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Warranties as signals under consumer moral hazard

Nancy A. Lutz*

This article examines the use of prices and warranties as signals of product quality to consumers who choose how to maintain their purchases. The seller's incentives are strongly affected by the interaction of quality and maintenance in determining product reliability. Two different assumptions about this interaction are made. A separating equilibrium in which high quality is signalled with a low warranty and low price is shown to be possible in both cases.

1. Introduction

The economic purpose of product warranties seems at first glance to be obvious: warranties provide insurance against unsatisfactory product performance. Warranties may also be a way for firms to communicate information about product quality. Provided that high-quality products are less expensive to warrant than low-quality products, the observable warranty could serve as a signal of unobservable product quality.

The empirical evidence on consumer product warranties, however, appears to be inconsistent with the use of warranties as signals. Bryant and Gerner (1978), Garvin (1983), Gerner and Bryant (1981), and Priest (1981) all report that most warranties offer only partial insurance, and high-quality products are not always (or even usually) sold with higher warranties than low-quality products. Spence (1977) offers a simple signalling model in which consumers are risk neutral and high warranties signal high quality. Grossman (1981) considers the use of warranties as both signals and insurance and finds a pooling equilibrium in which products are offered with full warranties regardless of their quality.

This article offers a signalling model of warranty provision which yields results consistent with the empirical evidence. The key feature of the model is the inclusion of consumer moral hazard. In the model, a risk-neutral monopolist produces a good of fixed and exogenous quality. This product is offered to a market of identical risk-averse consumers, and it can be bundled with a warranty of the monopolist's choosing. The probability that the product breaks down is a function of its quality and of the effort the consumer expends when using it. This consumer effort cannot be observed by the monopolist or any third party, so the warranty cannot be made conditional on the amount of effort expended; therefore, in choosing the warranty, the monopolist must take the moral hazard problem into account.

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results of the model show that warranty signalling does not imply that warranty and quality are positively correlated; high quality can be signalled with a low warranty.

In Section 2 I present the model and analyze the decisions faced by the consumer and the firm under the assumption that the consumer can observe product quality before purchase. The profit-maximizing price and warranty are determined; I find that a monopolist that produces a high-quality product might not choose a higher warranty than it would select if it produced a low-quality product. I then present the game played by the monopolist and the consumer when only the monopolist knows the quality of the product it sells. I define the equilibrium concept—sequential Nash equilibrium subject to a refinement of the consumer’s beliefs off the equilibrium path. I also discuss under what conditions the monopolist’s payoff in the game—its profit—is a monotonic function of the consumer’s beliefs about product quality. Whether it is monotone depends on how the consumer chooses his effort level as a function of his beliefs. This, in turn, depends critically on how quality and effort interact to determine the probability of product breakdown. Whether payoffs are monotonic affects equilibrium behavior.

In Section 3 I impose an assumption that guarantees payoff monotonicity and then solve the game. Under this assumption, no pooling equilibrium satisfying the refinement exists. A separating equilibrium satisfying the conditions may exist, and if it does, it is unique. In this equilibrium the monopolist producing high-quality signals, both through the warranty it offers with the product and through the price it charges for the product-warranty bundle. Whether the monopolist produces high or low quality, it offers only partial insurance against breakdown. If it produces high quality, it may signal that fact with a price-warranty contract that includes a less complete warranty than the one it would offer if it sold a low-quality product.

In Section 4 the assumption made in the previous section is reversed, so that the monopolist’s profits from offering any price-warranty contract can be a nonmonotonic function of the consumer’s beliefs about product quality. In this case there may be multiple pooling equilibria that satisfy the refinement. There may also be a unique separating equilibrium. In this separating equilibrium, the monopolist might “signal” if it produces low quality. Whether this is so in equilibrium, a high-quality item need not be sold with a more extensive warranty than a low-quality one. Section 5 expands on this and related conclusions.

Methodologically this article concerns a multivariate signalling model. Under the assumption made in Section 3, the game considered is similar to the one analyzed in Milgrom and Roberts’ (1986) article on price and dissipative advertising as signals of product quality. Both models feature the use of two signals, and both analyses rely on the employment of a refinement. Milgrom and Roberts use sequential elimination of dominated strategies and the intuitive criterion (Cho and Kreps, 1987) to reduce the set of equilibria to a single separating equilibrium. This technique can be used in Section 3 as well: using it permits the weakening of the conditions for the existence of an equilibrium, but changes none of my other results. This response to the existence of multiple equilibria would be far less successful in Section 4 because of the nonmonotonicity featured there; the intuitive criterion lacks “bite” under this condition.

2. Optimal warranties when quality is observable

In this section I present a model of warranty provision in the same spirit as Grossman’s, but with the addition of consumer moral hazard. Consider a consumer who must decide whether to purchase a product offered by a single firm. When the consumer’s utility from the purchase is a random variable, the consumer’s decision depends on three things: the product’s price, the expected utility from the purchase, and the consumer’s reservation utility.
There are many reasons why the consumer’s after-purchase utility could be a random variable. One is probabilistic product breakdown. The simplest way to model this is to assume that there are only two possible outcomes: the product performs as promised or it does not perform at all. A warranty then provides insurance against the second outcome. I assume that the probability that the product works, which I write as \( \pi \), is a function of two things: a parameter, \( q \), that represents the product’s intrinsic quality or durability and a variable, \( m \), that represents the level of maintenance and care in use provided by the consumer. The probability that the product works is then \( \pi(q, m) \in [0, 1] \). Product quality is assumed to be determined by nature and is thus exogenous to the firm. It takes one of two values, either “low” or “high.” Thus, \( q \in \{ L, H \} \), where \( L < H \). The probability that nature chooses high quality is \( r_N \in [0, 1] \). The marginal cost of production is constant and does not depend on quality; without loss of generality, I can normalize this cost to zero.

The actual quality of the product is observable to the firm, but not to the consumer, who must rely on his beliefs about quality when deciding both whether to purchase the product and what level of effort, \( m \), to expend in its use. He knows that quality must be either low or high, so his beliefs are given by \( r \), his subjective probability that the product is of high quality. Before he observes price or warranty, \( r = r_N \). He also knows how he can affect the probability that the product works by choosing a level of \( m \). I assume that \( \pi_m > 0 \) and \( \pi_m \leq 0 \); that is, increasing maintenance increases the probability of success, but at a nonincreasing rate. If maintenance were costless to the consumer, he would choose the maximum possible level of maintenance. If maintenance were observable (and verifiable) by the firm, it could write a contract obligating the consumer to take a specified level of effort as a condition of purchase, or equivalently, of the warranty. I assume that effort is both costly and unobservable; the firm will have to consider the effect of warranty insurance on the consumer’s incentive to take care.

The consumer who purchases the product chooses his effort level to maximize his utility. I can simplify this problem by assuming that the consumer must pay for the product in full when it is purchased. Restricting access to credit markets in this way implies a two-period structure for the model. In the first period, the consumer decides whether to purchase the product. In the second period, the consumer first chooses his effort level, the product then either works or breaks down, and the consumer enjoys a level of utility that depends on whether the product worked, the maintenance effort taken, and the warranty.

If the product works, the consumer enjoys utility \( U(\theta) - g(m) \), where \( \theta \) is the consumer’s monetary valuation of a working product, \( U(\cdot) \) is his utility function for money, and \( g(\cdot) \) is his disutility for effort. I assume that \( U' > 0 \), \( U'' < 0 \), \( g' > 0 \), and \( g'' > 0 \). Hence, the consumer is strictly risk averse and values the insurance provided by a warranty. I also assume that \( U(0) = 0 \).

