Pattern Bargaining as an Equilibrium Outcome†

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Abstract: Pattern bargaining is a negotiating strategy that is often employed by industry-wide unions in oligopolistic industries to set wages. However, formal explanations for its existence in equilibrium are rare. The conventional wisdom is that pattern bargaining “takes labor out of competition” and therefore softens bargaining between the union and firms, resulting in higher industry wide wages. However, this does not explain why firms agree to pattern bargaining. In this paper, we argue that the bargaining mechanism cannot be imposed upon the firms and that for pattern bargaining to survive over time as a negotiating strategy it must be agreed upon by all agents involved in the wage setting process. Thus, we analyze a model in which the agents negotiate over the bargaining mechanism, the order of the negotiations and the wages. We show that when side-payments are possible, pattern bargaining may be adopted in equilibrium, but this often occurs when pattern bargaining is the firms’ most preferred mechanism, not the union’s. We also show that when side-payments are not possible, there are cases in which all agents prefer pattern to sequential bargaining, but the union and the firms always disagree over the target firm and so pattern bargaining does not arise in equilibrium. Next, we show that the manner in which firms differ is critical, as when firms differ in non-labor costs, pattern bargaining can arise in equilibrium with the agents agreeing about the identity of the target firm and without the use of side payments. Finally, we show that when equilibrium is characterized by pattern bargaining, it may harm consumers and society. This provides an explanation as to how pattern bargaining can arise in equilibrium and why there is often strong political opposition to it.

†We have benefited from discussions and/or correspondence with Jeff Biddle, John Budd, Wally Mullin, João Montez, Daniel Seidmann, Paula Voos and seminar participants at Emory University and IIOC 2007.

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Pattern bargaining is a widely-used, sometimes controversial negotiating strategy that is often employed by industry-wide unions to set wages. If implemented perfectly, the union picks one firm to bargain with first (the target) and negotiates a wage. It then uses this outcome as a precedent, and makes take-or-leave-it demands to the remaining firms for similar concessions. Assuming that the firms agree, and they usually do, this practice results in a uniform wage rate across firms. Conventional wisdom is that this practice, often attributed in the United States to the United Auto Workers and the Teamsters, provides the union with countervailing power in its relationship with the powerful firms that employ their members. It is argued that by “taking labor out of competition” pattern bargaining helps the union secure higher wages than they would obtain otherwise. Controversy over its impact on wages recently led the Howard Government in Australia to outlaw this practice under the WorkChoices legislation (which became effective on March 27, 2006). It is the contention of union leaders that the government’s objective in doing so was to “irrevocably shift bargaining power in industrial relations to employers.”

Explanations of why pattern bargaining works and how it comes to be adopted are rare. This is surprising given that this strategy clearly plays an important role in wage determination in most OECD countries, often in important industries. The “taking labor out of competition” explanation is the one most commonly given, although, as far as we know, it has never been formalized. The basic idea is simple. Ordinarily, unionized oligopolistic firms have an incentive to bargain tough since any wage concessions that they can pry from the union will provide them with a competitive edge over their product

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1 What we have in mind is an industry in which several unionized oligopolistic firms negotiate wages with the same industry-wide union and then compete with each other in the product market. Thus, the wages that are being negotiated are paid to workers with similar skills in similar occupations. The U.S. automobile industry in which the United Auto Workers negotiates wages with Ford, Chrysler and General Motors or the U.S. airlines industry in which the Air Lines Pilot Association (ALPA) negotiates terms with the major airlines both fit into this framework nicely. Pattern bargaining is also used in other settings by employers to tie-down the wages of workers with different skill sets and occupations. For example, in the U.S. Airline industry pattern bargaining links the wages of pilots, maintenance workers, and workers in other crafts. Pattern bargaining is also used widely in state and local government negotiations with public sector employees with different skill sets (e.g., firefighters and police). In this paper, we restrict attention to pattern bargaining of the former type.

2 Related to pattern bargaining is the “me too” agreement in which some firms agree to accept the terms negotiated by other firms. For example, workers employed by Las Vegas casinos have their wages set through negotiations between their union and a multi-employer bargaining group. Casinos that are not members of the multi-employer group often sign “me too” agreements that bind them to accept those terms without additional negotiations.

3 See the discussion of pattern bargaining and the recent changes in Australian Labor Law on the Construction Forestry Mining Energy Union web site: www.cfmeu.asn.au/construction/research/secondwave/pattern.html.
market rivals. Pattern bargaining assures such firms that if they give in to the union’s wage demands, they will not have to worry that their will rivals will gain an advantage by being tougher during negotiations – after all, all firms wind up paying the same wage rate. Thus, pattern bargaining is viewed as one way to soften negotiations with oligopolistic firms. The problem with this rationale is that is does not explain why the firms are willing to accept such an arrangement.

In recent work, Marshall and Merlo (2004) provide a formal resolution to the first piece of this puzzle by showing in a model with heterogeneous unionized firms that an industry-wide union prefers pattern bargaining. In their model, the two firms produce substitute products and negotiate wages with the union. The firms face constant costs, but differ in labor productivity. Four different bargaining mechanisms are considered, with the Nash Bargaining Solution applied in each case. The union can bargain simultaneously over wages with both firms, they can bargain sequentially, they can use pattern bargaining that results in both firms paying the same wage, or they can use “pattern bargaining in labor costs” which results in wage rates that equalize marginal costs across the firms. In the latter three cases, the order of bargaining is important, so both possible orderings are considered. Marshall and Merlo derive two major results. First, they find that when firm-heterogeneity is weak, the union prefers pattern bargaining in wages; whereas when firm-heterogeneity is strong, the union prefers pattern bargaining in labor costs – thus, the union always prefers some type of pattern bargaining. Second, the union always chooses to use the efficient firm as the target. The first result is consistent with the perceived wisdom that pattern bargaining benefits union members. Marshall and Merlo argue that the second result is consistent with a stylized fact that unions almost never select the relatively unproductive firm as the target.

Though Marshall and Merlo show formally that the union prefers pattern bargaining, in their setting the firms always prefer an alternative bargaining mechanism. This is particularly true of the non-target firm. This firm should surely view the union’s take-it-or-leave it wage demand as non-credible. If the firm rejects the union’s wage demand, it will always be in the union’s interest to reopen negotiations,
effectively resulting in the same outcome that would be generated by sequential bargaining. Thus, it is hard to understand why the firms would ever agree to pattern bargaining.4

In this paper, we argue that the bargaining mechanism cannot simply be imposed upon the firms and that for pattern bargaining to survive as a long-run feature of the negotiation process it must be agreed upon by all agents on both sides of the labor market. It follows that if we are to truly understand pattern bargaining, we must examine conditions under which it emerges endogenously from a setting in which the agents negotiate not only over the wage rates but the bargaining mechanism as well. In order to do so, we develop a model that captures many of the essential features of the markets in which pattern bargaining has been so important (the US auto market serves as our primary motivator). In particular, we assume that the market is characterized by oligopolistic firms that produce substitute goods and that the firms’ production costs are subject to random shocks so that their relative competitive positions may change over time.5

Once the bargaining mechanism has been determined and productivity realized, our model then follows Marshall and Merlo (2004) in that there are two firms that face a linear demand curve, costs are constant, the firms differ in terms of labor productivity, the union’s goal is to maximize the wage bill and wages are determined by the Nash Bargaining Solution. However, our model differs from theirs in three key ways. First, the dynamic nature of the model and existence of the random shocks implies that the firms will prefer the bargaining mechanism that maximizes their expected profits over time. This implies

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4 Marshall and Merlo do recognize this issue and examine it in one of the latter sections of the paper. Building on an insight originally due to Williamson (1968), they extend their analysis to allow for a potential entrant that must use union labor if it enters the market. They then show that in such a setting there are entry costs such that the third firm would enter under sequential bargaining but stay out under pattern bargaining. Consequently, they argue that both the firms and the union may therefore prefer pattern bargaining because it serves as a barrier to entry. To make this point, they show that there exists a wage that would result in higher payoffs for both the union and the firms under duopoly than they would earn with sequential bargaining under triopoly. However, it is unclear whether or not this wage is an equilibrium wage with pattern bargaining. We conjecture that it is not and that if we restrict attention to equilibrium wages it is likely to be the case that the firms prefer pattern bargaining while the union prefers some other bargaining mechanism. We return to this issue and discuss it a bit further in footnote 13.

