Advances Topics

1. Consider the market for health insurance. Suppose there exist three different types of individuals. Type 1 individuals are very healthy, Type 2 individuals are moderately healthy and Type 3 individuals are unhealthy. There are twenty Type 1 individuals, thirty Type 2 individuals and ten Type 3 individuals. Type 1 individuals are willing to pay $2,000 for health insurance, Type 2 individuals are willing to pay $3,000 for health insurance and Type 3 individuals are willing to pay $4,000 for health insurance.

Assume that the health insurance industry is a monopoly in that there is only one firm in the industry. Suppose the expected medical costs of a healthy individual (Type 1) is $1,000, the expected medical costs of a moderately healthy individual (Type 2) is $2,000 and the expected medical costs of an unhealthy individual (Type 3) is $5,000.

What price will the monopolist charge for health insurance if the monopolist cannot differentiate the healthy individuals from the moderately healthy from the unhealthy? Who will end up obtaining health insurance? SHOW ALL CALCULATIONS AND PROVIDE BRIEF EXPLANATION.

Profits if Price=2000

\[(2K-1K)20+(2K-2K)30+(2K-5K)10=-10k\]

\[4000\]

Profits if Price=3000

\[(3K-2K)30+(3K-5K)10=+10k\]

\[3000\]

Profits if Price=4000

\[(4K-5K)10 = -10k\]

\[2000\]

So the monopolist should charge a price of $3000 and at that price only the moderately healthy and unhealthy types buy the insurance.
2. Mike Conlin and Stacy Dickert-Conlin live in East Lansing and are currently looking for a nanny for their children. Suppose there exist two different types of nannies in East Lansing. Type G is a good nanny who will play with the kids and Type B is a bad nanny who will watch Jerry Springer with the kids. There are thirty good nannies and seventy bad nannies. All good nannies have a reservation value of $25,000 per year and all bad nannies have a reservation value of $15,000 per year. (Therefore, the good nannies are not willing to work as a nanny for less than $25,000 per year and the bad nannies are not willing to work as a nanny for less than $15,000 per year. Perhaps the reason good nannies have a higher reservation value is because they can obtain jobs as teachers or at day care facilities.) All potential employers of nannies in East Lansing are willing to pay $30,000 for a good nanny and $20,000 for a bad nanny. Assume there are thousands of risk-neutral potential employers of nannies who cannot differentiate between a good nanny and a bad nanny.

a) Depict the supply and demand curves for nannies on the graph below. In equilibrium, how many nannies are employed in East Lansing and how much are they paid? SHOW CALCULATIONS.

If the wage is at least $25,000, then both the good and bad nannies are willing to take the job. All potential employers are willing to pay
\[30,000(30/100)+20,000(70/100) = $23,000.\]
Therefore, at a wage of $25k or more, the quantity demanded will be zero.

At a wage less than $25k and more than $20k, only the bad nannies will be on the market so potential employers are willing to pay $20k and quantity demanded will be zero.
At a wage of $20k, only the bad nannies will be on the market so potential employers are willing to pay $20k and quantity demanded will be \([0,\infty]\). Equilibrium wage is $20k and all 70 of bad nannies get jobs.

Price/Wage

b) Assume that everything remains the same as above except instead of being thirty good nannies and seventy bad nannies, there are sixty good nannies and forty bad nannies. How does this change the problem and the outcome in the nanny market? EXPLAIN AND SHOW CALCULATIONS.

If the wage is at least $25,000, then both the good and bad nannies are willing to take the job. All potential employers are willing to pay
\[30,000(60/100)+20,000(40/100) = $26,000.\]
Therefore, all 100 nannies are employed at a wage of $26,000 in equilibrium.
3. Consider the market for houses. Suppose there exist two different types of houses. Type S is a structural sound house and Type F is a structurally flawed house. There are sixty Type S houses and forty Type F houses. All owners of the structurally sound houses have reservation value $175,000 and all owners of the structurally flawed houses have reservation value $75,000. (Therefore, the owners of the structurally sound houses are not willing to sell their house for a price less than $175,000 and the owners of the structurally flawed houses are not willing to sell their house for a price less than $75,000.)

All potential buyers are willing to pay $200,000 for a structural sound house and $100,000 for a structural flawed house. Assume there are thousands of risk-neutral potential buyers and buyers cannot differentiate between a structurally sound and a structurally flawed house.

a. Depict the supply and demand curves for housing. How many houses are sold in equilibrium and at what price are the houses sold for?