The consumer’s utility in the event of a product breakdown depends on the level of warranty coverage. I assume that the consumer receives no utility from a broken product, and that the warranty takes the form of a monetary payment, \( w \in [0, \theta] \), to be made to the consumer if the product breaks. The consumer’s utility if the product fails can be written as \( U(w) = g(m) \). The consumer then expects his utility after the product is purchased to be equal to

\[
U(w) + s(r, m)\Delta(w) - g(m),
\]

(1)

---

1 Automobile warranties are the obvious example in which the provision of insurance by the producer against breakdown is conditional on maintenance. But note that the maintenance required is observable, either directly—oil changes and the like—or indirectly through inspection of the broken part.

2 Alternatively, the warranty could be modelled as providing some kind of repair. The two approaches are formally equivalent, since \( w \) could then be thought of as the consumer’s valuation of a repaired product.
where \( s(r, m) = r\pi(H, m) + (1 - r)\pi(L, m) \) is the consumer’s subjective probability that the product will work when he believes that the probability that the product is of high quality is \( r \in [0, 1] \). \( \Delta(w) = U(\theta) - U(w) \) is the difference in the utility between consuming a working and a broken product, given warranty \( w \). Note that if \( w = \theta \), the warranty fully insures the consumer against breakdown, while if \( w = 0 \), the warranty provides no insurance.

The consumer will choose a maintenance effort, \( m \), to maximize his expected utility after purchase. I write the effort expended to maintain a product believed to be of high quality with probability \( r \) under warranty \( w \) as \( m(r, w) \). Since expected utility is strictly concave in effort, \( m(r, w) \) is unique. Clearly, \( \partial m(r, w)/\partial w < 0 \), so an increase in the warranty implies a decrease in effort. This, of course, is the consumer moral hazard problem—more warranty insurance decreases the incentive for maintenance, which is a form of self-insurance.

The incentive for maintenance is also affected by what the consumer believes about the product’s quality, since the level of quality determines the marginal expected utility of changes in effort. We know that

\[
\frac{\partial^2 s(r, m)}{\partial r \partial m} = \pi_m(H, m) - \pi_m(L, m).
\]

Hence, if \( \pi_m < 0 \), effort decreases as quality increases for any given level of warranty, and if \( \pi_m > 0 \), effort increases as quality increases. Intuitively, this happens because in the first case, quality and effort are substitutes in determining the probability of success—the higher the beliefs, the lower the expected increase in the probability that the product works due to an increase in effort. In the second case, quality and effort are complements in determining the probability of success, so that higher beliefs increase the expected marginal effect of an increase in effort.

I can now address the consumer’s purchase decision. Assume that the consumer’s income is \( y \) in the purchase period and that there is no discounting. If the consumer pays price \( p \) for a product he believes to be of high quality with probability \( r \) under warranty \( w \), his expected total utility over the purchase period and the post-purchase period is equal to

\[
U(y - p) + U(w) + s(r, m(r, w))\Delta(w) - g(m(r, w)).
\]

The consumer will purchase the product with this warranty at this price if the total expected utility from purchasing the product is greater than or equal to the total utility from not purchasing it. Assuming that if no purchase is made, the utility in the second period equals zero, which is also the utility from a broken product that is not under warranty, the total utility of no purchase is just \( U(y) \). I can then solve for \( p^M(r, w) \), the consumer’s reservation price given beliefs \( r \) when the product is bundled with warranty \( w \):

\[
p^M(r, w) = y - h \{ U(y) - U(w) - s(r, m(r, w))\Delta(w) + g(m(r, w)) \},
\]

where \( h = U^{-1} \). This expression for \( p^M(r, w) \) will be positive and less than \( y \) (so the consumer’s budget constraint is nonbinding) as long as

\[
U(y) > U(w) + s(r, m(r, w))\Delta(w) - g(m(r, w))
\]

for all \( w \). This is true if and only if \( y \geq \theta \), as I shall assume. Differentiating both sides of (4) shows that the reservation price is strictly increasing in quality for all partial warranties and is independent of quality for a full warranty. Full insurance makes the consumer indifferent about the quality of the product. The reservation price is also an increasing and strictly concave function of the warranty.
Turning to the production side of the market, the monopolist’s profits will be a function of the exogenous quality of the product, the price and warranty it chooses, and the consumer’s beliefs. When choosing the price and warranty, the firm must consider both how the consumer draws conclusions about quality from price and warranty and what actions—purchase and effort—the consumer will take after reaching his conclusion. If the consumer believes \( r \) after observing \( p \) and \( w \), then if \( p \leq p^M(r, w) \) the firm earns a profit of \( p \) minus the expected warranty claims, which are defined as

\[
W(q, r, w) = (1 - \pi(q, m(r, w)))w.
\]

It earns nothing if \( p > p^M(r, w) \), because in this case the consumer does not purchase the product.

If quality is observable by consumers, the monopolist need not worry about how its choice of price and warranty will affect beliefs. The monopolist then maximizes profits at any warranty \( w \) by charging the consumer a price of \( p^M(0, w) \) if the product is of low quality and \( p^M(1, w) \) if the product is of high quality. Hence, the monopolist’s profit-maximization problem can be written in terms of only the choice of warranty:

\[
\max_w p^M(r, w) - W(q, r, w)
\]

where \( r = 0 \) if \( q = L \)

\( r = 1 \) if \( q = H \).

I have established that \( p^M(r, w) \) is strictly concave in the warranty. If it is also true that \( W(q, r, w) \) is a convex function of the warranty, then the monopolist’s profits are a strictly concave function of the degree of warranty coverage.\(^3\)

I shall write the unique warranty that solves this known profit-maximization problem as \( w^*(q) \) and the maximum profit the firm earns when its quality is known as \( \Pi_{\text{max}}(q) \).

An obvious question is whether the monopolist will offer a higher warranty if it produces a high-quality product than if it produces a low-quality product. To answer this question, we must examine how the marginal effects of an increase in \( w \) on the consumer’s reservation price and on the firm’s warranty costs depend on \( q \). While the consumer values a high-quality product over a low-quality product when both are bundled with the same warranty, \( \partial^2 p^M(r, w)/\partial w \partial r \) can be shown to be negative for all \( w < \theta \). An increase in warranty insurance is valued more when the product is of low quality and the risk of breakdown is higher. However, the effect of a change in observable quality on \( \partial W(q, r, w)/\partial w \) is indeterminate (no matter what the sign of \( \pi_{qm} \)); \( \partial^2 m(r, w)/\partial r \partial w \) is therefore indeterminate in sign, and so we do not know whether an increase in the warranty decreases consumer effort more if the product is known to be of low rather than high quality. This means that \( w^*(H) \) may be greater than or less than \( w^*(L) \).

As long as quality is not directly observable by the consumer, the firm will have to take into account how its choice of price and warranty affects the consumer’s beliefs. Since the consumer’s reservation price, \( p^M(r, w) \), is increasing in \( r \) for all partial warranties, for a given warranty, the firm can charge the consumer the highest price by convincing him that

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\(^3\) A sufficient, although not necessary, condition for \( \partial^2 W^2(q, r, w)/\partial w^2 \geq 0 \) is \( \pi_{mm}(q, m) \leq 0 \) and \( g''(m) \leq 0 \).
its product is of high quality. If effort, \( m(r, w) \), is also increasing in \( r \), then we can conclude that profits are monotonically increasing in beliefs. But if effort is decreasing in beliefs, then the consumer's reservation price and the expected warranty costs both increase as the belief that the product is of high quality strengthens. This means that profits—at least when quality and effort are substitutes—need not be monotonic in the consumer's beliefs about the quality of the product.

