Multidivisional Firms, Internal Competition and the Merger Paradox

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Abstract: The traditional approach to modeling mergers assumes that after the merger, the merged firms cooperate and maximize joint profits. This approach has generated a number of intuitively unappealing results such as (1) mergers are not beneficial for the merging parties in quantity-setting games (unless the vast majority of the industry is included in the merger; and, (2) outsiders always gain more than insiders in both quantity and price setting games. In this paper we take a different approach to mergers. We assume that after the merger, the parent company treats each insider as a separate division and plays the divisions off against one another in a manner that allows the parent firm to gain a strategic advantage over the outsiders. As a result, we are able to present a quantity-setting model of horizontal mergers in which insiders gain from the merger and outsiders lose. In addition, in our setting, mergers are socially beneficial and, if merging to monopoly is precluded by anti-trust concerns, then the optimal merger involves a relatively small number of firms. Finally, we show that while such a strategy would work in price-setting games as well, the parent company will not choose employ it, since it can always do better by treating the merger in a conventional manner (where the divisions cooperate and maximize joint profits).

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The theoretical literature on horizontal mergers is filled with many curious results. In quantity-setting games without cost synergies, mergers are not typically beneficial for the merging parties unless they include the vast majority of industry participants (Salant, Switzer, and Reynolds 1983). In contrast, outsiders (firms not involved in the merger) benefit from any merger. In price-setting games, mergers are always beneficial for the insiders (the firms that merge), but outsiders gain more than the insiders (Deneckere and Davidson 1985). This latter result makes it hard to explain how merger activity gets started, since a firm would always prefer to remain an outsider.1 Finally, it is difficult to write down a model in which mergers are beneficial and the optimal size of a merger includes only a small subset of industry participants.

We believe that the vast majority of economists are uncomfortable with at least some of these predictions – they just do not seem right. Most mergers, even in the absence of cost synergies, ought to be profitable since they provide the merging parties with increased market power that they should be able to exploit in a beneficial manner. In addition, it is hard to believe that outsiders gain when their rivals collude, especially in markets characterized by strategic substitutes.2 Finally, although it is intuitively plausible that firms would want to merge all the way to monopoly if the anti-trust authorities were willing to allow it, it appears that in many markets concentration remains well below what the anti-trust authorities seem to tolerate without igniting a wave of merger activity. Yet, providing a model consistent with these notions has proven difficult, giving rise to what has become known in the Industrial

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1 Kamien and Zang (1990) point out that this result makes it hard to prevent defections from a proposed merger even when the proposed merger is beneficial. In fact, in their setting, the stability problem is so severe that mergers occur only if there are very few firms in the market (for non-strategic mergers there must be no more than 7 firms with concave demand – with linear demand a merger occurs only when there are 2 firms in the market). In addition, for mergers to be beneficial, they show that at least half of the firms in the market must be included. Since DOJ guidelines effectively try to prevent any merger of more than 50% of the industry, it is surprising that we see any mergers in the US at all!

2 The empirical evidence on the impact of horizontal mergers on outsiders seems to support the view that outsiders are harmed (see Eckbo 1983, Stillman 1983, and Banerjee and Eckard 1998).
Organization literature as the “merger paradox.”

In this paper, we provide a model of horizontal mergers that generates intuitively appealing predictions. In particular, we provide a quantity-setting model in which mergers are beneficial for the merging parties but harmful for outsiders. Moreover, in our model, if merger to monopoly is precluded by anti-trust concerns, the optimal merger includes only a relatively small number of industry participants. Thus, our model yields predictions that match (what we believe to be) the commonly held view of how horizontal mergers affect equilibrium outcomes.

While previous models of horizontal merger have focused on the impact of reducing the degree of competition between the insiders on their profits and those of the outsiders, we take a different approach. We argue that horizontal mergers are beneficial for the merging parties because there are strategies available to a multidivisional firm (that are not available to single division firms) that can be exploited to increase profits above what the divisions would earn if they were entirely independent. The existence of these strategies provides an incentive for merger, even if the industry participants engage in Cournot competition.

The key difference between our approach to horizontal mergers and the traditional approach can be traced to the assumptions that we make about the internal structure of the multidivisional firm formed by the merger. Instead of assuming that the merged firms cooperate with each other and choose output to maximize joint profits, we assume that after the merger the new parent company exploits the strategic possibilities that are inherent from having distinct divisions. This is accomplished by delegating the output decision to the manager of each division and setting up an internal game in which the divisions compete

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3 Pepall, Richards, and Norman (1999) describe the merger paradox as follows: “What may be surprising to you is that it is, in fact, quite difficult to construct a simple economic model in which there are sizable profitability gains for the firms participating in a horizontal merger that is not a merger to monopoly. This difficulty is what we call the merger paradox.”
against one another. The parent company then structures the internal game in a manner that allows it to achieve a strategic advantage over the outsiders. This leads to higher joint profits for the insiders and lower profits for the outsiders. In this paper, we introduce one way of structuring the internal game, staggered competition, and show how a multidivisional firm can use such a strategy to gain a strategic advantage over its rivals.

To see how staggered competition works, consider a market in which \( N \) symmetric single division firms produce identical products and engage in Cournot competition. Now, suppose that a multidivisional firm consisting of \( M \) divisions forms when \( M < N \) of these firms merge. After the merger, we assume that the parent company gives each division great independence, assigns a manager to run each division and ties that manager’s compensation to the division’s profitability (so that each manager chooses output to maximize the profit of his division).\(^4\) The only exception to this independence is that we assume that the parent company may not treat all divisions in the same manner. In particular, we allow the parent company to stagger the output decisions of the divisions so that some divisions may be forced to set output after other divisions have committed to certain output levels. This effectively makes some divisions Stackelberg leaders within the multidivisional firm. Outside firms, while aware of the interior structure of the multidivisional firm, treat each division of the merged firm as a Cournot competitor. That is, after the merger, we have a game in which there is staggered competition within the multidivisional firm and Cournot

\(^4\) There is ample evidence that many multidivisional firms are run in this manner. For example, the different brands at Miller use different advertising agencies, signifying their independence (see Stewart 1994). In the hotel industry, the Bass Hotel chain (which owns Holiday Inn, Crowne Plaza, Staybridge Suites and other hotels) makes a point of emphasizing that its Crowne Plaza division has established its own independent reputation for quality (see “Bass PLC – Final Results” in Regulatory News Service December 8, 1999). Hotel franchises obviously have a great deal of autonomy. Moreover, Conlin (2000) finds that franchising (versus corporate owned hotels) creates greater competition for other hotels of the same brand. The importance of giving divisions their independence is stressed both in Williamson (1975) and Milgrom and Roberts (1992). For evidence that compensation for division managers is tied to the division’s profitability, see Murphy (1985), Milgrom and Roberts (1992), or Conyon (1997).
competition between the outsiders and the merged firm.

By staggering competition, the parent firm is taking advantage of the fact that all of the divisions are part of the same company. This allows the division managers to share information about production plans that would not be shared if the divisions were separate. The reason that the parent firm wants to stagger competition is that it makes the leading divisions more aggressive and allows the merged firm to capture a larger share of the market. The divisions that follow are necessary since they provide the commitment device that the leaders need in order to credibly expand their market share.

The increased aggression generated by staggered competition has several effects. The leading divisions earn higher profits than before while the followers that move last see their profits shrink. We show, however, that for most mergers the leading divisions gain more than the followers lose, so that joint profits for the insiders rise. These gains diminish as the number of insiders increases, since the merger works by taking market share from outsiders. It follows that staggered competition is most effective when the number of outsiders remains relatively large. This explains why the optimal merger contains a relatively small number of insiders. As for outsiders and society, we show that the increased aggression by the merged firm causes output by each outsider to fall while industry output rises. This implies that price falls, so that society gains, while outsiders see their profits fall.

In addition to providing at least a partial solution to the merger paradox, our results also fill a gap in the literature on divisionalization. Many multidivisional firms produce multiple products or services for the same market. The Marriott Corporation owns and operates Courtyard by Marriott, the Residence Inn and the Fairfield Inn hotel chains. The Miller Brewing Company offers no less than 20 different brands of beer while R.J. Reynolds Tobacco offers a wide variety of cigarettes. And then there is the auto industry, where General Motors, Chrysler, and Ford each have several divisions with each division offering a
large number of different models.