At a price above $175,000, risk-neutral buyers know that both types of houses will be put on the market so they are willing to pay $100,000($40/100)+$200,000($60/100)=$160,000. Therefore, the quantity demanded at a price above $175,000 is zero.

At a price between $75,000 and $175,000, buyers are willing to pay $100,000 because they know only structurally flawed houses are being put on the market. Therefore, the quantity demanded at a price above $100,000 is zero, at a price of $100,000 is (0,infinity) and at a price less than $100,000 is infinity. The demand curve is depicted above.

Only structurally flawed houses are sold and the price of these houses is $100,000.

b. If the buyers were risk-averse instead of risk-neutral, how would this affect the supply of and demand for houses? Depict on the attached graph.

It would not affect the supply and demand depicted above. The reason is that in the equilibrium above, the buyers know they are buying a flawed house and, therefore, do not incur any risk. The only thing it would affect in the analysis above is that at a price above $175,000 (when both types of houses are put on the market), buyers would be willing to pay less than their expected valuation of $160,000.

4. Suppose all companies in the automobile insurance industry offer a price (i.e., annual premium) of $1,200 to an 18 year old with no prior tickets that owns a 1990 Toyota Camry. This price is based on the fact that the expected level of claims for an 18 year old with no prior tickets that owns a 1990 Toyota Camry is $1,200 a year. Now suppose that Progressive Insurance Company discovers that the expected claims of an 18 year old with no prior tickets who owns a 1990 Toyota Camry is $1,500 if the 18 year old is male and $1,000 if the 18 year old is female. What should Progressive do? EXPLAIN IN DETAIL.

Assuming the other companies do not change their pricing policies, Progressive should charge 18 year old females $1,199 and charge 18 year old males more than $1,200. With this pricing policy, all 18 year old females would buy from Progressive and no 18 year old males would buy from Progressive. I would expect the other companies to change their pricing policies and, if the market is relatively competitive, 18 year old females’ insurance would cost around $1,000 and 18 year old males’ insurance would cost around $1,500.
5. A firm is deciding whether to produce a low quality product or a high quality product. If the firm selects to produce the low quality product, the firm would have a constant marginal cost of $2 and total fixed costs of $20. If the firm selects to produce the high quality product, the firm would have a constant marginal cost of $2 and total fixed costs of $50. If consumers know it is a low quality product, the firm faces a demand curve \(D_L\) and if consumers know it is a high quality product, the firm faces a demand curve \(D_H\). These demand curves are depicted on the graph below.

\[
\begin{align*}
\text{MR}_L & \quad \text{MR}_H \\
\end{align*}
\]

a) Suppose consumers can distinguish between a low quality and high quality product before purchasing the good. Will the firm produce the low quality or high quality product? What will be the firm’s profits? SHOW CALCULATIONS AND EXPLAIN.

The firm’s maximum profits if they produce a low quality product is
\[5(30)-2(30)-20 = 70.\]

The firm’s maximum profits if they produce a high quality product is
\[6(40)-2(40)-50 = 110.\]

Therefore, the firm should produce a high quality product.

b) Now suppose consumers cannot distinguish between a low quality and high quality product before purchasing the good. Will the firm produce the low quality or high quality product? What will be the firm’s profits? SHOW CALCULATIONS AND EXPLAIN.

This is almost identical to the problem we did in class. As I explained in class, the firm would end up producing a low quality product, consumers would expect a low quality product and the firm’s profits would be 70. If consumers were naïve and expected a high quality product, the firm would have incentive to produce a low quality product because they would increase their profits by 30 (50-20). Assuming that consumers are not naïve, they would expect a low quality product and given these expectations, the firm maximizes profits by producing a low quality product.

c) If consumers cannot distinguish between a low quality and high quality product before purchasing the good, what type of actions do you think the firm may take in order to signal to consumers the product’s quality? Explain what would be the likely result of taking these actions.

