The Displacement Effect of Reemployment Bonus Programs

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We develop a partial equilibrium matching model of the labor market in order to examine whether adoption of a reemployment bonus would displace workers not offered the bonus. We examine the displacement effect for (a) unemployment insurance (UI)-eligible workers who are offered but do not find a job in time to qualify for a bonus and (b) UI-ineligible workers who are never offered a bonus. The model predicts minimal displacement of the former group. But for the latter group, the model predicts an increase in unemployment duration of .2–.4 week and an increase in unemployment of up to 2 per thousand.

The reemployment bonus is a cash payment made to an unemployment insurance (UI) recipient who finds a job within a relatively short period of time after filing for UI benefits. Four randomized trials of the reemployment bonus have now been completed—in Illinois (Woodbury and Spiegelman 1987), New Jersey (Corson, Decker, Dunstan, and Gordon 1989), Washington State (Spiegelman, O'Leary, and Kline 1991), and Pennsylvania (Corson, Decker, Dunstan, and Kerachsky 1991)—and all have suggested that a reemployment bonus program could reduce the duration of insured unemployment without adverse consequences for workers who are offered a bonus.

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Nevertheless, an important concern surrounding the reemployment bonus is that it could reduce the number of steady-state jobs held by workers who are not offered a bonus. To take the extreme case, if each new job obtained by a bonus-offered worker were at the expense of a nonoffered worker, then the bonus’s effect would simply be distributional and there would be no real effect on total unemployment. This “displacement effect,” if large, would greatly decrease the attractiveness of the reemployment bonus.

The purpose of this article is to develop a partial equilibrium matching model of the labor market that allows us to investigate the nature and size of the displacement effect. We accomplish this by assuming that it takes time and effort for unemployed workers and firms with vacancies to find each other. Workers can increase the probability of finding employment by increasing their search effort, but increased search effort is costly. In equilibrium, workers choose a level of search activity that equates the expected gain from additional search with the marginal cost of search. The bonus program increases the expected payoff to search for bonus-offered workers and therefore results in an increase in their search effort. This increase in search activity has three effects. First, if we hold search effort by nonoffered workers constant, then there will be an immediate increase in the employment of bonus-offered workers. Some of the increased employment may come at the expense of nonoffered workers, but some will result simply because greater search activity allows the labor market to make better use of the existing search technology and therefore to fill more of the total available jobs. The increased employment of bonus-offered workers will eventually benefit nonoffered workers as well since when a worker needs to be replaced, both bonus-offered and nonoffered workers will be free to compete for the job vacancy. We refer to the increase in employment of bonus-offered workers due to their increased search effort as the “gross employment effect” of the bonus program. That some of the increase in employment of bonus-offered workers comes at the expense of nonoffered workers is referred to as a “displacement effect.”

The third effect of the bonus program is generated by the change in the search behavior of workers who are not offered a bonus. Since the increased search effort of bonus-offered workers will make it more difficult for nonoffered workers to find employment, the bonus program will generate spillover effects that change the search effort of nonoffered workers as well. We refer to this as the “rivalry effect” of the bonus program. We find that the rivalry effect differs between two classes of nonoffered workers. For UI-eligible workers it is essentially nonexistent; UI eligibles who fail to find a job in time to qualify for the bonus (including workers who have exhausted their UI benefits) change their search effort imperceptibly in response to the increase in bonus-offered workers’ search effort. In contrast, UI-ineligible workers (who are never offered the bonus) respond to bonus-
offered workers’ increased search effort by searching less hard, so for these latter workers the rivalry effect is an additional source of displacement.

The article is divided into five additional sections. In the first section we introduce a partial equilibrium matching model that is patterned after the work of Diamond (1982), Mortensen (1982), and Pissarides (1984). This model incorporates the central features of the Illinois reemployment bonus experiment, which offered a $500 cash bonus to UI-eligible workers who were reemployed within 11 weeks of filing for UI. In Section II, data gathered in the course of evaluating the Illinois experiment are used to infer values for the unobservable parameters of the model. The calibrated model is then solved in Section III assuming that the bonus program is in effect. This yields estimates of the displacement effect. In Section IV, we discuss how extensions of the model would affect our displacement estimates. In the final section, we begin by summarizing our results. We then argue that while our model is tractable, it is rich enough to capture the essential features of frictional unemployment and institutionally convincing enough to be useful in studying policy issues such as the displacement effects of various government programs. The analysis provided in this article is intended as an example.

I. The Model

To investigate the nature of the displacement effect, we need a model in which there are four classes of workers:

i) UI recipients who have been jobless for a short enough period of time that they would qualify for a bonus if they became reemployed,

ii) UI recipients who have been jobless so long that they would no longer qualify for a bonus if they became reemployed,

iii) former UI recipients who have exhausted their benefits, and

iv) jobless workers who were never eligible to receive UI benefits and were never offered a bonus.

In the Illinois experiment, UI-eligible workers were offered a bonus during the first 11 weeks of search, and UI benefits were exhausted after 27 weeks. Hence, group i corresponds to workers in their first 11 weeks of unemployment who receive UI benefits and can qualify for a bonus by finding a job. Group ii corresponds to workers in weeks 12–27 of search who are no longer offered the bonus but still receive UI if jobless. Group iii corresponds to workers who have been jobless for more than 27 weeks and have exhausted both their bonus offer and their UI benefits. Finally, group iv corresponds to workers who were never offered a bonus because they were never eligible for UI benefits (e.g., new entrants and reentrants to the labor force). We believe that specifying a model with these four groups of workers results in a model with enough institutional realism to offer convincing results.
In addition to specifying four groups of workers, the model must have two additional characteristics: (a) each jobless worker should choose search effort to maximize expected income, and (b) there must be an equilibrium level of unemployment. To capture a, we assume that it takes time and effort for firms with vacancies and unemployed workers to find each other. Jobless workers can reduce the time it takes to find employment by searching more intensely, although increasing one's search effort is costly. To capture b, it is necessary to assume that jobs do not last forever. That is, in each period some jobs dissolve (a job turns into a vacancy, creating unemployment) and new jobs form. In equilibrium, the rate of job creation and job dissolution are equal.

The fact that we employ an equilibrium model of the labor market differentiates our work from other studies of the Illinois experiment. For example, Mortensen (1987) and Levine (1989) each use a search model of labor supply to investigate the effect of the bonus program on the search behavior of bonus-offered workers. Their models do not consider labor demand (in that the distribution of wage offers is exogenous and does not change once the bonus program is implemented) or allow analysis of how changes in the behavior of bonus-offered workers affect the behavior and welfare of nonoffered workers. Our equilibrium model allows us to determine the direct effect of the bonus program (through its effect on bonus-offered workers' search behavior) as well as its indirect effect (through its effect on equilibrium unemployment and the search behavior of nonoffered workers).

The model is explained in two stages. First, we discuss the steady-state conditions and the search technology. These conditions describe the dynamics of the labor market including the manner in which jobs are created and destroyed. In addition, these conditions guarantee that the flows into and out of employment are equal (so that we have an equilibrium). Second, we solve the problem of a typical unemployed worker who must choose search effort to maximize expected lifetime income. To do so, we must first derive the expected income of workers who are employed and the expected income for unemployed workers. Search effort is then chosen to maximize the expected lifetime income of unemployed workers.\(^1\)

A. Steady-State Conditions

Let \( J \) denote the steady state number of jobs held in the economy and \( V \), the number of vacancies. Then, by definition, \( T \) denotes the total number of jobs available, where

\[
T = J + V. \tag{1}
\]

\(^1\) Throughout the article, we assume that workers search optimally. If this assumption is inaccurate, then, of course, the empirical relevance of our results is greatly reduced.
If we assume that each firm hires at most one worker, then $T$ also represents the number of firms in the economy. Alternatively, we could assume that each firm has several job opportunities but recruits and fills each job separately. For our purposes, these two assumptions are equivalent.\(^2\)

We measure time in 2-week intervals since UI claimants in Illinois are certified for 2 weeks of UI benefits at a time. With this in mind, let $L$ denote the total number of workers in the labor force (employed and unemployed), $U_i$, the number of UI-eligible unemployed workers who are in their $i$th period of search where $i \leq 14$ (these workers are eligible for UI benefits since they have been unemployed fewer than 28 weeks); $U_e$, the number of UI-eligible workers who have been unemployed more than 14 periods (so that they have exhausted their UI benefits); and $U_i$, the number of UI-ineligible unemployed workers. Then the number of bonus-offered workers at any time is $\sum_{i=1}^{14} U_i$, while $U_o = \sum_{i=1}^{14} U_i + U_e + U_i$ represents the number of nonoffered workers. Since all workers are either employed or unemployed, it follows that

$$L = j + \sum_{i=1}^{14} U_i + U_e + U_i = j + U,$$  \hspace{1cm} (2)

where $U$ is defined as total unemployment. (All notation is summarized in the Appendix.)