5 The assumption that costs are subject to random shocks that alter the competitive positions of firms is becoming increasingly common in many applied areas. For example, this assumption is a crucial component of the Mortensen and Pissarides (1994) model of search generated unemployment in macroeconomics, the Hopenhayn (1992) model of industry dynamics in industrial organization and the Melitz (2003) model of monopolistic competition in international economics.
that they will prefer a mechanism that rewards them when they are relatively efficient without penalizing them too much when they are at a competitive disadvantage. Second, although we follow Marshall and Merlo in assuming that the firms differ in labor productivity, we also consider the case in which firm heterogeneity is driven by differences in non-labor costs. Finally, as noted above, we expand the bargaining environment by allowing the agents to negotiate over the bargaining mechanism.

Our dynamic game consists of three stages. In the first stage, the agents negotiate over the bargaining mechanism; whereas the identity of the target firm is determined in second stage. Finally, in the third stage the agents participate in a repeated game of quantity competition in which each period’s wage rate is negotiated after the firms’ costs have been determined and publicly revealed. We find that in this setting there are important instances in which both the union and the firms prefer pattern bargaining over sequential bargaining, but that they always disagree about the order of negotiations. As a result, the extent to which the union can influence the identity of the target firm plays a crucial role in determining conditions under which pattern bargaining arises in equilibrium. However, contrary to the perceived wisdom, we find that this is most likely to occur when pattern bargaining is the preferred mechanism of the firms, not the union. Finally, we find that the source of firm-heterogeneity plays a crucial role in determining conditions under which pattern bargaining can be supported in equilibrium without side payments and the conditions under which consumers are harmed by such a bargaining mechanism. This last issue, which has (to the best of our knowledge) received no attention at all in the literature, is important in light of the recent Australian legislation outlawing pattern bargaining.

The remainder of the paper divides into five sections. In Section 2, in order to gain some insight into the forces driving our results, we look at pattern bargaining from a new perspective by analyzing an

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6 The fact that different assumptions about the source of firm heterogeneity yield different conclusions has been emphasized in Creane (2007).

7 We restrict attention to the two bargaining mechanisms most commonly observed: pattern and sequential bargaining. Marshall and Merlo also consider simultaneous bargaining and “pattern bargaining in costs.” As they show, simultaneous bargaining is always dominated, so that it cannot emerge in equilibrium. Similarly, “pattern bargaining in labor costs” always results a smaller joint surplus for the union and the firms than both sequential bargaining and pattern bargaining in wages. Thus, it cannot arise as an equilibrium outcome. We discuss this point in more detail in footnote 16 below.
extreme version of the Marshall-Merlo model in which the union has all the wage setting power. That is, rather than allow for negotiations, we start out by assuming that the union selects the wage paid by each firm and then the firms choose employment. In such a setting, pattern bargaining is equivalent to the union demanding the same wage from each firm; whereas we interpret a decision by the union to demand different wages as a preference for sequential bargaining. In this setting, contrary to both conventional wisdom and Marshall and Merlo (2004), the union prefers sequential and not pattern bargaining – after all, when the union has the ability to choose the price at which it supplies labor to the firms and the firms have the power to choose the number of workers to employ, the union is in exactly the same position as a monopolist in a product market that must choose whether to charge the same price to all consumers. As we know from the literature on price discrimination, the supplier always prefers to price discriminate when possible. As for the firms that demand labor, their total profits can be higher or lower with a uniform wage: they prefer the uniform wage when they are the more efficient firm but wage discrimination when they are the less efficient firm. Thus, it is clear that there are instances in which the firms will prefer pattern bargaining. Moreover, as we know from the price discrimination literature (see, for example, Varian 1985), the total surplus to be split between the union and the firms may be higher or lower with a uniform wage. This implies that there will be instances in which the firms will prefer pattern bargaining and can afford to bribe the union to accept this bargaining mechanism (bribes are unlikely in the classic model of price discrimination). Since the ability to make side payments plays a critical role in our analysis, we close section 2 by providing examples of side payments that have been used in markets in which pattern bargaining is prevalent.

The problem with the model analyzed in section 2 is that, since it does not allow for wage negotiations, it does not capture the firms’ strategic considerations that are inherent in the “taking labor out of competition” argument. We therefore turn to the general model in sections 3 and 4 (outlined above) which uses the Nash Bargaining Solution to determine wage rates. In section 3 we examine the agents’ preferences over bargaining mechanisms when, as in the Marshall-Merlo model, the firms differ only in labor productivity. Although not all results from section 2 generalize – for example, there are
now cases in which the union prefers a uniform wage – the general flavor of the results remains the same. In particular, we find that when side-payments are possible, pattern bargaining can only emerge as an equilibrium outcome when it is the firms’ preferred mechanism.

In section 3 we also provide intuition for our results by isolating two attributes of the bargaining mechanisms that influence the agents’ preferences. We argue that both the price discrimination aspect inherent in sequential bargaining (i.e., the more efficient firm pays a higher wage) and the ability to “take labor out of competition” under pattern bargaining benefit the union at the expense of the firms. When firm-heterogeneity is weak, the impact of the price discrimination aspect is small and thus the union prefers to take labor out of competition through pattern bargaining while the firms prefer sequential bargaining. But, when firm-heterogeneity is sufficiently strong, the price discrimination aspect is strong enough to reverse the agents’ preferences. These trade-offs are, of course, key in determining the equilibrium outcome of the negotiations over the bargaining mechanism. When the firms differ only in labor productivity, we find that the trade-offs are such that without side-payments pattern bargaining cannot emerge in equilibrium (Proposition 3.3).

In section 4 we extend the model to allow the firms to differ instead in non-labor costs and show that this alters the trade-offs such that pattern bargaining can be an equilibrium outcome even in the absence of side-payments (Proposition 4.3). In addition, we find that this model yields predictions that are more in accord with the conventional wisdom concerning pattern bargaining in that it is a mechanism that can benefit unions at the expense of firms. At the end of sections 3 and 4 we turn to an issue that has received surprisingly little attention in the literature – the impact of bargaining mechanism on consumer welfare – and show that when the firms differ in non-labor costs there are cases in which pattern bargaining is the equilibrium outcome even though consumer surplus would be higher with sequential bargaining (Proposition 4.4). Since the consumers could be interpreted as downstream firms that use the upstream firms’ output as an input, this provides us with a explanation for why pattern bargaining might be preferred all the agents negotiating wages while consumers and downstream firms lobby the government to ban its use. We conclude the paper in section 5 by summarizing our results.
2. Sequential Bargaining as Price Discrimination

In this section we introduce the basic model and then examine the outcome when the union is 
allowed to set the wages. To facilitate comparisons with Marshall and Merlo (2004), we adopt many of 
their assumptions concerning functional forms and use much of their notation.

There are two firms (a and b) that produce a homogeneous good and face the following linear 
demand curve: \( P = 1 - x_a - x_b \), where \( x_i \) denotes the output of firm \( i \).\(^8\) We assume that the workers at 
each firm are represented by the same industry-wide union. As for production, labor is the only input (we 
relax this assumption in the Section 4) but labor productivity at each firm is initially unknown and may 
change from one period to the next. It is common knowledge that in each period one firm (a) will be able 
to produce one unit of output with each worker hired; whereas the other firm (b) will get only \( t \leq 1 \) units 
of output from each worker. Thus, if we use \( \ell_j \) to denote employment for firm \( j \) then we have 
\( x_a = \ell_a \) and \( x_b = t\ell_b \). Note that firm a is the efficient firm whereas firm b is relatively inefficient. The 
identity of the more efficient firm is determined by nature at the beginning of each period, with each 
outcome equally likely. To ensure that both firms produce in equilibrium, we assume that \( t \) is sufficiently 
close to one (to be made precise below).

There are three stages to the game. In the first two stages, the union and the firms negotiate over 
the wage setting mechanism (pattern versus sequential) and, if the order matters, the identity of the target 
firm is determined (the efficient or the inefficient firm). The last stage consists of a repeated game in 
which, in each period, nature determines the firms’ productivities, these values are revealed to all players, 
the union and firms then apply the bargaining mechanism to determine wages and then, finally, the firms 
engage in quantity competition in the product market. Since a new, independent draw on firm 
productivities occurs in each period, the competitive positions of the firms are likely to change over time. 
The firms will keep this in mind when selecting a bargaining mechanism and thus, their goal will be to

\(^8\) For clarity we normalize the demand intercept in Marshall and Merlo (2004) to 1, and assume that the goods are 
perfect substitutes. The degree of substitutability plays no significant role in the results here or in Marshall and 
find the bargaining mechanism that maximizes their expected profits. We assume that the union’s goal is to maximize its expected wage bill.

