The Performance of Pivotal-Voter Models in Small-Scale Elections: Evidence from Texas Liquor Referenda

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Abstract

This paper explores the ability of pivotal-voter models to explain voter behavior in small-scale elections using data from Texas liquor referenda. The findings provide little support for the view that pivotal-voter models are a reasonable theory for understanding small-scale elections. Interestingly, this is not because they cannot explain the levels of turnout in our data, but rather because they cannot explain the size of the winning margins. The logic of pivotal-voter models implies that elections must be expected to be close even if there is a significant difference between the sizes of the groups or the intensity of their preferences. With even a relatively small number of eligible voters, elections that are expected to be close ex ante must end up being close ex post. However, in the data, winning margins are often significant.

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1 Introduction

The purpose of this paper is to shed light on the ability of *pivotal-voter models* to explain turnout in small-scale elections. According to this class of models, citizens are motivated to vote by the chance that they might swing the election. Citizens are assumed to rationally anticipate the probability that their votes will be pivotal and to vote if the expected "instrumental benefit" outweighs the cost of going to the polls. A positive level of turnout is assured in equilibrium, since if no citizen were expected to vote, any deviator would be pivotal with probability one.

Pivotal-voter models, as developed by Ledyard (1984) and Palfrey and Rosenthal (1983), (1985), form the basic framework for thinking about turnout in formal political theory. They are in many respects the simplest and most natural way of thinking about the problem. They require only that citizens be instrumentally motivated and that they have rational expectations, core assumptions of rational choice theory. They are relatively tractable and yield a number of interesting implications.[1]

Despite their theoretical appeal, it seems widely accepted that pivotal-voter models do not provide an empirically satisfactory theory of turnout in large-scale, single-issue elections (see, for example, Feddersen (2004) and Green and Shapiro (1994)). Palfrey and Rosenthal (1985) showed that when voters are uncertain about the preferences of their fellows, the critical cost levels below which citizens vote converge to zero as the number of citizens approaches infinity. Thus, in order for observed outcomes to be consistent with the model, the costs of voting for a large group of citizens must be minuscule. Not only does this seem unlikely, but if it were the case, then it is not clear how to explain the variation in turnout in large elections observed in the data.

Palfrey and Rosenthal's result does not, however, rule out the possibility that

[1] For example, Borgers (2004) uses a pivotal-voter model to show that when citizens are ex-ante identical, compelling citizens to vote is never desirable on welfare grounds (see also Ghosal and Lockwood (2004)). Campbell (1999) uses a pivotal-voter model to show how a policy outcome preferred by a small minority of the electorate can be implemented under majority voting if the preferences of that minority are strong or their costs of voting are low. Battaglini (2005) uses a pivotal-voter model to argue that when voting is costly, simultaneous elections may aggregate information better than sequential elections. Krisha and Morgan (2005) use a pivotal-voter model to show that the nature of voting costs impacts the extent to which majority rule elections aggregate information.

pivotal-voter models provide a reasonable theory for understanding small-scale elections and this seems to be the implicit justification for their continued use in theoretical work (see, for example, Borgers (2004)). With a relative small number of eligible voters, the equilibrium probability of being pivotal is large enough to motivate individuals with positive costs of voting to participate. While it would be cleaner to have a single model to explain turnout in all elections, there is no obvious reason to believe that the forces driving turnout in small-scale elections should be the same as those in large, single-issue elections. Moreover, the empirical relevance of small-scale elections is perhaps greater than is often assumed, because voters in national elections are often simultaneously voting on a host of local issues where the number of eligible voters is small.

To shed light on the performance of pivotal-voter models in small-scale elections, we use the data set on Texas liquor referenda assembled by Coate and Conlin (2004). The jurisdictions holding these elections range in size, with some being quite small. We focus on a subset of the smallest ones; namely, those with less than 900 eligible voters. The elections in the Coate-Conlin data set are ideal for testing pivotal-voter models because they are held separately from other elections, so that the only reason to go to the polls is to vote on the proposed change in liquor law. Moreover, the issues decided by the referenda are very similar across jurisdictions.

The findings of our study offer little to nurture the hope that pivotal-voter models offer an adequate theory for understanding voter behavior in small-scale elections. Interestingly, this is not because they cannot explain the levels of turnout in these elections, but rather because they cannot explain the size of the winning margins. The pivotal-voter logic implies that elections must be expected to be close even if there is a significant difference between the sizes of the groups supporting the candidates or the intensity of their preferences. With a very small number of eligible voters, elections that are expected to be close ex ante may end up not close ex post because of sampling error: for example, an unexpectedly large number of eligible voters may favor one side of the issue or, alternatively, a disproportionate number of eligible voters on one side of the issue may receive low voting cost realizations. As the number of eligible voters increases, this sampling error very quickly disappears and elections that are expected to be close ex ante will be close ex post. However, in our data set, winning margins can be significant even in the larger of our elections.

Our analysis begins with a description of the pattern of voter behavior in our data. We focus on *turnout* defined as the fraction of eligible voters who vote and *winning margin* defined as the absolute value of the difference between the percent of eligible voters who vote for and against the change. We document that turnout declines as a function of the size of the electorate. Winning margins also decrease with size, but there remains a significant fraction of relatively large elections with winning margins exceeding 0.10 or even 0.20.

We next illustrate the difficulty pivotal-voter models have in generating significant winning margins. We do this by simulating a parameterized pivotal-voter model. Restricting the number of eligible voters to between 50 and 500, our simulations indicate that our parameterized model can predict relatively large turnout in elections of this size. They also suggest that, when the number of eligible voters is less than 100, our model can often predict election outcomes where the winning margin is greater than not only 0.10 but also 0.20. However, the simulations also indicate that the probability of observing these types of large winning margins is close to zero as the number of eligible voters approaches 400.

To undertake the simulations, it is necessary to specify values for the parameters of our pivotal-voter model such as the payoffs to supporters and opposers of the referenda and the distribution of voting costs. While the simulations illustrate the difficulties the model has for the parameters we specify, they are open to the criticism that the results may reflect the particular parameters chosen rather than a general problem. To overcome this criticism, our next step is to structurally estimate our pivotal-voter model using the dataset on Texas liquor referenda. In this way, we obtain parameter values that best fit the data. Structurally estimating our pivotal-voter model is a challenging undertaking, given the complexity of the equilibrium conditions implied by the model.[2] Nonetheless, we are able to estimate the model for the elections in our data set.[3] As expected based on our simulations, we find that

[2] Evaluating the expected benefit of voting requires computing the equilibrium probability that a voter will be pivotal.