Equations (1) and (2) are simple accounting identities. We now turn to a description of the evolution of the labor market over time. There are 17 labor market states: employment (state $j$), UI-eligible/bonus-offered unemployment (one state each for $U_i$ with $i \leq 6$), UI-eligible/bonus nonoffered unemployment (one state each for $U_i$ with $i \in \{7, 14\}$), UI-exhausted/bonus nonoffered unemployment (state $U_e$), and UI-ineligible/bonus nonoffered unemployment (state $U_i$). Movements into and out of these states depend on the rates at which jobs are found and destroyed. Let $m$, denote the probability that a UI-eligible jobless worker in the $i$th period of search finds employment, and let $m_e$ and $m_i$ play the same roles for UI-exhaustees and UI-ineligible workers, respectively. Note that $m$, $m_e$, and $m_i$ are conditional reemployment probabilities — or conditional probabilities of creating a job match — often referred to as hazard rates. We use $s$ to denote the separation (or “breakup”) rate of jobs. That is, $s$ rep-

\(^2\) We are implicitly assuming constant returns to scale in recruiting. If the bonus program results in the opening of new plants or the expansion of existing plants (so that there is a massing of vacancies) and if there are increasing returns to scale in recruiting, then our model will overestimate the displacement effect. This follows from the fact that our model would overpredict the length of time required to fill an average vacancy. See Sec. IVB for a discussion of the effect of the bonus program on labor demand.
resents the probability that a job will dissolve at any point in time. The determination of \( m_{t-1} \) and \( m_t \) will be discussed in detail below, but for now they are treated as parameters. Finally, we define \( q \) to be the proportion of newly unemployed workers who are ineligible for UI.

The evolution of the labor market is depicted in Figure 1. Consider first the flows into each state of unemployment. In any given period there are \( f \) employed workers. At the end of the period, \( sf \) of these workers quit or are terminated from their jobs and must reenter the search process; a fraction \( q \) of these newly unemployed workers are ineligible for UI benefits, while the remainder \( (1 - q) \) are UI-eligible. Thus, the gross flow into state \( U_t \) is \( (1 - q)sf \), and the gross flow into state \( U_t \) is \( qsf \). Workers enter state \( U_t \) (with \( t \leq 14 \)) if and only if they were in their \((t - 1)\)st period of search in the previous period and failed to find a job. Thus, the flow into state \( U_t \) is \((1 - m_{t-1})U_{t-1} \). Workers eligible for UI exhaust their benefits if they fail to find employment in their fourteenth period of search. The flow into \( U_e \) is therefore equal to \((1 - m_{14})U_{14} \).

The flows out of each state are even easier to characterize. In each period, successful searchers in state \( U_t \) find employment and enter state \( f \), while unsuccessful ones move on to state \( U_{t+1} \). Thus, all jobless UI-eligible workers flow out of their current state. This implies that the flow out of \( U_t \) is simply \( U_t \). Jobless UI-exhaustees and UI-ineligible leave their states only when they find employment. Thus, the flow out of \( U_t \) equals \( m_t U_t \), and the flow out of \( U_t \) is \( m_t U_t \).

In a steady-state equilibrium, the flows into and out of each labor market state must be equal so that the unemployment rate and the composition of the unemployment pool do not vary over time. Equating these flows yields the following steady-state conditions:

\[
(1 - q)sf = U_t, \tag{3}
\]

\[
(1 - m_{t-1})U_{t-1} = U_t \quad \text{for } t = 2, \ldots, 14, \tag{4}
\]

\[
(1 - m_{14})U_{14} = m_t U_t, \tag{5}
\]

In terms of our notation, when a job \( f \) dissolves, it either turns into a vacancy \( V \) at the same firm or disappears at that firm and turns into a vacancy at another firm. In labor market terms, separations in this model are either frictional (the result of quits into unemployment or discharges) or structural (the result of permanent job loss in one sector that is compensated by growth in another sector). Separations in the model are never cyclically induced (i.e., they are never the result of a general decline in aggregate demand). Also, we do not model on-the-job search, since we ignore quits that do not contribute directly to unemployment (i.e., job changing). We are confident that this assumption does not bias our results: Since a worker must file for UI in order to be offered a bonus, and since voluntary quits are not immediately eligible for UI, the bonus program would not affect the rate of job changing.
and

\[ qSf = m_i U_i. \] (6)

If (3)–(6) hold, then \( f, U_i, U_e, \) and \( U_i \) will not vary over time.

B. The Search Technology

The conditional reemployment (or matching) probabilities, \( m_i, m_e, \) and \( m_i \), depend on the nature of the search process and the level of search effort expended by unemployed workers. In each period, jobless workers choose their level of search effort to maximize expected lifetime income. This search effort determines the probability that the worker contacts a firm and applies for a job (search effort may be fruitless). The firm hires the worker at a wage of \( w \) if it has a vacancy and if no other applications are filed. If more than one worker applies, the firm chooses randomly among all applicants. A worker's reemployment probability therefore depends on the probability of contacting a firm, the probability that the contacted firm has a vacancy, and the probability of getting the job over all other applicants.

Formally, let \( p_i \) denote the probability that, in any given period, a UI-eligible worker who has been unemployed for \( t \) periods contacts a firm (\( p_i \) and \( p_e \) play the same role for UI-exhaustees and UI-ineligibles, respectively). This contact probability can be interpreted as search effort since it can be increased only by searching more intensely. The determination of \( p_i \) is discussed in the next subsection; for now, we treat it as a parameter. Once a firm has been contacted, the probability that this randomly chosen firm has a vacancy is \( V/T \). Finally, if the firm does have a vacancy, the probability of receiving a job offer is \( 1/N + 1 \), where \( N \) denotes the number of other applications filed. Since each other worker either does or does not apply at the firm in question (i.e., there are only two possibilities), \( N \) is a random variable distributed according to a Poisson distribution with parameter \( \lambda \), defined as the average number of applications filed at each firm. Thus, the probability that the worker gets the job, conditional on having applied at a firm with a vacancy, is

\[ \text{Note that we do not allow an individual's duration of unemployment to influence the firm's choice among applicants. If the bonus increases the search effort of short-term unemployed workers more (as it does in our model) and short-term unemployed workers are preferred by firms, then the displacement effects of the program may be larger than our model predicts.}

\[ \text{If } p_i < 1, \text{ then } p_i \text{ represents a contact probability. If } p_i > 1, \text{ then we interpret } p_i \text{ as the number of firms contacted by the worker per period. For example, if } p_i = 1.5, \text{ then we assume that the worker contacts one firm with probability 1 and a second firm with probability 0.5. We ignore the possibility that any given worker may contact the same firm twice in any given period.} \]
\[
\sum_{N=0}^{\infty} \frac{1}{(N+1)N!} e^{-\lambda} \lambda^N = \frac{1}{\lambda} [1 - e^{-\lambda}].
\]

The product of these three probabilities yields the reemployment probability for each worker:

\[m_t = \frac{p_t V}{\lambda} \frac{V}{T} [1 - e^{-\lambda}] \text{ for } t = 1, \ldots, 14, \tag{7}\]

\[m_e = \frac{p_e V}{\lambda} \frac{V}{T} [1 - e^{-\lambda}], \tag{8}\]

\[m_i = \frac{p_i V}{\lambda} \frac{V}{T} [1 - e^{-\lambda}], \tag{9}\]

with

\[\lambda = \frac{1}{T} \left\{ \sum_{i=1}^{14} p_t U_t + p_e U_e + p_i U_i \right\}. \tag{10}\]

Note that equation (10) provides the mechanism for displacement in the model. As the search effort of bonus-offered workers (\(p_t\) for \(t \leq 6\)) increases in response to the bonus, the number of contacts per firm (the term in brackets in eq. [10]) increases. Hence, the probability of receiving a job offer from a contacted firm that has a vacancy falls.