There are several aspects of this set-up that are worth noting. To begin with, there is a single point in the game at which the bargaining mechanism is determined. That is, agents do not renegotiate the bargaining mechanism in each period. The basic idea that we are trying to capture is that negotiating over the bargaining mechanism is likely to be difficult and costly since the agents are likely to have very different preferences (e.g., the firms versus the union; the efficient firm versus the inefficient firm). Thus, the agents will not want to have to re-visit this issue in each period. Instead, since the firms and the union realize that they are in a long term relationship in which the relative competitive positions of the firms may change over time, they are likely to take a long-run view and try and select up front a bargaining mechanism that works well for all of them over time. A firm may be willing to put up with a bargaining mechanism which is not ideal for them when they are the efficient firm if they know that this same mechanism will work far better for them when they are at a competitive disadvantage.

The second aspect of our set-up worth noting is we are assuming that the wages are determined after each period’s productivity measures have been determined and revealed to all parties. Thus, wage setting takes place under complete information about the current period. Finally, note that since the firms compete in output after the wages have been determined, the firms effectively determine employment (as is the case in almost all labor negotiations).

We begin our analysis by describing the product market outcome. Straightforward calculations yield the Cournot output, employment, profit levels and wage bill as:

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\begin{align*}
(1) \quad x_a (w_a, w_b) &= \ell_a (w_a, w_b) = (1/3t)[t(1 - 2w_a) + w_b] \\
(2) \quad x_b (w_b, w_a) &= t\ell_b (w_b, w_a) = (1/3t)[t(1 + w_a) - 2w_b] \\
(3) \quad \pi_j (w_j, w_i) &= x_j (w_j, w_i)^2 \quad \text{for } j = a, b; \ i \neq j \\
(4) \quad \theta(w_a, w_b) &\equiv w_a \ell_a (w_a, w_b) + w_b \ell_b (w_b, w_a).
\end{align*}
\]
With $\pi_j$ denoting the profits earned by firm $j$ and $\theta$ denoting the wage bill. Following Marshall and Merlo, we restrict attention to the case in which $t \geq .5$.\footnote{This restriction is not needed for this section of the paper. We discuss the appropriate restriction on $t$ in footnote 11 below.}

As noted above, in any given period each firm is equally likely to be the efficient firm or the inefficient firm. Thus, ex ante, each firm’s expected profit in any given period is $0.5(\pi_a + \pi_b)$. It follows that the firms will be interested in selecting the bargaining mechanism that maximizes total per period profits. As for the union, since there is always one efficient firm and one inefficient firm, their goal will be to maximize the (certain) wage bill as defined in (4).

In this section we assume that the union has the power to select the wages paid by each firm. There are two possibilities: the union can demand the same wage from each firm (as in pattern bargaining), or, the union can select firm-specific wages. With the union having all of the wage-setting power, the order of negotiations is not relevant.

Maximizing (4) over $w_a$ and $w_b$ subject to (1)-(2) yields the union’s optimal firm-specific wages. We obtain $w_a = 1/2$ and $w_b = t/2$. Maximizing (4) subject to (1)-(2) and $w_a = w_b$ yields the union’s optimal common wage. We obtain $w_p = \frac{t(1+t)}{4(t^2 - t + 1)}$, where the $p$ subscript denotes that this is the “pattern” wage. For $t \in [.5,1]$, we have $w_b \leq w_p \leq w_a$; thus, since each firm’s profit is decreasing in its own wage and increasing in its rivals wage, the efficient firm prefers the pattern wage while the inefficient firm prefers the firm-specific wage structure. And, substituting these wages back into (1), (2) and (4) and comparing we find that the union is always better-off with the firm-specific wages.

None of these results should be surprising – they mimic standard results from the literature on price discrimination under monopoly (and do not depend upon the assumed functional forms). When the union selects a common wage, it settles on one that is between the two firm-specific wages. This benefits the efficient firm and harms the inefficient firm. And, a standard revealed preference argument is enough
to ensure that the union cannot be worse off when the wages are firm-specific. Thus, viewed from this perspective, one would never expect the union to prefer a common wage.

In our set-up, the agents settle on a bargaining mechanism before productivities are revealed. This is particularly important for the firms. They know that, over time, they are likely to be the efficient firm half the time and the inefficient firm the remainder of the time. Thus, they would prefer the mechanism that leads to the largest per period total profit. It is straightforward to show that with linear demand and constant costs, a common wage leads to higher total profit. It follows that in the initial stage, it will be the firms, not the unions, arguing for a common wage.

Whether the firms can convince the union to adopt a pattern wage structure depends upon the way in which the bargaining mechanism is chosen. We will not deal with that issue at this point. Instead, we will assume that side payments are possible in the initial stage so that the negotiations always lead to the bargaining mechanism that maximizes the joint surplus of the union and the firms. The type of side payments that we have in mind would be any payment made by the firms to the union (or vice versa) that would not affect the firms’ marginal costs of production. So, for example, any contribution made by the firms in the auto industry to the UAW’s “job fund” would qualify; as would a signing bonus paid to all union members who vote “yes” on a new proposed contract. Another example would be an agreement to change the pension formula that results in a more favorable retirement package for workers.\(^\text{10}\)

With linear demand, constant costs and both firms active in equilibrium, it is once again straightforward to show that the total surplus to be split by the union and the firms (i.e., \(\pi_e + \pi_i + \Theta\)) is always larger with a common wage (although this need not be true for other functional forms). Thus, the firms should be able to bribe the union to use set the pattern wage in each period. Note that in this case the outcome is consistent with pattern bargaining in that the firms wind up paying the same wage. However, this outcome emerges because the common wage is in the firms’ best interest and because a common wage generates more value to the agents than firm-specific wages.

\(^{10}\) We thank Paula Voos for suggesting these examples.
As we noted above, there is nothing new in these results. With the union setting the wage, these results are exactly what we expect given what we know about the impact of third-degree price discrimination. Moreover, since wages are not negotiated, this framework does not capture the forces that drive the “taking wages out of competition” argument. So, the reader might wonder why we bothered to present these results at all. The answer is that this analysis provides us with some simple intuition as to why, contrary to prevailing wisdom, an industry-wide union might prefer sequential bargaining. In addition, this suggests that the “taking wages out of competition” effect, which we expect leads the union to favor pattern bargaining, is not going to be the only force that determines the union’s preferences with respect to bargaining structure. And, finally, as we show in the remainder of the paper, the flavor of the results derived in this section survives when the model is extended to allow for wage negotiations.

3. Using Pattern Bargaining to Take Labor Out of Competition

In this section we extend the model so that wages are negotiated. We then examine the agents’ preferences over bargaining mechanisms under the assumption that wages are determined by the Nash Bargaining Solution (NBS). The non-cooperative foundations for the NBS are well known – it is the outcome of negotiations in which the agents trade offers until an offer is accepted (Rubinstein 1982).

When wages are negotiated, the order of bargaining may matter, so we have (potentially) four cases to consider. Since all four cases are algebra intensive, we sketch the solution method in the text and relegate the details to Appendix A where all analytic solutions are provided. We begin with pattern bargaining with firm \( j \) as the target. As is standard, we use backwards induction. If the union fails to reach an agreement with firm \( j \), firm \( i \) becomes a monopolist in the product market. Let \( w^m_i \) denote the wage negotiated by firm \( i \) when it is a monopolist and let \( \ell^m_i(w^m_i) \) and \( x^m_i(w^m_i) \) denote this firm’s labor demand and output at this wage. Then, according to the NBS, \( w^m_i \) maximizes the product of the union’s wage bill and the firm’s profit; that is \( w^m_i \) maximizes \( w\ell^m_i(w)[x^m_i(w)]^2 \) over \( w \). Turn next to the original negotiations between the union and firm \( j \). The wage bill evaluated at \( w^m_i \) serves as the union’s threat.
point during these negotiations, whereas firm $j$’s threat point is zero (since it does not produce if no agreement is reached). Since under pattern bargaining it is understood that both firms will end up paying the same wage, then using (3)-(4) and a straight-forward application of the NBS, the wage that the union settles on when using firm $j$ as the target solves

$$ \max_w \{ \theta(w, w) - w^m_j \ell^m_i (w^m_j) \} x_j(w, w)^2 $$

We use $w^{p(j)}$ to denote this wage, with the super-script denoting that this wage emerges under pattern bargaining with firm $j$ as the target. The solutions for $w^m_i$ and $w^{p(j)}$ are provided in the Appendix A.

