Learning to Teach Mathematics  
through Math Letter Exchanges with Fourth Graders

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Radical changes to the nature of teaching mathematics have posed serious challenges for mathematics teacher educators as they attempt to help prospective teachers learn to teach mathematics in ways they have not experienced before. One of the main challenges teacher educators face is that preservice teachers' past experience with school mathematics, more often than not, tend to promote a limited knowledge of and about mathematics, along with a negative view of the subject (Ball, 1990). In addition, their extended experience as students in traditional mathematics classrooms help them develop implicit and imitative pedagogical tendencies, such as habits of correcting, telling and supplying the answers (Feiman-Nemser, 1983).

Unfortunately, teacher education courses have been shown to be a weak intervention in preservice teacher education. Studies from the “teacher socialization” perspective highlight the potency of preservice teachers' early apprenticeship of observation in limiting the impact of the on-campus and field-based experiences offered in teacher preparation programs. In this literature there are many “examples of students interpreting the messages of teacher education courses in ways that reinforce the perspectives and dispositions they bring to the program, even when these interpretations involve a distortion of the intentions of teacher educators” (Zeichner & Gore, 1990, p. 337).

This research literature has made it clear that the traditional designs and instructional strategies used in teacher education courses, which rely on a “pedagogy of presentation,” have not had much success in altering the traditional images prospective teachers bring to their teacher preparation programs. It has become evident that a different design and pedagogy is needed if initial teacher preparation is to have a greater impact on prospective teachers' mathematics teaching. As a response, leading mathematics teacher educators have called for a “pedagogy of investigation” which relies on problem solving and reflective inquiry to “help prospective teachers learn what it is like to teach rather than learn how to teach. … [and] to think and inquire about teaching rather than to learn answers about teaching.” (Ball, Lampert, & Rosemberg, 1991, p. 7).

As a mathematics methods course instructor I have been inspired by such calls for reform and have drawn from recent literature on inquiry-based teacher education, such as case-based pedagogies and teacher inquiry methods, to help me design meaningful learning experiences for my students. Yet while these ideas are argued convincingly in the published world, some questions still remain: How can such an inquiry-oriented pedagogy be integrated within teacher preparation courses, in this case, a mathematics methods course? How might such investigative pedagogy help preservice teachers reconsider their preconceived notions and redirect them towards alternative ways of thinking and acting?

CONTEXT AND DESIGN OF STUDY

This study explored the nature and substance of preservice elementary teachers' learning in the context of an innovative version of the mathematics methods courses typically offered in the
Teacher Preparation Program at the University of British Columbia. This course engaged preservice elementary teachers in a weekly math letter exchange with fourth graders from a nearby school. These interactive experiences with students were meant to serve as a source for reflective inquiry for preservice teachers and as a context for their inquiry of mathematics and its teaching and learning. To bring their inquiries to closure, preservice teachers then wrote a case study of their learning experiences with students.

Provided with such an opportunity, what might preservice teachers learn? What would they make of such an experience? How would this experience contribute to their learning to teach mathematics? These were the questions that guided my research project. The main goals of this study, therefore, were to (a) understand and characterize preservice teachers' learning in this particular context, and to (b) get a sense of how such learning was encouraged and developed throughout the course.

Using the data of 13 participating preservice teachers' written letters to students, their journal reflections, and their final case study, I began the analysis of the data looking for incidents which indicated (either explicitly or implicitly) puzzlement, tensions, and difficulties raised by their interactions with students. This led me to notice patterns of similarity within and across the preservice teachers' data. Then, I grouped their tensions and puzzlements into three broad categories which spoke of preservice teachers' learning related to their: (a) posing of mathematical problems, (b) interpretations of the students' mathematical work, and (c) responding practices. Further analysis into these three categories revealed patterns of change in preservice teachers' discourse and practices. Next, I provide a snapshot of preservice teachers' learning in each of these categories by contrasting their earlier and later patterns of discourse, views, and practices. A more complete discussion of this work can be found in Crespo (1998).

