for infringement against patent \( i \), where \( V_{j0} = v - f_j \). Check that 
\( V_{AB} \geq V_{j0} \) if

\[
  f_j \geq [1 - (1 - \alpha)(1 - \beta)]v + 2L.
\]

This means that \( V_{AB} \geq V_{j0} \) for all \( f_j > v - L/p_i \) if and only if

\[
  v - \frac{L}{p_i} \geq [1 - (1 - \alpha)(1 - \beta)]v + 2L.
\]

This condition is harder to satisfy for \( i = A \) and \( p_i = \alpha \). Rearranging terms yields assump-
tion (A) in the text. It is easy to check that for any \( \alpha \geq 0 \) and \( \beta \geq 0 \), assumption (A) is 
more restrictive than (App-2). Hence, by assumption (A), the downstream firms prefers not 
buying any licenses rather than buying one license whenever the patentee has no incentive 
to sue exactly one patent. The lemma follows.

**Proof of Proposition 2.** Consider the best response \( f_i = \Lambda(f_j) \) for patentee \( i \) to license 
fee \( f_j \) of patentee \( j \). If \( f_j \leq \overline{f}_j \), then firm \( i \) prefers to set the highest fee that avoids 
litigation (which is at \( V_0 = V_i \)) rather than pricing in region \( i \) where the downstream firm 
buys the other firm’s license and challenges patent \( i \) since \( p_i(v - f_j) + L > p_i(v - f_j) - L \). 
For \( \overline{f}_j < f_j \leq p_jv + L/(1 - p_i) \), limit licensing occurs at the highest \( f_i \) that ensures \( V_j > V_i \).

This limit license dominates \( p_i(v - f_j) - L \) at \( f_j = \overline{f}_j \). The limit license increases in \( f_j \) while 
the expected profits from litigation in region \( i \) are decreasing. Hence, avoiding litigation 
is optimal. This also implies that setting the license fee that satisfies (7) with equality 
and entering litigation with probability \( 1/2 \) is always dominated by marginally cutting the 
license fee to avoid litigation with probability 1. Finally, in the third segment, the limit 
licensing fee satisfies \( V_j = V_{AB} \) which always exceeds patentee \( i \)’s expected profits when 
both patents are litigated since \( p_i v + L/(1 - p_j) > p_i(p_j^{1/2} + (1 - p_j))v - 2L \). The best
response for patent holder $i$ is the limit licensing strategy

$$f_i = \Lambda(f_j) = \begin{cases} 
  p_i(v - f_j) + L & \text{if } f_j \leq \bar{f}_j, \\
  [(1 - p_i)f_j - (p_j - p_i)v]/(1 - p_j) - \epsilon & \text{if } \bar{f}_j < f_j \leq p_jv + L/(1 - p_i), \\
  p_iv + L/(1 - p_j) & \text{otherwise}, 
\end{cases}$$

where $\epsilon > 0$ is an infinitesimally small number. From this follows that the unique Nash equilibrium is at the intersection of the respective first segment of each best response function, that is, at $(\bar{f}_A, \bar{f}_B)$. □

**Proof of Lemma 3.** Let $\Pi_i^k(f_i, f_j)$, $i \neq j$, denote firm $i$’s profits in region $k \in \{0, A, B, AB\}$:

$$\Pi_i^k(f_i, f_j) = G(V_k)f_i \text{ for } k \in \{0, j\}, \quad \Pi_i^i(f_i, f_j) = G(V_i)(p_i(v - f_j) - L) \quad \text{and} \quad \Pi_i^{AB}(f_i, f_j) = G(V_{AB})(p_ip_jv/2 + p_i(1 - p_j)v - L).$$

First, consider firm $i$’s best response function for $0 \leq f_j \leq \bar{f}_j$. Check that $\Pi_i^0(f_i, f_j) > \Pi_i^i(f_i, f_j)$ when $V_0 = V_i$ since

$$\Pi_i^0(p_i(v - f_j) + L, f_j) = G(v - p_i(v - f_j) - L - f_j)(p_i(v - f_j) + L)$$
$$= G((1 - p_i)(v - f_j) - L)(p_i(v - f_j) + L)$$
$$> G((1 - p_i)(v - f_j) - L)(p_i(v - f_j) - L).$$