C. Search Effort

Search effort is chosen to maximize expected lifetime income. It is clear from (7)–(9) that by searching harder an unemployed worker can increase the probability of reemployment. Of course, there is a cost associated with increased search effort. The optimal search effort results from equating the expected marginal benefit from an increase in search effort with its marginal cost.

To be precise, let \(V_t\) denote the expected lifetime income for a UI-eligible worker currently in the \(t\)th period of search (\(V_e\) and \(V_i\) play the same role for UI-exhaustees and UI-ineligibles, respectively). In addition, let \(V_{ui}\) represent the expected lifetime income for a UI-eligible worker who is currently employed at wage \(w\) (with \(V_{ui}\) playing the same role for UI-

\(^6\)This is equivalent to assuming that all job contacts are made by workers and that the underlying search technology is quadratic (see Diamond and Maskin [1979] for details).
ineligibles). Then, if we assume that search costs are convex, \( V_t, V_s, V_i, V_w, \) and \( V_{as} \) satisfy

\[
V_t = x - cp_t^* + \frac{1}{1 + r} \{ m_t (V_w + b) + (1 - m_t) V_{t+1} \}
\]

for \( t = 1, \ldots, 6, \) (11)

\[
V_t = x - cp_t^* + \frac{1}{1 + r} \{ m_t V_w + (1 - m_t) V_{t+1} \}
\]

for \( t = 7, \ldots, 13, \) (12)

\[
V_{14} = x - cp_t^* + \frac{1}{1 + r} \{ m_{14} V_w + (1 - m_{14}) V_s \},
\]

(13)

\[
V_e = -c_i p_t^* + \frac{1}{1 + r} \{ m_e V_w + (1 - m_e) V_s \},
\]

(14)

\[
V_i = -c_i p_t^* + \frac{1}{1 + r} \{ m_i V_{wi} + (1 - m_i) V_i \},
\]

(15)

\[
V_w = w + \frac{1}{1 + r} \{ s V_i + (1 - s) V_w \},
\]

(16)

and

\[
V_{as} = w + \frac{1}{1 + r} \{ s V_i + (1 - s) V_{as} \},
\]

(17)

where \( x \) denotes the biweekly UI benefit, \( z (>1) \) denotes the elasticity of search costs with respect to search effort, \( b \) denotes the bonus paid to bonus-offered workers when they find a job, and \( r \) is the biweekly interest rate. Equation (11) states that a bonus-offered unemployed worker currently collects UI benefits \( x \) and pays out search costs \( (cp_t^*) \). With probability \( m_t \) search is successful and the worker receives a bonus of \( b \) and begins working (so that expected lifetime income increases to \( V_w \)). With probability \( (1 - m_t) \) search is unsuccessful and the worker continues to search in the next period (so that expected lifetime income becomes \( V_{t+1} \)). These last two terms are discounted since they reflect future income start-

\(^7\) The assumption that the marginal cost of search increases with effort is consistent with the empirical finding that the net return to search is decreasing in effort (see, e.g., Barron and Gilley 1981; or Chirinko 1982). For the importance of this assumption, see Seater (1979).
ing in the next period. Equation (12) is the analogous condition for a worker who is still UI-eligible but is no longer offered a bonus. Equation (13) describes expected lifetime income for a UI-eligible worker who is about to exhaust his benefits, and (14) and (15) are the analogous conditions for UI-exhaustees and UI-ineligibles. Finally, (16) and (17) describe the situation faced by employed workers. Current income is \( \omega \) but with probability \( s \) the worker becomes unemployed and must begin searching for a new job (so that expected income drops to \( V_i \) for UI eligibles and \( V_i \) for UI-ineligibles). With probability \( (1 - s) \) the worker remains employed and continues to earn \( V_\omega \) if UI-eligible or \( V_{\omega \omega} \) if UI-ineligible.

Each unemployed worker chooses \( p \), the contact probability, to maximize expected lifetime income. Therefore, \( p \) can be interpreted as search effort. Applying Bellman’s principle of optimality and maximizing each expression yields the following optimal levels of search effort:* (the reader is reminded that the \( m \)’s are functions of the \( p \)’s through [7]–[10]):\(^9\)

\[
p_t = \left\{ \frac{m_t}{zc(1 + r)} \left[ V_\omega + b - V_{i+1} \right] \right\}^{\frac{1}{2}} \quad \text{for } t = 1, \ldots, 6, \tag{18}
\]

\[
p_t = \left\{ \frac{m_t}{zc(1 + r)} \left[ V_\omega - V_{i+1} \right] \right\}^{\frac{1}{2}} \quad \text{for } t = 7, \ldots, 13, \tag{19}
\]

\[
p_{t+1} = \left\{ \frac{m_{t+1}}{zc(1 + r)} \left[ V_\omega - V_i \right] \right\}^{\frac{1}{2}}, \tag{20}
\]

\[
p_s = \left\{ \frac{m_s}{zc(1 + r)} \left[ V_\omega - V_s \right] \right\}^{\frac{1}{2}}, \tag{21}
\]

and

\[
p_i = \left\{ \frac{m_i}{zc_i(1 + r)} \left[ V_{\omega \omega} - V_i \right] \right\}^{\frac{1}{2}}. \tag{22}
\]

* Bellman’s principle of optimality allows us to treat future search effort as fixed when deriving optimal current search effort. For example, in (11), since \( V_{i+1} \) depends on future search effort (and, more important, is independent of \( p_i \)), it can be treated as a parameter when maximizing \( V_i \) over \( p_i \). For a discussion of Bellman’s principle, see Dixit (1990).

\(^9\) In maximizing \( V_i \) over \( p_i \), we treat \( \lambda \) as a parameter. The rationale for this is that since each worker is small relative to the market, each individual can ignore his/her own effect on \( \lambda \).
To summarize, in each period unemployed workers choose search effort (as measured by $p$, the contact probability) to maximize expected lifetime income. The optimal values for search intensity are given by (18)–(22). These contact probabilities then determine the average number of applications filed at each firm ($\lambda$, as given in [10]) and the conditional reemployment probabilities (as given in [7]–[9]). Jobs are created as unemployed workers find firms with vacancies and production takes place. Finally, at the end of the period, a fraction $s$ of all jobs break up and the newly unemployed workers start searching for new jobs. The steady-state conditions (3)–(6) guarantee that the flows into and out of each employment state are balanced so that the unemployment rate and the composition of the unemployment pool (i.e., the distribution of $U_i$ over $i$) are time invariant.

The model consists of 69 equations (one each in [1]–[3], [5]–[6], [8]–[10], [13]–[17], and [20]–[22]; six each in [11] and [18]; seven each in [12] and [19]; 13 in [4]; and 14 in [7]) in 69 unknowns ($f, V, U, U_t$ for $t \leq 14, U_i, U_m, m_i, m_e, m_e, p, p_i, \lambda, V_e$ for $t \leq 14, V_e, V_1, V_2, V_3$, and $V_4$) and 10 parameters ($T, L, s, q, c, z, c, x, w$, and $r$). Once the model has been solved, the unemployment rate ($\mu$) and the expected duration of unemployment for UI-eligibles ($d$) and UI-ineligibles ($d_i$) can be calculated using (23)–(25):

$$\mu = \left\{ \sum_{i=1}^{14} U_i + U_m + U_e \right\} / L = \frac{U}{L},$$

(23)

$$d = \left\{ \sum_{i=1}^{14} m_i U_i + m_e^2 U_e \sum_{t=15}^{\infty} t(1 - m_e)^{t-15} \right\} / U,$$

(24)

and

$$d_i = \frac{1}{m_i}.$$

(25)

Equation (23) states that the unemployment rate is equal to total unemployment divided by the size of the labor force. The duration of unemployment for UI-eligibles is calculated by following one cohort of newly unemployed workers and calculating the average number of periods it takes these workers to find employment. For UI-ineligibles, expected duration is simply the reciprocal of the reemployment probability.