**LEARNING TO VALUE "PROBLEMATIC" MATHEMATICS PROBLEMS**

Preservice teachers' initial selection and adaptations of problems revealed their preference for unproblematic problems—problems which could be solved easily and quickly. Their early problem posing practices could be characterized by their attempts to remove potential difficulties and avoid students' errors. Their journal reflections suggested that by choosing such unproblematic problems preservice teachers intended to provide a pleasant experience for their students. Working on difficult problems and not being able to solve them or getting incorrect answers, in the eyes of these preservice teachers, would only serve to frustrate and “turn off” students from mathematics. Not surprisingly, they could be seen avoiding to pose “problematic” problems and simplifying problems they thought would be too difficult for their students to solve. Later interactions with students and journal reflections, however, revealed changes to preservice teachers' views and practices, which suggest their beginning to value “problematic” or non-trivial mathematical problems. Some of the ways in which this learning was revealed in the data included preservice teachers beginning to: (a) make more adventurous and less leading selections and adaptations, (b) rethink fun, interest, and enjoyment in mathematics, and (c) reconsider the role of ambiguity and errors in teaching and learning mathematics.

*Making More Adventurous and Less Leading Selections and Adaptations.* Preservice teachers' pattern of problem selection and adaptation became more adventurous and less leading later in their letter writing experience. Preservice teachers' selection of problems became more
“adventurous,” as David Cohen (1989) would say, in that these were less traditional types of problems (puzzle-like, exploratory, open-ended), extending beyond topics of arithmetic, and requiring more than computational facility. Furthermore, their adaptations to problems became less imposing or leading, and were meant to be helpful to students “without giving away the answer,” as one preservice teacher, “Marcia,” said.

Rethinking Fun, Interest, and Enjoyment in Mathematics. Their journal reflections also showed changes in preservice teachers' notions of what would make mathematics fun, interesting, and enjoyable to students. The students' work and feedback to the problems they were given began to challenge preservice teachers' assumptions about what kinds of problems students would be able and willing to solve. Problems which preservice teachers thought would be too difficult or frustrating to students often turned out to be enjoyable and attainable. Problems which preservice teachers thought would be easy or interesting to solve, were not always so for students. Therefore, preservice teachers' ideas as to what problems students might find enjoyable and interesting ceased to be taken for granted and began to be more explicitly investigated in their journals.

Reconsidering Ambiguity and Mathematical Errors. The more exploratory types of problems introduced and worked on during our mathematics methods classes also began to extend preservice teachers' ideas about the types of problems that they could pose to students. They began to see that certain types of problems could “teach kids to think about math, not necessarily as numbers and correct answers, but math as ideas,” as Marcia said. Preservice teachers' own mathematical and pedagogical explorations were also helping them reconsider their earlier assumptions. Some began to see ambiguity “as a valuable tool for further exploring mathematics” (Sally) and others began to realize, as Thea did, that becoming confused and making errors could lead to interesting and important mathematical investigations.

LEARNING TO SEE AND CONSTRUCT MEANING FROM STUDENTS' WORK

Preservice teachers' initial interpretations of their students' work focused mainly on its surface features. This was apparent both in their response letters to the students as well as in their journal entries. The few comments preservice teachers made about their students' work tended to highlight the merits of the students' work. They simply suggested their student “did a good job,” “was successful,” “made a careless mistake,” “was almost right” without attending to the details and the meaning of the student's mathematical work. Initially their discussions about the students' work were brief and evaluative. In short, preservice teachers' initial interpretations tended to be at a surface level and to make no inference or speculation about the student's underlying understanding of specific mathematical concepts and procedures.

In contrast, preservice teachers in this study were quite willing to speculate about, and make inferences (though quite often unsubstantiated) from the students' attitudinal comments (about math, teaching, learning, textbooks, school), their questions, and from the surface features of their writing (e.g., length of response, handwriting, spelling, neatness). Using students' brief comments as evidence—comments such as: “math in this class is prity isey (sic),” “as for math I need more practice,” “I like the teacher teaching me”—preservice teachers tended to make quick, evaluative, and sweeping generalizations about their student's mathematical attitude and ability. Rosa, Nilsa, and Linda, for instance, were convinced after receiving only one letter from their
student that their particular student “(was) not very good in math”, “(did) not enjoy math”, and “(was) at the bottom of her class.”

The introduction of an “interpretive tool” and their own mathematical investigations into the problems they were posing, however, began to help many of the preservice teachers to focus their attention onto, and delve into the meaning of, their students' mathematical work. Preservice teachers could be seen learning about students' work by: (a) raising questions about the meaning of students' work, (b) seeing beyond correctness, (c) making analytical interpretations, and (d) extending own understanding through interpretations.