$$= \Pi_i^i(p_i(v - f_j) + L, f_j).$$

Since $\Pi_i^i(f_i, f_j)$ is independent of $f_i$ it follows that the best response function for $0 \leq f_j \leq \bar{f}_j$ is continuous and given by $\Psi_i(f_j) = \min \{\Theta(f_j), p_i(v - f_j) + L\}$. Hence, for $\bar{f}_i \leq \Theta(\bar{f}_j)$, there exists a Nash equilibrium in which firms charge $\bar{f}_A$ and $\bar{f}_B$, respectively. Next assume $\bar{f}_j < f_j \leq p_jv + L/(1 - p_i)$. Define the local maximizer in region $j$ as

$$\hat{f}_j^i \equiv \arg \max_{f_A} \Pi_i^j(f_i, f_j) = f_iG(V_j) = f_iG((1 - p_j)(v - f_i) - L).$$

This maximizer satisfies the first-order condition

$$\hat{f}_j^i = \frac{G(V_j)}{(1 - p_j)g(V_j)}.$$
Note that the maximizer in this region does not depend on \( f_j \). Further note that for \((f_i, f_j)\) such that \( V_0 = V_j \), it holds that \( \Pi^i_i(f_i, f_j) = \Pi^i_j(f_i, f_j) \). Verify that for values \((f_i, f_j)\) such that \( V_i = V_j \), we get \( \Pi^j_i(f_i, f_j)|_{V_i = V_j} > \Pi^j_i(f_i, f_j)|_{V_i = V_j} \) if and only if \( f_i > p_i(v - f_j) - L \) or

\[
\frac{p_i - p_j}{1 - p_j} v + \frac{1 - p_i}{1 - p_j} f_j > p_i(v - f_j) - L
\]

\[
\iff f_j > \frac{p_j(1 - p_i)}{1 - p_ip_j} v - \frac{1 - p_j}{1 - p_ip_j} L
\]

which holds for any \( f_j > \hat{f}_j \). Hence, firm \( i \) also prefers to price slightly below the fee that yields \( V_i = V_j \) rather than setting \( f_i \) such that \( V_i = V_j \) or \( V_i > V_j \). Undercutting yields \( \Pi^i_i(f_i, f_j) \) whereas the two latter price points give \( \Pi^i_i(f_i, f_j)/2 + \Pi^i_i(f_i, f_j)/2 \) and \( \Pi^i_i(f_i, f_j) \), respectively. Hence, if \( \hat{f}_i \leq \Theta(\hat{f}_j) \), then firm \( i \)'s best response is either such that \( V_0 = V_j \) or strictly interior in region \( j \). From the concavity of \( \Pi^i_i(f_i, f_j) \) in \( f_i \) follows that \( \Psi_i(f_j) \) is continuous. In particular, if \( \hat{f}_i^j < \hat{f}_j \), then

\[
\Psi_i(f_j) = \max \left\{ v + L/p_j - f_j/p_j, \hat{f}_i^j \right\}
\]

otherwise,

\[
\Psi_i(f_j) = \min \left\{ \frac{p_i - p_j}{1 - p_j} v + \frac{1 - p_i}{1 - p_j} f_j - \epsilon, \hat{f}_i^j \right\}.
\]

Finally, consider \( f_j > p_j v + L/(1 - p_i) \). Check that \( \Pi^j_i(f_i, f_j)|_{V_j = v_{AB}} > \Pi^j_i(f_i, f_j)|_{V_j = v_{AB}} \) if and only if \( f_i > p_ip_j v/2 + p_i(1 - p_j)v - L \) or

\[
p_i v + L/(1 - p_j) > p_ip_j v/2 + p_i(1 - p_j)v - L
\]

\[
\iff p_ip_j v/2 + L/(1 - p_j) - L > 0
\]

which is always satisfied. Hence, the best response is continuous and lies in region \( j \),

\[
\Psi_i(f_j) = \min \left\{ p_i v + L/(1 - p_j), \hat{f}_i^j \right\}.
\]

Since for \( f_j > \hat{f}_j \), firm \( i \)'s best response function is in region \( j \) or where \( V_0 = V_j \), no further equilibrium exists. Finally, check that \( \hat{f}_A \leq \Theta(\hat{f}_B) \) follows from \( \hat{f}_B \leq \Theta(\hat{f}_A) \). Since \( \partial \Theta/\partial f > -1 \), we have \( \hat{f}_B - \hat{f}_A > \Theta(\hat{f}_A) - \Theta(\hat{f}_B) \) or \( \Theta(\hat{f}_B) - \hat{f}_A > \Theta(\hat{f}_A) - \hat{f}_B \). Thus, if \( \Theta(\hat{f}_A) - \hat{f}_B \geq 0 \), then \( \Theta(\hat{f}_B) - \hat{f}_A > 0 \). The lemma follows. ■
Proof of Lemma 4. The patent pool’s profits with individual licenses in the four regions of the license fee space from section 3 are given by

\[ \Pi^0 = G(V_0)(v - V_0), \Pi^{AB} = G(V_{AB})(v - 4L - V_{AB}), \]
\[ \Pi^k = G(V_k)(v - 2L - V_k), \text{ for } k \in \{A, B\}. \]