It seems intuitively reasonable that quality and effort work as complements in determining the probability that some products will work successfully. Such products would be more sensitive to the level of effort expended if they were of high rather than low quality. Low-quality units would be very likely to break down no matter how much care was taken; using them with great care would make breakdown only slightly less likely. There is no reason to believe, however, that quality and effort are complements for all products. High-quality units may easily be less sensitive to effort levels: they may withstand abuse more readily than low-quality products. If this is true, then quality and effort are substitutes in determining the probability of breakdown; profits from the sale of a product of fixed but unobservable quality may well be nonmonotonic in the consumer's belief about product quality.

Whether quality and effort are substitutes or complements, multiple equilibria are to be expected. Almost any contract might be supportable as an equilibrium strategy. To draw more specific conclusions about how warranties are provided under asymmetric information, we would like to limit the set of equilibria. At the same time, we would like to know which—if any—of the sequential equilibria are "reasonable." Equilibria in games of this sort strongly depend on out-of-equilibrium beliefs, and certain kinds of beliefs may seem unreasonable. One way to refine the definition of sequential equilibrium is to establish a definition of reasonable: restricting attention to only those equilibria supported by reasonable beliefs can strongly limit the set of equilibria of the game.

A number of refinements of sequential equilibrium have been developed in the literature. In this article I shall use a refinement based on equilibrium concepts defined by Farrell (1983) and Grossman and Perry (1986). It was employed in a two-type signalling game by Gertner, Gibbons, and Scharfstein (1988). In this model the refinement can be defined as follows:

**Definition: The Farrell-Grossman-Perry refinement.** An equilibrium in which the monopolist earns profits of \( \Pi^L \) if it produces a low-quality product, and \( \Pi^H \) if it produces a high-quality product, satisfies the Farrell-Grossman-Perry refinement if and only if there does not exist a contract \((p, W)\) such that

(a) \( p \leq p^M(1, \bar{w}) \) while \( \Pi^H < p - W(H, 1, \bar{w}) \) and \( \Pi^L > p - W(L, 1, \bar{w}) \)

or such that

(b) \( p \leq p^M(0, \bar{w}) \) while \( \Pi^H > p - W(H, 0, \bar{w}) \) and \( \Pi^L < p - W(L, 0, \bar{w}) \)

or such that

(c) \( p \leq p^M(r^N, \bar{w}) \) while \( \Pi^H < p - W(H, r^N, \bar{w}) \) and \( \Pi^L > p - W(L, r^N, \bar{w}) \).

The Farrell-Grossman-Perry refinement removes all equilibria that are vulnerable to what are termed consistent deviations—an equilibrium is "unreasonable" if a consistent deviation away from it exists. In this two-type model there are three kinds of consistent deviations. A consistent high-quality separating deviation is a contract, \((\bar{p}, \bar{w})\), that yields
the monopolist higher than equilibrium profits if and only if the monopolist produces high quality, as long as the consumer would conclude upon observing \((p, w)\) that the monopolist must offer high quality. Part (a) of the definition defines this kind of consistent deviation. The second type of deviation defined by Part (b) is a consistent low-quality separating deviation. This type of deviation would yield the monopolist producing low-quality products higher than equilibrium profits—and if the monopolist produces high quality, lower than equilibrium profits—if the consumer would be convinced after observing this contract that the product is of low quality. The third type of deviation is a consistent pooling deviation, which is defined in Part (c). Such a deviation yields higher than equilibrium profits, regardless of quality, if the consumer maintains his original belief that the product is of high quality with probability \(p^M\).

In the next section I shall analyze the signalling game under the assumption that \(\sigma_m > 0\), so that quality and effort are complements and the firm’s profits are increasing in consumer beliefs. I shall establish that no pooling equilibrium exists, and that there is at most one separating equilibrium satisfying the Farrell-Grossman-Perry refinement. I shall then give conditions under which the unique separating equilibrium exists. In Section 4 I shall analyze the game under the assumption that quality and effort are substitutes.

### 3. Warranties when consumer effort and unobservable quality are complements

When quality and effort are complements in determining the probability of breakdown, profits to the firm are monotonically increasing in beliefs, and the game played by the firm and the consumer is a variation on the signalling games familiar in the literature. The variation is that the consumer conditions his beliefs on two observables—price and warranty—rather than on a single variable, which makes this a multivariate signalling model similar to the one studied by Milgrom and Roberts (1986).

There are both multiple pooling and multiple separating equilibria for this game. This is the usual result in signalling models, but the plethora of equilibria makes it impossible to draw any conclusions about the effect of asymmetric information on warranty provision. Applying the Farrell-Grossman-Perry refinement removes this multiplicity. To begin with, no pooling equilibrium satisfies the refinement, as the following proposition demonstrates.

**Proposition I.** Let \((p, w)\) be a contract supportable as a pooling equilibrium for this game. Then there exists a contract \((\bar{p}, \bar{w})\) that is a consistent high-quality separating deviation away from the equilibrium.

This refinement can be compared to the intuitive criterion developed by Cho and Kreps (1987), which can also be used to limit the set of equilibria in signalling games. The intuitive criterion requires that out-of-equilibrium beliefs put no weight on types that have no incentive to deviate no matter what consumers would conclude from observing the deviation. The refinement used here requires that out-of-equilibrium beliefs put no weight on types that would have no incentive to deviate if these beliefs put no weight on these types. An equilibrium to this game would fail to satisfy the intuitive criterion if there existed a contract \((\bar{p}, \bar{w})\) such that

\[
\begin{align*}
\text{(a)} & \quad \bar{p} \leq p^M(1, \bar{w}) \quad \text{while} \quad \Pi^H < \bar{p} - W(H, 1, \bar{w}) \quad \text{and} \quad \Pi^L > \max_{\bar{p}} \bar{p} - W(L, r, w)) \\
\text{(b)} & \quad \bar{p} \leq p^M(0, \bar{w}) \quad \text{while} \quad \Pi^H > \max_{\bar{p}} \bar{p} - W(H, r, \bar{w}) \quad \text{and} \quad \Pi^L < \bar{p} - W(L, 0, \bar{w}).
\end{align*}
\]

Any equilibrium that fails to satisfy the intuitive criterion also does not satisfy the Farrell-Grossman-Perry refinement. If \(\partial p^M(r, w)/\partial r > 0\) and \(\partial W(q, r, w)/\partial r < 0\), then the firm’s profits are increasing in consumer beliefs, \(r\), and any equilibrium that fails to satisfy Part (a) of the refinement also does not satisfy the criterion. In this case, no equilibrium fails to satisfy Part (b) of the refinement or Part (b) of the criterion, and the only difference between the Farrell-Grossman-Perry refinement and the intuitive criterion is that some equilibria satisfying the criterion may not satisfy Part (c) of the refinement.
Proof. I need to show that there exists a \((\bar{p}, \bar{w})\) such that

\[
\begin{align*}
(i) & \quad \bar{p} \leq p^M(1, \bar{w}), \\
(ii) & \quad p - W(H, r^N, w) < \bar{p} - W(H, 1, \bar{w}), \quad \text{and} \\
(iii) & \quad p - W(L, r^N, w) > \bar{p} - W(L, 1, \bar{w}).
\end{align*}
\]

Rearranging and combining (ii) and (iii) yields the following condition on \((\bar{p}, \bar{w})\):

\[
(iv) \quad W(L, r^N, w) - W(L, 1, \bar{w}) < p - \bar{p} < W(H, r^N, w) - W(H, 1, \bar{w}).
\]

Suppose that \(w < \theta\). Set \(\bar{w} = w\), the pooling equilibrium warranty. Then if we can find a price \(\bar{p} \leq p^M(1, w)\) such that (iv) is satisfied, we are done. Since \(\pi_{qm} > 0\), \(\partial m(r, w)/\partial r > 0\) as well, and these two facts taken together determine that \(\partial^2 W(q, r, w)/\partial q \partial r < 0\).