Raising Questions about the Meaning of Students' Work. Preservice teachers' initial observations of their students' mathematical work focused on its surface features (e.g., correctness, spelling, and neatness). Initial journal entries scarcely and briefly referred to the meaning of the students' work. Students' unexpected and unclear responses, however, began to raise the curiosity in preservice teachers about what their students' work meant. For instance, receiving students' inexplicit, brief, and unclear mathematical work encouraged preservice teachers to raise questions about the meaning of such work. Preservice teachers, for instance, began to ask: “Does he know why he got 3 or why he added it to the 5?” (Rosa); “Do they know what showing your work means?” (Carly); “I am worried she did not understand the problem in the end” (Megan).

Seeing Beyond Correctness. Unexpected work from the students also encouraged preservice teachers to look for and see more than the correctness in students' work. They began to use the students' communication of their work as another source of evidence for students' mathematical understanding. Preservice teachers' comments about their students' work began to read: “She did a good job explaining,” (Linda) ”I would have liked to see his rough work,” (Megan) “I can finally follow her thinking,” (Carly) “She did not say why she did what she did” (Miriam). These indicate that preservice teachers had begun attending to the length, clarity, and explicitness of the students' written communication as other clues for mathematical understandings.

Making Analytical Interpretations. Preservice teachers' later interpretations of their students' work turned more toward the details and the meaning in students' work, even when such work was brief and not obvious. This means that students' correct work ceased to be assumed as evidence of understanding and students' incorrect solutions were not dismissed as careless mistakes or signs of confusion. Instead, students' mathematical work began to be closely examined and discussed at length in the journals with particular attention to the mathematical details and meanings. A “descriptive-interpretive” journal writing tool provided in our class provided a useful format for helping preservice teachers organize and focus their reflections upon the students' work. In addition, the introduction of unfamiliar and exploratory types of problems and the opportunity for collective and individual explorations of these problems began to help preservice teachers make more analytical interpretations of their students' mathematical work. Their own mathematical investigations of the problems they would pose to their students raised preservice teachers' interest, confidence, and resources to delve into the meaning of their students' work.

Extending Own Understanding through Interpretations. Close examination of their students' work also helped preservice teachers extend their own mathematical and pedagogical understandings. It helped preservice teachers become familiar with students' communication and
explanations of their mathematical work. It also gave preservice teachers further insights into the problems they posed, that is insights into: the mathematical concepts involved, the students' initial interpretations of the problem, the subsequent questions and difficulties that could arise, and alternative solutions and problem solving strategies for the problems they had posed.

LEARNING TO INTERROGATE HIDDEN MESSAGES IN THEIR “TEACHERLY TALK”

Preservice teachers' responses and feedback to students' mathematical work initially focused on the overall correctness of the student's answer. These responses were mainly evaluative of the students' work. For instance, preservice teachers tended to respond by praising the correct answers and by correcting the wrong ones. Interestingly, these kinds of responses, often associated with the immediacy of classroom interactions between teachers and students, also pervaded the slower paced letter writing interactions between preservice teachers and their student writers. In later interactions, however, preservice teachers' responses to their students' work became more deliberately constructed and to focus on more than the correctness of the students' answers.

Writing their responses to students encouraged preservice teachers to carefully consider a response to their students' work. At times writing out their response in their letters helped preservice teachers notice the underlying messages they were sending to students. Other times it was when revisiting and revising their letter exchanges that preservice teachers were able to problematize their responses to their students. For others, it was much later, when writing a case study of their learning experience that they identified and reflected on their potentially damaging responses to students. Some of the ways in which preservice teachers can be seen recognizing hidden messages in their responses to students included: (a) problematizing praise, (b) questioning the practice of correcting, and (c) questioning own lack of questioning.

Problematizing Praise. Preservice teachers who received correct work from their students became aware and began to problematize praising as a response to students' answers. Carly, for example, realized that indicating to students that their answers were right discouraged students from relying on their own sense making and from exploring other answers and solution strategies to problems. Linda and Megan also became aware of their tendencies to consistently respond with praising comments to one of the two students with which they were exchanging letters. They noted substantial differences in the quality and length of the responses they gave to the students they perceived as high and low achievers. The implicit and dangerous messages of such responses became apparent to Linda and Megan when they reflected on the effects that such responses had on their students' mathematical attitudes and performances. The practice of praising also became problematic to other preservice teachers when they found themselves praising students' effort and their incomplete and often incorrect work.