Offer a warranty or some other type of credible guarantee ensuring the consumers it is a high quality product.
6. Suppose you own Starbucks Coffee and live in Seattle, Washington. You are deciding the compensation to offer Fred Spartan, the manager of your store next to Michigan State University. Suppose Fred can work 40 hours a week or 60 hours a week. Fred’s opportunity cost of working the additional 20 hours a week (from 40 hours to 60 hours) is $20,000 per year. Also, suppose Fred can obtain a job at Brueger’s Bagels where he works 40 hours a week and that job pays $40,000 per year. (Assume that Fred is indifferent between a job at Brueger’s Bagels and managing the Starbucks if the compensation and hours are the same.)

You are far too busy to monitor whether Fred is working 40 hours a week or 60 hours a week but you do observe the store’s revenue. You know that total annual revenue from the Starbucks next to MSU is likely to be higher if Fred works 60 hours a week. Assume that total annual revenue from the store is either $100,000 or $200,000. If Fred works 40 hours a week, total revenue will be $100,000 with probability .6 and $200,000 with probability .4. If Fred works 60 hours a week, total revenue will be $100,000 with probability .2 and $200,000 with probability .8. (Basically, Fred working more hours increases the probability that revenue will be $200,000 rather than $100,000.)

a) If you pay Fred a wage of $40,000, what would be your expected profits from the store next to MSU? Assume that your only cost is Fred’s wage. Show Calculations.

\[0.4\times200000 + 0.6\times100000 - 40000 = 100000.\]

b) How can you structure Fred’s contract to induce him to work 60 hours a week? (Don’t provide numbers, just explain the intuition.)

Compensate Fred more if total revenue is 200,000 compared to 100,000 and have it enough more to induce Fred to take the job. You must also have Fred’s expected compensation be enough to induce him to take the job at Starbucks.

c) What is the optimal contract to offer Fred in order to maximize your expected profits from the store? Assume that Fred is risk-neutral and that your only cost is Fred’s wage. Show Calculations and Explain.

Let \(w_{200}\) be Fred’s wage if total revenue is 200,000 and \(w_{100}\) be Fred’s wage if total revenue is 100,000.

Fred takes job and works 60 hours if his expected compensation is at least $60,000 and if his expected compensation from working 60 hours is a minimum of $20,000 more than his expected compensation from working 40 hours. This is the same thing as saying that the following two inequalities must be satisfied.

\[0.8w_{200} + 0.2w_{100} - 20,000 \geq 0.4w_{200} + 0.6w_{100}\]
\[0.8w_{200} + 0.2w_{100} \geq 60,000\]

Suppose both equations above hold with equality (the second one must hold with equality if you are maximizing profits). Solving for the wages, \(w_{200} = 70,000\) and \(w_{100} = 20,000\). Fred’s expected compensation would be \(0.8\times70k + 0.2\times20k = 60,000\) and Starbucks expected revenue would be \(0.8\times200000 + 0.2\times100000 = 180,000\). Therefore, expected profits are \(180,000 - 60,000 = 120,000\) which is greater than the profits from Fred being on a fixed salary of $40,000.

A more intuitive way to think about it is to realize that the probability of obtaining $200,000 in revenue increases by .4 if Fred works hard. Therefore, in order for Fred to work 60 hours (assuming he takes the job), you would have to compensate him at least $50,000 more because \(0.4\times50,000 = 20,000\) which is his marginal cost of working 60 hours rather than 40 hours. (This is based on the assumption that Fred is risk neutral.)

Basically, any compensation scheme where the expected wage is 60,000 and \(w_{200} \geq w_{100} + 50,000\) is optimal. In the real world where people are risk averse, you would want to expose the employee to as little risk as possible while still providing him/her with the proper incentives. If the employee is very risk adverse, you might be better off paying the employee a fixed wage and having the employee exert low effort.
7. NBA basketball player Tyreke Evans, who plays for the Sacramento Kings, likes to drive fast and has an automobile insurance policy. He pays a $7,000 annual premium for his insurance policy. The policy stipulates that if Evans gets in an accident, he must pay a $1,000 deductible if he incurs an accident and 30% of the repair costs. Evans is deciding whether to drive 70 miles per hour or 120 miles per hour. He believes that the probability of getting in an accident is 0.1 if he drives 70 miles per hour and 0.3 if he drives 120 miles per hour. He expects the repair costs to be $10,000 if he gets in an accident going 120 miles per hour and expects the repair costs to be $5,000 if he gets in an accident going 70 miles per hour. Note that Tyreke Evans pays 30% of these repair costs.

a) Suppose Tyreke Evans decides to drive 120 miles per hour. What is the minimum benefit Tyreke Evans must derive from driving 120 compared to 70 miles per hour? Show calculations.