Let

\[
\bar{p} = p - \frac{1}{2} \{ W(L, r^N, w) - W(L, 1, w) + W(H, r^N, w) - W(H, 1, w) \}.
\]

Then \(\bar{p} < p\) and \(p < p^M(r^N, w)\), so \(\bar{p} < p^M(1, w)\). Condition (iv) on \((\bar{p}, \bar{w})\) is met by construction. Hence, \((\bar{p}, \bar{w})\) is a consistent high-quality separating deviation.

Now suppose that \(w = 1\). Because this warranty provides full insurance,

\[
W(q, r^N, 1) = W(q, 1, 1).
\]

Since \(\partial W(H, 1, w)/\partial w\) is greater than \(\partial W(L, 1, w)/\partial w\) when \(w = 1\), we know that there exists a \(\bar{w}\) such that \(W(L, 1, 1) - W(L, 1, \bar{w}) < W(H, 1, 1) - W(H, 1, \bar{w})\). Since \(\partial W(H, 1, w)/\partial w\) is also greater than \(\partial p^M(1, w)/\partial w\) for all \(\bar{w} < 1\), we know that for some such \(\bar{w}\) there exists a \(p\) such that Conditions (i) and (iv) are met. Q.E.D.

Any given pooling equilibrium can fail to satisfy the refinement for several reasons, since there are three kinds of consistent deviations. The proposition establishes that no pooling equilibrium can satisfy the refinement because a consistent high-quality separating deviation always exists. If the pooling equilibrium includes a partial warranty, this deviation is a contract combining the pooling warranty with a lower price; the firm will deviate to this contract if and only if its product is of high quality. To establish the existence of this deviation, I used the assumption of complementarity: any given increase in effort will decrease total warranty costs, and the decrease is greater if quality is high rather than low. Effort increases if the firm convinces the consumer that it sells a high-quality product without changing the warranty. The firm would be willing to cut the price from the pooling price to convince consumers of this, but it is willing to cut price by less if its product is of low quality than it would if its product were of high quality. Hence, there is a price cut that the firm will take to convince consumers if and only if it produces a high-quality good.

Since the refinement removes all pooling equilibria, I can restrict the search for equilibria satisfying the refinement to separating equilibria. There are multiple separating equilibria for the game, but in all of them the monopolist offers the low-quality product with warranty \(w^*(L)\) at price \(p^M(0, w^*(L))\), exactly the contract that would be offered for this product if its quality were observable. The easiest way to see why this contract must be offered is to consider the profit earned by the monopolist who offers low quality in a separating equilibrium. Such a monopolist cannot earn more than \(\Pi_{\max}(L)\), since the consumer correctly infers the quality of its product by observing the equilibrium contract. But offering its equilibrium contract cannot yield profits of less than \(\Pi_{\max}(L)\) either: if it did yield less the monopolist would deviate to \((p^M(0, w^*(L)), w^*(L))\), since it can always earn at least \(\Pi_{\max}(L)\) with this contract. The separating equilibrium payoff to the firm if the product is of low quality must then be equal to \(\Pi_{\max}(L)\), and \((p^M(0, w^*(L)), w^*(L))\) is the unique contract yielding this profit when the product is known to be of low quality.
Multiple separating equilibria are still possible because there is a potential plethora of contracts that can be supported as separating equilibrium strategies to be offered if the monopolist’s product is of high quality. We need to find which, if any, of these equilibria satisfy the Farrell-Grossman-Perry refinement.

We need not be concerned with consistent separating deviations for the low-quality firm—they cannot exist for any of these equilibria since the monopolist will earn \( \Pi_{\text{max}}(L) \) in equilibrium if it produces low quality. That leaves us with consistent separating deviations for the high-quality firm, and consistent pooling deviations. I will ignore the latter for the moment and explain how to check the set of separating equilibria for the first kind of deviation.

Fix a contract offered by the monopolist who produces high quality in a separating equilibrium. If there exist no consistent separating deviations, then the contract must solve the following problem:

\[
\begin{align*}
\max_{p,w} & \quad p - W(H, 1, w) \\
\text{subject to} & \quad (a) \quad p \leq p^M(1, w) \\
& \quad (b) \quad p - W(L, 1, w) \leq \Pi_{\text{max}}(L).
\end{align*}
\]

If the contract did not solve this problem, then any contract that did would be a consistent high-quality separating deviation. Solutions to (6) maximize profits if the consumer is convinced that the product is of high quality and if our attention is restricted to the set of contracts from which the monopolist would not deviate if it produced low quality. For any given warranty, \( w \), profits are maximized by charging the highest price satisfying the constraints, namely, the minimum of \( \{p^M(1, w), W(L, 1, w) + \Pi_{\text{max}}(L)\} \).

There are, in fact, four price-warranty contracts that are possible solutions to this problem, depending on whether one or the other or both of the constraints are binding at the optimum. Table 1 displays the four possibilities.

If only the first constraint is binding, the solution is \((p^M(1, w^*(H)), w^*(H))\)—the same contract that would be optimal if the consumer could directly observe the product’s high quality. In this case this first-best contract is incentive compatible; the low-quality firm will not attempt to mimic the high-quality firm.

The remaining three possibilities can only occur when the first-best contracts are not incentive compatible, so if the firm produces high quality, it must deviate from its profit-maximizing contract to convince consumers about its quality. If only the second constraint binds, the solution is \((\Pi_{\text{max}}(H) + W(L, 1, w^S), w^S)\), where \( w^S \) maximizes profits subject to the second constraint:

\[
w^S = \arg\max_w \Pi_{\text{max}}(L) + W(L, 1, w) - W(H, 1, w).
\]

**TABLE 1 Contracts Solving (6)**

<table>
<thead>
<tr>
<th>Contract</th>
<th>Solves (6) Under These Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>((p^M(1, w^<em>(H)), w^</em>(H)))</td>
<td>(p^M(1, w^<em>(H)) - W(L, 1, w^</em>(H)) \leq \Pi_{\text{max}}(L))</td>
</tr>
<tr>
<td>((\Pi_{\text{max}}(H) + W(L, 1, w^S), w^S))</td>
<td>(p^M(1, w^<em>(H)) - W(L, 1, w^</em>(H)) &gt; \Pi_{\text{max}}(L), \quad p^M(1, w^S) \geq \Pi_{\text{max}}(L) + W(L, 1, w^S))</td>
</tr>
<tr>
<td>((p^M(1, w^S), w^S))</td>
<td>(p^M(1, w^<em>(H)) - W(L, 1, w^</em>(H)) &gt; \Pi_{\text{max}}(L), \quad p^M(1, w^S) &lt; \Pi_{\text{max}}(L) + W(L, 1, w^S), \quad w^S &lt; w^*(H))</td>
</tr>
<tr>
<td>((p^M(1, w^S), w^*))</td>
<td>(p^M(1, w^<em>(H)) - W(L, 1, w^</em>(H)) &gt; \Pi_{\text{max}}(L), \quad p^M(1, w^S) &lt; \Pi_{\text{max}}(L) + W(L, 1, w^S), \quad w^S &gt; w^*(H))</td>
</tr>
</tbody>
</table>
Since $\partial^2 W(H, 1, w)/\partial w^2 > \partial^2 W(L, 1, w)/\partial w^2 > 0$, while $\partial W(H, 1, w)/\partial w$ is less than $\partial W(L, 1, w)/\partial w$ at $w = 0$ but greater than $\partial W(L, 1, w)/\partial w$ when $w = 1$, there is a unique $w^S$, which is determined by the tangency between the $W(H, 1, w)$ and $W(L, 1, w)$ curves. The two remaining solutions occur when both constraints bind. The third is the left-hand-side intersection between the two constraints: the contract is $(p^M(1, w^3), w^3)$, where $w^3 = \min \{ w | p^M(1, w) = \Pi_{\max}(L) + W(L, 1, w) \}$. This contract is preferred to the fourth possible solution when $w^S < w^*(H)$, since this is true if and only if $w^S < w^3$, and by definition $w^S$ maximizes profits subject to the second constraint. The fourth and final contract that can solve (6) is the right-hand-side intersection between the two constraints: the contract is $(p^M(1, w^4), w^4)$, where $w^4 = \max \{ w | p^M(1, w) = \Pi_{\max}(L) + W(L, 1, w) \}$. This contract is preferred to the third possible solution when $w^S > w^*(H)$.