It follows straight from the discontinuity at \( V_k = V_{AB} \) that license fees in region \( AB \) are never optimal. Let \( V_0^* = v - F^{**} \) denote the argument that maximizes \( \Pi^0 \). It then follows from the definition of the profits that if \( F^{**} \leq \bar{f}_A + \bar{f}_B \), then any \( f_A + f_B = F^{**} \) maximizes the pool’s global profits. Further let \( V_i^* = (1 - p_i)(v - \bar{f}_j) - L \) denote the argument that maximizes \( \Pi^i \). This implies that \( V_A^* = V_B^* \) such that \( \tilde{f}_A \) and \( \tilde{f}_B = (\beta - \alpha)v/(1 - \alpha) + (1 - \beta)\tilde{f}_A/(1 - \alpha) \) are a license fee pair that – if interior – maximizes the pool’s profit in regions A and B. The maximizer in regions A and B is thus either an interior solution \((\tilde{f}_A, \tilde{f}_B)\) or a boundary solution \((\alpha v + L/(1 - \beta), \beta v + L/(1 - \alpha))\). The next step is to show that if \( F^{**} > \bar{f}_A + \bar{f}_B \), that is the local maximizer in region 0 is at \((\bar{f}_A, \bar{f}_B)\), then \( \tilde{f}_A > \bar{f}_A \). From profit maximization, it follows that \( V_0^* > V_B^* \) or

\[ \tilde{f}_A > \frac{F^{**} - \beta v + L}{1 - \beta} = \frac{\bar{f}_A + \bar{f}_B}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \bar{f}_A. \]

Hence, for any \( F^{**} > \bar{f}_A + \bar{f}_B \), we get \( \tilde{f}_A > \bar{f}_A \). This means that if \( F^{**} > \bar{f}_A + \bar{f}_B \), there are two potential global maximizers, \((\bar{f}_A, \bar{f}_B)\) or the maximizer in region A and B. Check that at \( L = 0 \) the global profit function is continuous at \( V_0 = V_A = V_B \). Thus, it follows from \( F^{**} > \bar{f}_A + \bar{f}_B \), \( \tilde{f}_A > \bar{f}_A \) and the concavity of the profit functions that there exists a \( L'' > 0 \) such that if \( L < L'' \), then the global maximizer is the local maximizer of regions A and B. Finally, \( L'' < L' \) is implied by the fact that \( \bar{F}^* \leq F^{**} \) holds if \( L \leq L' \)

Proof of Proposition 5. (i) Profit ranking: If \( L \geq L' \), then \( F^{**} < \bar{f}_A + \bar{f}_B \) and the patent pool’s total license fee is \( F^{**} \) independent of whether it sells package or individual licenses. Both arrangements yield the same profit. If \( L'' < L < L' \), the pool charges \( \min \{ F^{**}, \bar{F}^{**} \} \) for a package license and \( \bar{f}_A + \bar{f}_B < \min \{ F^{**}, \bar{F}^{**} \} \) with individual licenses. Without litigation, the pool’s profits are maximized at \( F = F^{**} \). Hence, package licensing strictly dominates. Finally, suppose \( L \leq L' \). Further suppose \( \bar{F}^{**} \geq F^{**} \) such that with package licensing, the pool charges \( F^{**} \). In the case where a pool with individual licenses charges
fees at the corner solution \((\alpha v + L/(1 - \beta), \beta v + L/(1 - \alpha))\), the downstream firm gets \(V_{AB}\) and package licensing strictly dominates since \(G(V_0^*)(v - V_0^*) > G(V_{AB})(v - 2L - V_{AB})\) which holds due to the fact that \(V_0^*\) maximizes \(G(V)(v - V)\). In the case where a pool with individual licenses charges \(\widehat{f}_A\) and \(\widehat{f}_B\), the downstream firm gets \(V_B^* < V_0^*\) and package licensing dominates since \(G(V_0^*)(v - V_0^*) > G(V_B^*)(v - 2L - V_B^*)\). Finally, consider \(F^{**} > \overline{F}^{**}\) such that with package licensing, the pool charges \(\overline{F}^{**}\) and the downstream firm gets \(V_{AB}\).