We close this section by introducing one more variable. As already noted, there are two classes of workers who are not offered the bonus: all UI-ineligibles ($U_i$) and UI-eligibles who fail to find a job within the first 6 periods ($\sum_{t=1}^{14} U_t + U_0$). As we show below, the bonus program affects
these two groups in fundamentally different ways. To examine the effect on the group as a whole, we use \( U_n \) to represent all nonoffered workers. That is,

\[
U_n = U_i + \sum_{i=7}^{14} U_i + U_e.
\]  \hspace{1cm} (26)

II. The Solution Algorithm

To determine the effect of the bonus program, we need to solve the model for \( b = 0 \) and \( b = 500 \) and compare the unemployment rates and conditional reemployment probabilities. To accomplish this, we need estimates of the parameters of the model. Unfortunately, the cost parameters \( (c, c_i, \text{ and } z) \) and total jobs available \( (T) \) are not directly or easily observable. We therefore proceed in two steps. First, since we are focusing on the Illinois bonus experiment, we use data from before the experiment took place to infer values for \( c, c_i, \text{ and } T \), taking \( z \) as given. That is, for an arbitrarily chosen value of \( z \), we determine what values of \( c, c_i, \text{ and } T \) would be consistent with the data collected before the Illinois bonus experiment was implemented. We then fix these values and solve the model with \( b = 500 \) to determine the effect of the program. For each value of \( z \), the model predicts a different bonus-induced change in the duration of unemployment, so we solve the model for a range of values of \( z \) and choose the value of \( z \) that yields the actual effect of the program.

A. Step 1

We begin with the observable parameters. In the Illinois experiment, the average 2-week wage earned by reemployed workers was $500, and the average 2-week UI-benefit equaled $245. In addition, roughly 60% of all unemployed workers in Illinois during the bonus experiment were ineligible for UI, and the average spell of insured unemployment for controls was 22.7 weeks. We therefore set \( w = 500, x = 245, q = .6, \text{ and } d = 22.7 \text{ weeks} \) (or 11.35 2-week periods). In addition, we begin by setting \( b = 0 \).

For values of \( s \) (the separation rate) and \( r \) (the discount rate), we turn to the literature. Research by Ehrenberg (1980), Clark and Summers (1982), and Murphy and Topel (1987) suggests that \( s \) falls somewhere in the range of \( .006 \text{ to } .014 \) (the mean appears to be about \( .01 \) with a standard error of \( .004 \)). We therefore focus on the case in which \( s = .01 \) but also demonstrate that similar results are obtained when \( s = .006 \) or \(.014 \). For \( r \), we consider values from \(.002 \text{ to } .02 \). This translates into annual discount rates ranging from 5% to 67% and therefore should include most relevant values.

Next, we infer \( d \), by using estimates of the relationship between the potential duration of UI benefits and the expected duration of unemployment. Using data obtained during the Illinois experiment, Woodbury (1991)
estimates that eligibility for 1 additional week of UI benefits increases the expected duration of unemployment by about .4 week (this value is based on unadjusted estimates of the conditional probability of reemployment). This estimate is higher than that obtained by Katz and Meyer (1990), who report values in the range of .16-.2. Since $d = 22.7$, using Woodbury's estimate puts $d_i$ at 12.3 weeks, while the lower bound from Katz and Meyer puts $d_i$ at 18.5. Given that Woodbury's estimate is based on the Illinois data, we set $d_i = 12.3$ to compute our main results. However, to ensure that our results are robust, we have also carried out the analysis for $d_i = 18.5$ (as implied by Katz and Meyer's estimate). As we report in Section IV.D, similar results are obtained in both cases.

Finally, to infer values of $c$, $c_i$, and $T$, we adopt the following procedure. First, we note that the model is homogeneous of degree zero in $T$ and $L$, so that we may set $L = 100$ without loss of generality. Next, we set $x$, $\omega$, $q$, $d$, $d_i$, $r$, and $s$ as described above and choose arbitrary values for $T$ and $z$. By treating $d$ as a parameter, we are then able to add (24) and (25) to the model and solve the system treating $c$ and $c_i$ as endogenous. This gives us values of $c$ and $c_i$ that are consistent with the preexperimental data for each $(T, z)$ combination. Finally, we note that as $T$ varies with $L$ fixed, the model generates different values for $V$ and $U$. Evidence provided by Abraham (1983) suggests that $U$ tends to be close to $2V$. Our model predicts that $U = 2V$ when $T$ is approximately 96.25 (the actual value depends on the other parameters). Therefore, we have carried out the analysis for $T$ ranging from 95 to 97.5 (so that $U$ ranges from 1.5$V$ to $3V$). We also note that with these values of $T$, $L$, and $s$, the model predicts an equilibrium unemployment rate in the range of 4.6%-10.1%, which seems appropriate given the time period in which the experiment was conducted. While the values for $c$ and $c_i$ are sometimes sensitive to the values chosen for $T$, $r$, and $s$, as we show below, our overall results concerning displacement are remarkably robust.

B. Step 2

Step 1 yields values of $c$ and $c_i$ for each vector $(z, s, r)$. In step 2, we use these values to estimate the effect of the bonus program. To do so, we continue to assume that $\omega = 500$ and $x = 245$ (the importance of the assumption that the wage is not affected by the bonus is discussed in sec. IV.A below). We then set $b = 500$, choose values for $z$, $s$, and $r$, and impose the values of $c$ and $c_i$ implied by the step 1 computations. The model can then be solved for all endogenous variables including the unemployment rate ($\mu$) and the expected durations of unemployment for insured and uninsured workers ($d$ and $d_i$). Since the model yields a different bonus effect for each value of $z$, we solve the model using various values of $z$ until it yields the observed bonus effect, which was .714 week (Davidson
and Woodbury 1991). Hence, for each vector \((s, r)\), we obtain a value of \(z\) that makes the model’s predicted bonus effect equal to the observed outcome of the Illinois experiment.

### III. Results

#### A. Search Effort and Reemployment Probabilities

It is useful to describe the predicted effects of the bonus program on search effort and reemployment probabilities for three groups of workers: bonus-offered (all of whom are UI-eligible), UI-eligible nonoffered workers (UI-eligible workers who have been unemployed long enough that the bonus offer has expired), and UI-ineligible nonoffered workers (who never had a bonus offer). Figures 2 and 3 show details of the case where \(r = 0.008\), \(s = 0.01\), and \(T = 96.25\).

As figure 2 indicates, the search effort of unemployed workers offered the bonus increases (that is, \(p_i\) through \(p_6\) increase) since the bonus offer increases the expected return to search during the first 6 periods of unemployment (see eq. [18] and the predicted search effort functions plotted with large dots in fig. 2). The relative size of the increase in search effort increases from about 12% in period 1 to about 18% in period 6. That is, newly unemployed workers respond less than those for whom the bonus offer is about to expire because newly unemployed workers face a longer time horizon over which to search and (possibly) collect the bonus. As the spell of unemployment grows longer, workers begin to worry about the bonus offer expiring and respond by searching with greater effort.

The effect of the bonus program on the search effort of workers who are not offered the bonus (that is, UI-eligible workers who have been searching for longer than 6 periods and all UI-ineligible workers) is more subtle. This rivalry effect involves two opposing forces. First, the increased search effort of bonus-offered workers increases the expected number of job applicants at each firm (i.e., \(λ\) rises), making it harder for nonoffered workers to gain employment and lowering their reemployment probability (see eqq. [7]–[10]). This lowered reemployment probability tends to reduce the search effort of nonoffered workers (see eqq. [19]–[21]). Second, the gaps between expected lifetime income if employed and if unemployed increase for nonoffered workers—that is, \(V_{w1} - V_1\) (for \(t > 6\)), \(V_{w} - V_r\), and \(V_{w1} - V_1\), all increase. This happens for two reasons: (i) expected lifetime income while unemployed—\(V_r\) (for \(t > 6\)), \(V_e\), and \(V_r\)—all decrease because with the bonus program in effect, it is harder for nonoffered workers to find a job and expected unemployment duration is longer; and (ii)

---

10 The estimated bonus effect of 0.14 week is for workers who were ineligible for Federal Supplemental Compensation (FSC). This estimate is less than the bonus effect reported by Woodbury and Spiegelman (1987), which is for FSC-eligibles and FSC-ineligibles combined. See Davidson and Woodbury (1991) for details.
$V_w$ increases because when UI-eligible workers lose their jobs, they are now offered the bonus (in eq. [16] for $V_w$). $V_i$ is now larger because it includes the bonus—see in turn eq. [11]). Note that $V_w - V_i$ (for $t > 6$) and $V_w - V_e$ increase by more than $V_w - V_i$ since $V_w$ does not increase as a result of the bonus offer (UI-ineligible workers are not offered the bonus). In any event, the increase in these gaps tends to increase the search effort of nonoffered workers (see eq. [19]–[22]).