Turn next to sequential bargaining under the assumption that the union negotiates with firm $j$ first. Let $w^{s(j)}_j$ denote the wage that comes out of these initial negotiations – note that the super-script refers to the fact that this wage is the result of sequential bargaining with firm $j$ as the target while the sub-script signifies that this is the wage paid by firm $j$. Then, when firm $i$ negotiates with the union, the NBS solution wage that emerges solves

$$ \max_w \{ w^{s(j)}_j \ell_j (w^{s(j)}_j, w) + w \ell_j (w, w^{s(j)}_j) - w^{s(j)}_j \ell^m_j (w^{s(j)}_j) \} x_i(w, w^{s(j)}_j)^2 $$

Note that if the union fails to reach an agreement with firm $i$, firm $j$ becomes a monopolist in the product market, but must still pay the (already) negotiated wage of $w^{s(j)}_j$. This explains why the union’s threat point is given by $w^{s(j)}_j \ell^m_j (w^{s(j)}_j)$. Let $w^{s(j)}_i (w^{s(j)}_j)$ denote the solution to (6). As before, the super-script refers to the fact that this wage emerges under sequential bargaining with firm $j$ as the target; whereas the sub-script signifies that this is the wage to be paid by firm $i$. It is important to note that $w^{s(j)}_i$ is increasing in $w^{s(j)}_j$ with a slope less than one. The reason for this is straight-forward: an increase in firm $j$’s wage increases the surplus to be split by firm $i$ and the union which triggers an increase in firm $i$’s wage. However, $w_i$ does not increase by the full increase in $w_j$ since this would completely undo the competitive advantage just gained by firm $i$ (there is also a stability argument that restricts the slope to be below one).
Finally, we turn the initial negotiations between firm $j$ and the union in order to describe how $w^s(j)$ is determined. If the union and firm $j$ fail to reach an agreement, firm $j$ does not produce (and therefore earns nothing) while firm $i$ becomes a monopolist and the union and firm $i$ settle on a wage of $w^m_i$. This implies that the union’s threat point is given by $w^m_i \ell^m_i (w^m_i)$ and $w^s(j)$ solves

$$\max_w \{w^s_j (w_i^s (w)) + w^s_i (w_i^s (w), w) - w^m_i \ell^m_i (w^m_i) \}, x_j (w, w^s_i (w))^2$$

Note that in negotiating over its own wage, firm $j$ takes into account how the outcome will affect the bargaining between firm $i$ and the union. This is the aspect of wage bargaining in oligopolistic industries that pattern bargaining is meant to eliminate. The analytic solutions for $w^s_j (w)$ and $w^s_i (w_i^s (w))$ are provided in Appendix A.

With the wages determined, we can use (3) and (4) to calculate the union’s wage bill and the firms’ profits. In addition, summing these values allows us to determine the surplus to be split between the agents (producer surplus). To assess the preferences over the different bargaining mechanisms for the agents, we restrict attention to the case in which $5 \leq t \leq 1$.11 This leaves us with the same model analyzed by Marshall and Merlo and allows for a direct comparison of our results. As they demonstrate, with the firms differing only in labor productivity, the order of bargaining does not matter when sequential bargaining is used. Thus, we have three bargaining mechanisms to consider. To facilitate the exposition that follows, we often use S to denote sequential bargaining and P(j) to denote pattern bargaining with firm $j = E, I$ (for efficient and inefficient, respectively) as the target.

We start with the union’s preferences, which follow from Figure 1a. The first result is not surprising: under pattern bargaining, the union always prefers to bargain with the more efficient firm first:

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11 We make this assumption to facilitate comparison to Marshall and Merlo (2004) who restrict attention to $t > \frac{1}{2}$. However, it is possible to show that there is an interior equilibrium in which both firms produce for any $t$ greater than (roughly) $1/3$. The explicit expression for $w^{(i)}$ derived from the first order condition is not defined at $t = \frac{1}{2}$ -- but it is straightforward to verify that at $t = \frac{1}{2}$ there exists a $w$ that satisfies the first order condition ($w = \frac{1}{4}$). This wage is the explicit solution to the first order condition when the limit is taken as $t$ approaches $\frac{1}{2}$. It is not difficult to check that the equilibrium is well behaved (i.e., stability conditions are satisfied) for $t \in (1/3, 1/2]$. Intuitively, we want to restrict $t$ such that the union would prefer to have both firms produce. The qualitatively nature of our results do not change by expanding the range of $t$ – in fact, pattern bargaining becomes more likely for low $t$. 

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\( \theta(w^{p(a)}, w^{p(a)}) \geq \theta(w^{p(b)}, w^{p(b)}) \) for \( t \in [0.5, 1] \). This follows from the fact that the efficient firm generates a larger surplus and can therefore afford to pay a higher wage than its counterpart. As we show in the next section, this result generalizes to situations in which the firms differ in non-labor costs as well – it is always in the interest of the union to use the more efficient firm as the target. The second result that is evident from Figure 1a is that when the degree of firm-heterogeneity is weak \((0.7338 \leq t \leq 1)\), the union prefers pattern over sequential bargaining regardless of which firm is used as the target. When firm-heterogeneity is moderate \((0.5785 \leq t \leq 0.7338)\), the union still prefers pattern bargaining to sequential, as long as the target is the efficient firm. It is only when the firms differ dramatically in labor productivity \((0.5 \leq t \leq 0.5785)\) that the union prefers to bargain sequentially.

Marshal and Merlo (2004) provide intuition for the union’s preference ordering and it is the main result of their paper. However, we offer a somewhat different explanation for these results. As we showed in the previous section, the price discrimination aspects of sequential bargaining benefit the union at the expense of the firms. However, the “taking labor out of competition” aspect of pattern bargaining results in higher average wages in the industry. This follows from the fact that firms are more willing to agree to wage increases when they know that their rival’s wage will rise by the same amount that their wage is increasing (and, as we saw above, with sequential bargaining any increase in \( w_j \) leads to an increase in \( w_i \) of smaller magnitude). Thus, sequential bargaining benefits the union by allowing for different wages across firms but harms the union by generating lower average wages. The benefit from wage discrimination is not particularly important when the firms are similar, so that with weak heterogeneity the latter force dominates and the union prefers pattern bargaining. The advantages from wage discrimination grow as the firms become less alike, and this effect dominates when firm-heterogeneity is strong. As a result, when the firms are quite dissimilar, the union prefers sequential bargaining.

The problem with the Marshall-Merlo result is evident from Figure 1b where the firms’ aggregate profits \((\Pi)\) are depicted for each of the bargaining mechanisms. As we noted above, the firms’ goal is
to select the bargaining mechanism that maximizes its expected profits, which, given our set-up, is equivalent to maximizing total industry profits. To begin, it is clear that with pattern bargaining, the firms always prefer the inefficient firm to be the target. So, even if we restrict our attention to pattern-bargaining, there is disagreement between the union and the firms over the nature of the bargaining process. When we include sequential bargaining, the possibility of agreement seems even less likely. Figure 1b indicates that when firm-heterogeneity is weak (.7267 ≤ t ≤ 1), the firms prefer sequential bargaining over any form of pattern bargaining. Note, however, that when firm-heterogeneity is moderate or dramatic (.5 ≤ t ≤ .7267), the firms prefer pattern bargaining. More specifically, their first choice is always pattern bargaining with the inefficient firm as the target – that is, P(I). The fact that the firms prefer that the weak firm serves as the target should not be surprising – since the inefficient firm generates a smaller surplus, using it as the target results in a lower wage and higher profits.\footnote{Note that with sufficient heterogeneity (.5 ≤ t ≤ .529) the firms will prefer any sort of pattern bargaining over sequential bargaining.}

The explanation for the firms’ preferences is similar to that given above for the union. The price discrimination inherent in sequential bargaining harms the firms; whereas the ability to take labor out of competition through pattern bargaining also harms the firm (due to the higher average wages). When the negative aspects from sequential bargaining are dominate, the firms prefer pattern bargaining; otherwise, they prefer sequential bargaining. If the firms are similar, the impact of price discrimination is small and the firms prefer sequential bargaining. As the difference between the firms grows, the impact of price discrimination increases so that the firms eventually switch and prefer pattern bargaining.