From our analysis in the proof of Lemma 4 we know that the interior maximizer in regions A and B satisfies

\[
\widehat{f}_A > \frac{F^{**}}{1 - \beta} - \frac{\beta v + L}{1 - \beta}.
\]

Since \(F^{**} > \overline{F}^{**}\) the minimum value the RHS can take is

\[
\frac{\overline{F}^{**}}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \frac{(1 - (1 - \alpha)(1 - \beta))v + 2L}{1 - \beta} - \frac{\beta v + L}{1 - \beta} = \alpha v + \frac{L}{1 - \beta}.
\]

Hence, \(\widehat{f}_A > \alpha v + L/(1 - \beta)\) and the boundary solution in regions A and B holds. This implies that package licensing dominates since \(G(V_{AB})(v - V_{AB}) > G(V_{AB})(v - 2L - V_{AB})\).

(ii) Total welfare ranking: Suppose \(0 \leq L \leq L''\). Total welfare with package licensing is welfare superior to individual licenses if

\[
G(\max\{V_{AB}, V_0^*\})v \geq G(\max\{V_B^*, V_{AB}\})(v - 2L)
\]

which always holds due to \(V_0^* > V_B^*\). Consider \(L'' < L \leq L'\) where a pool with individual licensing charges \(\overline{F}^{*} = \overline{f}_A + \overline{f}_B\). Individual licensing is welfare superior if

\[
G(V_0(\overline{F}^{*}))(v) \geq G(\max\{V_{AB}, V_0^*\})v = G(\max\{V_0(\overline{F}^{**}), V_0^*\})v
\]

which always holds since \(\overline{F}^{*} \leq \min\{\overline{F}^{**}, F^{**}\}\). Finally, for \(L > L'\) a patent pool charges \(F^{**}\) in total licensing fees both with individual and package licenses, and welfare is the same.

**Proof of Lemma 5.** To ensure that the threat of litigation is credible, we submit the analysis to the condition

\[
\frac{L}{v} \leq \frac{\alpha}{1 + \alpha n}(1 - \alpha)^n,
\]

which can be derived in a similar way to condition (A) in the proof of Lemma 1. Rank all license fee offers in increasing order. Buying the \(l\) lowest ranked licenses and litigating
against the remaining \( n - l \) patents yields

\[
V(l) = (1 - \alpha)^{n-l}(v - \sum_{r=1}^{l} f_r) - (n - l)L.
\]

Litigating against patent \( i \) and buying the license of patent \( j \) is never optimal when \( f_j > f_i \). It involves the same litigation cost and results in higher expected license fees. In order to show that \( V(l) \) is concave in \( l \), we prove that (i) if \( V(i) > V(i+1) \), then \( V(i+1) > V(i+2) \) and (ii) if \( V(i) > V(i-1) \), then \( V(i-1) > V(i-2) \). Check that \( V(i) > V(i+1) \) if

\[
\alpha v \geq f_{i+1} + \alpha \sum_{r=1}^{i} f_r - \frac{L}{(1 - \alpha)^{n-i-1}}
\]

and \( V(i+1) > V(i+2) \) if

\[
\alpha v \geq f_{i+2} + \alpha \sum_{r=1}^{i+1} f_r - \frac{L}{(1 - \alpha)^{n-i-2}}.
\]

The first condition implies the second since

\[
f_{i+2} - f_{i+1} + \alpha f_{i+1} + \frac{L}{(1 - \alpha)^{n-i-1}} - \frac{L}{(1 - \alpha)^{n-i-2}} > 0.
\]

Next verify that \( V(i) > V(i-1) \) if

\[
\alpha v \geq f_{i} + \alpha \sum_{r=1}^{i-1} f_r - \frac{L}{(1 - \alpha)^{n-i}}
\]

and \( V(i-1) > V(i-2) \) if

\[
\alpha v \geq f_{i-1} + \alpha \sum_{r=1}^{i-2} f_r - \frac{L}{(1 - \alpha)^{n-i+1}}.
\]

Again the first condition implies the second since

\[
f_{i} - f_{i-1} + \alpha f_{i-1} + \frac{L}{(1 - \alpha)^{n-i+1}} - \frac{L}{(1 - \alpha)^{n-i}} > 0.
\]

From this the lemma follows.