[3] Computational constraints are kept manageable by our decision to focus on jurisdictions with

the estimated model matches turnout reasonably well, but predicts much smaller winning margins than are observed in the data.

As a final step to illustrate the poor performance of our pivotal-voter model in explaining voter behavior in our data, we compare its performance with that of a simple alternative model - the *intensity model* - based on the idea of expressive voting. We find that this very simple model explains voter behavior better than our considerably more sophisticated pivotal-voter model.

The organization of the remainder of the paper is as follows. Section 2 discusses some related literature. Section 3 describes the institutional details concerning the referenda that we study and presents the data on turnout and winning margins. Section 4 outlines our parameterized pivotal-voter model. Section 5 simulates the model and establishes the difficulty it has in generating large winning margins. Section 6 structurally estimates our model and describes the estimation results. Section 7 introduces the intensity model and compares its performance with that of our pivotal-voter model. Section 8 summarizes the results and discusses their implications for future research on voter turnout.

2 Related literature

To date, despite their popularity in theoretical work, there has been very little research using field data that directly tests the performance of pivotal-voter models. This reflects both the difficulty of finding appropriate data and the analytical difficulties of implementing the models. Indeed, to our knowledge, the only other paper to have even attempted the task is Hansen, Palfrey and Rosenthal (1987). They use data on school budget referenda and make strong assumptions to undertake the estimation. In particular, they assume that the population is equally divided between supporters and opposers of the proposed budget and that both sides have identical benefits from their preferred outcomes. Consistent with these simplifying assumptions, Hansen, Palfrey and Rosenthal estimate the parameters of their model using only referenda with "close" outcomes and focus only on total turnout. As we

less than 900 eligible voters.

do, they find that their model estimates match observed levels of turnout reasonably well. However, our analysis imposes neither the equal division nor the identical benefits assumption, which allows us to estimate the parameters of our model using *all* election outcomes (close and non-close). This permits us to analyze our model's predictions regarding not only total turnout but also winning margin.

There are a number of papers that use laboratory experiments to test the performance of pivotal-voter models.[4] However, most of these papers assume that voters have homogeneous voting costs as in Palfrey and Rosenthal (1983). Under this assumption, the model has multiple mixed strategy equilibria which makes testing difficult. In an interesting recent contribution, Levine and Palfrey (2007) adopt the more realistic heterogeneous voting cost set-up of Ledyard (1984) and Palfrey and Rosenthal (1985). In a series of experiments involving up to 50 voters, they find that the main comparative static predictions of their pivotal-voter model are observed in the experimental data. In particular, turnout is decreasing in the number of participants. Their model matches turnout reasonably well, although there is under-voting for smaller electorates and over-voting for larger electorates. Moreover, a model which adopts the quantal response equilibrium concept fits the data even better. Our findings in this paper do not directly contradict these conclusions. We also find that the comparative static predictions of our pivotal-voter model are bourne out in the data. Moreover, our simulations indicate that our pivotal-voter model can explain both total turnout and large winning margins for jurisdictions with less than 100 eligible voters. The problem is that it cannot explain large winning margins when the number of eligible voters approaches 400. Thus, our paper simply raises the question of whether Levine and Palfrey's pivotal-voter model would suffer from the same problem in their experiments if the number of eligible voters were significantly increased.

Our paper complements the recent work of Coate and Conlin (2004). Using the same data set, Coate and Conlin structurally estimate the parameters of a model of voter turnout which assumes that individuals are motivated to vote by

[4] See, for example, Schram and Sonnemans (1996), Cason and Mui (2005), Grosser, Kugler and Schram (2005), and Grosser and Schram (2006).

the ethical desire to do their part to help their side win.[5] They show that this *group rule-utilitarian model* fits the data well and performs better than the simple intensity model of voter turnout discussed above. While interesting, it is natural to wonder how more standard pivotal-voter models would fare in this environment and this is the issue we take up here. Unfortunately, we are unable to provide direct comparisons of all three models because Coate and Conlin's analysis makes the simplifying assumption of a continuum of voters. This must be dispensed with here because it implies that no voter can be pivotal. However, the intensity model is sufficiently simple that it can be easily estimated with either a finite number of voters or a continuum, so we are able to compare its performance with that of our pivotal-voter model.[6]

3 Texas liquor referenda

3.1 Institutional background[7]

Chapter 251 of the Texas Alcoholic Beverage Code states that "On proper petition by the required number of voters of a county, or of a justice precinct or incorporated city or town in the county, the Commissioners' Court shall order a local election in the political subdivision to determine whether or not the sale of alcoholic beverages of one or more of the various types and alcoholic contents shall be prohibited or legalized in the county, justice precinct, or incorporated city or town". Thus, citizens can propose changes in the liquor laws of their communities and have their proposals directly voted on in a referendum. Such direct democracy has a long history in Texas, with local liquor elections dating back to the mid-1800s.

The process by which citizens may propose a change for their jurisdiction is relatively straightforward. The first step involves applying to the Registrar of Voters for a petition. This only requires the signatures of ten or more registered voters in the jurisdiction. The hard work comes after receipt of the petition. The applicants

[5] This model is based on the ideas of Harsanyi (1980) and Feddersen and Sandroni (2006).

[6] Estimating the group rule-utilitarian model with a finite number of voters is computationally infeasible.

[7] This discussion of the institutional details draws on Coate and Conlin (2004).

must get it signed by at least 35% of the registered voters in their jurisdiction and must do this within thirty days.[8] If this hurdle is successfully completed, the Commissioners' Court of the county to which the jurisdiction belongs must order a referendum be held. This order must be issued at its first regular session following the completion of the petition and the referendum must be held between twenty and thirty days from the time of the order. All registered voters can vote and if the proposed change receives at least as many affirmative as negative votes, it is approved.

Citizens may propose changes for their entire county, their justice precinct, or the city or town in which they reside. The state is divided into 254 counties and each county is divided into justice precincts.[9] Accordingly, a justice precinct lies within the county to which it belongs. By contrast, a city may spillover into two or more justice precincts. If only part of a city belongs to a particular justice precinct that has approved a change, then that part must abide by the new regulations. However, if the city then subsequently approved a different set of regulations, they would also be binding on the part contained in the justice precinct in question. Effectively, current regulations are determined by the most recently approved referendum.