Lemma 3.1: When the firms differ in labor productivity

A. If the firms are relatively homogenous (.5785 ≤ t ≤ 1), the wage bill is maximized by pattern bargaining with the efficient firm as the target. Otherwise, the wage bill is maximized by sequential bargaining.
B. If the firms are relatively homogenous \((.7267 \leq t \leq 1)\), then aggregate profits are maximized by sequential bargaining. Otherwise, aggregate profits are maximized by pattern bargaining with the inefficient firm as the target.

From Lemma 3.1 it is clear that no common bargaining mechanism is the first choice for both sides. For \(t < .5785\), the union prefers S while the firms prefer P(I). For \(t > .7267\) the firms prefer sequential bargaining but the union prefers P(E). Finally for \(t \in (.5785, .7267)\), both sides prefer pattern bargaining, however they disagree as to the identity of the target.

3.1 Pattern bargaining as an equilibrium outcome

Since the union and the firms’ preferences are so different, it is useful to look at how the surplus to be split by the agents, producer surplus (PS), varies with the bargaining mechanism. This is shown in Figure 1c. When firm-heterogeneity is weak to moderate \((.6502 \leq t \leq 1)\), Figure 1c indicates that the PS is largest with sequential bargaining. In this case, it is hard to imagine how pattern bargaining could ever arise in equilibrium. Figure 1c also indicates that when firm-heterogeneity is strong \((.5 \leq t \leq .6502)\) producer surplus is largest when the firms get their preferred outcome: P(I). This suggests that pattern bargaining is likely to emerge only when it is the outcome desired by the firms, not the unions. These results are entirely consistent with those presented in section 2 when we assumed that the union selects the wage. We summarize the results with

**Lemma 3.2:** When the firms differ in labor productivity, if \(t \in (.6502, 1)\) then producer surplus is maximized by sequential bargaining. Otherwise, producer surplus is maximized by pattern bargaining with the inefficient firm as the target.

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13 Marshall and Merlo (2004) do not examine producer surplus. However, the issue does come up implicitly in the final section of their article when they discuss potential entry by a third firm. As discussed in footnote 4, Marshall and Merlo show that in the presence of potential entry, there is a common wage that would make both the union and the two incumbents better-off than they would be with sequential bargaining (this common wage blocks entry while sequential bargaining allows the third firm to enter). They then argue that pattern bargaining would therefore be in the interest of the union and the two incumbents. The fact that such a wage exists merely shows that producer surplus would be higher with a common wage than with sequential bargaining. What needs to be shown is that this common wage is also an *equilibrium* wage with pattern bargaining.
The explanation for the relationship between bargaining structure and producer surplus can be explained as follows. As with the firms and the union, there are two forces at work. Since sequential bargaining leads to lower average wages, one should expect sequential bargaining to generate a greater surplus – lower wages imply greater output and a larger surplus to be shared. However, with sequential bargaining the inefficient firm winds up paying a lower wage rate than it would under pattern bargaining. This implies that under sequential bargaining, the inefficient firm has a greater market share and thus, a greater fraction of total output is produced inefficiently. Consequently, even though sequential bargaining leads to lower average wages, total output can be lower with sequential bargaining. When the firms are similar, the first effect dominates and producer surplus is higher with sequential bargaining. As firm-heterogeneity become more pronounced, the loss in output from inefficient production grows, so that producer surplus eventually becomes larger with pattern bargaining.

To characterize equilibrium, we must now describe the three-stage game that the agents face. In the first stage, the agents select the bargaining mechanism. If pattern bargaining is selected, then in the second stage with probability $q \in [0,1]$ nature assigns the union the power to determine the identity of target firm. We therefore use $q$ as a measure of the union’s ability to influence the selection of the target. For example, if $q = 1$, then the union always chooses the target whereas if $q = 0$, then the firms always choose the target. By varying $q$ we can consider all possible stage two outcomes and investigate the manner in which the union’s ability to influence the selection of the target affects the likelihood of pattern

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14 The efficient wage (the one which maximizes producer surplus and welfare) is equal to zero.

15 In the next section, we show that with positive non-labor costs this market shifting effect leads to an additional inefficiency. The logic is as follows. Since workers have zero opportunity costs and duopolistic competition implies that price is above marginal cost, efficiency is determined by the bargaining mechanism’s impact on total output. In our current setting, the less efficient firm is less efficient because it produces less output for a given number of workers. One way to see this is to note that, since workers have zero opportunity costs, the inefficiency is that given the number of workers employed, output could increase if a worker was transferred from the less efficient to the more efficient firm. In the next section, where we allow for heterogeneity in non-labor costs, the less efficient firm is less efficient because it produces less and uses more resources per unit of output. As a result, there is another inefficiency that arises holding fixed the number of workers employed -- if one worker is transferred to the more efficient firm, output would increase and fewer non-labor resources would be used.

16 As we discussed in footnote 7, we do not consider pattern bargaining in costs since it leads to a smaller joint surplus than sequential bargaining and/or pattern bargaining in wages. The reason for this is that pattern bargaining in costs combines the worst aspects of the other two mechanisms (in terms of generating a large joint surplus) – by softening negotiations it leads to higher average wages than sequential bargaining and by equalizing marginal costs across firms it results in a less efficient distribution of production than pattern bargaining in wages.
bargaining arising in equilibrium. Finally, with the target set, in the last stage the agents engage in a repeated game of quantity-competition in the product market in which the firms’ productivity levels vary (but are always publicly revealed) from period to period.

As benchmark we first consider the outcomes when side payments can be used to influence the bargaining choice. This in turn can be divided into two cases. In the first case, we assume that side payments can be used to influence both the bargaining mechanism and the selection of the target. In the second case, we assume that the agents are unable to contract on the target. In the first case, we expect the agents to select the mechanism that maximizes producer surplus. Thus, Figure 1c is sufficient to predict the outcome of the bargaining process (though the direction of the side payments depends on \( q \), the probability that the union selects the target). From Lemma 3.2 it follows that

**Proposition 3.1:** Suppose that the firms differ in labor productivity and that side payments can be used to influence both the bargaining mechanism and the target firm, then if the firms are sufficiently homogenous (.6502 \( \leq r \leq 1 \)), sequential bargaining is the equilibrium outcome. Otherwise, equilibrium is characterized by pattern bargaining with the inefficient firm as the target.

If side payments cannot be used to influence the selection of the target, then the value of \( q \) becomes important. To see this, consider what occurs in the two extreme cases. First, assume that the union always has the power to select the target (i.e., \( q = 1 \), a case that seems to be consistent with the institutions in several markets). Given the union’s preference, sub-game perfection dictates that if pattern bargaining is selected in stage one, then the union always selects the efficient firm as the target. However, Figure 1c indicates that producer surplus with sequential bargaining always exceeds its value under pattern bargaining when the efficient firm is the target. Thus, when \( q = 1 \) and side payments can only be

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17 See the discussion at the end of section 2 for examples of the kinds of side payments that we have in mind.

18 Values of \( q \in (0,1) \) are consistent with a commonly observed phenomenon associated with pattern bargaining in that the target firm and its characteristics change over time (i.e., sometimes the union will select the most efficient firm as the target while at other times it will select a mid-level firm as the target).
used in stage one, S is the only outcome. Note that in this case, if $t > .5786$, then since the union prefers $P(E)$, we would expect side payments to flow from the firm to the union. That is, if they could, the firms would bribe the union in order to obtain sequential bargaining.

Now turn to the other extreme in which firms select the target (i.e., $q = 0$). In the sub-game in which pattern bargaining is selected in stage one, the firms always select the inefficient firm as the target. Given this, the relative values of producer surplus in Figure 1c indicate that if $t \in (.6502, 1)$, S is the equilibrium outcome. Moreover, the side-payments go from the union to the firm whenever $t > .7267$. Thus, when $q = 0$ and $t$ is sufficiently high, the union would be willing to bribe the firms in order to obtain sequential bargaining. Finally, if $t \in (.5, .6502)$, $P(I)$ is the equilibrium outcome. It is worth noting that by Proposition 3.1, assigning the firms the power of selecting the target firm raises producer surplus.