**Proof of Proposition 6.** First, we show that patent holders prefer to limit license as claimed in the text. If the patent holder \( j \) charges a higher fee than the limit license fee in
the text, the downstream firm challenges his patent. In this case, the patent holder only receives a return if his patent is upheld by the court. His share of the total upstream profit is determined by how many other patents are upheld. Let $\Pr\{k|n-l^*\}$ denote the probability that $k$ out of the $n-l^*$ remaining litigated patents are upheld. Then the expected profit from inducing litigation is

$$\alpha \frac{n-l^*}{k+1} \left( v - \sum_{i \in \mathcal{L}} f_i \right) - L = \frac{1 - (1 - \alpha)^{n-l^*+1}}{n-l^*+1} \left( v - \sum_{i \in \mathcal{L}} f_i \right) - L.$$

Since patent holder $j$’s expected market share is always less than $\alpha$, it follows that limit licensing always dominates. Second, in order to show that condition (13) always holds, re-write it as

$$v(1 - (1 - \alpha)^n) + nL \geq n \frac{L + \alpha v}{1 + \alpha(n-1)}.$$

The LHS increases faster in $L$ than the RHS. If this condition holds for $L = 0$, then it must hold for all $L \geq 0$. At $L = 0$, this condition holds if

$$\alpha \leq \frac{1 - (1 - \alpha)^n}{1 + (1 - \alpha)^n(n-1)} \equiv \Upsilon(\alpha).$$

Check that $\Upsilon(0) = 0$, $\Upsilon(1) = 1$ and

$$\frac{\partial \Upsilon}{\partial \alpha}(\alpha = 0) = \frac{(1 - \alpha)^n - 1}{1 + (1 - \alpha)^n(n-1)} = 1.$$

Furthermore, we have

$$\frac{\partial^2 \Upsilon}{(\partial \alpha)^2} = \frac{(1 - \alpha)^{n-2}n^2(n-1)}{[1 + (1 - \alpha)^n(n-1)]^3} \left[ (n+1)(1 - \alpha)^n - 1 \right].$$

It thus holds that there exists an $\alpha'$, with $0 < \alpha' < 1$ such that $\Upsilon$ is convex in $\alpha$ for $\alpha \leq \alpha'$ and concave otherwise. It follows that $\Upsilon(\alpha) \geq \alpha$ for all $\alpha \in [0, 1]$.

The last point to show for this proposition is the existence of an upper bound on $n$ below which patent pools charge higher fees. Let $f^*(n)$ denote the Nash equilibrium license fee with $n$ independent patent holders when only the demand margin binds (see proof of Proposition 1). Total license fees with individual patent holders are $n \min \left\{ f^*(n), \bar{f}^*(n) \right\}$ whereas the patent pool charges $\min \left\{ \bar{F}^*(n), F^{**} \right\}$. From $F^{**} < nf^*(n)$ and (13) follows that individual patent holders charge lower fees if $n\bar{f}^*(n) < F^{**}$. Since $n\bar{f}^*(n)$ is increasing.
and approaching $v$ as $n$ becomes large while $F^{**} < v$ is not affected by the number of patents, this condition must fail to hold when $n$ is sufficiently large. ■

**Proof of Proposition 7.** For notational convenience, consider the optimal licensing of the downstream firm when $n$ independent firms charge the same $f_i = f$. From our analysis in Lemma 5 follows that if $\phi(k) < f \leq \phi(k - 1)$, then the optimal number of licenses bought is $l^* = k - 1$ where

$$\phi(k) = \frac{(1 - \alpha)^{k-n}L + \alpha v}{1 + \alpha(k - 1)}$$

with $\partial \phi / \partial k < 0$ for $k \geq 1$. If $f \leq \phi(n)$, $l^* = n$. If $f > \phi(1)$, $l^* = 0$. In the following, we show that any $f_i = f > \phi(1)$ and $F \in [F^*, F^{ILR}]$ can be part of a Subgame Perfect equilibrium of the independent licensing game. If $n = 2$, $F^{ILR} = F^*$. If $n > 2$, it holds that $F^* < F^{ILR} < F^{**}$. Due to $f > \phi(1)$, the best option, apart from buying the package license, is to enter litigation against all $n$ patentees. However, for $F^{ILR} \leq F^{**}$, this option is dominated by purchasing the pool’s package license. As argued in the main text, deviations are not profitable if $F/n \geq \bar{g}$ where

$$\bar{g} = v - \frac{v - F + (n - 1)L}{(1 - \alpha)^{n-1}} = \phi(1) - \frac{F^{**} - F}{(1 - \alpha)^{n-1}}.$$