Our computed results indicate that for UI-eligible nonoffered workers, the two effects essentially cancel each other out for all relevant parameter values so that the search effort of these workers—$p_i$ (for $t > 6$) and $p_e$—does not change by any significant amount when the bonus program begins. That is, the increases in $V_w - V_i$ (for $t > 6$) and $V_w - V_e$ almost exactly offset the effect of the lowered reemployment probability on search effort. Note that the change in search effort by these workers is very slight and much less than the increase by bonus-offered workers (in fig. 2, the dashed line is only slightly different than the solid line in periods 7 and higher).

For UI-ineligible nonoffered workers, in contrast, the second effect is quite small and the first effect dominates. Thus, the search effort of UI-ineligible workers ($p_i$) falls (by about 0.6%) when the bonus program begins (see the predicted search effort functions plotted with squares in fig. 2). Because UI-eligible and UI-ineligible nonoffered workers respond differently to the increased search effort by bonus-offered workers, the bonus generates displacement among UI-ineligible workers but not among UI-eligible nonoffered workers.

The increased search effort of UI-eligible bonus-offered workers trans-
Fig. 3.—Predicted reemployment probabilities with and without reemployment bonus program.

lates into higher reemployment probabilities for these workers (see the reemployment probability function denoted by the large dots in fig. 3). But for UI-eligible workers who have been unemployed long enough that their bonus offer has expired (that is, for longer than 11 weeks or 6 periods), the fact that their search effort has not changed results in a lower probability of finding a job. That is, the rivalry for jobs created by the greater search effort of bonus-offered workers produces an environment in which the likelihood of UI-eligible nonoffered workers finding a job falls even though they are searching just as hard as before. For UI-ineligible nonoffered workers, a decrease in search effort and the increased rivalry for jobs both lead to a lower reemployment probability (see the reemployment functions denoted by squares in fig. 3).

B. Duration and the Level of Unemployment

Changes in reemployment probabilities induced by the bonus program affect the expected duration and level of unemployment of various groups of workers. Policy issues suggest the importance of focusing on the bonus program's effect on (i) the expected duration of unemployment of UI-eligible and UI-ineligible workers and (ii) the level of unemployment overall, for UI-ineligible workers, and for workers not offered the bonus (i.e., the sum of UI-eligible nonoffered workers and UI-ineligible workers).

Our calibration technique involves choosing \( z \) (the elasticity of search cost with respect to search effort) such that our model's predicted change in \( d \) (the expected duration of unemployment for UI-eligible workers) matches the change observed in the Illinois experiment. Therefore, the
Table 1
Bonus-induced Changes in Unemployment Duration and Unemployment, Job Matching Model with Rivalry Effect

<table>
<thead>
<tr>
<th></th>
<th>$s = .006$</th>
<th></th>
<th>$s = .010$</th>
<th></th>
<th>$s = .014$</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$T = 96.25$</td>
<td>$T = 97.5$</td>
<td>$T = 95$</td>
<td>$T = 96.25$</td>
<td>$T = 97.5$</td>
</tr>
<tr>
<td>$\Delta d_d$</td>
<td>-.714</td>
<td>-.714</td>
<td>-.714</td>
<td>-.714</td>
<td>-.714</td>
</tr>
<tr>
<td>$\Delta d_i$</td>
<td>.416</td>
<td>.286</td>
<td>.333</td>
<td>.269</td>
<td>.225</td>
</tr>
<tr>
<td>$\Delta U_d$</td>
<td>-.010</td>
<td>-.031</td>
<td>-.038</td>
<td>-.053</td>
<td>-.065</td>
</tr>
<tr>
<td>$\Delta U_i$</td>
<td>.071</td>
<td>.050</td>
<td>.093</td>
<td>.077</td>
<td>.065</td>
</tr>
<tr>
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<td>.015</td>
<td>-.008</td>
<td>.000</td>
<td>-.018</td>
<td>-.031</td>
</tr>
<tr>
<td>$z$ (for $s = .002$)</td>
<td>1.087</td>
<td>1.122</td>
<td>1.164</td>
<td>1.196</td>
<td>1.226</td>
</tr>
<tr>
<td>$\beta$ (for $s = .008$)</td>
<td>1.149</td>
<td>1.204</td>
<td>1.238</td>
<td>1.280</td>
<td>1.319</td>
</tr>
<tr>
<td>$z$ (for $s = .020$)</td>
<td>1.269</td>
<td>1.357</td>
<td>1.378</td>
<td>1.447</td>
<td>1.489</td>
</tr>
</tbody>
</table>

Note.—$s =$ biweekly separation rate; $T =$ total available jobs; $\Delta d =$ change in expected duration of unemployment for UI-eligible workers (in weeks); $\Delta d_i =$ change in expected duration for UI-ineligible workers (in weeks); $\Delta U =$ change in total number of unemployed workers (= change in unemployment rate); $\Delta U_d =$ change in number of UI-eligible unemployed workers; $\Delta U_i =$ change in number of unemployed workers not offered the bonus; $z =$ parameter in the search-cost function; $\beta =$ biweekly interest rate. Estimates of $\Delta d, \Delta d_i, \Delta U, \Delta U_d,$ and $\Delta U_i$ are nearly invariant to changes in $s$; figures shown are for $s = .008$. Computed values of $z$ vary with respect to $\beta$ as shown. In the absence of the bonus, unemployment ($U$) equals 4.6 when $s = .006$, 7.43 when $s = .01$, and 10.1 when $s = .014$. Also in the absence of the bonus, $d_d$ is set to 22.7 weeks and $d_i$ to 12.3 weeks.
rows in table 1 labeled $\Delta d$ all have the same entry ($-0.714$). The value of $z$ that produces this result is listed for each combination of parameters. Note that this value depends on the interest rate and the separation rate, with increases in either resulting in a higher value for $z$.

Since UI-ineligible workers respond to the bonus by reducing search effort, their expected duration of unemployment rises (see the rows labeled $\Delta d$ in table 1). The results suggest that UI-ineligible workers' expected duration increases by between 0.202 and 0.414 weeks. The increase in expected duration for UI-ineligible workers is usually less than half as large as the decrease for UI-eligible workers (the absolute value of $\Delta d_i/\Delta d$ is between 0.283 and 0.580). Note that although our estimate of $z$ varies with $r$, none of the model's other predictions depends on $r$. Therefore, all entries in tables 1–4 hold for all relevant values of the interest rate.

Regarding unemployment, the model predicts that the bonus program reduces equilibrium unemployment (see the rows labeled $\Delta U$ in table 1). The number of UI-ineligible unemployed workers does increase (see the rows labeled $\Delta U_i$ in table 1), as a result of both their own reduced search efforts and the increased search efforts of bonus-offered workers. But the number of all nonoffered unemployed workers ($U_n$, the sum of UI-eligible nonoffered and UI-ineligible workers) decreases for most relevant values of $r$, $s$, and $T$—see the rows labeled $\Delta U_n$ in table 1. The reason is that the increased search effort of bonus-offered workers results in increased employment (see below) and a drop in the number of UI-eligible nonoffered workers that exceeds the rise in the number of UI-ineligible unemployed workers ($U_i$).

The predicted changes in unemployment are small: the decrease in total unemployment ($\Delta U$) never exceeds 0.1% (note that in several cases about one-half of this reduction comes from the fall in unemployment of nonoffered workers). Also, the increase in unemployment of UI-ineligible workers never exceeds 0.102%. These quantitative effects are insensitive to changes in the parameters. Note also that, because $U$ can be interpreted as the unemployment rate (since $L = 100$), the decreases in unemployment imply equal decreases in the unemployment rate. Finally, changes in unemployment for bonus-offered and UI-eligible workers can be inferred from table 1 by subtracting $\Delta U_n$ and $\Delta U_i$, respectively, from $\Delta U$.