Tyreke’s expected cost associated with going 120 miles per hour is .3(1,000+.3*10,000) = 1,200.
Tyreke’s expected cost associated with going 70 miles per hour is .1(1,000+.3*5,000) = 250.
If Tyreke chooses to drive 120 miles per hour, it must be the case that he benefits a minimum of 1200-250 = 950.

b) What does this have to do with moral hazard? Explain.

Moral hazard is a situation where one party to a contract can take a hidden action that benefits him or her at the expense of another party. In this case, by driving fast Tyreke is benefitting himself at the expense of his automobile insurance company.

c) Is Tyreke Evans’ incentive to drive 70 miles per hour rather than 120 miles per hour influenced more by increasing his deductible by $150 or by increasing the percent he pays of the repair costs from 30% to 40%? Explain.

Tyreke’s expected cost associated with going 120 miles per hour would increase by .3*150 = $45 if the deductible increases by $150.
Tyreke’s expected cost associated with going 120 miles per hour would increase by .3*.1*10000 = $300 if the percent he pays of the repair costs increases by 10%.
Tyreke’s expected cost associated with going 70 miles per hour would increase by .1*150 = $15 if the deductible increases by $150.
Tyreke’s expected cost associated with going 120 miles per hour would increase by .1*.1*5000 = $50 if the percent he pays of the repair costs increases by 10%.

The deductible increase results in the marginal cost of driving 120 miles per hour to increase $30 more than the increase in the marginal cost of driving 70 miles per hour. The percentage increase from 30% to 40% results in the marginal cost of driving 120 miles per hour to increase $250 more than the increase in the marginal cost of driving 70 miles per hour. Therefore, the percent increase in repair costs provides more incentive for Tyreke to drive 70 miles per hour.
8. The quality of a firm’s product depends on the effort of the design department and the production department. The quality of the output determines the price for which the product can be sold. For each unit of effort undertaken by either department, profits increase by $3 million. (For example, if the design department exerts 2 units of effort and the production department exerts 3 units of effort, company profits increase by 5*3M=$15M.) Let the cost per unit of effort be $1.5M in either department and assume this cost is borne entirely by the specific department. Because the effort level is unobservable to management, management has instituted a profit sharing plan whereby design and production each get one-third of the profits as compensation.

For simplicity, assume the design department can choose between either 0, 1, 2 or 3 units of effort and the production department can choose between either 0, 1, 2 or 3 units of effort. Suppose the design and production departments choose their effort levels at the same time.

a) Depict the normal form of the game described above.

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b) Does either department have a dominant strategy? Explain.

The dominant strategy for the design department is zero units of effort.
The dominant strategy for the production department is zero units of effort.

c) Identify the Nash Equilibrium (Equilibria) of the game. Explain.

(0 units, 0 units)

d) Could you recommend a different compensation structure so that the design and production teams would have “better” incentives – resulting in higher company profits?

There are many possible answers here. One is for the firm to offer both departments slightly more than $4.5M in compensation if firm profits are $19M.
9. Robert Shapiro and Alan Dershowitz are currently lawyers who each have their own practice. Robert Shapiro makes $8 million ($8M) a year at his own practice while Alan Dershowitz makes $8M a year at his own practice. They each work 60 hours a week at their own practice. They are thinking about entering a law partnership. If they establish a partnership, Robert Shapiro believes he can generate $9M if he works 60 hours a week and believes he can generate $5M if he works 40 hours a week. If they establish a partnership, Alan Dershowitz believes he can generate $9M if he works 60 hours a week and believes he can generate $5M if he works 40 hours a week. For each of them, the opportunity cost of working the additional 20 hours a week (from 40 hours to 60 hours) is $2.5M per year. Suppose Robert Shapiro and Alan Dershowitz simultaneously choose the number of hours to work. Also assume that if they do enter a partnership, they evenly split the revenues generated. Use backward induction to determine the most likely outcome to this game.

Identify the Subgame Perfect Equilibrium and provide a detailed explanation as to why this is the Subgame Perfect Equilibrium.

What does this have to do with moral hazard?

If the partnership is formed, the normal form game in terms of Robert Shapiro and Alan Dershowitz choosing 40 hours or 60 hours is depicted below.