The traditional explanation for divisionalization is that these firms are filling price
niches by offering a wide variety of products targeted at different classes of consumers. This
product differentiation argument can take us only so far though, since many firms offer
multiple products that appear to be either identical or extremely close substitutes for one
another. For example, it is not uncommon to find more than one hotel owned by the same
parent company located at the same exit of an interstate highway.\footnote{An example of this can be found near the intersection of Boardwalk and Victor’s Way in Ann Arbor, Michigan where a Courtyard by Marriott and a Fairfield Inn are located so close to one another that they share a parking lot. Less than a quarter of a mile away there sits a Residence Inn as well. All three hotels are owned by Marriott and offer similar accommodations at similar prices. Moreover, the Mobile Travel Guide (2000) rates all three as two star hotels, suggesting that there is little difference in the quality of the accommodations. Conlin (2000) finds evidence that there is strong competition between different brands within a hotel corporation.} In the beer industry, Miller separates its brands into several different categories. While it is clear that there is an
important distinction between the company’s premium beers and its specialty brews, the
distinction within each class of beers is harder to discern. Of course, the best examples of
this practice can be found in the auto industry where “twin” products abound. Chrysler
Corporation offers the Plymouth Voyager and the Dodge Caravan, two mini-vans that are
virtually identical that sell for similar prices while Ford offers the Taurus along with its twin,
the Sable (produced in its Mercury division).

Work by Baye, Crocker, and Ju (1996), Schwartz and Thompson (1986) and
Veendorp (1991) provides a second rationale for divisionalization aimed at explaining why
firms would want to offer identical products through different divisions. The gist of the
argument is as follows. Suppose that we start out with an oligopolistic industry comprised of
\( N \) symmetric Cournot competitors so that each firm’s share of industry profits would be \( 1/N \).
Suppose now that one firm were to start a new division that produces the same product as all
other firms in the industry. Suppose further that, as in our model, the new multidivisional
firm grants each division its independence and rewards its manager based on the profitability of his division. Then the industry would now consist of \(N+1\) competing units and the multidivisional firm’s share of industry profit would be \(2/(N+1)\). The increase in the number of industry participants drives down industry profit, but the multidivisional firm now earns a greater share of those profits. Baye, Crocker, and Ju (henceforth, BCJ) show that with Cournot competition, the second effect dominates and the firm gains from divisionalization. This result can be viewed as a byproduct of the merger paradox – since it does not pay to merge under Cournot competition, it should pay to divisionalize. Schwartz and Thompson (1986) show that incumbents can use divisionalization similarly so as to prevent entry.

Unfortunately, these analyses leave at least one issue concerning divisionalization unresolved. In these models, the new divisions are created by already existing firms. This results in an increase in the number of competing units in the industry. While it is true that some firms do create new divisions (e.g. Chrysler created its Plymouth division in 1928), it is often the case that parent companies add new divisions by merging with or acquiring a rival (e.g. Chrysler acquired its Dodge division in the same year that it created Plymouth by buying out Dodge Brothers, Inc.). When such a merger or acquisition occurs, the number of competing units in the industry does not change. Because of this, in these models, if there are more than two firms, then the merger of two single division firms has no effect on the equilibrium outcome if each division remains active.\(^6\) Thus, these analyses cannot explain why multidivisional firms would ever want to add new active divisions via merger.\(^7\)

Our model complements the work by BJC and Schwartz and Thompson by providing such an explanation. Firms merge in order to take advantage of strategies that are available

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\(^6\) It is important to remember that in both models the divisions are completely independent so that if the number of active divisions does not change, the equilibrium does not change.

\(^7\) In BCJ the number of competing units actually decreases with a merger. That is, a merger in the BCJ model would cause some divisions to be shut down.
only to multidivisional firms. In particular, in our model, firms merge in order to stagger competition and become more aggressive. Of course, a firm could also stagger competition by creating a new division, but merging is a superior alternative since creating a division increases the number of competing divisions while a merger leaves that number unchanged. Thus, our model does what previous explanations of divisionalization cannot do; it explains why a multidivisional firm might want to add a new division via merger or acquisition.

Finally, our paper is also related to the literature on strategic contracting. In this literature, it is argued that firms can structure contracts that they use to compensate managers in a manner that allows them to achieve a strategic advantage over their rivals.\(^8\) In a recent paper, Ziss (1999) combines insights from the contracting and divisionalization literatures and shows that in the absence of uncertainty, firms prefer to use contracting – that is, the optimal structure of the firm includes only one division. Furthermore, Ziss (2001) shows that when all firms use optimal contracts, mergers may be beneficial but only if all but one of the merged firms is shut down.\(^9\) It follows that if firms were free to use any compensation scheme and there is no uncertainty, we would not expect to observe multidivisional firms. In our model, we restrict attention to a particular type of compensation – the one most commonly used – and show the combination of this compensation scheme and internal competition can explain the profitability of mergers and the existence of multidivisional firms even in the absence of uncertainty.

The paper divides into four additional sections. In the next section, we show how staggered competition can work to the benefit of a multidivisional firm under the assumption that there are two stages of production within the merged firm. We show that for all \(N \geq 4\),

\(^8\) The classic reference in this area is Fersthman and Judd (1987).

\(^9\) It is also worth noting that in Ziss’ framework outsiders gain from merger. Thus, the literature on strategic contracting cannot explain why outsiders would oppose a merger by others in their industry.
there is a merger with two-stage staggered competition that benefits the insiders.\textsuperscript{10} We also show that such mergers harm outsiders and lower industry price. In fact, we show that when the merged firm chooses the number of leading divisions to maximize profits, the industry price is minimized. We close section 2 by showing that two-stage staggered competition can only explain mergers that involve a relatively small number of industry participants.

In section 3, we consider the issue of optimal division structure. We do so by allowing the multidivisional firm to structure production in any manner that it chooses to. That is, we allow the merged firm to choose the number of stages of production, the number of leaders in each stage, and the number of participating firms in order to maximize profit per division. We show that if there are $M$ firms involved in the merger, the multidivisional firm will choose to have $M$ stages of production so that all divisions make their production decisions sequentially. We show that such mergers always benefit the merging firms, harm outsiders and lower industry price.

In section 4 we turn to price games and focus on the difference between staggered competition in the presence of Cournot and Bertrand competition. We argue that although staggered competition works in price games as well, the parent company will never choose to adopt such a strategy. The reason for this is that under price competition the parent company can do better by controlling the prices charged by each division and setting them to maximize joint profits. This might provide at least a partial explanation as to why some companies retain control of their divisions while others allow their divisions great independence.

In section 5 we summarize and discuss our results. One particular point that we try and emphasize is that there is a more general way to view our results. We argue that mergers are beneficial in our model because the parent firm is able to play its divisions off against one

\textsuperscript{10} In independent work, Huck, Konrad and Muller (2001a) provide a similar argument for horizontal mergers and derive these same results. They do not consider endogenous mergers nor do they address any
another in a manner that benefits them as a whole. This is accomplished by creating a game within a game in which the divisions compete against one another. The parent firm then structures the internal game in a manner that generates higher average profit across its divisions. Staggered competition is just one example of the way in which the parent firm can alter the nature of the competition between its divisions and benefit.

2. Two-Stage Staggered Competition: Creating a Game with a Game

Consider an industry that consists of \( N \) symmetric single-division quantity-setting firms. Each firm faces a constant marginal cost of production, which is normalized to zero without loss of generality. The inverse demand curve for the industry is assumed to be linear and is given by \( P = A - bQ \), where \( Q = \sum q_i \) so that \( Q \) denotes industry output and \( q_i \) denotes the output of firm \( i \). Then it follows that the initial Cournot equilibrium price \( (P_c) \), industry output \( (Q_c) \), per firm output \( (q_c) \) and per firm profit \( (\pi_c) \) are given by

\[
P_c = \frac{A}{N+1}, \quad Q_c = \frac{AN}{b(N+1)}, \quad q_c = \frac{A}{b(N+1)} \quad \text{and} \quad \pi_c = \frac{A^2}{b(N+1)^2}.
\]

Now, suppose that a subset of \( M \) firms decides to merge. This creates a single firm with \( M \) independent divisions. Following BCJ and Schwartz and Thompson, we assume that the parent company rewards the manager of each division based on his division’s profit. Thus, the manager of each division chooses output in order to maximize his division’s profits. We refer to each division within the merged firm as an \textit{insider}, while each firm that is not involved in the merger is referred to as an \textit{outsider}.

We do \textbf{not} assume that the merger gives the multidivisional firm a first-mover

of the issues that we explore in sections 3 and 4 of our paper.
advantage with respect to the outsiders.\textsuperscript{11} That is, the multidivisional firm and the N-M outsiders are still assumed to be Cournot competitors. However, we do allow the divisions within the merged firm to stagger their output decisions. For now, we assume that the merged firm divides its divisions into two groups. All divisions in group 1 set their output simultaneously \textit{before} the group 2 divisions are allowed to move. Once these outputs are set, the group 2 divisions observe these choices and then make their own production decisions simultaneously. While outsiders cannot observe the output decisions of either the group 1 or group 2 divisions, we assume that they are aware of the structure of the multidivisional firm and that they know the number of divisions in each group. We use $T_1$ to represent the number of group 1 divisions and assume that the merged firm chooses $T_1$ in order to maximize its total profit. However, $T_1$ cannot exceed $M - 1$, since there must be at least one insider in group 2 (otherwise, all insiders would move simultaneously, resulting in the original Cournot outcome).