The solution to the maximization problem (6), which I write as $(p^*, w^*)$, is clearly unique. Hence, I have established that there is at most one equilibrium satisfying the Farrell-Grossman-Perry refinement—an equilibrium in which the monopolist offers a high-quality product with the contract $(p^*, w^*)$ and the low-quality product with the contract $(p^M(0, w^*(L)), w^*(L))$. However, we are not certain that this pair of contracts is supportable as a separating equilibrium, and we must also determine whether or not there are consistent pooling deviations away from the equilibrium. These issues are addressed in the proof of the next proposition, which I leave to the Appendix.

**Proposition 2.** Suppose that $\sigma_{qm} > 0$. Then, there is a unique contract $(p^*, w^*)$ solving (6). There exists a unique equilibrium satisfying the refinement if and only if two conditions are met:

(i) \[ p^* - W(H, 1, w^*) \geq \max_w p^M(0, w) - W(H, 0, w) \]
and $\bar{w}$ such that

(ii) \[ p^* - W(H, 1, w^*) < p^M(r^N, \bar{w}) - W(H, r^N, \bar{w}) \]
and

\[ \Pi_{\max}(L) < p^M(r^N, \bar{w}) - W(L, r^N, \bar{w}). \]

This is a separating equilibrium in which the contract $(p^*, w^*)$ will be offered if the product is of high quality and $(p^M(0, w^*(L)), w^*(L))$ will be offered if the product is of low quality. If the necessary and sufficient conditions are not met, no equilibrium satisfies the refinement.

What then is the effect of asymmetric information on warranty provision in this game in which consumer maintenance effort and quality are complements? Comparing the optimal contracts under observable quality with the separating equilibrium contracts yields some answers. To begin with, it is possible that asymmetric information has no impact on warranty provision. If $(p^*, w^*)$ equals $(p^M(1, w^*(H)), w^*(H))$, then when quality is unobserved the monopolist offers its product, whether of high or low quality, with the same contract as it would employ if quality were observable. And, in any case, asymmetries in information about quality can affect only the contract offered if the product is of high quality: a low-quality product is always offered with the contract $(p^M(0, w^*(L)), w^*(L))$.

These results do not support the view that only higher warranties signal higher quality. It is the entire price-warranty contract that serves as a signal of product quality to consumers. The high-quality signalling warranty $w^*$ may be greater or less than $w^*(H)$, the optimal observed high-quality warranty. It can also be more or less extensive than $w^*(L)$, which is the equilibrium warranty offered with a low-quality product. We do know that $p^*$, the price in the signalling contract, cannot be greater than $p^M(1, w^*(H))$ if $w^*$ is less than $w^*(H)$.

An example illustrates that $w^* < w^*(H)$ is a possible outcome. Let $\sigma(m, q)$ be equal
to $m \cdot q$ and let $L = .1$. Assume also that $U(x) = \sqrt{x}$, $g(m) = m^2$, $y = 36$, $\theta = 4$, and $H = 1$. Then,

$$
\frac{\partial W(H, 1, w)}{\partial w} = \frac{3\sqrt{W}}{4}, \quad \frac{\partial W(L, 1, w)}{\partial w} = \frac{3(12 + \sqrt{W})}{40},
$$

and

$$
\frac{\partial p^M(1, w)}{\partial w} = \left(6 - \sqrt{w} - \frac{(2 - \sqrt{w})^2}{4}\right) \left(\frac{1}{\sqrt{w}} - \frac{(2 - \sqrt{w})}{2\sqrt{w}}\right).
$$

Solving for $w^S$ shows that $w^S = 16/9$; $\frac{\partial p^M(1, w)}{\partial w}$ evaluated at $w^S$ is greater than one, implying that $w^S < w^*(H)$.

We have a clearer understanding of how information affects the payoffs to the firm and its consumers. Consumers always know the quality of any product they purchase; if that quality cannot be observed, they correctly infer it from the price-warranty contract. If they are offered a high-quality product, however, they may be better off when quality is unobservable than they would be if offered an observably high-quality product or a low-quality (observable or not) product. When the unobservably high-quality product is offered with $(\Pi_{\max}(L) + W(L, 1, w^S))$ in equilibrium, the consumer improves upon his reservation utility by purchasing the product. The intuition is clear: the monopolist signals its product's high quality by forgoing profits, offering a nonoptimal warranty at a price lower than the consumer's reservation price. It is the pricing that makes the consumer better off than he would be if he could observe quality. Of course, just because the high-quality monopolist is forgoing profits to signal quality does not mean that signalling makes the consumer better off than he would be if he could observe quality. Of course, just because the high-quality monopolist is forgoing profits to signal quality does not mean that signalling makes the consumer better off than he would be if he could observe quality. Of course, just because the high-quality monopolist is forgoing profits to signal quality does not mean that signalling makes the consumer better off than he would be if he could observe quality. Of course, just because the high-quality monopolist is forgoing profits to signal quality does not mean that signalling makes the consumer better off than he would be if he could observe quality.

All of these results are contingent on two assumptions: first, that quality and effort are in fact complements, and second, that an equilibrium satisfying the Farrell-Grossman-Perry refinement exists. In the next section I shall address the first assumption by analyzing the game when quality and effort are substitutes. As we shall see, this game has some very different equilibria. I have established the necessary and sufficient conditions for the second assumption to be met in the proof of Proposition 2. It should also be noted that an alternative approach to refining sequential equilibrium—the combination of removing strategies from the game by using iterated weak domination and applying the Cho-Kreps (1987) intuitive criterion—yields exactly the same unique equilibria under weaker conditions for existence: Condition (i) of Proposition 3 must still hold, but (ii) can be dropped.5

4. Warranties when consumer effort and unobserved quality are substitutes

Now suppose that $\pi_{gm} < 0$, so that consumer maintenance and product quality are substitutes affecting the probability of product success. If consumers cannot observe quality, the firm's incentives are more complicated than those discussed in the previous section. There is still a possible incentive to convince consumers that the product is of high quality, because consumers are willing to pay more if convinced of this. But now there is also a possible incentive to convince consumers that the product is of low quality, since doing so would minimize warranty costs by leading the consumer to choose a large amount of effort.

---

5 See Lutz (1986) for the application of this method to this game. The key to the equivalence between refinements in this game is the monotonicity of profits in $r$, consumer beliefs. Profits may be nonmonotonic under the assumption in the next section that $\pi_{gm} < 0$: in this case equilibria satisfying the intuitive criterion may still be subject to consistent separating deviations.
These offsetting incentives can be expected to complicate equilibrium behavior. A pooling equilibrium might be supported by off-the-equilibrium-path beliefs that make it too costly or impossible for the monopolist to reveal low quality. There may be separating equilibria in which the monopolist will earn observable-quality profits if it produces high quality, but might choose to earn less than observable-quality profits if its product is low quality in order to signal that quality. There may also be separating equilibria in which the monopolist always earns less profit than it could if its quality were observable.