Importantly for our purposes, liquor referenda are typically held separately from other elections. Section 41.01 of the Texas Election Laws sets aside four dates each year as uniform election dates. These are the dates when presidential, gubernatorial, and congressional elections are held. In addition, other issues are often decided on these days such as the election of aldermen and the approval of the sale of public land or bond issuances. Elections pertaining to these other issues may occur, but rarely do, on dates other than uniform election days. Liquor referenda, in contrast, do not typically occur on uniform election dates. This reflects the tight restrictions placed by Chapter 251 on the timing of elections.[10]

[8] Prior to 1993, the number of signatures needed was 35% of the total number of votes cast in the last preceding gubernatorial election.

[9] The number of justice precincts in a county range from 1 to 8.

[10] Interestingly, the Texas state government voted in 2001 to require liquor law referendum votes to occur on one of the four uniform election dates. This was to avoid the costs of holding referenda separately.

	Jurisdictions
Number of referenda	144
Referenda that passed	65
Jurisdiction characteristics	
Voting age population	370 (200)
Fraction of baptists	52% (11)
Located in an MSA	44% (50)
Incorporated city or town	95% (22)
Referendum characteristics	
Beer/wine	46% (50)
Off-premise	40% (49)
Off- and on-premise	15% (35)
More liberal than county	42% (49)
Held on weekend	68% (47)
Turnout	0.54 (0.19)
Winning margin	0.17 (0.16)

Table 1: Summary Statistics (standard deviations in parentheses)

3.2 Data

Coate and Conlin (2004) assembled data on 366 local liquor elections in Texas between 1976 and 1996 where there were no other issues on the ballot and prior to the election the voting jurisdictions prohibited the retail sale of all alcohol. The Texas Alcoholic Beverage Commission provided the details of the referenda as well as the election results. This data was augmented with information from the U.S. Census and Churches & Church Membership in the U.S.. The voting age population in these 366 jurisdictions averaged 4,112 and ranged from 43 to 81,904.

As noted in the introduction, we focus on the subset of these elections in which the number of eligible voters (citizens over the age of 18) is less than 900. This focus is necessitated by the computational difficulties of structurally estimating our parameterized pivotal-voter model. These difficulties increase rapidly with the number

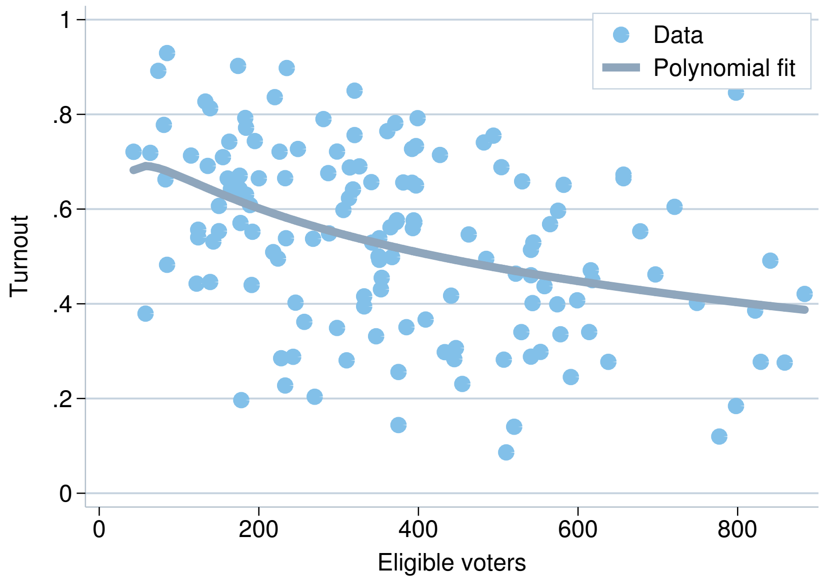

Figure 1: Voter turnout as a percent of eligible voters in the data

of eligible voters. There are 144 referenda in which the number of eligible voters is less than 900 and the referendum passed in 65 of these elections. Table 1 provides summary statistics for the 144 jurisdictions. The average number of eligible voters is 370. The fraction of baptists in the population is 52% and the fraction of jurisdictions located in an MSA is 44%. Those jurisdictions that are not incorporated cities or towns are all justice precincts.

While all the elections involved eliminating restrictions imposed on the sale of alcohol, they differed in the scope of reform. *Beer/wine referenda* permit only the sale of beer and wine. *Off-premise referenda* allow the sale of all liquor for consumption off-premise (i.e., no bars). *Off- and on-premise referenda* are the most permissive, allowing both off- and on-premise consumption of all liquor. Table 1 indicates that the majority of referenda were either beer/wine or off-premise. Furthermore, 42% of these referenda are proposing a more liberal policy than prevailing in the rest of the county and 68% of the elections are held on the weekend.

Table 1 indicates that average turnout in our sample is 0.54. Figure 1 provides a

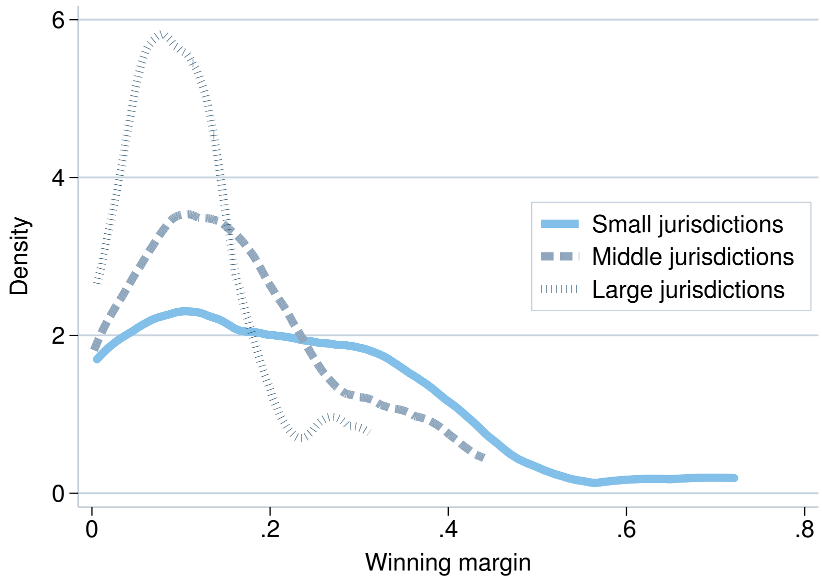

Figure 2: Distribution over winning margin in the data

closer look at the relationship between jurisdiction size and turnout. The scatter plot represents the percent turnout in each jurisdiction and the line is a polynomial fit across these observations. This line indicates that turnout decreases with population size.