Formally, let $O$ denote the set of outsiders, $G_1$ denote the set of insiders in group 1 and $G_2$ denote the set of insiders in group 2. Then:

(a) Each outsider chooses $q_i$ to maximize its profit given its conjectures about $q_j$ for all $j \in G_1$, $q_h$ for all $h \in G_2$ and the output of the other outsiders,

(b) Each group 1 insider chooses $q_j$ to maximize its division’s profit given its conjectures about $q_i$ for all $i \in O$ and the output choices of all other group 1 outsiders.

\textsuperscript{11} Daughety (1990) considers an industry consisting of $M$ leaders and $N - M$ followers and investigates the impact of a merger between 2 followers under the assumption that the merger leads to a new firm that is a leader. Our approach differs from Daughety’s in at least two important ways. First, in Daughety’s model the number of leaders and followers are specified exogenously. Second, Daughety assumes that a merger transforms two followers into a leader. In our model, there are no industry-wide leaders, there are only leaders within the merged firm. Moreover, the number of leading divisions arises endogenously as the multidivisional firm staggers competition in order to maximize profits. See also Huck, Konrad and Mueller (2001b) who show that in a model with Stackelberg leaders and followers, mergers between two leaders or two followers typically are not advantageous but mergers between a leader and a follower typically
insiders. Each group 1 insider also takes into account how its output choice will affect the production plan of each group 2 insider.

(c) After observing $q_j$ for all $j \in G_1$, each group 2 insider chooses $q_h$ to maximize its division’s profit given its conjectures about $q_i$ for all $i \in O$ and the output choices of all other group 2 insiders.

(d) The merged firm chooses $T_1$, the number of group 1 firms, to maximize its profit.

(e) In equilibrium, each outside firm and each division of the merged firm have correct conjectures.

By dividing its divisions into two groups, allowing one group to move first and tying each division manager’s compensation to his division’s profit, the merged firm makes its group 1 divisions more aggressive than they would be otherwise. Each group 1 division increases production, knowing that the group 2 insiders will respond by cutting back production. Moreover, since the group 1 insiders are only concerned about the profits earned in their own division, they produce more than they would if they had to worry about the adverse effects of their own production decisions on the other divisions in their firm. This aggressiveness by the group 1 insiders increases the market share of the merged firm and, as we show below, may increase profit.

To derive the equilibrium, we start with the group 2 insiders. Each group 2 insider’s profit is given by:

$$
\pi_h = q_h \{ A - b[q_h + \sum_{i \in O} q_i + \sum_{j \in G_1} q_j + \sum_{l \in G_2, l \neq h} q_l] \}
$$

Maximizing (1) with respect to $q_h$ and applying symmetry ($q_h = q_2$ for all $h \in G_2$) yields the following equation that defines $q_2$, the output of a typical group 2 insider, as a function of

increase their joint profits.
the output produced by each group 1 insider and the firm’s conjecture about the output of each outsider. We obtain

\[
q_2 = \frac{A - b\left[\sum_{i \in O} q_i + \sum_{j \in G} q_j \right]}{b(M - T_i + 1)}
\]

Turn next to the outsiders, where each firm’s profit is given by

\[
\pi_i = q_i \left\{ A - b\left[ q_i + \sum_{\ell \in G, \ell \neq i} q_\ell + \sum_{j \in G} q_j + \sum_{h \in G_2} q_h \right] \right\}
\]

Differentiating (3) with respect to \( q_i \) and then applying symmetry (\( q_i = q_o \) for all \( i \in O \)) allows us to solve for \( q_o \), the output of a typical outsider, as a function of firm i’s conjecture about the output of all insiders. We obtain

\[
q_o = \frac{A - b\left[\sum_{j \in G_1} q_j + \sum_{h \in G_2} q_h \right]}{b(N - M + 1)}
\]

Finally, consider the group 1 insiders. Their profits are given by

\[
\pi_j = q_j \left\{ A - b\left[ q_j + \sum_{\ell \in G_1, \ell \neq j} q_\ell + \sum_{i \in O} q_i + \sum_{h \in G_2} q_h (q_j, q_{-j}) \right] \right\}
\]

where \( q_j \) is a vector consisting of the output choices of all group 1 insiders other than firm \( j \) and \( q_{h}(q_j, q_{-j}) \) is given by (2). If we differentiate (5) with respect to \( q_j \), then substitute for \( q_2 \) from (2) and apply symmetry \( (q_j = q_i \) for all \( j \in G_1) \) we obtain

\[
q_1 = \frac{A - b\sum_{i \in O} q_i}{b(T_i + 1)}
\]

where \( q_1 \) denotes the output of a typical group 1 insider.

We can now use (2) to substitute for \( q_h \) in (4) and then solve (4) and (6) for \( q_1 \) and \( q_o \). These expressions can then be substituted back into (2) to obtain the equilibrium value for \( q_2 \). We obtain
Comparing (7) and (8) with the per firm output in the original Cournot equilibrium ($q_C$), we see that for any merger (i.e., for any $M > I$ and $T_I \geq 1$), $q_1 > q_C > q_o = q_2$. Thus, as expected, staggered competition makes the leaders more aggressive and causes the followers and outsiders to reduce output.

Substituting $q_o$, $q_1$ and $q_2$ back into the demand curve and profit functions yields the equilibrium price and profit levels for each division.

$\pi_1 = \frac{A^2(M - T_I + 1)}{b[N + 1 + T_I(M - T_I)]^2}$

$\pi_o = \pi_2 = \frac{A^2}{b[N + 1 + T_I(M - T_I)]^2}$

If we compare (9) and (11) with their counterparts from the original Cournot equilibrium, we see immediately that any merger lowers price and harms outsiders (as long as $T_I \geq 1$). The rationale for these two results is the same. By allowing some firms to lead, the parent company makes the leading divisions more aggressive than they would be otherwise. This aggression increases the market share of the merged firm, to the detriment of outsiders. In addition, the increase in output by the leading divisions more than compensates for the reduction in output by followers and outsiders, resulting in a lower equilibrium price.

The fact that any merger reduces the price underscores just how unconventional our approach to modeling horizontal mergers is. Using the traditional approach that assumes the
parent company sets all output levels to maximize joint profits, Farrell and Shapiro (1990) show that in the absence of cost synergies any merger must increase price. Our approach shows that if the parent company plays the divisions off against one another in order to increase profits, price can fall and society may benefit as a result.

The fact that mergers harm outsiders means that there is an additional benefit from staggered competition. If mergers increase the profits of outsiders, then firms that were considering merging would have to worry about triggering entry in the post-merger equilibrium. However, firms that merge to stagger competition have no such worries – if entry is unprofitable before the merger, it is even less profitable afterwards.

To see how the merger affects the multidivisional firm, we must now calculate profit per division for the merged firm. Total profit is given by $\pi_M = T_1\pi_1 + (M - T_1)\pi_2$. Dividing this expression by $M$, the number of divisions in the merged firm, allows us to calculate profit per division. We obtain

\[
\frac{\pi_M}{M} = \frac{A^2[T_1(M - T_1) + M]}{Mb[N + 1 + T_1(M - T_1)]^2}
\]

Before we compare this value with the profit earned in the original Cournot equilibrium, we must solve for the equilibrium value of $T_1$. The merged firm’s goal is to choose $T_1$ to maximize profit per division.\(^{12}\) In the Appendix, we show that (12) is maximized when $T_1 = M/2$ (see result A.1 of the Appendix). Thus, the merged firm wants to divide its divisions such that half of them lead and half of them follow. Evaluating (12) at $T_1 = M/2$ leaves us with the equilibrium profit per division for the merged firm. We obtain

\[
\frac{\pi_M}{M} = \frac{A^2[(M / 4) + 1]}{b[N + 1 + (M / 2)^2]^2}
\]

\(^{12}\) With $M$ fixed, maximizing profit per division is the same as maximizing total profit.
Comparing (13) with the profits in the original Cournot equilibrium, we find that as long as \( N \geq 4 \), there is some value for \( M \) that makes the merger profitable for insiders. However, the merger is only beneficial if it involves a relatively small number of firms (the restriction on \( M \) is given in Proposition 1 below). For example, for \( N = 10 \), a merger of up to 4 firms is beneficial but a merger of 5 firms is not. For \( N = 100 \), mergers are only beneficial if 26 or fewer firms are involved. The reason for this result is that the merger is beneficial only if the increase in aggression lowers the total output produced by the outsiders by a significant amount. But, this can only happen when the number of outsiders is large.