All of these kinds of equilibria can exist in the same game. But limiting our attention to only those equilibria that satisfy the Farrell-Grossman-Perry refinement removes most of these equilibria from consideration. I begin by applying the refinement to the game's separating equilibria.

By definition, a separating equilibrium in which the firm offers a warranty \( w^q \) if it produces quality \( q \) cannot satisfy the refinement if there are consistent separating deviations away from the equilibrium. This means that a separating equilibrium with contracts \((\hat{p}^H, \hat{w}^H)\) and \((\hat{p}^L, \hat{w}^L)\) can only satisfy the refinement if

\[
(\hat{p}^H, \hat{w}^H) = \arg \max_{p, w} p - W(H, 1, w)
\]

subject to

(a) \( p \leq p^M(1, w) \)

(b) \( p - W(L, 1, w) \leq \hat{p}^L - W(L, 0, \hat{w}^L) \),

since otherwise a consistent high-quality separating deviation will exist. At the same time, the separating equilibrium contracts must solve

\[
(\hat{p}^L, \hat{w}^L) = \arg \max_{p, w} p - W(L, 0, w)
\]

subject to

(a) \( p \leq p^M(0, w) \)

(b) \( p - W(H, 0, w) \leq \hat{p}^H - W(H, 1, \hat{w}^H) \),

since otherwise a consistent low-quality separating deviation will exist.

A separating equilibrium to this game satisfies the refinement if and only if (a) the equilibrium contracts simultaneously solve (7) and (8), and (b) there are no consistent pooling deviations. There is at most a single separating equilibrium satisfying the refinement.

**Proposition 3.** There is a unique pair of contracts \((\hat{p}^H, \hat{w}^H)\) and \((\hat{p}^L, \hat{w}^L)\) that simultaneously solves (7) and (8). These contracts are supportable in a separating equilibrium satisfying the refinement if and only if

(i) \( \hat{p}^H > \max_w p^M(0, w) - W(H, 1, w) \),

(ii) \( \hat{p}^L > \max_w p^M(0, w) - W(L, 1, w) \), and

(iii) there is no contract \((p^M(r^N, w^p), w^p)\) such that

(a) \( w^p \in \Omega = \{ w : \delta W(L, r^N, w) / \delta w \geq \delta p^M(r^N, w) / \delta w \geq \delta W(H, r^N, w) / \delta w \} \),

(b) \( p^M(r^N, w^p) = W(L, r^N, w^p) > \hat{p}^L \),

(c) \( p^M(r^N, w^p) = W(H, r^N, w^p) > \hat{p}^H \).

**Proof.** See the Appendix.

Conditions (i) and (ii) establish that a separating equilibrium exists in which \((\hat{p}^q, \hat{w}^q)\) is the contract offered when quality is \( q \). Condition (iii) establishes that there are no consistent pooling deviations from the equilibrium.
This separating equilibrium differs from the one discussed in Section 3 in two interesting ways. First, it need not be the high-quality monopolist who manipulates its contract to convince consumers of its quality, while the low-quality good is sold at \((p^M(0, w^*(L)), w^*(L))\). If \(\Pi_{\text{max}}(H) < p^M(0, w^*(L)) - W(H, 0, w^*(L))\) and \(\Pi_{\text{max}}(L) > p^M(1, w^*(H)) - W(L, 1, w^*(H))\), this familiar pattern still holds: the profit-maximizing contracts are not incentive compatible because the low-quality monopolist would copy the high-quality profit-maximizing contract. But if these two inequalities are reversed, it is the low-quality producer who manipulates its equilibrium contract since the high-quality producer would mimic the low-quality profit-maximizing contract: the high-quality good would be sold at \((p^M(1, w^*(H)), w^*(H))\). The firm may signal regard-}

less of quality; this will happen when \(I_{\text{Imax}}(L) < p^M(1, w^*(H)) - W(L, 1, w^*(H))\) and \(I_{\text{Imax}}(H) > p^M(0, w^*(L)) - W(H, 0, w^*(L))\). In this case, the profit-maximizing contract is not incentive compatible no matter what the quality of the product.

Second, in this case high quality is signalled with a warranty, \(w^H\), which is more extensive than \(w^*(H)\), the warranty the firm would offer if this quality were observable. Low quality is signalled with a warranty, \(w^L\), which is less than \(w^*(L)\). In fact, if \(w^L \leq p^M(1, \theta) - W(L, 1, \theta)\) and \(w^H \leq p^M(0, 0) - W(H, 0, 0)\), then \(w^H = \theta\) and \(w^L = 0\), so the low-quality product is offered with no warranty and the high-quality product is sold with a full warranty. This relationship holds because \(\partial W(L, r, w)/\partial w\) is greater than \(\partial W(H, r, w)/\partial w\) for all \(w\) and \(r\). If the second constraint in (7) is binding at \((\beta^H, \hat{w}^H)\) and at \((\beta^L, \hat{w}^L)\), then \(w^H\) must be greater than \(w^*(H)\). In the same way, if the second constraint in (8) is binding at these contracts, then \(w^L\) must be less than \(w^*(L)\). Recalling that \(w^*(L)\) may be greater than \(w^*(H)\), we can see that \(w^L\) may be greater than \(w^H\).

In the previous section, I showed that under the assumption that quality and effort are complements, no pooling equilibrium satisfied the refinement; there was always a consistent separating deviation away from any such equilibrium. This result does not hold if quality and effort are substitutes. The next proposition gives both necessary and sufficient conditions for a pooling equilibrium to satisfy the refinement.

**Proposition 4.** Necessary conditions for a pooling equilibrium on a contract \((p^p, w^p)\) which yields payoff \(\Pi^q = p^p - W(q, r^N, w^p)\) if the monopolist produces quality \(q\), to satisfy the refinement are (i) \(p^p = p^M(r^N, w^p)\), (ii) \(w^p \in \Omega = \{ w : \partial W(L, r^N, w)/\partial w \geq \partial p^M(r^N, w)/\partial w \geq \partial W(H, r^N, w)/\partial w \}\), and (iii) \(\Pi^q \geq \Pi^q\) for all \(q\). Sufficient conditions are (i), (ii), and (iv) \(\Pi^q \geq \Pi_{\text{max}}(q)\) for all \(q\).

**Proof.** See the Appendix.

Although only a single separating equilibrium can satisfy the refinement, multiple pooling equilibria may do so. The existence of one kind of equilibrium does not rule out the existence of the other. There might be a separating equilibrium and a pooling equilibrium, both satisfying the refinement, such that \(\Pi^L = \Pi^L\) and \(\Pi^H > \Pi^H\), or a pair of equilibria with \(\Pi^L > \Pi^L\) and \(\Pi^H = \Pi^L\), or a pair that are payoff-equivalent. If more than two equilibria satisfy the refinement, they are all pooling equilibria.

In pooling equilibria consumers pay more for a low-quality product than they would be willing to pay if they could observe quality: they pay \(p^M(r^N, w^p)\), which is greater than \(p^M(0, w^p)\). But they are actually no worse off than they would be if quality were observable since they choose effort level \(m(r^N, w^p)\), which is less than \(m(0, w^p)\). In the same way, if
the product is of high quality, consumers pay less when they cannot infer quality than they
would be willing to pay if that quality were observable. But they also expend more effort
and are, overall, no better off.

The firm, on the other hand, whether it produces high or low quality might be able to
earn higher profits from pooling than it could if its quality were observable. Not surprisingly,
if this is true, then pooling will result—but this is not a necessary condition for the existence
of a pooling equilibrium.