C. Employment Effects and Displacement

The results to this point suggest that the numbers of bonus-offered and (usually) nonoffered unemployed workers fall, and the overall unemployment rate falls, as a result of the bonus program. The reason is that the bonus program leads to increased search effort of bonus-offered workers, which leads in turn to more of the total jobs available in the economy being filled rather than left vacant—referring back to identity (1), $J$ rises and $V$ falls correspondingly.
Table 2
Employment Changes for UI-eligible and UI-ineligible Workers, Model with Rivalry Effect (per 100,000 in Labor Force)

<table>
<thead>
<tr>
<th></th>
<th>$s = 0.006$</th>
<th>$s = 0.010$</th>
<th>$s = 0.014$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T = 96.25$</td>
<td>$T = 97.5$</td>
<td>$T = 96.25$</td>
</tr>
<tr>
<td>Unemployed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>workers</td>
<td>213</td>
<td>212</td>
<td>353</td>
</tr>
<tr>
<td>UI-ineligible</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>workers</td>
<td>-124</td>
<td>-82</td>
<td>-168</td>
</tr>
</tbody>
</table>

Note: $s$ = biweekly separation rate; $T$ = total available jobs. Estimates are nearly invariant to changes in the biweekly interest rate ($r$); figures shown are for $r = 0.03$.

Our motivating question is whether the bonus program results in any displacement—that is, an increase in unemployment among groups of workers not offered the bonus. Two kinds of displacement can be examined in this model. The first would be represented by an increase in the number of unemployed workers who are not offered the bonus ($U_u$, the sum of workers whose bonus offer has expired and UI-ineligibles who are never offered the bonus). It is clear from table 1 that $U_u$ usually falls as a result of the bonus program. But there is one case in which $U_u$ increases—specifically, when total available jobs ($T$) and the separation rate ($s$) are both low. Slack demand (low $T$) and low turnover (low $s$) both characterize a recession, so our model implies that a bonus program is most likely to displace nonoffered workers during a recession and may be a rather unattractive policy for increasing employment during a period of slack demand.

A second kind of displacement would be an increase in the number of UI-ineligible workers—that is, unemployed workers who never receive either a bonus offer or UI benefits. It is clear from table 1 that the model predicts that the bonus program always results in displacement of UI-ineligible workers. To examine the size of this second displacement effect, we display in table 2 the employment gains of UI-eligible workers and the employment losses of UI-ineligible workers that result from the bonus program. The gain for UI-eligible workers is the gross employment effect referred to in the introduction, and the loss for UI-ineligible workers is a displacement effect. Although the changes in employment depend on the values of the parameters, the results are remarkably robust. Note that higher separation rates increase both the absolute and relative effect of the bonus program: As $s$ increases, the employment gains of UI-eligible workers rise and the ratio of employment gains of UI-eligible to employment losses of UI-ineligibles rises (this ratio ranges from 1.7 to 3.2, and is also increasing

\[ \text{The employment gains of UI-eligible workers are defined as } \Delta U - \Delta U, \text{ divided by the number of jobs held by UI-eligible workers, } (1-q)J. \]
in $T$, the total number of jobs available). Intuitively, higher values of $s$ imply faster job turnover and a higher equilibrium unemployment rate. Moreover, it is when unemployment is high that increases in search activity have the biggest effect on the economy. Therefore, the increase in search activity has a bigger effect on the economy when jobs are not very enduring. In all cases, however, the bonus program’s effects on employment are rather small, never exceeding an increase of 500 per 100,000 for UI-eligible workers or a decrease of 190 per 100,000 for UI-ineligible workers.

Our findings on displacement can be summarized as follows. The bonus program, which results in improvements for bonus-offered workers, leads to displacement of nonoffered workers as a whole (the sum of workers whose bonus offer has expired and UI-ineligibles) only when demand is slack and separation rates are low—that is, under conditions characteristic of a recession. (Even when such displacement occurs, the change in nonoffered unemployment due to the bonus program is trivial.) But UI-ineligible workers taken alone (i.e., unemployed workers who never receive UI benefits or a bonus offer) clearly experience displacement as a result of the bonus program. And again, displacement of these workers is greatest in relative terms under conditions characteristic of a recession—slack demand and low turnover.

D. Displacement and Rivalry

In most cases, our model predicts that the bonus program will not displace UI-eligible nonoffered workers, although in all cases there is some displacement of UI-ineligibles. There are two reasons for this. First, although increased search effort by bonus-offered workers reduces somewhat the reemployment probability of all nonoffered workers, it also improves the performance of the economy by increasing employment (with increased search activity, the economy is able to fill more of the total available jobs in each period). These newly filled jobs will eventually break up and produce vacancies that can be filled either with bonus-offered or nonoffered workers. This naturally benefits all unemployed workers. Second, increased search effort by bonus-offered workers triggers a reduction in search effort by UI-ineligibles. This rivalry effect increases the displacement of UI-ineligible workers.

Since the rivalry effect is indirect and requires nonoffered workers to appreciate subtle effects of the bonus program, it is important to know the extent to which our results are driven by this force. Whether the rivalry effect is important can be appraised by solving the model with the bonus program in effect holding the search efforts of nonoffered workers fixed at their prebonus levels. Tables 3 and 4 are equivalent to tables 1 and 2 but omit the rivalry effect. A comparison of these tables makes it clear that the rivalry effect is small, but not insignificant. For example, the effect of the bonus program on expected duration of unemployment is reduced by anywhere from 14% to 17% when the rivalry effect is omitted. The
Table 3
Bonus-induced Changes in Unemployment Duration and Unemployment, Job Matching Model without the Rivalry Effect

<table>
<thead>
<tr>
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<th>s = .006</th>
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<th></th>
<th>s = .014</th>
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<tbody>
<tr>
<td></td>
<td>T = 96.25</td>
<td>T = 97.5</td>
<td>T = 95</td>
<td>T = 96.25</td>
<td>T = 97.5</td>
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<tr>
<td>Ad</td>
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<td>-.697</td>
<td>-.676</td>
<td>-.689</td>
<td>-.693</td>
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<tr>
<td>Ad</td>
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<td>.243</td>
<td>.279</td>
<td>.226</td>
<td>.198</td>
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<tr>
<td>∆UT</td>
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<td>∆UT</td>
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<td>∆UT</td>
<td>.010</td>
<td>-.015</td>
<td>-.009</td>
<td>-.025</td>
<td>-.038</td>
</tr>
</tbody>
</table>

NOTE.—s = biweekly separation rate; T = total available jobs; Ad = change in expected duration of unemployment for UI-eligible workers (in weeks); Ad = change in expected duration for UI-ineligible workers (in weeks); ∆UT = change in total number of unemployed workers (= change in unemployment rate); ∆UT = change in number of UI-eligible unemployed workers, ∆UT = change in number of unemployed workers not offered the bonus. Estimates of Ad, Ad, ∆UT, ∆UT, and ∆UT are nearly invariant to changes in the biweekly interest rate (r); figures shown are for s = .008. In the absence of the bonus, unemployment (UT) equals 4.6 when s = .006, 7.43 when s = .001, and 10.1 when s = .014. Also in the absence of the bonus, d is set to 22.7 weeks and d to 12.3 weeks.

Implication is that the gross employment effect is the main force that drives our displacement results. \footnote{A further caveat that should be made is that our model implies an average duration of job vacancies of about 4–8 weeks (depending on T, with the duration of vacancies increasing in T). This is somewhat longer than available empirical estimates of the average duration of a job vacancy. Holzer’s (1989) estimates suggest a mean vacancy duration of 1.2–3.1 weeks (depending on occupation) in a relatively slack labor market, and Woodbury’s (1992) estimates suggest a mean vacancy duration of 2.4–5.3 weeks (depending again on occupation) in a relatively tight labor market. Whether this difference between the implications of our model and existing empirical estimates should be a cause for concern is unclear. First, the empirical estimates of vacancy duration are higher for skilled than for unskilled occupations, and UI-eligible workers tend to be more skilled. Second, during the mid-1980s, the average duration of insured unemployment was 21%–24% higher in Illinois than in most other states (U.S. Department of Labor, various issues), suggesting that vacancy durations could also be higher than average there. The vacancy durations implied by our model, then, may not greatly exceed the vacancy durations that existed in northern Illinois during the mid-1980s.}

IV. Extensions and Sensitivity

Our model includes a number of assumptions that are used to keep the analysis tractable. In this section, we discuss extensions of the model that relax some of the more troubling assumptions and report how the results are affected.