Table 1 also reports that the average winning margin is 0.17. Figure 2 depicts the distribution of winning margins. The various lines show the kernel density estimates of the winning margins in three sub-groups of jurisdictions of equal size. Figure 2 indicates that there is considerable variation in winning margins across jurisdictions and that this variation decreases with jurisdiction size.

4 A pivotal-voter model

We now present a simple parameterized pivotal-voter model, tailored to the referendum context. The model is of the same general form as presented by Palfrey and Rosenthal (1985), except that we assume all citizens face a non-negative cost of voting and that citizens are uncertain as to the precise number of voters which favor and oppose the proposal in question.

A community is holding a referendum. There are n citizens, indexed by $i \in \{1, ..., n\}$. These citizens are divided into supporters and opposers of the proposal. Supporters obtain a benefit b from the proposed change, while opposers incur a loss x. Each citizen knows whether he is a supporter or an opposer, but not the number of citizens in each category. All citizens know the probability of a randomly selected individual being a supporter is μ.

Citizens must decide whether to vote in the referendum. If they do, supporters vote in favor and opposers vote against. Each citizen i faces a cost of voting c_i where c_i is the realization of a random variable uniformly distributed on $[0, c]$. Citizens observe their own voting costs, but only know that the costs of their fellows are the independent realizations of $n - 1$ random variables.

The only benefit of voting is the instrumental benefit of changing the outcome. Since the probability of being pivotal depends upon who else is voting, voting is a strategic decision. Accordingly, the situation is modelled as a *game of incomplete information* in which nature chooses the number of supporters and citizens' voting costs. Then, each citizen, having observed his own voting cost, decides whether or not to vote. If the number of votes in favor of the referendum is at least as big as the number against, the proposed change is approved.

A *strategy* for a citizen i is a function which for each possible realization of his voting cost specifies whether he will vote or abstain. The equilibrium concept is *Bayesian-Nash equilibrium* - each citizen must be happy with his strategy given the strategies of the other citizens and his statistical knowledge concerning the distribution of supporters and voting costs. Following Palfrey and Rosenthal (1985), we look for a *symmetric* equilibrium in which supporters and opposers use common strategies. With no loss of generality, we can assume that supporters and opposers use "cut-off" strategies that specify that they vote if and only if their cost of voting is below some critical level. Accordingly, a symmetric equilibrium is characterized by a pair of numbers γ_s^* and γ_o^* representing the cut-off cost levels of the two groups.

To characterize the equilibrium cut-off levels, consider the decision of some citizen i. Suppose the remaining $n-1$ citizens are playing according to the equilibrium strategies; i.e., supporters (opposers) vote if their voting cost is less than γ_s^* (γ_o^*). Let $\rho(v_s, v_o; \gamma_s^*, \gamma_o^*)$ denote the probability that v_s of the $n-1$ individuals vote in support and v_o vote in opposition when they play according to the equilibrium strategies. We show how to compute this below. Recall that the referendum passes if and only if at least as many people vote for as against the proposed change. Thus, if citizen i is a supporter, he will be pivotal whenever v of the $n-1$ other individuals vote in opposition and $v-1$ vote in support. In all other circumstances, his vote does not impact the outcome. Accordingly, the expected benefit of i voting is[11]

$$\sum_{v=1}^{n/2} \rho(v-1, v; \gamma_s^*, \gamma_o^*)b. \tag{1}$$

Individual i will wish to vote if this expected benefit exceeds his cost of voting. Accordingly, in equilibrium

$$\sum_{v=1}^{n/2} \rho(v-1, v; \gamma_s^*, \gamma_o^*)b = \gamma_s^*. \tag{2}$$

If citizen i is an opposer, he will be pivotal whenever v of the $n-1$ other individuals vote in opposition and v vote in support. In all other circumstances, his vote does not impact the outcome. Accordingly, the expected benefit of i voting is

$$\sum_{v=0}^{n/2-1} \rho(v, v; \gamma_s^*, \gamma_o^*)x. \tag{3}$$

In equilibrium, we have that:

$$\sum_{v=0}^{n/2-1} \rho(v, v; \gamma_s^*, \gamma_o^*)x = \gamma_o^*. \tag{4}$$

Equations (2) and (4) give us two equations in the two unknown equilibrium variables γ_s^* and γ_o^*. The exogenous parameters of the model are the probability that each individual is a supporter μ, the benefit to supporters b, the loss to opposers x, and the upper bound of the cost distribution c. Existence of an equilibrium pair γ_s^* and γ_o^* for any given values of the exogenous parameters is not an issue (see Ledyard

[11] This assumes that n is even. The case in which n is odd requires obvious modifications.

(1984) and Palfrey and Rosenthal (1985)), but there might in principle be multiple solutions. While Borgers (2004) has shown that there is a unique equilibrium when $b = x$ and $\mu = 1/2$ (see his Proposition 1), there is no general uniqueness result in the literature.

To compute equilibria we need to know the function $\rho(v_s, v_o; \gamma_s^*, \gamma_o^*)$. Let $P(s)$ denote the probability that s of the $n-1$ other citizens are supporters. This is given by:

$$P(s) = \binom{n-1}{s} \mu^s (1-\mu)^{n-1-s}. \tag{5}$$

If there are s supporters, the probability that $v_s \in \{1, ..., s\}$ vote in support is

$$\binom{s}{v_s} (\frac{\gamma_s^*}{c})^{v_s} (1 - \frac{\gamma_s^*}{c})^{s-v_s}. \tag{6}$$

Similarly, the probability that $v_o \in \{1, ..., n-1-s\}$ vote in opposition is

$$\binom{n-1-s}{v_o} (\frac{\gamma_o^*}{c})^{v_o} (1 - \frac{\gamma_o^*}{c})^{n-1-s-v_o}. \tag{7}$$

Thus, the probability that v_s vote in support and v_o vote in opposition is

$$\rho(v_s, v_o; \gamma_s^*, \gamma_o^*) = \tag{8}$$
$$\sum_{s=v_s}^{n-1-v_o} \binom{s}{v_s} (\frac{\gamma_s^*}{c})^{v_s} (1 - \frac{\gamma_s^*}{c})^{s-v_s} \binom{n-1-s}{v_o} (\frac{\gamma_o^*}{c})^{v_o} (1 - \frac{\gamma_o^*}{c})^{n-1-s-v_o} P(s).$$

Because (2) and (4) are a system of nonlinear equations, with no reduced form solution, equilibria can only be found numerically. This is a computationally difficult problem. There is no algorithm guaranteeing that *a* solution of a system of linear equations can be found, let alone *all* solutions. Our approach is to use a standard root-computation routine to find a solution from a user-given starting point.