5. Conclusion

We have seen that the empirical evidence about the relationship between warranties
and product quality is consistent with the use of warranties as signals of product quality
when consumer moral hazard is present. In particular, I have shown that high quality can
be signalled by a low warranty. While the empirical evidence has led some authors to
conclude that signalling is not an important explanation for warranty provision, these results
demonstrate that this conclusion rests on a false premise about how warranties can signal
product quality.6

Cooper and Ross (1985) have offered an alternative explanation for the stylized facts
about warranty provision which also depends on moral hazard. In their model the consumer
chooses maintenance without being able to observe quality directly. The product’s quality
is not exogenously given, but is chosen by the firm. This is a model of double moral hazard,
and a warranty affects the firm’s choice of quality as well as the consumer’s maintenance.
This model is in some ways closely related to the one presented here. In both, the consumer
chooses maintenance to maximize his utility given his conjecture about the product’s quality.
One difference between the models is the way the consumer forms this conjecture. In the
Cooper-Ross model, quality choice is a hidden action, and the consumer’s conjecture is
based on the warranty because the firm’s incentive to build quality is based on the warranty.
In my model, the level of quality is hidden information, and the conjecture is based on the
price–warranty contract because the firm’s profits are based on the contract and the product’s
quality. The relationship between quality and effort in reducing the probability of product
failure is important in each of these models for the same reason. When quality and effort
are complements, the consumer increases his effort when he conjectures that the product
is of higher quality. Higher-quality products therefore carry lower warranty costs. When
quality and effort are complements, the consumer lowers effort when he conjectures that
the product is of lower quality. Higher-quality products might then have higher warranty
costs. More empirical work will be necessary to determine whether warranties serve as
signals or as incentives for the production of quality products.

The predictions of this model depend in an important way on how product quality
and consumer effort interact. Whether pooling equilibria exist, and whether the monopolist’s
price-warranty contract might be expected to change if quality became observable, depend
in part on whether consumer effort is more productive for high- or low-quality goods.
Hence, the role of warranties will have to be addressed on a market-by-market basis, because
these derivatives reflect technological factors about the product; quality and effort can be
expected to be complements in some markets and substitutes in others.

Information about how quality and effort determine the likelihood that the product
works may also be important in addressing policy questions. If quality and effort are com-
plements in equilibrium, consumers infer product quality even when they cannot observe
it. Because they value both signals (high warranty/low price) they are at least as well off
when they cannot observe quality as they would be if they could do so before purchase.

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There is little reason to expect that consumers are somehow exploited through warranties, and good reason to expect that any manufacturer of a product of unobservably high quality is interested in ways of making that quality observable. When quality and effort are substitutes, consumers are again at least as well off under any equilibrium as they would be if they had complete information about product quality. But they may not be able to infer product quality from the equilibrium price and warranty contract, and the monopolist may have no incentive to make any level of product quality observable.

Appendix

The proofs of Propositions 2, 3, and 4 follow.

**Proof of Proposition 2.** There are three conditions which must be met for a contract \((p, w)\) to be supportable as a sequential separating equilibrium strategy for a high-quality producer, given that the low-quality producer offers \((p^M(0, w^*(L)), w^*(L))\). First, the consumer must be willing to purchase the product at \((p, w)\) if he is convinced that it is of high quality; this implies that \(p \geq p^M(1, w)\). Second, if the firm produces low quality, it must weakly prefer to offer the contract \((p^M(0, w^*(L)), w^*(L))\) and reveal its quality rather than offering \((p, w)\) and being falsely taken to be a high-quality producer; this implies that \(p - W(L, 1, w) \leq \Pi_{\text{max}}(L)\). Finally, the monopolist producing high quality must earn higher profits by offering \((p, w)\) and revealing its quality than it could earn at any other contract given the relationship between contracts and consumer beliefs. A necessary condition for this to be true is that

\[
p - W(H, 1, w) \geq \max_w p^M(0, w) - W(H, 0, w).
\]

This last condition also guarantees that the monopolist will not deviate to \((p^M(0, w^*(L)), w^*(L))\), the separating equilibrium contract offered with a low-quality product.

The two constraints in (6) guarantee that any contract solving this problem satisfies two of these three requirements. Hence, \((p^*, w^*)\) will be part of a sequential equilibrium satisfying the Farrell-Grossman-Perry refinement if no consistent pooling deviations exist, and if

\[
p^* - W(H, 1, w^*) > \max_w p^M(0, w) - W(H, 1, w),
\]

so that the third of the necessary conditions is met. Consistent pooling deviations are defined by Part (c) of the refinement, which gives Condition (ii). Q.E.D.

**Proof of Proposition 3.** I begin by defining

\[
B_H(\Pi^L) = \max_{p, w} p - W(H, 1, w)
\]

subject to \(p \leq \min \{p^M(1, w), \Pi^L + W(L, 1, w)\}\) and

\[
B_L(\Pi^H) = \max_{p, w} p - W(L, 0, w)
\]

subject to \(p \leq \min \{p^M(0, w), \Pi^H + W(H, 0, w)\}\).

If there exists a \(\Pi^H = B_H(B_L(\Pi^H))\) such that \(\Pi^H = B_H(\Pi^L(\Pi^H))\), then there is no consistent separating deviation from an equilibrium in which the monopolist earns \(\Pi^H\) by selling the quality \(q\) product, with \(\Pi^L = B_L(\Pi^H).\) I show that such \(\Pi^H\) and \(\Pi^L\) exist and are unique.

Solving for \(B_H(\Pi^L)\) yields

\[
B_H(\Pi^L) = \begin{cases} 
\Pi^L + W(L, 1, \theta) - W(H, 1, \theta) & \text{if } 0 \leq \Pi^L < p^M(1, \theta) - W(L, 1, \theta), \\
\Pi^L + W(L, 1, \bar{w}(\Pi^L)) - W(H, 1, \bar{w}(\Pi^L)) & \text{if } p^M(1, \theta) - W(L, 1, \theta) \leq \Pi^L \leq p^M(1, w^*(H)) - W(L, 1, w^*(H)), \\
\Pi_{\text{max}}(H) & \text{if } \Pi^L > p^M(1, w^*(H)) - W(L, 1, w^*(H)),
\end{cases}
\]

where \(\bar{w}(\Pi^L) = \max \{w : p^M(1, w) - W(L, 1, w) = \Pi^L\}.\)
Note that \( B_H(\Pi^L) \) is a continuous and weakly increasing function of \( \Pi^L \), mapping \([0, \Pi_{\max}(L)]\) into \([0, \Pi_{\max}(H)]\). It is piecewise differentiable, and \( 0 \leq dB_H(\Pi^L)/d\Pi^L \leq 1 \).

Solving for \( B_L(\Pi^H) \) yields

\[
B_L(\Pi^H) = \begin{cases} 
\Pi^H & \text{if } 0 \leq \Pi^H < p^M(0, 0), \\
\Pi^H + W(H, 0, w(\Pi^H)) & \text{if } p^M(0, 0) \leq \Pi^H \leq p^M(0, w^*(L)) - W(H, 0, w^*(L)), \\
\Pi_{\max}(L) & \text{if } \Pi^H > p^M(0, w^*(L)) - W(H, 0, w^*(L)),
\end{cases}
\]

where \( w(\Pi^H) = \min\{w : p^M(0, w) - W(H, 0, w) = \Pi^H\} \).

Note that \( B_L(\Pi^H) \) is a continuous and weakly increasing function of \( n_H \), mapping \([0, \Pi_{\max}(H)]\) into \([0, \Pi_{\max}(L)]\). It is piecewise differentiable, and \( 0 \leq dB_L(\Pi^H)/d\Pi^H \leq 1 \).