Extending the analysis to the intermediate cases in which $q \in (0,1)$ is straightforward, since we simply take a weighted average of the payoffs. For $t \in (.6502,1)$, both sides prefer sequential bargaining regardless of $q$. If $t \in (.5,.6502)$, then the outcome depends on whether, given $t$, producer surplus with sequential bargaining exceeds $q \cdot \text{PS}^{(a)} + (1 - q) \cdot \text{PS}^{(b)}$ where the super-script $p(j)$ indicates that pattern bargaining has been adopted with firm $j$ as the target. Since for any $t < .6502$, producer surplus with the efficient firm as the target is strictly greater than producer surplus with sequential bargaining, there always exists a $q$ arbitrarily close to 0 such that producer surplus with pattern bargaining is greater than with sequential bargaining.

**Proposition 3.2:** Suppose that the firms differ in labor productivity and that side payments can be used to influence the bargaining mechanism but not the target, then if the firms are sufficiently homogenous ($.6502 \leq t < 1$), sequential bargaining is the equilibrium outcome. In addition, when the firms are sufficiently heterogeneous, ($.5 \leq t \leq .6502$) there exists a $q^* \in (0, 1)$ such that for $q > q^*$ there is sequential bargaining; otherwise equilibrium is characterized by pattern bargaining with the inefficient firm as the target.
Proposition 3.2 indicates that pattern bargaining can still arise in equilibrium, but only when the firms have sufficient influence on the selection of the target.

We now turn to the case in which side payments are not possible. We assume that in the first stage, knowing how the target will be selected, the agents simultaneously announce their choice as sequential (S) or pattern (P). However, it is impossible to implement pattern bargaining if all agents do not agree to adopt it. Thus, we assume that if all agents select P, then pattern bargaining is adopted; otherwise, sequential bargaining is the outcome.

Again we first consider the two extreme cases: the union always chooses the target \((q = 1)\) or the firms always choose the target \((q = 0)\). When the union chooses the target, the efficient firm is always selected. Figure 1a indicates that the union prefers S to P(E) for all \(t < .5786\); whereas Figure 1b indicates that the firms prefer S to P(E) for all \(t > .5291\). Thus, S is always the outcome. When the firms choose the target, the inefficient firm is always selected. Figure 1a indicates that the union prefers S to P(I) as long as \(t < .7338\); whereas Figure 1b indicates that the firms prefer S to P(I) for all \(t > .7267\). Thus, in these two extreme cases, sequential bargaining is the only possible equilibrium outcome.

We now consider intermediate values for \(q\). In the first stage, the agents know that if pattern bargaining is chosen the efficient firm will be selected as the target with probability \(q\). As a result, the payoff to the union when P is the outcome is \(q\theta_p(a) + (1-q)\theta_p(b)\) (recall that the super-script \(p(j)\) indicates that pattern bargaining has been adopted with firm \(j\) as the target). On the other hand, under sequential bargaining the union earns \(\theta^S\) (since the order of bargaining does not matter). The payoff for the firms is calculated in an analogous fashion. It should be clear that for \(t \geq .7267\) and \(t \leq .5785\), S will be the outcome. In the former case, the firms prefer S even when they get to choose target firm with probability one; whereas in the latter case it is the union that insists on S regardless of \(q\).

For \(.5785 \leq t \leq 7267\), the firm will only choose P if \(q\) is sufficiently high; whereas the firms will choose P only when \(q\) is sufficiently low. Let \(q_U(q_F)\) denote the value of \(q\) that equates the expected wage bill (expected total profit) with pattern bargaining with the wage bill (total profit) from sequential
bargaining. Then there exists a value of $q$ such that both the union and the firms prefer $P$ over $S$ iff $q_U \leq q_F$. However, when the firms differ only in labor productivity we find that for all $t$ in this range $q_U > q_F$, so that there is no value for $q$ such that both the union and the firms prefer pattern bargaining! Thus, when side payments are not possible, we conclude that in the Marshall-Merlo framework we would always expect sequential bargaining to be chosen in equilibrium. It follows that the fact that the union may prefer pattern bargaining is not sufficient to explain how it emerges as an equilibrium outcome.

**Proposition 3.3:** When the firms differ in labor productivity if side payments are not possible, then there is sequential bargaining for all $t$.

To summarize our results so far, in the Marshall-Merlo framework, Proposition 3.3 indicates that without side payments, pattern bargaining can never emerge as an equilibrium outcome. Propositions 3.1 and 3.2 indicate that when side payments are possible, equilibrium may be characterized by pattern bargaining, but only when the inefficient firm is used as the target. Moreover, from Lemma 3.1, pattern bargaining only emerges in equilibrium when it is the preferred bargaining mechanism of the firms.

We now turn to an issue that has received surprisingly little attention in the literature – the impact of bargaining structure on consumer welfare and total welfare. There are several features of our set-up that simplify this analysis considerably – the good is homogenous, workers have zero opportunity costs and the firms have zero non-labor costs. This implies that both consumer welfare and overall welfare are completely determined by the level of total output (the wage bill is simply a transfer from the firms to the union). Specifically, since the equilibrium price is always greater the resource costs (which is zero), an increase in output here always results in an increase in both measures of welfare.

The results are shown in Figure 1d and are summarized in our next Proposition:

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19 We revisit this issue in the next section when we allow for positive non-labor costs. In that case, consumer welfare and total welfare need not move in the same direction when the bargaining mechanism changes.
**Proposition 3.3:** Suppose that the firms differ in labor productivity; then if \( t \in (.6502, 1) \),

consumer welfare and total welfare are both maximized by sequential bargaining.

Otherwise, consumer welfare and total welfare are maximized by pattern bargaining with the inefficient firm as the target.

A careful examination of Propositions 3.1-3.3 reveals that the only time that pattern bargaining emerges in equilibrium it is also the bargaining mechanism that maximizes both consumer welfare and total welfare. This calls into question the motivation behind the Howard Government’s decision to outlaw the practice in Australia. We return to this issue in the next section where we show that with differences in non-labor costs, equilibrium may be characterized by pattern bargaining in situations in which it is an inefficient mechanism.

Finally, for future reference, we note the following corollary to Lemma 3.2 and Proposition 3.4.

**Corollary 3.1:** When the firms differ in labor productivity, consumer surplus and total welfare are maximized by the same bargaining mechanism that maximizes producer surplus.

This follows from the fact that, for the equilibrium values here, producer surplus is increasing in output (although we show in the next section that this may not hold when the firms differ in non-labor costs). There are two reasons for this. First, equilibrium outputs are below the levels that maximize producer surplus. Second, because workers have no opportunity costs and the firms face no additional costs, the distribution of output across firms plays no role in determining welfare. Hence, whichever bargaining mechanism leads to the highest output level also results in the greatest producer surplus and total surplus.

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20 This result depends on the structure of the product market. For example, if the union faced a competitive market downstream (so the firms are price takers both in the wage and the output price), then producer surplus would be maximized.
4. Differences in Non-Labor Costs

We now extend the model to allow the firms to differ in non-labor costs. We do so by assuming that each time that firm \( b \) (the inefficient firm) produces a unit of output it must incur a non-labor cost of \( c \). Firm \( a \), on the other hand, faces no additional per unit costs. In order to highlight the importance of this extension, we set \( t = 1 \) so that this is the only difference across the firms. This implies that firm \( a \)'s marginal cost of production is \( w_a \); whereas firm \( b \)'s marginal cost is \( w_b + c \). As we show, this assumption alters the trade-offs faced by the firms and the unions and makes it possible for pattern bargaining to emerge as an equilibrium outcome in even in the absence of side payments.

With this set-up, Cournot competition in the product market leads to the following equilibrium outcomes (both firms are active in equilibrium under all relevant bargaining mechanisms provided that \( c \) is sufficiently low – that is, \( c \leq .342 \)).

\[
\begin{align*}
\ell_a(w_a, w_b) &= \frac{1}{3}[1 - 2w_a + (w_b + c)] \\
\ell_b(w_b, w_a) &= \frac{1}{3}[1 + w_a - 2(w_b + c)]
\end{align*}
\]

with the firms’ profits and the union’s wage bill still given by (3) and (4), respectively. Furthermore, the NBS wages under the different bargaining mechanisms are still defined by (5)-(7), with the analytic solutions provided in the Appendix A.