Finally, from (9) we see that price is minimized when \( T_1 = M/2 \), so that when the merged firm maximizes profit per division, it also minimizes the industry price.\(^{13}\) These results are summarized in Propositions 1 and 2 below (the values of \( M \) satisfying the inequality in Proposition 1 are illustrated in Figure 1).

**Proposition 1**: With two-stage staggered competition, any merger harms outsiders. In addition, if \( N \geq 4 \) the merger is beneficial if \( 4(N + 1)(N + 1 - 2M) > M^3 \).

**Proposition 2**: With two-stage staggered competition, any merger is socially beneficial. In addition, the merged firm’s optimal division structure minimizes the industry price.

It is worth emphasizing that the strategy of staggering the output decisions of its divisions is what makes the merger work. Without this strategy, mergers that do not include the vast majority of the firms would not be beneficial for the firms involved (see Salant, Switzer, and Reynolds 1983). As we have already noted, the merger makes the leading

\(^{13}\)Daughety (1990) finds a similar result in his model (see footnote 11 for a description of his model). He finds that industry price is minimized when half of the firms lead and the other half follow. Our results would be the same if there were no outsiders in our model.
divisions more aggressive – they increase output because doing so will cause insiders, who observe the leaders’ output before choosing their own output, to cut output. Outsiders, knowing the structure of the merged firm and knowing that the leading divisions will increase output, respond to the merger by cutting production as well. As a result, the divisions that lead gain while the divisions that follow suffer losses. But, provided that the number of outsiders is relatively large, the leaders gain more than the followers lose, making the merged firm better off. It is important to emphasize that the merged firm could not achieve this outcome without staggering competition. Any attempt to expand output in some divisions would not be viewed as credible by the outsiders. Thus, the followers are necessary, since their presence allows the leading divisions to credibly commit to expanding production.

An alternative explanation of the benefit of staggering competition is as follows. Consider first a traditional merger of $M$ firms in which the parent company controls the output of each division and sets output to maximize the joint profits. When such a merger occurs, it is optimal to reduce output in each division in order to move closer to the collusive outcome. This reduction in output benefits outsiders and makes the merged firm a softer competitor. As a result, the merger is not beneficial, unless it includes a vast majority of the firms in the industry. Now compare this outcome with what happens under two-stage staggered competition. Under staggered competition, the leading divisions respond by increasing output, because they know that by doing so they can force the followers to cut production. This increase in output harms outsiders and makes the multidivisional firm a tougher competitor. As a result, the merged firm gains.

It is worth emphasizing that the multidivisional firm does not change the nature of its competition with the outsiders – after the merger the industry is still characterized by Cournot competition. But, by staggering competition, the merged firm does change the nature of the competition between its divisions. By creating this new game within a game, the parent
company is able to structure the internal game in a way that benefits the parent company.

We close this section by considering endogenous mergers. If they could, the firms would prefer to join forces and merge all the way to monopoly. However, we assume that this outcome is precluded by anti-trust concerns. So, instead, we assume that the firms merge so that profit per division is maximized. At that point, adding another division would not be appealing since it would lower the profit earned by the average insider. Maximizing (13) over $M$, we find that for all $N \geq 1$, the optimal number of divisions is given by

$$M_2 = \frac{1}{3}[2\sqrt{19+3N} - 8]$$

(see Result A.1 of the Appendix). Figure 1 shows how the optimal size of the merged firm varies with $N$. As is clear from the Figure, merging in order to stagger competition across two stages can only explain mergers of a relatively small number of firms (for $N = 10$, $M_2 = 2$; for $N = 100$, $M_2 = 9$). The reason is that staggered competition works by structuring the internal competition in a way that causes outsiders to cut output. This is only beneficial if there are a sufficient number of outsiders left in the industry after the merger. Thus, our explanation of mergers is consistent with the stylized fact that most mergers involve only small firms.

3. Optimal Division Structure

In this section, we allow the multidivisional firm to stagger competition within the firm in any manner that it chooses to. The firm may choose the number of divisions to acquire (through merger), the number of stages of competition and the number of divisions that must make their output decisions in each stage. We use $M$ to denote the number of divisions, $K$ to denote the number of stages of competition within the firm and $T_i$ to represent the number of divisions that make their output decisions in stage $i$ (for $k = 1,...,K$). The only restrictions that we place on $M$, $K$, and $T_i$ are that $T_i \geq I$ for all $i$, $M \geq K$ and $\sum_i T_i = M$. 

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Market competition is structured in much the same manner that it was in section 2. The multidivisional firm forms and then chooses its internal structure. Outsiders are assumed to be aware of the internal structure of the merged firm. Competition within the multidivisional firm is staggered, with all firms that move after stage \( i \) has been completed observing the output of the \( T_i \) firms that produce in that stage. Outsiders take the output of each division in the merged firm as given, so that the competition between the merged firm and the outsiders is still consistent with Cournot.

As in section 2, we let \( O \) denote the set of outsiders and use \( G_i \) to represent the set of insiders that move at stage \( i \). It follows that firm \( j \)’s profit is given by

\[
\pi_j = q_j \{ A - b \left[ \sum_{i=0}^{\infty} q_i + \sum_{h=0}^{k} \sum_{r \in G_h} q_r \right] \}
\]

If firm \( j \) is an insider that moves at stage \( k \), it chooses \( q_j \) to maximize \( \pi_j \) given

1. the output levels that it has already observed (\( q_r \) for all \( r \in G_h \) with \( h < k \));
2. its conjectures about the output levels by all outsiders and the other stage \( k \) insiders; and,
3. taking into account how its output choice influences the output decisions by all insiders that move after stage \( k \).

If we maximize (14) with respect to \( q_j \) for each outsider and stage-\( k \) insider and then apply symmetry, we obtain the following expressions for the output levels (see result A.2 of the Appendix)

\[
q_o = \frac{A - b \sum_{j=0}^{\infty} T_j q_j}{b[N - M + 1]}
\]

\[
q_k = \frac{A - b[(N - M)q_o + \sum_{j=0}^{k-1} T_j q_j]}{b(T_k + 1)} \quad \forall k = 1, \ldots, K
\]

where \( q_o \) is the output of a typical outsider and \( q_k \) denotes the output of a stage-\( k \) insider \( k \).
Solving (15) and (16) for $q_o$ and $q_k$ (for $k = 1, \ldots, K$) yields (see result A.3 of the Appendix)

\begin{align*}
(17) \quad q_o &= \frac{A}{b[N - M + H_k]} \\
(18) \quad q_k &= \frac{AH_k}{b[N - M + H_k]H_k} \quad \forall k = 1, \ldots, K
\end{align*}

where $H_k = \prod_{j=1}^{k}(1 + T_j)$. Note that for $k = K$, $q_k = q_0$. Thus, as in section 2, the insiders that move last produce the same output per division as each outsider. Moreover, as we would expect, if $T_i = T_j$ and $i < j$, then $q_i > q_k$. That is, insiders that move earlier in the process produce more output (or, are more aggressive), all else equal.

Substituting (17) and (18) back into the inverse demand curve allows us to solve for the industry price. We obtain

\begin{equation}
(19) \quad P = \frac{A}{N - M + H_k}
\end{equation}

It follows that profit for each outsider is given by

\begin{equation}
(20) \quad \pi_o = \frac{A^2}{b[N - M + H_k]^2}
\end{equation}

For the merged firm, total profits are given by $\pi_M = \sum_{k=1}^{K} T_k q_k$. Substitution from (18) and (19) allows us to write profit per division as

\begin{equation}
(21) \quad \frac{\pi_M}{M} = \frac{A^2 (H_k - 1)}{bM[N - M + H_k]^2}
\end{equation}

The multidivisional firm’s goal is to choose the number of divisions to acquire and the structure of those divisions in order to maximize (21). In other words, the firm’s goal is to choose $M$, $K$, and $T_k$ (for $k = 1 \ldots K$) to maximize (21) subject to the constraints that $M \geq K$. 