A. Wage Determination

The Illinois bonus experiment had virtually no effect on the earnings after reemployment of workers who were offered the bonus (Woodbury and Spiegelman 1987, table 5). This provides at least one defense for our
Table 4
Employment Changes for UI-eligible and UI-ineligible Workers, Model without the Rivalry Effect (per 100,000 in Labor Force)

<table>
<thead>
<tr>
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<th>s = .010</th>
<th>s = .014</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>T = 96.25</td>
<td>T = 97.5</td>
<td>T = 98.5</td>
</tr>
<tr>
<td>UI-eligible</td>
<td>196</td>
<td>207</td>
<td>232</td>
</tr>
<tr>
<td>workers</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>workers</td>
<td></td>
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</tr>
</tbody>
</table>

Note.—s = biweekly separation rate; T = total available jobs. Estimates are nearly invariant to changes in the biweekly interest rate (r); figures shown are for r = .008.

treatment of wages as exogenous. Nevertheless, it seems important to verify whether the empirical finding that wages are unaffected by the bonus program is consistent with a matching model in which wages are endogenous. Accordingly, in the working paper precursor to this article (Davidson and Woodbury 1990), we present an extension of our model in which wages are determined through negotiation once a job contact is made. This is accomplished by introducing profit functions for the firm similar to the value functions that describe expected lifetime income for workers (eqq. [11]–[17]). There are two profit functions. One calculates expected lifetime profit for a firm employing a worker and paying a wage w, while the other calculates expected lifetime profit for a firm with a vacancy. Following Diamond (1982), we then assume that the negotiations result in a wage that splits the surplus generated by the job evenly.

The main finding in this extended model is that wages vary little when they are treated as endogenous (they increase only slightly for all workers, with the wages of bonus-offered workers rising slightly more). As noted above, this is consistent with empirical findings from the Illinois experiment. With only a small change in wages, we would not expect the results in the extended model to differ much from those obtained above, and they do not. In general, the effect on expected duration of unemployment is slightly higher, and the displacement effect is slightly higher in that there are more cases in which unemployment of workers not offered the bonus rises as a result of the program. Otherwise, the predicted effect of the program is remarkably similar. We conclude that our fixed wage assumption is largely innocuous.

B. Labor Demand

By assuming that T is unaffected by the bonus program, we are ignoring the effect that the program might have on labor demand. In this sense, our model is only a slight improvement on the partial-partial search models that ignore both firm behavior and equilibrium conditions. Using the model described in subsection A above, it is possible to extend the model even
further to make T endogenous. Following Pissarides (1990), this can be done by assuming that firms create vacancies until the expected lifetime profit from doing so equals the associated fixed cost. Although this greatly complicates the analysis, it is clear that such an extension would reduce the predicted displacement effect. Since the bonus increases the search effort of bonus-offered workers and reduces overall unemployment, the bonus also increases the expected return to a created vacancy (since vacancies become easier to fill). Therefore, T should rise as a result of the bonus program. These newly created available jobs benefit all workers and reduce unemployment even further (holding all parameters including z fixed, the displacement effect is decreasing in T). We conclude that endogenizing labor demand would reduce the predicted displacement effect.

C. Job Separations

Because the bonus program makes it easier to fill vacancies, it might affect the job separation rate, s. In particular, the bonus program could make firms more prone to terminate workers for cause and less prone to create conditions that would lower the probability of voluntary quitting. Terminations and voluntary quits create vacancies that are costly to fill, and because the bonus program reduces these costs, they (and s) should rise as a result of the program. In contrast, if the bonus program increases labor demand, as argued in subsection B above, s might fall as firms retain workers who would have otherwise separated. It seems likely, however, that the first effect would dominate and the bonus program would increase the separation rate.

An increase in s affects our results in two ways. First, with greater labor market turnover, there is more scope for the bonus program to work. This is clear from tables 1–4 where the program’s effect rises with s. However, increasing s increases the equilibrium unemployment rate, and this harms all workers. Therefore, we would expect the duration of unemployment to fall when s rises, but the displacement effect might actually rise. To check this intuition, we resolved the base case of the model (r = .008, s = .01, T = 96.25) assuming that s increases as a result of the bonus. (This involves imposing an arbitrary higher value of s in step 2 of the solution algorithm.) We first confirmed that our intuition concerning the effect of the duration of unemployment and displacement was correct. We then calculated the increase in s that would be necessary to result in complete displacement. We found that an increase in s of about 5.5% does the job (i.e., s must increase from .01 to .01035). With a bonus-induced increase

13 The bonus program would be unlikely to affect other types of separation; in particular, the program would not affect any employer-initiated separation that results mainly from demand shifts or technological change. Examples would be temporary layoffs (which result in vacancies that can be filled virtually without cost) and permanent layoffs (which create no vacancies).
in $s$ of 3.5%, the bonus program reduces the expected duration of unemployment of UI-eligibles ($d$) by 1.135 weeks (compared with .714 in table 1) and lowers the expected duration of unemployment of UI-ineligibles ($d_1$) by .013 weeks (compared with an increase of .269 in table 1). Regarding displacement, employment of UI-eligible workers increases by only 194.5 jobs and employment of UI-ineligible workers declines by 194 jobs—displacement is complete.

Whether the bonus program could induce an increase in $s$ as large as 3.5% is a matter of speculation. We would note that the changes in unemployment that are predicted by the model are well below 3.5%, but the predicted changes in expected duration of unemployment ($d$ and $d_1$) are generally in excess of 3.5%. A bonus-induced change in $s$ as large as 3.5% cannot be ruled out.

D. Sensitivity

Tables 1–4 show that our results are robust with respect to changes in $s$, $r$, and $T$. However, one other key parameter that is not directly observed is $d_1$, the expected duration of unemployment for UI-ineligibles. We noted above that in calculating $d$, we use Woodbury's (1991) estimate of the effect of the potential duration of UI benefits on expected duration of unemployment and that this is roughly double the estimate obtained by Katz and Meyer (1990). We therefore solved the base case ($r = .008, s = .01, T = 96.25$) following the same calibration technique using $d_1 = 18.5$ weeks, as implied by Katz and Meyer's estimate. Doing so yields results remarkably similar to those reported above: $d$, rises by .267 weeks (compared with .269 from table 1), and the changes in unemployment are virtually the same (although the estimated value of $z$ rises from 1.280 to 1.406). With respect to displacement, using $d_1 = 18.5$ weeks implies that UI-eligible employment rises by 352 while UI-ineligible employment falls by 159 (compared with 351 and 138, respectively, from table 2). We conclude that our results are quite insensitive to the assumed value of $d_1$.

V. Conclusion

We have used an institutionally rich trade frictions model to examine the effect of a reemployment bonus program on the search effort, expected duration of unemployment, and the level of unemployment of various groups of workers. An important distinction we draw is between UI-eligible workers (who begin with a bonus offer and lose it if they remain unemployed beyond a specified qualification period) and UI-ineligible workers (who never get a bonus offer).

The model's results suggest that under a reemployment bonus program, bonus-offered workers would increase their search effort and experience higher reemployment probabilities as a result. Workers eligible for UI for whom the bonus offer has expired do not alter their search activity by any
significant amount, but the competition for jobs generated by the greater search effort of bonus-offered workers results in a lower reemployment probability for these workers. For UI-ineligible workers (who are never offered a bonus), search effort falls in response to the bonus program, as does the probability of reemployment. These results are summarized in figures 2 and 3.

How do these changes in search effort and reemployment probabilities affect expected durations of unemployment? For UI-eligible workers taken as a whole, the expected duration of unemployment (\(d\)) falls by about .7 week. But for UI-ineligible workers, who are never offered the bonus and who reduce their search effort in response to the bonus program, expected duration of unemployment (\(d_i\)) rises by between .2 and .4 week (see the results displayed in table 1).

Regarding unemployment levels, we focus on overall unemployment (\(U\)), unemployment of UI-ineligible workers (\(U_i\)), and unemployment of all workers not offered the bonus (\(U_o\), the sum of workers whose bonus offer has expired and UI-ineligibles). Overall unemployment and (for most cases we examine) unemployment among all workers not offered the bonus both fall in response to the bonus program. The number of unemployed UI-ineligible workers, however, increases (see again table 1). In contrast to the bonus-induced changes in expected duration of unemployment, which appear significant, the bonus-induced changes in unemployment predicted by the model are quite small.