5 Simulations

To illustrate the difficulty pivotal-voter models have in explaining large winning margins, we now simulate our model.[12] To do so, we fix the number of eligible voters, select arbitrary parameter values (b, x, μ, c), and compute cut-off levels (γ_s^*, γ_o^*) which

[12] As a theoretical matter, it is clear that winning margins (as we have defined them) must converge to zero as the number of voters goes to infinity. This follows immediately from the fact that turnout is going to zero. The question for our purposes is how fast this happens and to answer this we need to simulate.

satisfy the equilibrium conditions (2) and (4).[13] We then simulate election outcomes by randomly drawing a particular realization of preferences and cost shocks. Thus, for each citizen, the probability of being a supporter is μ and each citizen's voting cost is drawn from the uniform distribution on $[0, c]$. These draws together with the equilibrium cut-offs then determine turnout for and against the proposed change. Each (aggregate) draw determines a sample with a pattern of election outcomes. By repeating the procedure numerous times and averaging across all samples we can compute the predicted distributions for total turnout and winning margins implied by the model.

Table 2 provides summary statistics from these predicted distributions for different vectors of parameter values and varying the number of eligible voters. Our choice of parameter values was guided by the results of the structural estimation to be reported in the next section. In particular, we normalize the upper bound on voting costs c equal to one and choose baseline values for b and x that are roughly equal to the mean estimated values of b/c and x/c.[14] The summary statistics indicate that (for all parameter values selected) expected total turnout is reasonably large except when the probability of a citizen being a supporter and the number of eligible voters are relatively large. As for winning margins, Table 2 indicates that for all parameter values selected the probability of an election outcome with a winning margin greater than 0.10 or 0.20 is close to zero if the number of eligible voters exceeds 400.

This creates a problem for our pivotal-voter model because winning margins greater than 0.10 or 0.20 are not uncommon in the data. This is clear from Figure 2. For a more disaggregated look, note that in the 48 smallest jurisdictions (with eligible voters ranging between 43 and 246) the fraction of referenda with a winning margin greater than 0.10 and 0.20 is 0.69 and 0.50, respectively. For the next 48 jurisdictions (with eligible voters ranging between 249 and 433) these fractions are 0.65 and 0.27, while for the 48 largest jurisdictions (with between 438 and 884 eligible voters), these fractions are 0.43 and 0.12, respectively. Thus, while it is true

[13] We obtain these cut-off levels $(\gamma_{sj}^*, \gamma_{oj}^*)$ by performing a grid search and, for all simulations, this search obtained a unique $(\gamma_{sj}^*, \gamma_{oj}^*)$ pair.

[14] These mean estimated values are described in Table 4 below.

(a) Baseline: $\mu = 0.5,\ b = x = 17.5,\ c = 1$

Eligible voters	Turnout	Margin > 0.10	Margin > 0.20
50	0.99	0.52	0.18
100	0.79	0.29	0.03
200	0.62	0.08	0.01
400	0.49	0.01	0
500	0.46	0	0

(b) Larger b**, smaller** x**:** $\mu = 0.5,\ b = 24,\ x = 12,\ c = 1$

Eligible voters	Turnout	Margin > 0.10	Margin > 0.20
50	0.62	0.71	0.33
100	0.41	0.51	0.6
200	0.26	0.16	0
400	0.17	0	0
500	0.14	0	0

(c) Larger μ**:** $\mu = 0.6,\ b = x = 17.5,\ c = 1$

Eligible voters	Turnout	Margin > 0.10	Margin > 0.20
50	0.76	0.59	0.24
100	0.55	0.39	0.05
200	0.38	0.14	0
400	0.26	0.01	0
500	0.23	0	0

(d) Even Larger μ**:** $\mu = 0.75,\ b = x = 17.5,\ c = 1$

Eligible voters	Turnout	Margin > 0.10	Margin > 0.20
50	0.39	0.70	0.24
100	0.25	0.40	0.01
200	0.15	0.05	0
400	0.09	0	0
500	0.08	0	0

Table 2: Simulations of turnout and winning margins under different parameter configurations

that the fraction of elections with large winning margins decreases in the size of the jurisdiction, the drop is nowhere near fast enough to match the dramatic drop suggested by the simulations.

6 Estimation

Our simulations suggest that a dramatic decrease in the probability of large winning margins occurs with a relatively small number of voters. While we believe this will hold for all values of our parameters, our simulations are necessarily based on specific parameter values. Therefore, we proceed to structurally estimate our model, which allows us to infer the optimal parameter values from the election outcomes.

6.1 Identification

Before describing in detail the estimation procedure, it is worth briefly discussing what features of the data allow us to identify the coefficients associated with our model's parameters. Recall that our model is characterized by just four parameters: the supporters' benefit b; the opposers' loss x; the probability that a citizen is a supporter μ; and the upper bound of the uniform cost distribution, c. We will consider how changes in the values of the parameters impact the outcome predicted by the model.

To simplify the discussion, assume that all jurisdictions share the same values of b, x, μ, and c. Note first that only the relative values of b, x, and c matter; that is, when b, x, and c are multiplied by the same factor, the equilibrium values of γ_s and γ_o are also scaled by the same factor, and the probability of observing a referendum outcome (the number of votes for and against) conditional on the equilibrium cut-off levels is independent of the scaling factor. Therefore, under the simplifying assumption that all jurisdictions are identical, c is not identified, and we can normalize c to 1.

Next observe that the combined *magnitude* of parameters b and x effects overall turnout, because increasing the benefits from winning the election increases the incentives to vote, and therefore turnout. What is left is the determination of the *difference* $b-x$ and μ. Both $b-x$ and μ are identified from the observed difference between turnout for and turnout against. We are able to separately identify $b-x$ and μ because: (i) a change in $b-x$ has different effects than a change in μ on turnout; and (ii) the differential effects of changes in $b-x$ and μ vary with the size of the jurisdiction and total turnout. For example, the difference between votes for and against provides more information on μ relative to $b-x$ when a jurisdiction has high total turnout and a large number of eligible voters. Hence, variation in closeness of the electoral outcome across jurisdictions with different levels of turnout and different numbers of eligible voters facilitate the identification of μ from $b-x$.