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and $\sum_{k}^{K} T_{k} = M$. To do so, we begin by differentiating (21) with respect to $T_{i}$, making use of the fact that $\frac{\partial M}{\partial T_{i}} = 1$ and $\frac{\partial H_{k}}{\partial T_{i}} = \frac{H_{k}}{1 + T_{i}}$. We obtain the following first-order-condition for $T_{i}$

$$
\frac{H_{k}}{1 + T_{i}} M[N - M + H_{k}] - (H_{k} - 1)[N - M + H_{k} + \frac{2M}{1 + T_{i}}[H_{k} - 1 - T_{i}]] = 0
$$

Solving for $1 + T_{i}$ yields

$$
1 + T_{i} = \frac{H_{k}M[N - M - H_{k} + 2]}{(H_{k} - 1)[N - 3M + H_{k}]}
$$

Since the right-hand-side of (22) is independent of $i$, there is a symmetric solution to the system of first-order conditions. That is, $T_{1} = T_{2} = \ldots = T_{K} = T$, so that the merged firm wants to have the same number of leaders at each stage of production. This symmetry feature is inherited from two-stage staggered competition, where we saw that the merged firm would maximize profit per division by having half of its divisions lead and the other half follow.

Setting $T_{k} = T$ for all $k$ allows us to simplify profit per division. Making use of the fact that $M = TK$, we now have

$$
\frac{\pi_{M}}{M} = \frac{A^{2}[(1 + T)^{\frac{M}{T}} - 1]}{bM[N - M + (1 + T)^{\frac{M}{T}}]}^{2}
$$

The following proposition describes the optimal division structure for the merged firm (the proof of Proposition 3 can be found in the Appendix).

**Proposition 3:** The merged firm’s profit per division is maximized when there is a single firm producing at each stage in the production process.

Intuitively, the merged firm chooses to stagger competition in order to make the leading
divisions more aggressive. Having more than one leader choose output at the same time dampens that aggressiveness, making it harder for staggered competition to have its desired effect. It follows that the multidivisional firm will have the same number of stages of staggered competition as it has divisions.

If we set $T = 1$ in (19), (20) and (23), we can rewrite price, profit for each outsider and profit per division for the merged firm as

$$P = \frac{A}{N-M+2^M}, \quad \pi_o = \frac{A^2}{b[N-M+2^M]^2}$$

and

$$\frac{\pi_M}{M} = \frac{A^2[2^M-1]}{bM[N-M+2^M]^2}$$

Comparing these values with their counterparts in the original Cournot equilibrium, we see immediately that most of the results reported in Propositions 1 and 2 continue to hold when the merged firm uses the optimal division structure. Any merger lowers price and harms outsiders (since $2^M > M + 1$ for all $M > 1$). In addition, for $N \geq 4$, there is some merger that is beneficial for the merging parties. Only the second half of Proposition 2 fails to generalize. With two-stage staggered competition we found that when the merged firm chose the number of leaders to maximize profit per division, the industry price would be minimized. However, with general staggered competition, industry price would continue to fall if $M$ were increased beyond the level that maximizes profit per division. Thus, while mergers to stagger competition still lower price, with general staggered competition they no longer minimize the industry price. These results are summarized in Propositions 4 and 5.

**Proposition 4:** With general staggered competition, any merger harms outsiders. In addition, if $N \geq 4$, the merger is beneficial for insiders if $M < \hat{M}$ where $\hat{M}$ satisfies $M[N-M+2^M]^2 = (2^M - 1)(N + 1)^2$. 

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Proposition 5: With general staggered competition, any merger is socially beneficial.

We close this section, by considering endogenous mergers. We assume that the multidivisional firm chooses $M$ to maximize profit per division. The value of $M$ that maximizes (24) is depicted by $M^*$ in Figure 1. As with two-stage staggered competition, the optimal value of $M$ is relatively small. Thus, the merged firm will stop acquiring rivals when the industry is still quite competitive. As before, this is due to the fact that staggered competition works only if the number of outsiders in the industry is relatively large.

4. Price Games

Results derived in quantity-setting games do not always generalize to price-setting games. Nowhere is this more evident than in the literature on horizontal mergers. While mergers in quantity-setting games rarely benefit the merging firms, mergers are always beneficial in price-setting games (Deneckere and Davidson 1985). The difference in the outcomes stems from the fact that outsiders react to mergers in fundamentally different ways in these two settings. In quantity-setting games, outsiders respond by increasing output and this harms the merged firm by driving down the industry price. In contrast, in price-setting games, the outsiders respond to the merger by increasing price and this benefits the merged firm by increasing the demand for its product.

In this section, we show that although staggered competition works in price games, the merged firm will never employ this strategy since it can always do better by setting each division’s price so as to maximize joint profit. The main reason for this result is that while moving first is advantageous in a quantity-setting game (since you can force your competitor to cut output), moving first puts a firm at a disadvantage in a price-setting game (since it allows your competitor to undercut you). Thus, the merged firm would rather have both of
its divisions set price at the same time. We illustrate this in a simple 4-firm example that
captures the flavor of the results obtained in general price-setting games.

Following Deneckere and Davidson (1985), we assume that demand for firm \( i \)'s
product is given by

\[
q_i = A - p_i - \gamma \left[ p_i - \frac{1}{N} \sum_{j=1}^{4} p_j \right]
\]

With this demand curve, firm \( i \)'s demand depends on its own price and the difference
between its own price and the average price charged in the industry. The \( \gamma \geq 0 \) term is a
substitutability parameter. When \( \gamma = 0 \) the goods are unrelated and as \( \gamma \) approaches infinity,
the goods become perfect substitutes. As for costs, we continue to maintain our assumption
that production costs are zero.

With the demand curve in (25), it is straightforward to show that in the original
symmetric Bertrand equilibrium each firm’s price \( P_B \), output \( q_B \) and profit \( \pi_B \) are given
by

\[
P_B = \frac{4A}{8 + 3\gamma} ; \quad q_B = \frac{A(4 + 3\gamma)}{8 + 3\gamma} ; \quad \text{and,} \quad \pi_B = \frac{4A^2(4 + 3\gamma)}{(8 + 3\gamma)^2} .
\]

Now, suppose that two of the firms merge. In a conventional merger, the parent firm
would choose the price charged by each division to maximize joint profits. Let \( \hat{p}_i \) denote
the price charged by each insider and use \( \hat{p}_o \) to represent the price charged by each outsider.
Then, from Deneckere and Davidson (1985), we know that the prices charged by the firms
after a conventional merger are given by

\[
\hat{p}_i = \frac{(8 + 7\gamma)A}{4\gamma^2 + 18\gamma + 16} \quad \text{and} \quad \hat{p}_o = \frac{(8 + 6\gamma)A}{4\gamma^2 + 18\gamma + 16} .
\]

Comparing these prices with the original Bertrand prices, we find that \( \hat{p}_i > \hat{p}_o > p_B \). Thus,
all firms increase their prices due to the merger with the insiders raising their prices by more.
These price increases lead to higher profits. From Deneckere and Davidson (1985), the new
profits are given by \( \hat{\pi}_l = \left( \frac{(8 + 7\gamma)A}{4\gamma^2 + 18\gamma + 16} \right) \left( \frac{2 + \gamma}{2} \right) \) and, \( \hat{\pi}_o = \left( \frac{(8 + 6\gamma)A}{4\gamma^2 + 18\gamma + 16} \right) \left( \frac{4 + 3\gamma}{4} \right) \).

It is straightforward to show that \( \hat{\pi}_o > \hat{\pi}_i > \pi_B \). All firms benefit from the merger with the outsiders free riding off of the insiders and earning higher profits.

What if the merged firm had chosen to stagger the pricing decisions of its two divisions? Would such a strategy have been beneficial and would it have allowed the merged firm to earn higher profits than it earns under the conventional approach to mergers? To answer these questions, we now solve for the equilibrium prices assuming that price competition within the merged firm is staggered. This means that when the leading division sets its price, it takes into account the fact that the following division will be able to observe and react to whatever price it chooses. The two outside firms are aware of the structure of the merged firm, but the competition between the insiders and the outsiders remains consistent with standard Bertrand competition.