Both expressions increase in $F$ but the slope of $\bar{g}$ is larger since $n > (1 - \alpha)^{n-1}$. Let $F^{ILR}$ denote the value of $F$ such that $F/n = \bar{g}$ while for $F \leq F^{ILR}$ deviations are not profitable. Next we show that $F^* \leq F^{ILR} < F^{**}$. First, check that $\bar{g}$ takes a higher value than $F/n$ at $F = F^*$. This follows since $\bar{g}(F^*) - F^{**}/n$ increases in $L$ and at $L = 0$ this difference is $[\alpha n + (1 - \alpha^n - 1)v/n > 0$ for all $\alpha, n$. Second, check that $F/n$ takes at least as high a value as $\bar{g}$ at $F = F^*$. We get

$$\frac{\partial (F^* - \bar{g}(F^*))}{\partial L} = 1 - \frac{1 - \alpha(n-1)^2}{(1 - \alpha)^{n-1}} \geq 0$$

as the expression takes value 0 at $n = 2$ and increases in $n$. Furthermore, at $L = 0$, $F^* - \bar{g}(F^*) \geq 0$ if and only if $(1 - \alpha)^2 - (1 - \alpha)^n(1 + \alpha(n - 2)) \geq 0$. Check that the LHS takes value 0 at $n = 2$ and increases in $n$ if and only if $\ln(1 - \alpha)(1 + \alpha(n - 2)) + \alpha < 0$ which holds since the expression equals $\alpha + \ln(1 - \alpha) < 0$ at $n = 2$ and decreases in $n$. It follows that for $n = 2$, $F^*/n = \bar{g}(F^*)$ and $F^{ILR} = F^*$ is the unique intersection. If $n > 2$, $F^*/n > \bar{g}(F^*)$ and $F^* < F^{ILR} < F^{**}$. Finally, note that with asymmetric stake holdings, the
pool could not do any better as $F^{ILR}$ is constrained by the firm with the smallest stake. Hence, symmetric stakes are optimal. ■

**Proof of Proposition 8.** Consider $F = nf \geq \bar{F} = n\phi(n)$. From our previous analysis it follows that buying all individual licenses or the same priced pool package is dominated by infringing on at least one license and entering litigation. Given the downstream firm’s optimal number of license purchases $l^*(f)$, a patentee expects profits of

$$E\Pi_i(f) = \frac{l^*(f)}{n} f + \frac{n - l^*(f)}{n} \alpha \sum_{k=0}^{n-l^*(f)-1} Pr\{k|n-l^*(f)-1\} \frac{v - l^*(f)f}{k+1} - L,$$

where the optimal number of licenses bought is $l^* = k - 1$ if $\phi(k) < f \leq \phi(k-1)$. For $f \leq \phi(n)$, profits are $E\Pi_i(f) = f$. The higher is $f$, the lower the number of purchased licenses and the more litigation. For $f \in [\phi(l+1), \phi(l)]$, $l$ licenses are purchased and the profit increases in $f$ since $\partial E\Pi_i/\partial f = l(1 - \alpha)^{n-l}/n > 0$. This slope is 1 for $l^* = n$ but strictly less than 1 for higher values of $f$ and lower values of $l^*$. This implies $E\Pi_i(f) < f$ for any $f > \phi(n)$ or $l^* < n$. If $0 < l^* < n - 1$, the optimal deviation for an individual licensor is to shave the license fee $f$ and avoid litigation with certainty. This deviation yields $f$ and since $E\Pi_i(f) < f$ for $l^* < n - 1$ it is always profitable. If $f > \phi(1)$ and $l^* = 0$, the optimal deviation to avoid litigation requires that the downstream prefers to buy from the deviator rather than litigate. The limit litigation fee is then $g = \phi(1)$. Since

$$E\Pi_i(l^*(f) = 0) = \alpha \sum_{k=0}^{n-1} Pr\{k|n-1\} \frac{v}{k+1} - L < g = \phi(1) = \alpha v + \frac{L}{(1 - \alpha)^{n-1}},$$

development is profitable for any $f > \phi(n)$. For $f \leq \phi(n)$, there is no litigation and an individual patentee has no incentive to reduce its license fee. Hence, in any equilibrium it has to hold that $F = nf \leq \bar{F}$. ■
References