Variation of displacement with the level of unemployment is an important feature of the results. One type of displacement that we have considered—displacement by bonus-offered workers of all workers who are not offered the bonus—seems at first not to be a problem: unemployment of all nonoffered workers (\(U_o\)) rises in only one case we examine. But the case in which unemployment of nonoffered workers does increase as a result of the bonus program is troubling—nonoffered workers are displaced when total available jobs and the separation rate are both low, that is, under conditions characteristic of the trough of a recession.\(^{14}\)

Another type of displacement is equally important—displacement by UI-eligible workers of UI-ineligible workers (unemployed workers who never receive either a bonus offer or UI benefits). The model predicts that the bonus program always results in displacement of UI-ineligible workers (\(\Delta U\); is always positive in table 1). Admittedly, the first of these two displacement effects (when it exists) is small, and the second is always small. As table 2 shows, displacement of UI-ineligible workers as a result of the bonus program never exceeds 190 per 100,000, or 1.9 per thousand. Nevertheless, it is clear that adoption of a reemployment bonus program would not yield a Pareto improvement.

\(^{14}\) The separation rate peaks at the onset of a recession, then falls once employers have shed labor and reached their desired (lower) employment level.
That is, 30%-60% of the gross employment effect of the bonus program is offset by displacement of UI-ineligible workers (see table 2). In the likely absence of compensation schemes, it may be difficult to justify the displacement of any UI-ineligible workers (who have weaker work histories and could be at a disadvantage) by UI-eligible workers (who generally have more experience and stronger work histories).

Are there possible ways of overcoming the displacement problem? As long as the reemployment bonus program is part of the Unemployment Insurance system, as has been proposed, the answer is no. In short, the displacement effects of a reemployment bonus program, and the finding that displacement is greatest under conditions characteristic of the trough of a recession, may make the reemployment bonus a rather unattractive policy alternative.

We believe that the work described here has implications both for modeling the effects of various labor market programs and for future data-gathering efforts. Clearly, a trade frictions model of the kind developed here could be used to investigate the features of an optimal Unemployment Insurance system—the optimal benefit amount, the optimal potential duration of benefits, and whether intensive job search assistance would be efficient. The approach could also be used to investigate the effects of programs not related to the UI system—subsidized training programs and targeted job tax credits, for example. The model is relatively simple in structure but it is rich enough to capture important features of the labor market, such as job turnover and search behavior, that are slighted in other classes of models. Also, since it is an equilibrium model, it is capable of capturing both direct program effects and program-induced feedback effects.

If investigations using the trade frictions approach are to be convincing, however, it is important that estimates of the parameters underlying the model be convincing. We had to supply several such parameters from sources beyond the administrative data that were gathered to evaluate the Illinois experiment. Some of the parameters that we had to supply do not seem to pose significant troubles. The sensitivity checks that we performed with respect to the interest rate (\(r\)), the separation rate (\(s\)), and the expected duration of unemployment for UI-ineligibles (\(d_u\)) suggest that our results vary little in response to rather wide swings in these parameters. But we were able to rely on existing empirical work to peg all three of these parameters within some reasonable range.

In contrast, there is far less certainty about the other parameters that we had to supply—\(c\) and \(c_i\) (the parameters of the search cost functions for UI-eligibles and UI-ineligibles), \(z\) (the elasticity of search cost with respect to search effort), and \(T\) (total jobs available, or \(j + U\)). It is especially troubling that there exist no empirical estimates of \(c\), \(c_i\), or \(z\) with which to compare the estimates derived from our solution algorithm because our results are sensitive to variations in these parameters. For example,
the apparently high duration of job vacancies implied by our model (see n. 12 above) could reflect a set of \( c, c_i, \) and \( z \) values that we have been unable to peg more realistically for lack of data. It seems clear that empirical work providing direct evidence on these theoretically important parameters would pay significant dividends—such estimates would be both intrinsically interesting and useful to further applied work along the lines we have pursued here.

Appendix

Summary of Notation

\[ \begin{align*}
  b & = \text{reemployment bonus amount} \\
  c & = \text{parameter of the cost of search function \( (zcp^{2-1} = \text{the marginal cost of search effort for a worker in the \( t \)th period of search}) \)} \\
  d & = \text{expected duration of unemployment of UI-eligible workers} \\
  d_i & = \text{expected duration of unemployment of UI-ineligible workers} \\
  j & = \text{steady-state number of jobs} \\
  L & = \text{total labor force \( (J + U) \)} \\
  m_e & = \text{conditional probability of reemployment in period \( t \) for a UI-eligible worker \( (t < 7 \) for bonus-offered workers, \( t > 6 \) for nonoffered workers) } \\
  m_e & = \text{conditional probability of reemployment for a worker who has exhausted UI benefits} \\
  m_i & = \text{conditional probability of reemployment for a UI-ineligible worker} \\
  p & = \text{search effort (probability of contacting a firm) in period \( t \) for a UI-eligible worker \( (t < 7 \) for bonus-offered workers, \( t > 6 \) for nonoffered workers) } \\
  p_e & = \text{search effort of a worker who has exhausted UI benefits} \\
  p_i & = \text{search effort of a UI-ineligible worker} \\
  q & = \text{proportion of newly unemployed workers who are not eligible for UI benefits} \\
  r & = \text{biweekly interest rate} \\
  s & = \text{biweekly separation rate (rate at which jobs break up)} \\
  T & = \text{total available jobs \( (J + V) \)} \\
  U & = \text{total number of unemployed workers} \\
  U_e & = \text{number of UI-eligible workers in their \( t \)th period of search \( (t < 7 \) for bonus-offered workers, \( t > 6 \) for nonoffered workers) } \\
  U_e & = \text{number of unemployed workers who have exhausted their UI benefits} \\
  U_i & = \text{number of UI-ineligible unemployed workers (these workers were never eligible for UI benefits) } \\
  U_n & = \text{total number of unemployed workers not offered the bonus (the sum of \( U_t \) for \( t > 6 \), \( U_e \), and \( U_i \) )} \\
  V & = \text{steady-state number of vacancies} \\
  V_e & = \text{the expected lifetime income for a UI-eligible worker in the \( t \)th period of search \( (t < 7 \) for bonus-offered workers, \( t > 6 \) for nonoffered workers) } \\
\end{align*} \]
\( V_e \) = the expected lifetime income for an unemployed worker who has exhausted UI benefits

\( V_s \) = the expected lifetime income for a UI-ineligible worker

\( V_{se} \) = the expected lifetime income for an employed worker who would be eligible for UI if unemployed

\( V_{se} \) = the expected lifetime income for an employed worker who would not be eligible for UI if unemployed

\( \omega \) = the wage earned by all employed workers

\( x \) = unemployment benefits

\( z \) = the elasticity of search costs with respect to search effort

\( \lambda \) = the average number of applications filed at each firm

\( \mu \) = the unemployment rate \((U/L)\)

**Values of the Key Variables in the Base Case**

\((r = .008, s = .01, T = 96.25)\)

With \( b = 0 \):

\[ f = 92.57540, \]
\[ V = 3.67455, \]
\[ \lambda = .29027, \]
\[ U_i - U_s \text{ decreases monotonically from .37030 to .25836}, \]
\[ U_i - U_{se} \text{ decreases monotonically from .23861 to .12088}, \]
\[ U_i = .90757, \]
\[ U_i = 3.22163, \]
\[ m_1 - m_6 \text{ increases monotonically from .065542 to .076465}, \]
\[ m_2 - m_4 \text{ increases monotonically from .079468 to .11753}, \]
\[ m_5 = .11753, \]
\[ m_6 = .17241, \]
\[ p_1 - p_6 \text{ increases monotonically from 1.97798 to 2.30764}, \]
\[ p_7 - p_4 \text{ increases monotonically from 2.39825 to 3.54705}, \]
\[ p_7 = 3.54702, \text{ and} \]
\[ p_8 = 5.20326. \]

With \( b = 500 \):

\[ f = 92.6288, \]
\[ V = 3.62122, \]
\[ \lambda = .29544, \]
\[ U_i - U_s \text{ decreases monotonically from .37052 to .24679}, \]
\[ U_i - U_{se} \text{ decreases monotonically from .22497 to .11542}, \]
\[ U_i = 0.88730, \]
\[ U_i = 3.29815, \]
\[ m_1 - m_6 \text{ increases monotonically from .072405 to .088441}, \]
\[ m_2 - m_4 \text{ increases monotonically from .078162 to .11511}, \]
\[ m_5 = .11511, \]
\[ m_6 = .16851, \]
\[ p_1 - p_6 \text{ increases monotonically from 2.22272 to 2.71499}, \]
\[ p_7 - p_4 \text{ increases monotonically from 2.39949 to 3.53364}, \]
\[ p_c = 3.53364, \text{ and } \]
\[ p_s = 5.17304. \]

References