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SAME AS

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If they enter the partnership, the Nash equilibrium of this game is for both Robert Shapiro and Alan Dershowitz to choose 40 hours. Therefore, a subgame perfect equilibrium is for both to not enter a partnership, work 60 hours and make $8M. This is an example of what is called the “free rider problem.” It results in no partnership forming (depending on the payoffs) even when there is a net benefit associated with the partnership. There is a net benefit because both would make $9M working 60 hours in the partnership and only $8M working 60 hours in separate practices. This is a moral hazard problem because it is difficult to contract on effort.
10. There are 10 buyers of a particular good and each demand one unit of the good. The good can be either low quality or high quality. All ten risk-neutral buyers value the low quality good at $100 and the high quality good at $200. There is physically only one firm that sells the good but the firm can be one of two types. A Type I firm produces a high quality good with probability .9 while a Type II firm produces a high quality good with probability .5. The firm knows what type it is but the buyers cannot observe the type. Both types of firms have constant marginal costs of $20 and this cost is the same whether it produces a high quality or a low quality good. Suppose the buyers cannot tell the difference between a high quality good and a low quality good at the time of purchase. The firm cannot differentiate a high quality from a low quality either but knows the probability that the good is of high quality. (This probability depends on whether the firm is Type I or Type II.)

Suppose Type I signals that it is a Type I firm by offering a warranty of \( W \). Let the warranty be such that the buyer can only collect the warranty if the good is of low quality. If the buyer collects the warranty, he still keeps the low quality good. For simplicity, assume all ten risk-neutral buyers are willing to pay their expected valuation for the good and that this willingness to pay does not depend on the warranty. (This is obviously not the case in the real world but it simplifies the problem. 😊)

Calculate the minimum value of \( W \) that enables the Type I firm to credibly signal that it is the type of firm that produces a high quality product with probability .9. Provide an explanation of these calculations.

Each buyer is willing to pay 100*.1+200*.9=190 if he thinks it is the Type I firm and is willing to pay 100*.5+200*.5=150 if he thinks it is the Type II firm. Therefore, the marginal benefit (from each consumer) to the firm receives from being perceived a Type I firm is 40. The marginal cost of signaling thru a warranty that the firm is Type I is .1*W for the Type I firm and .5*W for the Type II firm. To “discourage” Type II from offering the warranty in order to be perceived as Type I, it must be that .5*W ≥ 40 or W ≥ 80.
11. You own a software company and plan to hire a salesperson. You know there exists two types of salespeople: a “good” type who would make annual sales of $4 million ($4M) and an “average” type who would make annual sales of $3M. The individuals you interview know whether they are a “good” type or an “average” type but you cannot differentiate between them based on the interview. Suppose your compensation plan is to guarantee the salesperson $50,000 annually and then pay 4% commission on all sales. (Thus, if a salesperson made sales of $1M, her total compensation would be $50,000 + .04*1M = $90,000.) The minimum expected compensation the “good” and “average” types are willing to accept is $105,000 (perhaps because they are able to obtain a different job that pays this amount).

You would prefer to hire a “good” type and, with that intent, have decided to require the salesperson you hire to annually attend a training session in Detroit, Michigan. Suppose the training session does not benefit the salesperson in terms of productivity/expected sales but does provide a potential “screen”. The salesperson opportunity cost associated with attending is $10,000 per week. This is the opportunity cost for the “good” type as well as the “bad” type.

a) What is the minimum number of weeks the training session must be in order to provide an effective screen and ensure that you hire a “good” type? Show calculations and explain.

A “good” type will make $50,000 + .04*(4M) = $210,000 so the “good” type’s marginal benefit of obtaining the job is $210,000 - $105,000 = $105,000.

An “average” type will make $50,000 + .04*(3M) = $170,000 so the “average” type’s marginal benefit of obtaining the job is $170,000 - $105,000 = $65,000.

Therefore, the minimum number of weeks in order to provide an effective screen is 7 weeks because then the marginal cost would be $70,000 and the “average” type would prefer not to take the job (while the “good” type is willing to go for the 7 weeks of training).

b) What is the maximum number of weeks the training session must be in order for the “good” type to accept your sales job? Show calculations and explain.

The maximum number of weeks a “good” type will attend the training session is 10 weeks because the marginal benefit of $105k would be greater than the marginal cost of $100k. If it were 11 weeks the marginal cost would be greater than the marginal benefit.