With \( c > 0 \), the outcome from sequential bargaining now depends on which firm is used as the target. Thus, we now have four different mechanisms to consider. This is the first indication that the source of firm-heterogeneity may be important. Moreover, this outcome seems more intuitive, as one would normally expect the order of negotiations to matter.

For this case, the analogs of Figures 1a-d are complex so that it is easier to follow the ensuing discussion by referring to a description of the agents’ preference orderings. These are provided in Appendix B. As is clear from the orderings of the wage bills, the union prefers pattern bargaining over

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21 The condition binds for when there is pattern bargaining with the efficient firm.
22 In addition, the monopoly outcome also differs with cost differences while productivity differences have no affect. These results suggest that the functional form for productivity differences may have some peculiar properties to it.
sequential bargaining for all $c$.\textsuperscript{23} Thus, when the firms differ only in non-labor costs, the “taking labor out of competition” effect associated with pattern bargaining always dominates the advantages from price discrimination that arise under sequential bargaining. This result is consistent with the conventional wisdom about pattern bargaining as well as the basic message of Marshal and Merlo – the union prefers pattern bargaining. The orderings also indicates that, as in the Marshall-Merlo framework (and for the same reasons), the union always prefers to bargain with the efficient firm first (regardless of whether sequential or pattern bargaining is chosen).

Turning to the firms, we find that their preferences are consistent with the case of productivity differences discussed in section 3 (see Appendix B). That is, they prefer sequential bargaining when firm heterogeneity is weak ($c \leq .263$) but prefer pattern bargaining when firms are sufficiently heterogeneous ($c \geq .263$). The intuition follows previous sections – for the firms there are both positive and negative aspects of sequential bargaining. The positive aspect is that it results in lower average wages. The negative aspect, which dominates when the firms are sufficiently heterogeneous, is that they are harmed by the price discrimination inherent in sequential bargaining. We also see that, as before and for the same reasons, the firms always prefer that the inefficient firm carry out their negotiations first.

**Lemma 4.1:** When the firms differ in non-labor costs

**A.** The wage bill is always largest under pattern bargaining with the efficient firm as the target.

**B.** If the firms are relatively homogenous ($0 \leq c \leq .263$), then aggregate profits are maximized by sequential bargaining. Otherwise, aggregate profits are maximized by pattern bargaining with the inefficient firm as the target.

\textsuperscript{23} This result is partly the artifact of examining interior solutions in all cases, which requires that $c < .342$ (this restriction is required under pattern bargaining with the efficient firm as the target). If we allow for corner solutions, then there exists a small region of the parameter space when firms are sufficiently heterogeneous in which sequential bargaining with the efficient firm as the target is preferred by the union to pattern bargaining with the efficient firm as the target.
There are two important similarities between the results reported in this lemma and those summarized in Lemma 3.1. First, in both cases, there are parameter values such that all agents prefer pattern bargaining to sequential bargaining – when the firms differ in productivity, this occurs when \( t \) is moderate (\( .5785 \leq t \leq 1 \)); whereas with differences in non-labor costs this occurs with sufficient firm-heterogeneity (\( c > .263 \)). Second, in both cases the agents disagree about the target. When the firms differ in productivity, the disagreement over the identity of the target is strong enough that it becomes impossible to get pattern bargaining in equilibrium without side payments. As we show below, this is no longer the case when the firms differ in non-labor costs.

### 4.1 Pattern bargaining in equilibrium

To explore the conditions necessary for pattern bargaining to arise in equilibrium, we once again begin by examining producer surplus (see the orderings in Appendix B). As in Section 3, producer surplus is maximized when the inefficient firm is the target since this results in lower wages. Moreover, producer surplus is maximized under sequential bargaining when the firms are similar, but pattern bargaining generates a larger surplus when the firms are significantly different.

**Lemma 4.2:** When the firms differ in non-labor costs, if \( c \in [0, .1965) \), then producer surplus is maximized by sequential bargaining with the inefficient firm as the target. Otherwise, producer surplus is maximized by pattern bargaining with the inefficient firm as the target.

From Lemmas 4.1 and 4.2 we conclude that for much of the parameter space, the firms’ first choice also maximizes producer surplus. Specifically, if \( c \leq .1965 \) or \( c \geq .263 \) producer surplus and aggregate profits are maximized by the same bargaining mechanism (sequential bargaining in the former case, pattern bargaining in the latter case and in both instances, the inefficient firm serves as the target). In addition, when \( c \geq .263 \) all agents have pattern bargaining as their first choice – unfortunately, the unions and the firms do not agree as to the target. Lemma 4.2 implies that in this case, if side payments
could be used to influence the target they would flow from the firm to the union to ensure the selection of the inefficient firm.

For the intermediate case (.1965 \leq c \leq .263), the firms’ payoff is no longer maximized by the bargaining mechanism that maximizes producer surplus – the firms prefer sequential bargaining while pattern bargaining maximizes producer surplus. As for the union, it prefers pattern bargaining, but with the efficient firm as the target. However, for most of this region .2052 \leq c \leq .263, producer surplus is maximized by both sides’ second choice – pattern bargaining with the inefficient firm as the target, suggesting that this may arise as the equilibrium outcome.

We now consider how pattern bargaining may arise under various settings. We begin as before with the strongest case: when side-payments can be used to influence both the bargaining mechanism and the target. The result follows directly from Lemma 4.2.

**Proposition 4.1:** Suppose that the firms differ in non-labor costs and that if side payments can be used to influence both the bargaining mechanism and the target. Then if the firms are sufficiently homogenous (0 \leq c < .1965), sequential bargaining with the inefficient firm as the target is the equilibrium outcome. Otherwise, equilibrium is characterized by pattern bargaining with the inefficient firm as the target.

If we compare Propositions 3.1 and 4.1 we see that when side payments are possible and the agents can contract on the target firm, the two models to yield almost identical conclusions. To further explore the importance of the target firm, we now relax the assumption that the agents can contract on its identity. As before, we start with the extremes in which either the union or the firms gets to select the target firm with certainty.

Consider first the case in which the union selects the target (q = 1). By sub-same perfection, the efficient firm must always bargain first. Given this we see (from Appendix B) that whenever c > .1523, pattern bargaining maximizes producer surplus; otherwise sequential bargaining is optimal. We can
divide this into two sub-cases: when \( c \geq .2642 \) the firms and the union both prefer pattern bargaining, so that side payments are not needed; whereas when \(.1523 \leq c \leq .2642 \), the union must bribe the firms in order to secure pattern bargaining. Note that this latter case is consistent with the conventional wisdom – it is the union that prefers pattern bargaining and it must find some way to convince the firms to accept this mechanism. Finally, if \( c \leq .1523 \), the side payments flow in the opposite direction with the firms bribing the union in order to secure sequential bargaining.

Turning now to the case when the firms select the target (\( q = 0 \)), sub-game perfection dictates that the inefficient firm must always be the target. Thus, with side payments we expect to see (from Appendix B) pattern bargaining as the outcome whenever \( c > .1965 \); otherwise, equilibrium should be characterized by sequential bargaining. It follows that, as in Section 3, if we give the firms the ability to select the target we always get the bargaining mechanism that maximizes producer surplus (see Lemma 4.2). For the case in which \( c \leq .1965 \), side payments flow from the firms to the union (to secure sequential bargaining); whereas when \(.1965 \leq c \leq .2630 \), side payments flow from the union to the firms (to obtaining pattern bargaining). For the case with \(.2630 \leq c \leq .3127 \), since both sides would prefer pattern bargaining over sequential, no side payments are required. In the last case, in which \( c \geq .3127 \), the firms must bribe the union to secure pattern bargaining.

Turn next to the intermediate cases in which \( q \in (0,1) \). When the firms are sufficiently heterogeneous \( (c > .1965) \), things are straightforward since pattern bargaining arises both when the firms and the union select the target. Hence, equilibrium is always characterized by pattern bargaining with the side payments, if they are needed, flowing from the union to the firms.\(^{24}\) Similarly if \( c < .1523 \), sequential bargaining is the outcome for all \( q \) and if side payments are needed, they flow from the firms to the union. Note that both cases accord with the conventional wisdom – side payments are either used by the union to

\[^{24}\text{There is one small exception to this -- if the firms are strongly heterogeneous (}\( c > .3127 \), the side payments could flow from the firms to the union (since the union prefers sequential whenever the firms select the target).}\]
Proposition 4.2: Suppose that the firms differ in non-labor costs and that side payments can be used to influence the bargaining mechanism but not the target. Then if the firms are sufficiently homogenous \((c < .1523)\), sequential bargaining is the outcome. If the firms sufficiently heterogeneous \((c > .1965)\), pattern bargaining is the outcome. For intermediate cases \((.1523 < c < .1965)\), there exists a \(q^* \in (0, 1)\) such that for \(q < q^*\) sequential bargaining is the outcome; otherwise equilibrium is characterized by pattern bargaining.