Thus, under the assumption that all jurisdictions have identical b, x, μ, and c, identification of these parameters is possible (after normalizing the cost of voting) provided that sufficient variation exists across jurisdictions in eligible voters and referendum outcomes. In our empirical implementation, we allow the four parameters to have different values by having them depend on exogenous jurisdiction characteristics. The coefficients associated with these jurisdiction specific variables are identified from the variation in electoral outcomes across jurisdictions with different exogenous characteristics.

6.2 Estimation procedure

We assume that each of our 144 small jurisdictions is characterized by a distinct quadruple of parameters (μ_j,b_j,x_j,c_j) but that each parameter is determined by jurisdiction and referenda specific characteristics in a common way. Specifically, for each jurisdiction j, we assume that:

$$b_j = \exp\left(\boldsymbol{\beta}^b \cdot \mathbf{z}_j^b\right) \tag{9}$$

$$x_j = \exp\left(\boldsymbol{\beta}^x \cdot \mathbf{z}_j^x\right) \tag{10}$$

$$\mu_j = \frac{\exp\left(\boldsymbol{\beta}^\mu \cdot \mathbf{z}_j^\mu\right)}{1+\exp\left(\boldsymbol{\beta}^\mu \cdot \mathbf{z}_j^\mu\right)} \tag{11}$$

$$c_j = \exp\left(\beta^c \cdot z_j^c\right) \tag{12}$$

where $(\boldsymbol{\beta}^b, \boldsymbol{\beta}^x, \boldsymbol{\beta}^\mu, \beta^c)$ are vectors of coefficients to be determined and $(\mathbf{z}_j^b, \mathbf{z}_j^x, \mathbf{z}_j^\mu, z_j^c)$ are vectors of observable jurisdiction and referenda characteristics. More specifically,

$\mathbf{z}_j^b = \mathbf{z}_j^x =$(1, off-premise, off- and on-premise, incorporated city or town, more liberal than county), $\mathbf{z}_j^\mu$ = (1, fraction baptist, MSA), and $z_j^c =$(election on weekend). The functional forms for b_j, x_j and c_j are selected so that they are positive and the functional form for μ_j is selected to ensure that it lies between zero and one. The formulation also embodies the normalization that c_j equals 1 when the election is not held on the weekend.

The task is to estimate the coefficients ($\boldsymbol{\beta}^b$, $\boldsymbol{\beta}^x$, $\boldsymbol{\beta}^\mu$, β^c). For each jurisdiction j we observe the votes for and against the proposed change, together with the total number of eligible voters which we denote respectively as v_{sj}, v_{oj}, and n_j. The choice of ($\boldsymbol{\beta}^b$, $\boldsymbol{\beta}^x$, $\boldsymbol{\beta}^\mu$, β^c) determine b_j, x_j, μ_j and c_j and these determine via equations (2) and (4) a set of M_j equilibria in each jurisdiction j, which we denote $\{\gamma_{sj}^m, \gamma_{oj}^m\}_{m=1}^{M_j}$.

Each equilibrium implies a probability distribution over election outcomes. In particular, the probability of observing data (v_{sj}, v_{oj}) conditional on the citizens voting according to the cut-off levels $(\gamma_{sj}, \gamma_{oj})$ is given by (8). While it is impossible to know *a priori* whether there are multiple equilibria, if there is multiplicity problems arise because there is not a unique mapping from the parameter space to the likelihood of observable events. We resolve this issue by ignoring multiplicity, that is, selecting the first equilibrium the computation routine finds, using the midpoint of the strategy space as a starting point of the routine.[15]

Assuming this equilibrium selection rule, denote the selected equilibrium $(\gamma_{sj}^{m*}, \gamma_{oj}^{m*})$. Then, equation (8) defines the likelihood of observing a referendum outcome *condi-*

[15] This is equivalent to assuming that the "true" equilibrium selection follows the same procedure we used. The estimator remains consistent under this assumption because the equilibrium computation routine is deterministic. The selection procedure we chose is arbitrary only to the extent that multiplicity is prevalent. We investigated this issue and found that multiplicity does not arise very often. We thoroughly searched for equilibria using a grid of parameter vectors centered around our estimates. For all of these parameter vectors, we found multiplicity in less than ten percent of the jurisdictions. At the estimated values of the parameters, we found only 7 jurisdictions displaying multiple equilibria. Recent literature has developed different techniques that propose consistent estimators that explicitly take into account the equilibrium multiplicity and use the data generating process to infer the equilibrium selection, (see, for example, Aguirregabiria and Mira (2007), Bajari, Hong, Krainer and Nekipelov (2006), Berry, Ostrovsky, and Pakes (2005), Moro (2003), and Tamer (2003)). None of these procedures are directly applicable to our setup. We experimented with assuming a parametric equilibrium selection rule, and tried to estimate the parameters of this rule together with those of the model. We found it computationally unfeasible, as it requires the computation of all equilibria of the model. We decided not to pursue this issue in light of the fact that multiplicity is not prevalent in our model.

tional on the equilibrium cut-off levels $(\gamma_{sj}^{m*}, \gamma_{oj}^{m*})$. The likelihood function is therefore

$$L(\boldsymbol{\beta}^b, \boldsymbol{\beta}^x, \boldsymbol{\beta}^\mu, \beta^c) = \prod_j \rho(v_{sj}, v_{oj}; \gamma_{sj}^{m*}, \gamma_{oj}^{m*}). \tag{13}$$

The estimation strategy is the following: first, guess a vector of coefficient values ($\boldsymbol{\beta}^b$, $\boldsymbol{\beta}^x$, $\boldsymbol{\beta}^\mu$, β^c). Then compute equilibrium critical cost levels for each jurisdiction (based on the equilibrium selection rule stated above), and the probability of observing the jurisdiction election outcome conditional on such thresholds. These probabilities determine the likelihood of observing the data given by $L(\boldsymbol{\beta}^b, \boldsymbol{\beta}^x, \boldsymbol{\beta}^\mu, \beta^c)$. Finally, find the coefficient values that maximize $L(\boldsymbol{\beta}^b, \boldsymbol{\beta}^x, \boldsymbol{\beta}^\mu, \beta^c)$.

The sums in equations (2), (4), and (8) take progressively longer to compute as the number of eligible voters increases, making the computation of the equilibria extremely time consuming in large jurisdictions, even using Normal approximations to the Binomial distributions. This explains why we limited our sample to the 144 observations with the smallest number of eligible voters.[16]

6.3 Coefficient estimates

Table 3 contains the coefficient estimates that maximize the likelihood function, with bootstrapped standard errors in parentheses, while Table 4 provides the average values of the model's exogenous variables implied by these estimates.