Following the same approach laid out in sections 2 and 3, we find that under staggered competition the prices charged by the follower \( (p_F) \), the two outsiders \( (p_o) \) and the leader \( (p_L) \) are given by the terms \( p_F = p_o = \frac{2A[4\gamma^2 + 104\gamma + 64]}{6\gamma^3 + 320\gamma^2 + 512\gamma + 256} \) and

\[
p_L = \frac{4A(4 + 3\gamma)(8 + 7\gamma)}{6\gamma^3 + 320\gamma^2 + 512\gamma + 256}.
\]

As in the quantity-setting game, the follower and the outsiders employ the same strategy (i.e., \( p_F = p_o \)). Comparing these prices with those charged in the original Bertrand equilibrium, we find that \( p_L > p_F = p_o > p_B \). With staggered competition, the leader increases price to move the industry closer to the collusive outcome. This price increase results in higher demands for the follower and outsiders’ products and leads them to increase price as well, but not by as much. Thus, the follower and the outsiders are both able to undercut the leader’s price.
These higher prices lead to higher profits. But, since the follower and outsiders undercut the leader, they all earn more than the leader. Formally, equilibrium profits for the leader, follower and outsider are given by

\[
\pi_L = \frac{2A^2[4 + 3\gamma][8 + 7\gamma][119\gamma^3 + 472\gamma^2 + 608\gamma + 256]}{[61\gamma^3 + 320\gamma^2 + 512\gamma + 256]^2}
\]

\[
\pi_F = \pi_o = \left[ \frac{A(41\gamma^2 + 104\gamma + 64)}{61\gamma^3 + 320\gamma^2 + 512\gamma + 256} \right]^2 (4 + 3\gamma)
\]

Comparing these values with each other and their counterparts in the original Bertrand equilibrium we find that \(\pi_F = \pi_o > \pi_L > \pi_B\). Staggered competition works in the price game, since both divisions of the merged firm earn more than they would in the Bertrand equilibrium, but it works in a different manner than it did in the quantity setting game. In the quantity setting game, the leaders benefit while the followers and outsiders lose. The merged firm gains because the leaders gain more than the followers lose. In the price-setting game, the leader is at a disadvantage and winds up being undercut by the follower and outsiders. Although the leader gains, it gains much less than the followers and outsiders. Thus, the primary beneficiaries of staggered competition are the followers and the outsiders.

Although staggered competition increases the profits earned by the multidivisional firm, it is not a strategy that the merged firm will choose to use. The firm can always do better by simply treating the merger in a conventional manner. Comparing the profits under staggered competition with those earned by choosing prices to maximize joint profits we obtain \(\tilde{\pi}_o > \tilde{\pi}_f > \pi_o = \pi_F > \pi_L\). Since moving first puts the leading division at a disadvantage, the merged firm does better by having all its divisions move simultaneously. In addition, since prices are strategic complements, the firm would rather have its divisions set prices to maximize joint profits rather than the profit earned by each individual division.
Our results imply that a price-setting multidivisional firm will wish to exert greater corporate control over its divisions than a similar firm that competes in a quantity-setting market. Quantity-setting firms benefit by granting their divisions great independence, basing managerial compensation on division profitability and staggering competition within the firm. In contrast, price-setting firms benefit by setting all prices centrally and choosing them to maximize joint profits. If we adopt the Kreps-Scheinkman (1983) approach and view quantity-setting markets as those in which prices are more flexible than output, then the auto industry provides an example of a quantity-setting market in which multidivisional firms grant their divisions great independence. If we view price-setting markets as those in which prices are set and then firms produce to order, the fast food industry would provide an example of a market in which firms should be unwilling to allow divisions to set prices on their own. This appears to be the case with MacDonalds and Burger King, where prices charged by corporate owned units are not set by each unit, but rather are controlled by the parent company (Lafontaine 1995).

5. Discussion

In this paper, we have provided a quantity-setting model in which horizontal mergers are beneficial for the merging parties but harmful for firms that are not included in the merger. We believe that, a priori, most economists probably suspect that this is the way in which mergers actually work, even in the absence of scale economies. Yet, providing such a model has proven to be quite difficult. As far as we know, ours is the only quantity-setting model of horizontal mergers that contains these features.\textsuperscript{14}

Our approach to modeling horizontal mergers is fundamentally different to the

\textsuperscript{14} As we noted earlier, Huck, Konrad and Mueller (2001a) also provide a quantity-setting model with this feature based on similar reasoning.
approach taken in the traditional Industrial Organization literature. Instead of focusing on how collusion by a subset of firms affects the equilibrium outcome, we have focused instead on how the new merged firm can structure competition between its own divisions in order to increase profits. We have provided an example of a strategy that could be used by multidivisional firms to gain a strategic advantage over its rivals. According to this strategy, some divisions within the firm are allowed to choose output before other divisions. We have shown that this makes the leading divisions more aggressive and that this benefits the insiders by allowing the multidivisional firm to capture a larger share of the market. We have also shown that while such a strategy also works with Bertrand competition, the multidivisional firm can do better by setting the prices charged in each division to maximize joint profits. It follows that while it is optimal to delegate the output decision to the divisions under quantity competition, it is better to control all decisions centrally under price competition.

There are two natural questions that can be raised concerning our results. The first has to do with generality. We have used a model with linear demand, constant costs and identical goods. It is only natural to wonder if our results generalize to more complex settings. It should be fairly obvious that the assumption of identical products can be relaxed without altering anything. Staggered competition is a strategy that should work for quantity-setting multidivisional firm even if their divisions produce differentiated products. Thus, a firm that creates or acquires a new division to fill a price niche should be able to earn larger profit by staggering competition than it would by allowing all divisions to set output simultaneously. As for the linear demand, constant cost assumption we note that the intuition behind our results in sections 2 and 4 are based on the features of quantity competition, price competition and the differences between simultaneous and sequential games. There is no reason a priori to expect these results to disappear in a more general setting. The results that may hinge on linearity are those derived in section 3. It is not at all clear that the parent
company would want the same number of divisions to move at each stage with more general conditions on demand. This is an issue that needs to be addressed in future research.

The second question about our results has to do with their descriptive power. Do multidivisional firms really stagger the output decisions of their divisions? This is a difficult question to answer, since the decisions involved in production process are not observable. One possible way to distinguish between our approach and the traditional approach to mergers would be to compare their predictions and look to the empirical literature on mergers for guidance. The traditional approach predicts that, in the absence of cost synergies, mergers will harm insiders, benefit outsiders and harm society. In addition, after a merger we should observe the output of all insiders fall while the outsiders expand production. In our framework, insiders and society gain from mergers while outsiders lose. As for output responses, outsiders should decrease production while the response by insiders should be mixed with some divisions expanding while others shrink. As we noted in the introduction, the empirical literature does seem to support the prediction that outsiders are often harmed by merger (see footnote 2) and the fact that mergers occur at all seems to support the prediction that they are beneficial for insiders. As for the welfare consequences of mergers, the evidence is mixed with clear examples of welfare changes in both directions (Federal Trade Commission 1999). We know of no evidence on how mergers affect the market shares of insiders and outsiders. But, we suspect that most economists would be quite surprised by evidence showing that mergers increase the market share of outsiders at the expense of participating firms!

Another approach to resolving this issue would be to look at how multi-divisional firms are actually run. There are two basic points that we have tried to make by introducing

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15 Cost synergies cannot explain all mergers – there are clear cases of horizontal mergers that occurred in industries that are not characterized by convex costs (Office of Fair Trading 1999).
the notion of staggered competition -- that it may be in the interest of the parent firm to play its divisions off against one another to gain a strategic advantage over outsiders and that it may not be in the parent firm’s interest to treat all divisions in the same manner. By favoring some divisions over others or by having some divisions lead others in some dimension, the parent company may be able to structure the internal game in a way that allows them to achieve higher joint profits and capture a larger share of the market.

There is ample evidence that some firms do, in fact, encourage competition across divisions. Seagram’s, Procter and Gamble (Schwartz and Thompson 1986), Mitsubishi (Nikkei Weekly 1994), Estee Lauder Cosmetics (Low 1996), and JWP Inc. (a provider of high-tech services, Forbes 1992) are companies that all foster internal competition. In each case, the goal is the same -- to make their divisions more aggressive in order to increase the profit earned by the parent company. As we have shown, staggered competition is one way to structure the game within the firm that creates more aggressive divisions. There is also some evidence that firms do indeed treat their divisions differently. One example comes from the market for printers where Hewlett Packard has found it necessary to treat their laser and ink-jet divisions differently in order for each to thrive (Christensen and Overdorf 2000). Another example comes from the auto industry, where sales for twin models often differ dramatically.\textsuperscript{16} There can be no doubt that the difference in sales reflects differences in consumer tastes, but some may be due to differential treatment by the parent companies across divisions (in terms of timing, marketing, and so on).\textsuperscript{17}

\textsuperscript{16} The Ford Taurus always outsells its twin, the Mercury Sable. In fact, in most years Ford sells more than twice as many Taurus as Sables (Ward’s Automotive Yearbook, various years). A similar pattern can be found for other auto producers. For example, at Chrysler, the twin (or triplet in the case of Plymouth’s Concorde, Dodge’s Intrepid, and Eagle’s Vision) version produced by the Dodge division seems to always outsell the versions produced by Plymouth or Eagle.