By Brouwer's fixed-point theorem, the composite function \( B_H(B_L(\Pi^H)) \) has a fixed point, since it is continuous and maps \([0, \Pi_{\max}(H)]\) into itself. Let \( \tilde{\Pi}^H \) be a fixed point of the composite function. The fixed point is unique: since \( dB_H(\Pi^L)/d\Pi^L \leq 1/(dB_L(\Pi^H)/d\Pi^H) \), \( B_H(B_L(\Pi^H)) \) is less than \( \Pi^H \) for any \( \Pi^H > \tilde{\Pi}^H \) and greater than \( \Pi^H \) for any \( \Pi^H < \tilde{\Pi}^H \).

There is a unique \((\hat{\beta}^H, \hat{\nu}^H)\) such that \( \hat{\beta}^H - W(H, 1, \hat{\nu}^H) = \tilde{\Pi}^H, \hat{\beta}^H \leq p^M(1, \hat{\nu}^H), \) and \( \hat{\beta}^H - W(L, 1, \hat{\nu}^H) \geq \tilde{\Pi}^L \).

We prove this by noting that if there is a second contract, \((\tilde{\beta}^H, \tilde{\nu}^H)\), such that

\[
\tilde{\beta} - W(H, 1, \tilde{\nu}) = \tilde{\Pi}^H, \tilde{\beta} \leq p^M(1, \tilde{\nu}^H),
\]

and \( \tilde{\beta} - W(L, 1, \tilde{\nu}^H) \leq \tilde{\Pi}^L \), then any convex combination of the two contracts, say \((p, w)\), would satisfy \( p \leq p^M(1, w) \) and \( p - W(H, 1, w) \leq \tilde{\Pi}^L \), while \( p - W(H, 1, w) \) would be greater than \( \tilde{\Pi}^H \). This is true because \( p^M(1, w) \) is increasing and strictly concave, while \( W(q, 1, w) \) is increasing with \( \partial W(H, 1, w)/\partial w \).

But then \( (p, w) \) would show that \( \tilde{\Pi}^H \neq B_H(\tilde{\Pi}^L) \), and this is a contradiction. The parallel argument shows that there is a unique contract \((\hat{\beta}^L, \hat{\nu}^L)\) such that \( \hat{\beta}^L - W(L, 0, \hat{\nu}^L) = \tilde{\Pi}^L, \hat{\beta}^L \leq p^M(1, \hat{\nu}^L), \) and \( \hat{\beta} - W(H, 0, \hat{\nu}) \leq \tilde{\Pi}^H \).

I have not yet established that a separating equilibrium satisfying the refinement can exist; I have so far only proven that if it does exist, the monopolist’s equilibrium strategy is unique. We need to see if \((\hat{\beta}^H, \hat{\nu}^H)\) and \((\hat{\beta}^L, \hat{\nu}^L)\) are supportable as separating equilibrium strategies. If they are so supportable, we need to see if there are any consistent pooling deviations away from the equilibrium.

By construction, \( \tilde{\Pi}^H \geq \hat{\beta}^L - W(L, 1, \hat{\nu}^L) \) and \( \tilde{\Pi}^L \geq \hat{\beta}^H - W(L, 1, \hat{\nu}^H) \), so we know that there is a separating equilibrium in which \((\hat{\beta}^H, \hat{\nu}^H)\) is the contract offered if the product is of high quality and \((\hat{\beta}^L, \hat{\nu}^L)\) is the contract offered if the product is of low quality as long as consumer beliefs give the monopolist no incentive to deviate to some out-of-equilibrium contract. Out-of-equilibrium payoffs are minimized, since \( \partial p^H(r, w)/\partial r > 0 \) and \( \partial m(r, w)/\partial r < 0 \), when beliefs are structured in the following way:

\[
r(p, w) = 0 \quad \text{for all } (p, w) \quad \text{such that } \quad p > p^M(0, w) \\
\text{and } \quad p > \hat{\beta}^H, \quad w \neq \hat{\nu}^H
\]

\[
r(p, w) = 1 \quad \text{for all } (p, w) \quad \text{such that } \quad p \leq p^M(0, w) \\
\text{and } \quad p > \hat{\beta}^L, \quad w \neq \hat{\nu}^L.
\]

Thus, \((\hat{\beta}^H, \hat{\nu}^H)\) and \((\hat{\beta}^L, \hat{\nu}^L)\) can be supported as a separating equilibrium if and only if

\[
\tilde{\Pi}^H > \max_w p^H(0, w) - W(H, 1, w)
\]

and

\[
\tilde{\Pi}^L > \max_w p^H(0, w) - W(L, 1, w).
\]

Given that \((\hat{\beta}^H, \hat{\nu}^H)\) and \((\hat{\beta}^L, \hat{\nu}^L)\) are supportable as separating equilibrium strategies, what remains to be shown is that there is no pooling deviation from this equilibrium if and only if there is no contract \((p^H(r^N, w^N), w^N)\) where \( w^N \in \Omega = \{w : \partial W(L, r^N, w)/\partial w \geq \partial W(H, r^N, w)/\partial w \} \) such that

\[
p^H(r^N, w^N) - W(L, r^N, w^N) > \tilde{\Pi}^L,
\]

while \( p^H(r^N, w^N) - W(H, r^N, w^N) > \tilde{\Pi}^H \).
The condition is clearly necessary; any such \((p^M(r^N, w^p), w^p)\) contract is a consistent pooling deviation.

The condition is sufficient because, for any \((p, w)\), there is a \(w^p \in \Omega\) such that profits to the monopolist from offering \((p, w)\) when the consumer holds beliefs \(r^N\) are no greater than the profits from offering \((p^M(r^N, w^p), w^p)\), regardless of quality, when the consumer holds the same beliefs. \(Q.E.D.\)

Proof of Proposition 4. Conditions (i) and (ii) are necessary and sufficient for there to be no consistent pooling deviations from the equilibrium. Condition (iv) guarantees that there is no consistent separating deviation from the equilibrium, since there is no contract \((p^M(0, w), w)\) such that \(p^M(0, w) - W(L, 0, w) > \Pi^L\) or contract \((p^M(1, w), w)\) such that \(p^M(1, w) - W(H, 1, w) > \Pi^H\).

What remains to be shown is that a consistent separating deviation will exist if \(\Pi^L < \Pi^L\) or if \(\Pi^H < \Pi^H\). If \(\Pi^L < \Pi^L\) while \(\Pi^H \geq \Pi^H\), then it follows that \(B_L(\Pi^H)\) must be greater than \(\Pi^L\), since \(B_L\) is nondecreasing in \(\Pi^H\) and \(\Pi^L = B_L(\Pi^H)\). This means that there is a consistent, low-quality separating deviation from the pooling equilibrium. If \(\Pi^H < \Pi^H\) and \(\Pi^L \geq \Pi^L, I\) conclude that \(B_H(\Pi^L) > \Pi^H\). But then there is a consistent high-quality separating deviation from the pooling equilibrium.

Suppose finally that \(\Pi^H \leq \Pi^H\) and \(\Pi^L \leq \Pi^L\), with one of these inequalities being strict. Unless \(B_L(\Pi^H) = \Pi^L\) while \(B_H(\Pi^L) \leq \Pi^H\), a consistent separating deviation from the equilibrium will exist. If \(\Pi^H < \Pi^H\), then \(B_H(B_L(\Pi^L)) > \Pi^H\). But this means that if \(B_L(\Pi^H) \leq \Pi^L\), it must also be true that \(B_H(\Pi^L) > \Pi^H\), since \(B_H\) is nondecreasing in \(\Pi^L\). Similarly, if \(\Pi^L < \Pi^L\), then \(B_L(\Pi^L) > \Pi^L\) — but this means that \(B_L(\Pi^H)\) must be greater than \(\Pi^L\) while \(B_H(\Pi^L) \leq \Pi^H\). Thus, a consistent separating deviation from the pooling equilibrium must exist. \(Q.E.D.\)

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