The point estimates in Table 3 suggest that the probability an individual is a supporter does not depend appreciably on the fraction of baptists in the jurisdiction or on whether the jurisdiction is located in an MSA. The coefficient estimates associated with the benefit and loss parameters, b and x, suggest that the effect of the jurisdiction-specific characteristics on preferences are often quite large and similar for supporters and opposers. Both supporters and opposers benefit more from their preferred outcome if the jurisdiction holding the election is an incorporated city or town. Less intuitively, both groups' benefits are less for off- and on-premise referenda compared to off-premise referenda and beer/wine referenda (the omitted category). The average supporter's benefit and opposer's loss almost double when

[16] We used an implementation of the simulated annealing method developed by Bill Goffe (see Goffe et al. (1992)) to maximize (13) on a 16-processor supercomputer.

Parameter/Variable ($\ln L: -5694.21$)		Estimate	Marginal Effect
μ:	Fraction of baptists	-0.058 (0.188)	-0.015
	Located in an MSA	-0.089 (0.072)	-0.022
	Constant	0.062 (0.097)	
b:	Off-premise	0.182 (0.086)	2.85
	Off- and on-premise	-0.642 (0.232)	-7.89
	Incorporated city or town	1.819 (0.354)	13.7
	More liberal than county	0.199 (0.068)	3.15
	Constant	0.875 (0.405)	
x:	Off-premise consumption	0.097 (0.082)	1.56
	Off- and on-premise	-0.589 (0.253)	-7.58
	Incorporated city or town	1.791 (0.340)	14.0
	More liberal than county	0.361 (0.062)	5.90
	Constant	0.886 (0.370)	
c:	Held on weekend	-0.172 (0.085)	-16.0

Table 3: Pivotal voter model estimates (standard errors in parentheses)[17]

the jurisdiction is a city/town and are approximately fifty percent less for an off- and on-premise referendum than a beer/wine referendum. The benefit and loss parameters increase slightly if the referendum passing results in the jurisdiction having a more liberal alcohol policy than in the rest of the county. In terms of the cost of voting, the coefficient estimates suggest that having an election on a weekend decreases average voting costs by approximately 16 percent.

The average values of the model's exogenous variables implied by the estimates (Table 4) indicate that, on average, the probability a citizen is a supporter is one-half (i.e., $\mu = .50$). The average benefit of supporters is slightly less than the loss of opposers, which means that the average probability a supporter votes is a little less than that of an opposer (0.516 versus 0.530). These probabilities are quite large because both the average benefit to supporters and the loss to opposers are approximately 35 times greater than the average voting costs.

[17] For the "Fraction of baptists" variable, the marginal effect column reports the percent change in the value of parameter μ given a one percent change in the fraction of baptists. For all other

Parameter	Mean estimate
Probability supporter μ	0.500 (0.011)
Supporters' benefit b	15.52 (4.81)
Opposers' benefit x	15.90 (5.12)
Upper bound on cost c	0.892 (0.074)
Supporters that vote $\frac{\gamma_s}{c}$	0.516 (0.167)
Opposers that vote $\frac{\gamma_o}{c}$	0.530 (0.174)

Table 4: Mean parameter estimates (standard errors in parentheses)

The implied values of the model's exogenous variables (Table 4) can be interpreted in dollar terms if we assign a value to the cost of voting. For example, suppose that we guess that the average voting cost across all jurisdictions is \$10. Then, this implies that $c/2 = 0.892/2 = \$10$. This ties down the units in dollar terms since $0.892 = \$20$ or $1 = \$20/(0.892) = \$22.42$. It follows that the average value of b is $\$22.42(15.52) = \$347.98$ and the average value of x is $\$22.42(15.90) = \$356.50$. These numbers are on the large side, but within the range of plausibility.

6.4 Model performance

We can now redo the simulation exercise from the previous section using the actual jurisdiction sizes and their estimated parameters. Thus, we use the coefficient estimates in Table 3 to calculate the implied values of the model's parameters (b_j, x_j, μ_j, c_j) for each jurisdiction and compute a pair of cut-off levels $(\gamma^*_{sj}, \gamma^*_{oj})$ that satisfies the equilibrium conditions (2) and (4). Then we simulate election outcomes for all 144 jurisdictions to obtain the predicted distributions for total turnout and closeness implied by the model.

Table 5 compares average actual turnout with the average predicted from the simulations. It does this for all sample observations and for various smaller subsamples based on jurisdiction size. The table indicates that our pivotal-voter model

variables, which are all dummies, the entry is the percent change in the value of the parameter with respect to the change from zero to one in the independent variable.

Eligible voters n	N. of obs.	Data	Pivotal-voter model
$n < 247$	48	0.62	0.65
$247 < n < 434$	48	0.55	0.51
$434 < n < 900$	48	0.43	0.40
All sample ($n < 900$)	144	0.54	0.52

Table 5: Average turnout as a percentage of eligible voters: model vs. data

is reasonably accurate in predicting the average total turnout of 0.54 for the entire sample as well as the average turnouts for the equally-sized subsamples.

Turning to winning margins, Figure 3 depicts the distributions of actual and predicted winning margins obtained from our simulations. The solid line is the predicted kernel density estimate of the distribution from the model, while the dashed line is a kernel density estimate of the distribution in the data. It is evident from Figure 3 that the model predicts much smaller winning margins than are observed in the data.

7 A simple alternative model

Given the failure of our pivotal-voter model to correctly predict winning margins, it is natural to wonder whether a simpler model based on the idea of citizens voting for non-instrumental reasons might fit the data better. To investigate this we study the comparative performance of a simple expressive model of voting.