\textsuperscript{17} Additional evidence can be found in the literature on franchising where there are two unresolved questions that may be related to our results. To begin with, there is the issue of why some firms operate stores directly while franchising others. Typically, company-run stores and privately held franchises differ in size, pay scale, and other important attributes, indicating that the parent company may desire asymmetry
Hotel franchising provides another example in which individual units maximize their own profits while directly competing with other independent units of the same corporation (as well as corporate-owned units). Although there are strong theoretical explanations for franchising (e.g., monitoring problems and informational asymmetries), these explanations are weaker at explaining the observation that franchises compete with corporate-owned stores in the same market. Moreover, Conlin (2000) finds evidence that new franchised-owned hotels have a much larger impact on the prices charged by other franchises of the same brand than do new corporate-owned hotels and that the effect is negative and significant. He also finds that the franchises of different brands within the same corporation directly compete with each other and that, once again, the effect from another franchise is greater than the affect from another corporate-owned hotel. Thus, Conlin’s work shows that units directly compete with each other and it suggests that there maybe asymmetric treatment of units (franchised versus corporate-owned) both within brand and across brands.

There have also been some indications in the literature on strategic interaction that treating firms asymmetrically may have some advantages. The most notable example of this can be found in Salant and Shaffer (1999) who show that in two-stage games in which firms first invest to lower marginal cost and then engage in Cournot competition, asymmetry is both socially and privately optimal. The basic idea is fairly simple. Suppose that we start out in a symmetric Cournot equilibrium with industry price \( P^* \). Suppose now that we lower the marginal cost of one firm and raise the marginal cost of another firm. Suppose further that the increase in marginal cost for the second firm is chosen so that in the new equilibrium across outlets that provide their product. Second, there is strong empirical evidence that the company run stores tend to be larger than privately held franchises. This second result is considered a paradox, since it conflicts with the predictions from agency theory (Lafontaine 1992, Lafontaine and Slade 1997 and Conlin 2000). We note that our model would provide a rationale for this stylized fact if we assume that company-run stores are treated as leaders within the firm with the franchises relegated to the role of followers. Whether or not this is actually the case is an open question.
industry price is once again $P^*$. Then, in the new equilibrium the low-cost firm will produce more than before while the high-cost firm will produce less. It follows that the total cost of production will be lower, implying that social surplus must be higher than it was in the original symmetric equilibrium.

In a very different model, Daughety (1990) also finds that asymmetry may be socially beneficial. In his model, although the firms are identical in all respects, some firms are exogenously specified as Stackelberg leaders while the others follow. He shows that having the same number of followers and leaders maximizes social welfare. This means that society benefits when some firms are given an advantage over their otherwise identical rivals.\(^{18}\)

Finally, we close by pointing out that structured competition is but one example of the way in which a parent company can alter the internal competition between its divisions in a beneficial manner. There are a myriad of other ways that the parent company can use delegation, information manipulation and structured incentives to achieve an objective that it could not achieve otherwise. For example, it should be clear that compensating the managers of some divisions based on market share while compensating others based on division profitability will make some divisions more aggressive than others and lead to a similar outcome. Another example can be found in the literature on cooperative R&D. Consider a market in which $N$ firms compete in a two-stage game of the following form. In stage one, each firm invests in cost reducing R&D. In stage two, once costs have been determined and observed, the firms then engage in Cournot competition. Now, suppose that a subset of these firms decide to set up a research joint venture (RJV) in which they share successful cost-reducing R&D. Thus, a subset of the firms share R&D success in stage one, but all firms

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\(^{18}\) See also Katz and Shapiro (1986), who show that if a Research Joint Venture consists of a sufficiently large number of firms, then it is not in the group's interest to treat all members symmetrically. In particular, when an innovation occurs, it is not optimal for the innovation to be disseminated to all members.
continue to compete in Cournot fashion in stage two. DeCourcey (2000) has recently analyzed a game of this nature for an international market in which the RJV insiders are domestic firms. She shows that the RJV benefits the insiders and harms outsiders (foreign firms). Intuitively, by agreeing to share their R&D success, the insiders reduce their own incentives to engage in R&D activity. In equilibrium, insiders spend less on R&D but, since they share all R&D success, they still end up with a lower marginal cost than they would have without the RJV. The lower cost makes the insiders more aggressive in the Cournot game and harms outsiders.

Now, suppose that we change the set-up of this game in the following manner. Suppose that a merger occurs in which parent company treats the merged firms as separate divisions, forces them to compete with each other in output, but forces them to share their first-stage R&D success. Then this merger will produce exactly the same outcome as the RJV examined in DeCourcy (2000). That is, the merged firm will benefit while the outsiders are harmed. In this instance, the parent company changes the nature of the internal competition by forcing rival divisions within the firm to share valuable information.

The research joint venture example works in a fundamentally different way than our model of staggered competition. With staggered competition the parent company forces its divisions to choose output sequentially, and then plays them off against each other. This makes some divisions more aggressive, resulting in higher profits for the multidivisional firm. In the R&D example, the parent company forces the divisions to share information about R&D in a manner that increases joint profits. This alters the incentives to undertake R&D and increases joint profits. Yet, the basic mechanism at work in both cases is the same. In each case, the parent company sets up an internal game and structures that game in a way that benefits the firm as a whole. One of the points that we have tried to emphasize in this paper is that the ability to control the game within the firm can provide an incentive for the
quantity-setting firms to merge. Whether there is another way to play the divisions off against one another that is superior to staggered competition is an area for future research.
Appendix

Result A.1: Our goal is to show that profit per division, as given in (12), is maximized when
\( T_1 = M/2 \) and \( M = \frac{2}{3} [\sqrt{3N + 19} - 4] \). The first-order condition for \( T_1 \) is given by

\[
(M - 2T_1)[1 + N - T_1(M - T_1) - 2M] = 0
\]

There are three roots to this equation: \( T_1 = (M/2) \) and \( T_1 = \frac{1}{2} [M ± \sqrt{M^2 - 4(N + 1 - 2M)}] \).

To show that the first root is the relevant one, we must consider two cases.

Case 1: If \( N > (1/4)[M^2 + 8M - 4] \), then the second and third roots are imaginary. In this case, we must compare profit per division evaluated at \( T_1 = M/2, I \) and \( M - I \). However, since profit per division is the same when \( T_1 = I \) or \( M - I \), it is sufficient to compare (12) when evaluated at \( T_1 = M/2 \) with its value when evaluated at \( T_1 = I \). Evaluating (12) at \( T_1 = M/2 \), we obtain

\[
\frac{\pi M}{M} (T_1 = M/2) = \frac{4A^2(M^2 + 4M)}{Mb(M^2 + 4N + 4)^2}
\]

Evaluating (12) at \( T_1 = I \), we obtain \( \frac{\pi M}{M} (T_1 = I) = \frac{A^2(2M - 1)}{Mb(N + M)^2} \). It is straightforward to show that the former exceeds the latter if and only if \( N \geq M - 1 + \frac{1}{2} \sqrt{M(2M - 1)(M + 4)} \), which is a weaker condition than our initial requirement that \( N > (1/4) [M^2 + 8M - 4] \). Thus, in case 1, \( T_1^* = M/2 \).

We now substitute \( M/2 \) into (12) for \( T_1 \) and maximize profit per division over \( M \). We obtain \( M^* = \frac{2}{3} \sqrt{19 + 3N} - \frac{8}{3} \). With this value for \( M \), our initial requirement that \( N > (1/4)[M^2 + 8M - 4]\) holds if \( N^2 + 6N + 5 > 0 \) which is true for all \( N \).

Case 2: If \( N < (1/4)[M^2 + 8M - 4] \), then all three roots are real. However, since (12) is the same when evaluated at the second and third roots, it is sufficient to compare profit per
division when \( T_i = M/2 \) and \( T_1 = T_\cdot \) where \( T_\cdot = \max\{1, \frac{1}{2}[M - \sqrt{M^2 - 4(N + 1 - 2M)}]\} \).

Evaluating (12) at \( T_1 = T_\cdot \) we obtain

\[
\frac{\pi}{M}(T_1 = T_\cdot) = \max \left\{ \frac{A^2}{4Mb[N + 1 - M]}, \frac{A^2(2M - 1)}{Mb(N + M)^2} \right\}
\]

If we compare this expression with \( \frac{\pi}{M}(T_1 = M/2) \) we find that the former is always larger than the latter (for all relevant values of \( N \)). Thus, in this case, \( T_1^* = T_\cdot \).

Finally, if we maximize \( \frac{\pi}{M}(T_1 = T_\cdot) \) over \( M \), we obtain \( M^* = \frac{1}{8}[3 + \sqrt{9 + 16N}] \).

But, this case is only valid if \( N < (1/4)[M^2 + 8M - 4] \). Substituting \( M^* \) into this expression leaves us with \( 225N^2 - 220N - 164 < 0 \), which holds only if \( N < 1.473 \). It follows that case 2 can be ruled out for all relevant values of \( N \).