Questioning the Practice of Correcting. Their responses to students' incorrect work—correcting and supplying the answer—were a source of much deliberation and reflection for preservice teachers throughout their letter writing experience. Very early in their journals preservice teachers began questioning and investigating the effects of pointing out and correcting mistakes in their students' work. Furthermore, all the preservice teachers who responded by supplying the correct answers regretted having done so in their very next journal entries. Interestingly, the underlying messages of their corrective responses were more easily recognized
and problematized by these preservice teachers than the implicit messages in their praising responses to their students' correct work.

Questioning Own Lack of Questioning. Imposing their answers and ways of solving mathematics problems, many preservice teachers could see, was disrespectful and discouraging to students' sense making. However, responding without pointing towards the solution was not an obvious choice for them. On many occasions, it was after they had already sent their responses to students, that they began questioning their lack of questioning of the students' work. These are some examples of preservice teachers' comments after supplying students with the answers: “I wished I had asked him how he did this, but I forgot,” (Carly); “I'm not sure how she did this, … I didn't think to ask her,” (Sally); “Had I posed the following problem instead, John would have had to do the same math, but would have had to think more deeply about his answer and he may have been more certain about himself and his own knowledge” (Marcia). For others, like Linda, it was in retrospect that they realized “the importance of questioning beyond superficial levels,” and regretted not having asked more “how” and “why” questions to their students.

ROLE OF INTERACTIVE EXPERIENCES WITH STUDENTS IN SUPPORTING THE LEARNING OF MATHEMATICS AND MATHEMATICAL PEDAGOGY

A math letter writing experience associated with a mathematics methods course offered a rare opportunity to preservice teachers in this study. They were engaged in a sustained interaction with school students while attending their on-campus classes. This sort of course-related field experience, while enthusiastically advocated in the literature, as Carter and Anders (1996) point out, has not been systematically studied. In fact, I found very few research articles on this topic and very few have been written since the 1970s. The present research study, therefore, contributes to this literature.

In particular, this study provides further evidence of the transformational influence that working closely with students has on the learning of not only experienced teachers (as the CGI studies have found), but on preservice teachers as well. Preservice teachers in this study took their students' data very seriously. They sought and worked very hard to understand their students' mathematical work and communication. The students' data, in turn, provided them with much insight and deliberation related to their understandings of mathematics, students as learners, and pedagogical practices. This demonstrates that preservice teachers' attention and concerns can be focused away from “self-concerns”—concerns with classroom survival, managerial, and disciplinary concerns—even in contexts which are closely related to classroom practice.

Different from traditional interactions between students and teachers, written letter interactions afforded preservice teachers no concerns for managerial or disciplinary issues and no school and curricular pressures. It, however, brought to the foreground concerns for students' abilities to communicate their thinking. The structural features of their written interactions with students, therefore, made it possible for preservice teachers to engage in mathematical and pedagogical inquiries and to focus their reflections on issues of mathematics teaching and learning. In particular preservice teachers in this study focused on issues related to: communicating mathematically, writing in mathematics, understanding students' mathematical thinking, posing good questions and practicing good questioning.
Written interactions with students provided opportunities for mathematical inquiry alongside and often intertwined with preservice teachers' pedagogical inquiries. Writing the solutions to the problems the students posed to them provided one such opportunity. These, in turn, were often taken as pedagogical opportunities for modeling to students how they might respond and make their mathematical work and thinking more explicit in writing. In addition, the students' responses served to, in some occasions, engage preservice teachers in further mathematical explorations of the problems they had posed. There were many occasions when preservice teachers' pedagogical inquiries—examining and trying to make sense of a students' response—led to further examination of the mathematics involved in a particular problem.

Students' responses were also very influential catalyst to preservice teachers' learning. Preservice teachers took their students' data very seriously yet oftentimes they could not make sense of it or had more questions about it than answers. Students' unexpected, unclear, and brief responses to their questions were a source of much deliberation for preservice teachers. They served to challenge some of the preservice teachers' prior assumptions about students and mathematics learning and helped them make new assumptions, some of which turned out to be false and were eventually revised.

References