The expressive view asserts that citizens vote not to impact the outcome but to express their preferences (see, for example, Brennan and Lomasky (1993)). It seems natural to assume that people care more about expressing their preferences the more intensely they feel about an issue. This suggests what Coate and Conlin (2004) refer to as the *intensity model* which works within the same basic environment as a

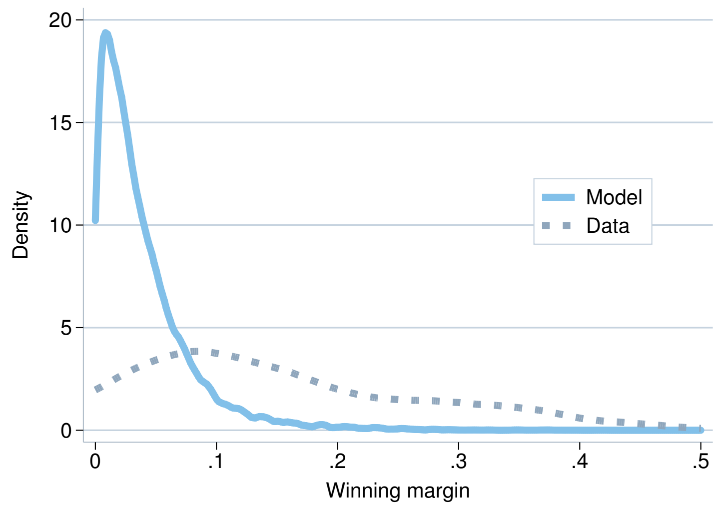

Figure 3: Distributions over winning margin, model vs. data

pivotal-voter model but assumes that supporters vote if their voting cost is less than $\gamma_s = \alpha b$, while opposers vote if their voting cost is less than $\gamma_o = \alpha x$, where $\alpha > 0$. Here, the parameter α measures the strength of citizens' desire to express themselves through voting. Under this specification, the probability that a supporter votes is the probability that γ_s exceeds his voting cost, which is $\gamma_s/c = \alpha b/c$. Similarly, the probability that an opposer votes is $\gamma_o/c = \alpha x/c$. Note that, unlike a pivotal-voter model, the propensity of a supporter (opposer) to vote does not depend on the propensity of an opposer (supporter) to vote.

We assume that the parameters b_j, x_j, μ_j, and c_j depend on the same jurisdiction characteristics as in our pivotal-voter model (see equations 9-12). In addition, we assume that α_j is constant across jurisdictions. Observe that this specification does not allow any of the parameters to vary with the size of the jurisdiction. Therefore, this model predicts turnout to be a constant percentage of the electorate's size, *ceteris paribus*. By choosing this simple specification, we make it more difficult for the intensity model to match the data (recall from Figure 1 that turnout decreases with the size of the electorate). Also observe that we can only identify the aggregate expressive benefits to supporters and opposers which are $\alpha_j b_j$ and $\alpha_j x_j$. The effect on voting behavior of an increase in the desire of citizens to express themselves (i.e., an increase in α_j) can be mimicked by an increase in how strongly they feel about the issue (i.e., a simultaneous increase in b_j and x_j). This is not problematic given that our primary goal is to compare the fit of this model with that of our pivotal-voter model.

The estimation procedure is similar to that for our pivotal-voter model. To economize on scarce journal space, we omit a discussion of the coefficient estimates and focus directly on the predictive power and relative performance of the two models[18]. As expected, the intensity model does much worse than the pivotal-voter model in predicting average total turnout. It significantly underpredicts total turnout in small jurisdictions and slightly overpredicts in the larger jurisdictions. However, while it still predicts closer elections than are observed in the data, the intensity model does a much better job than the pivotal-voter model at predicting the distribution of actual winning margins. This advantage means that the intensity model, despite its relative simplicity, is not out-performed by the pivotal-voter model. Indeed, the maximum log-likelihood value of the intensity model is *greater than* the maximum log-likelihood value of the pivotal-voter model (-4,567 compared to -5,694). Using the likelihood-ratio based statistic for non-nested models proposed by Vuong (1989), this implies that there is no support for the hypothesis that the pivotal-voter model is closer to the true data generating process than the intensity model.

Furthermore, if we allow citizens' desire to express themselves to vary with jurisdiction size by specifying $\alpha_j =$(number of eligible voters in jurisdiction $j)^{\beta}$, the intensity model fits total turnout significantly better (without compromising its fit of closeness). The null hypothesis that the two models are equally close to the true data generating process can then be rejected at the ten percent significance level against the alternative hypothesis that the intensity model is closer. This suggests that the negative correlation between turnout and size is not due to the strategic nature of the voting decision that is implied by the pivotal-voter model, but is the result of unexplained factors that make voter participation more likely in smaller communities (such as stronger peer pressure or sense of community).

[18] Detailed results can be found in Coate, Conlin and Moro (2004), or from the authors upon request.

8 Conclusion

Pivotal-voter models form the basic framework for thinking about turnout in formal political theory. While it seems widely conceded that these models do not provide an empirically satisfactory theory of turnout in large-scale, single-issue elections, the hope has remained that they might explain voter behavior in small-scale elections. It is this hope that seems to implicitly justify the continued use of such models in theoretical work.

The results of this paper provide little to nurture this hope. While our parameterized pivotal-voter model can explain turnout in small-scale Texas liquor elections, it cannot explain the large winning margins that are common in these elections. The logic of pivotal-voter models implies that elections must be expected to be close even if there is a significant difference between the expected sizes of the groups who support the options to be voted on or the intensity of their preferences. With a very small number of voters, elections that are expected to be close ex ante may end up not close ex post because of sampling error. As the number of voters increases, this sampling error very quickly disappears and elections that are expected to be close ex ante will be close ex post. This difficulty in explaining large winning margins allows our pivotal-voter model to be outperformed by the intensity model - a simple expressive voting model which just assumes that people are more likely to vote the more intensely they feel about the issue.

We see two possible responses to our findings. First, in the spirit of Levine and Palfrey (2007), one might defend pivotal-voter models by noting that their key comparative static predictions are correct. In the data, turnout is decreasing in the size of the electorate and winning margins tend to be smaller in larger jurisdictions. The prediction concerning the effect of electorate size on winning margins is not at all obvious and we find it somewhat remarkable that it is indeed true in the data. Given this, it seems overly harsh to claim that such models have no value in understanding voter behavior. It may also be that, as in Levine and Palfrey's experiments, the difficulties that the models have in matching the level of winning margins could be attentuated by adopting the quantal response equilibrium concept. This is something that future work should look at.

The second response is to abandon pivotal-voter models in favor of an alternative approach. Using the same data set, Coate and Conlin (2004) have shown that a model of voter turnout in which individuals are motivated to vote by ethical reasons, fits the data well, and outperforms the intensity model. Relative to the intensity model, this model does better at explaining both the variation in turnout and the winning margin. Transitivity does not strictly speaking apply, because Coate and Conlin's analysis makes different assumptions, most notably a continuum of voters. Nonetheless, the combined results of the two papers certainly suggest that an ethical approach may be a more promising way of understanding voter behavior than pivotal-voter models.

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