We conclude that profit per division is maximized when \( T_1 = M/2 \) and \( M = \frac{2}{3}[\sqrt{3N + 19} - 4] \).

**Result A.2:** Our goal is to show that with general staggered competition the first-order conditions for profit maximization can be solved to obtain (15) and (16) in the text.

We begin with the insiders that move last (in stage \( K \)). These firms have already observed the output choices of all insiders that move in stages \( I \) through \( K - 1 \) (we use \( q_I \) through \( q_{K-1} \) to denote these values) and they have conjectures about the output choices of all outsiders (\( q_o \)). Profit for such a firm is given by

\[
\pi_r = q_r \{ A - b[q_r + \sum_{j \in G_k,j \neq r} q_j + \sum_{i=1}^{K-1} T_i q_i + (N - M)q_o] \}
\]

If we differentiating \( \pi_r \) with respect to \( q_r \), set the expression equal to zero, apply symmetry
(so that \( q_r = q_K \forall r \in G_K \)) and then solving for \( q_K \), we obtain

\[
q_K = \frac{A - b\left[\sum_{i=1}^{K-1} T_i q_i + (N - M) q_o \right]}{b(T_K + 1)}
\] (A.2)

Now, consider the insiders that move at stage \( K - 1 \). For these firms, profit is given by

\[
\pi_s = q_s \{ A - b[q_s + \sum_{j \in G_{K-1}, j \neq s} q_j + \sum_{i=1}^{K-2} T_i q_i + T_K q_K + (N - M) q_o ] \}
\]

where \( q_K \) is given by (A.2). If we use (A.2) to substitute for \( q_o \), differentiate \( \pi_s \) with respect to \( q_s \), set the expression equal to zero, apply symmetry (so that \( q_s = q_{K-1} \forall s \in G_{K-1} \)) and solve for \( q_{K-1} \) we obtain

\[
q_{K-1} = \frac{A - b\left[\sum_{i=1}^{K-2} T_i q_i + (N - M) q_o \right]}{b(T_{K-1} + 1)}
\] (A.3)

For later use, we use (A.3) to substitute for \( q_{K-1} \) in (A.2). This allows us to write \( q_K \) as

\[
q_K = \frac{A - b\left[\sum_{i=1}^{K-2} T_i q_i + (N - M) q_o \right]}{b(T_K + 1)(T_{K-1} + 1)}
\] (A.3')

Comparing (A.2) with (A.3) we see that a clear pattern emerges. If we were to continue this approach for each stage, we would obtain the following expression for the output produced by a typical stage \( j \) insider

\[
q_j = \frac{A_j}{b(T_j + 1)} \quad \text{for} \quad j = 1, \ldots, K
\] (A.4)

where \( A_j \equiv A - b\left[\sum_{i=1}^{j-1} T_i q_i + (N - M) q_o \right] \). Note that (A.4) is (16) in the text.

Solving recursively (as in A.3'), we find that a stage \( j \) insider’s output can also be written as

\[
q_j = \frac{A_i}{b \prod_{i=1}^{j}(1 + T_i)} \quad \text{for} \quad j = 1, \ldots, K
\] (A.5)
where $A_i = A - b(N - M)q_0$.

Now, turn to the outsiders. For each outsider, profit is given by

$$\pi_h = q_h \left\{ A - b[q_h + \sum_{i=1}^{\mathcal{K}} T_i q_i + \sum_{j \in \mathcal{O}, j \neq h} q_j] \right\}$$

If we differentiate $\pi_h$ with respect to $q_h$, set the derivative equal to zero, apply symmetry (so that $q_h = q_o \ \forall h \in \mathcal{O}$) and solve for $q_o$ we obtain

$$q_o = \frac{A - b \sum_{j=1}^{\mathcal{K}} T_j q_j}{b(N - M + 1)}$$

which is (15) in the text.

**Result A.3:** Our goal is to show that (15) and (16) in the text can be solved to obtain (17) and (18). We begin by using (A.5) to substitute for $q_j$ in (A.6). We then solve (A.6) for $q_o$. We obtain

$$q_o = \frac{A}{b} \left\{ \frac{\sum_{i=1}^{\mathcal{K}} \frac{T_i}{\prod_{j=1}^{i} (1 + T_j)} - 1}{\sum_{i=1}^{\mathcal{K}} \frac{T_i}{\prod_{j=1}^{i} (1 + T_j)} - 1 \{N - M\} - 1} \right\}$$

To simplify (A.7), we note that

$$\sum_{i=1}^{\mathcal{K}} \frac{T_i}{\prod_{j=1}^{i} (1 + T_j)} = \frac{1}{\prod_{j=1}^{\mathcal{K}} (1 + T_j)} \sum_{i=1}^{\mathcal{K}} T_i \prod_{j=i+1}^{\mathcal{K}} (1 + T_j).$$

It follows that

$$\sum_{i=1}^{\mathcal{K}} \frac{T_i}{\prod_{j=1}^{i} (1 + T_j)} - 1 = \frac{-1}{\prod_{j=1}^{\mathcal{K}} (1 + T_j)}.$$

If we know define $H_i \equiv \prod_{j=1}^{i} (1 + T_j)$ we can write this as

$$\sum_{i=1}^{\mathcal{K}} \frac{T_i}{\prod_{j=1}^{i} (1 + T_j)} - 1 = \sum_{i=1}^{\mathcal{K}} \frac{T_i}{H_i} - 1 = \frac{-1}{H_K}.$$
\[ q_o = \frac{A}{b[N-M+H_K]} \]

which is (17) in the text. Substituting (A.8) back into (A.5) and solving for \( q_k \) yields (18) in the text.

**Result A.4:** Our goal is to prove Proposition 3. We begin by noting that

\[
\frac{\partial H_K}{\partial M} = H_K \ell n(1+T) > 0 \quad \text{for all } T \geq 1,
\]

and that \( \text{sign} \frac{\partial H_K}{\partial T} = \text{sign}\{1 - M \frac{1+T}{T} \ell n(1+T)\} < 0 \)

for all \( T \geq 1, M \geq 2. \)

We begin by differentiating (24) with respect to \( T \). We find that

\[
\text{sign} \left\{ \frac{\partial \pi_M}{\partial T} \right\} = \text{sign} \left\{ \frac{\partial H_K}{\partial T} [N-M-H_K + 2] \right\}
\]

Since \( \frac{\partial H_K}{\partial T} < 0 \), it follows that if \( N-M-H_K + 2 > 0 \) for all \( T \geq 1 \), then the optimal \( T = 1 \).

But, since \( H_K \) is decreasing in \( T \), if \( N-M-H_K + 2 > 0 \) when \( T = 1 \), then it is true for all larger values of \( T \) as well. Setting \( T = 1 \), this condition becomes \( N-M-2^M + 2 > 0 \). It follows if \( N-M-2^M + 2 > 0 \) then \( T^* = 1 \). Otherwise, \( T^* \) solves \( N-M-(1+T)^M + 2 = 0 \).

If we let \( T(M) \) denote the solution to this latter expression, then we may write the optimal value for \( T \) as \( T^* = \max\{T(M),1\} \).

Now, substitute \( T^* \) back into (24) and differentiate with respect to \( M \). We obtain

\[
\text{sign} \left\{ \frac{\partial \pi_M}{\partial M} \right\} = \text{sign} \left\{ \frac{\partial H_K}{\partial M} [N-M-H_K + 2] - (H_K-1)[N-3M+H_K] \right\}
\]

We are now in position to show that \( T^* = 1 \). Suppose not; that is, suppose that \( T^* = T(M) > 1 \).
1. Then, by the definition of $T(M)$, the first term on the right-hand-side vanishes (since $T(M)$ is defined to set $N - M - H_K + 2 = 0$). This means that the first order condition for $M$ reduces to $N - 3M + H_K = 0$. But, if $N - 3M + H_K = 0$ and $N - M - H_K + 2 = 0$, it must be the case that $M = (N + 1)/2$. Substituting this value for $M$ back into the condition $N - M - H_K + 2 = 0$ and solving for $T$ yields $T = M$. But when $T = M$ we have all insiders moving simultaneously (which gives us the original Cournot equilibrium)! We know that we can do better than this with two-stage staggered competition, so this cannot be profit maximizing behavior. Thus, it cannot be the case that $T^* > 1$. 
References


*Ward’s Automotive Yearbook*, Intertec Publishing (Southfield, MI).


Notes: Mergers are beneficial with two-staged staggered competition if $M < M$. $M_2$ denotes the optimal value for $M$ with two-stage staggered competition. $M^*$ denotes the optimal value for $M$ with general staggered competition.