Comparing Propositions 3.2 and 4.2 we see that there are some important distinctions in the results generated by the two models. First, when firm-heterogeneity is tied to differences in non-labor costs, pattern bargaining can arise for all \(q\) (when \(c \geq .1965\)) – this is not possible when the firms differ only in labor productivity. Second, Proposition 4.2 and Lemma 4.1 indicate that with differences in non-labor costs pattern bargaining can arise in equilibrium when it is the union’s (but not the firms’) preferred mechanism (when \(.1965 < c < .263\)); again, this is not possible with differences in labor productivity. Finally, with differences in non-labor costs pattern bargaining can emerge in equilibrium even though sequential bargaining would yield greater producer surplus (when \(.1523 < c < .1965\) and \(q\) is sufficiently large). In contrast, with productivity differences we saw that whenever equilibrium is characterized by pattern bargaining, it is also the mechanism that maximizes producer surplus, consumer surplus and total welfare.

\(^{25}\) Recall that expected producer surplus under pattern bargaining is \(q \cdot PS_p(a) + (1 - q) \cdot PS_p(b) \equiv E[PS_p^\circ]\); whereas expected producer surplus under sequential bargaining is \(q \cdot PS_s(a) + (1 - q) \cdot PS_s(b) \equiv E[PS_s^\circ]\). In this case, the analysis turns on the fact, conditional on the union selecting the target, producer surplus would have been greater if pattern bargaining was the mechanism; but, conditional on the firms selecting the target, sequential bargaining is superior. That is, pattern bargaining is selected if \(E[PS_p^\circ] > E[PS_s^\circ]\); that is, if \(q(PS_p(a) - PS_p(b)) > (1 - q)(PS_s(b) - PS_s(a))\). Let \(q^*\) be the \(q\) such that \(q(PS_p(a) - PS_p(b)) = (1 - q)(PS_s(b) - PS_s(a))\); then if \(q > q^*\), pattern bargaining maximizes producer surplus. For values of \(c\) close to \(.1523\), \(PS_p(a) \approx PS_s(a)\) and \(PS_p(b) < PS_s(b)\) so that \(q^*\) approaches 1. Similarly, for values of \(c\) close to \(.1965\), since \(PS_p(a) > PS_s(a)\) and \(PS_p(b) \approx PS_s(b)\), so that \(q^*\) approaches 0. Further, since \((PS_p(a) - PS_p(b))\) is increasing in function of \(c\) while \((PS_s(b) - PS_s(a))\) is a decreasing function of \(c\), \(q^*\) is decreasing in \(c\).
We now complete the analysis by considering the (perhaps most empirically relevant) case in which side payments are not possible. As we argued above, since the non-target firm can always reject the union’s take-it-or-leave-it offer, we require that both sides select P in stage 1 (with $q$ taken as given) for pattern bargaining to be implemented in equilibrium. We begin with the two extreme cases. If the union has complete control over the target ($q = 1$), the efficient firm will be selected as the target. We already know that the union will opt for pattern bargaining (Lemma 4.1). Turning to the firm, we see that for strong heterogeneity ($c > .2646$) the firms also would agree to pattern bargaining. This represents the first example that we know of in which pattern bargaining can emerge as an equilibrium outcome without the use of side payments.

Turning to the case in which the firms can select the target ($q = 0$), we know that they will always select the inefficient firm. In this case (from Appendix B), the union would select pattern bargaining as long as the firms are not too different ($c < .3127$, see Appendix B). The firms, on the other hand, prefer pattern bargaining whenever $c > .2630$; otherwise, they prefer sequential bargaining. This implies that pattern bargaining will be the equilibrium outcome for all $c \in [.2630, .3127]$.26

Extending the analysis to allow for all possible $q$, it follows for $c \in [.2646, .3127]$ pattern bargaining arises regardless of the target firm. That is, for all $q$, pattern bargaining is the equilibrium outcome. Further, for $c \geq .2630$, there always exists a range of values for $q$ such that pattern bargaining. For $c < .2630$, sequential bargaining is the outcome. Summarizing, we have

**Proposition 4.3:** Suppose that the firms differ in non-labor costs and that side payments are not possible, then if firms are sufficiently homogeneous ($c < .2630$), there is sequential bargaining. For $c \in [.2630, .2646]$, there exists a $q^*$ such that we get

---

26 It is worth noting that the equilibrium wage solutions provided in Appendix A are sufficient to allow for consideration of the general model in which the firms differ in both labor productivity and non-labor costs. We have examined this model and the results are qualitatively identical to those described in this section. The key result remains: for a significant portion of the parameter space the trade-offs are such that all agents prefer pattern bargaining to sequential bargaining. Thus, as long as the firms differ in non-labor costs, pattern bargaining can emerge in equilibrium even when side payments are not possible.
pattern bargaining if \( q < q^* \); otherwise we get sequential bargaining. For \( c \in [0.2646, 0.3127] \), equilibrium is always characterized by pattern bargaining.

Finally, for \( c \geq 0.3127 \), there exists a \( q^* \) such that pattern bargaining is the equilibrium outcome for all \( q > q^* \); otherwise equilibrium is characterized by sequential bargaining.

The contrast between the cases in which \( c \in [0.2646, 0.2630] \) and \( c \geq 0.3127 \) is worth highlighting. In the former case, we get pattern bargaining only if \( q \) is sufficiently low. This is due to the fact that for these values of \( c \), the union always prefers pattern bargaining (regardless of the target) while the firms prefer pattern bargaining only if they have sufficient control over the target. In contrast, in the latter case, we only get pattern bargaining if \( q \) is sufficiently high. This follows from the fact that when the firms are sufficiently heterogeneous the firms always prefer pattern bargaining but the union will only agree to pattern bargaining when it has sufficient control over the target.

It is also worth noting that for \( c \in [0.2052, 0.2630] \) pattern bargaining with the inefficient firm as the target does not occur in equilibrium even though it is the mechanism that would maximize producer surplus. This can be viewed as a commitment (or an incomplete contracting) problem. Specifically, if the union could select the target firm (\( q = 1 \)), it could generate a Pareto improvement by contracting to select the inefficient (efficient) firm if pattern (sequential) bargaining is chosen in the first stage. In such a case, the union and the firms would prefer this outcome to the “status quo” in which equilibrium is characterized by sequential bargaining with the efficient firm as the target.\(^{27}\) While our current set-up is not rich enough to allow for this outcome, this argument suggests that if the space of negotiations were increased to allow the agents to negotiate over the mechanism and the target jointly, then pattern bargaining might arise in a larger portion of the parameter space. We formalize this concept in Appendix C where we extend our model to allow for an initial stage in which agents can propose both a bargaining

\(^{27}\) Note that if the firms choose the target (\( q = 0 \)), no such Pareto improving contract exists since the firms can secure their most preferred outcome (sequential bargaining with the inefficient firm as the target) in the status quo.
mechanism and a target firm. If all agents approve the proposal, then the bargaining mechanism is implemented; otherwise, the agents revert to the three stage game analyzed above. We show that in this slightly richer bargaining environment pattern bargaining would arise in equilibrium for all $c > .2052$ (see Proposition C.1).

4.2 Consumer Surplus and Total Welfare

As far as consumers are concerned (see Appendix B), they prefer the bargaining structure that leads to the lowest output price. This means that all they are interested in is aggregate output – the distribution of output is not important. While discussing the impact of bargaining structure on producer surplus in Section 3 we noted that sequential bargaining always leads to lower average wages and greater aggregate output. This holds true with productivity differences as well. Thus, consumers always prefer sequential bargaining over pattern bargaining regardless of the order of the negotiations.\footnote{For the same reason consumers prefer that the inefficient firm be the target firm.} Thus, even though producer surplus is greater with pattern bargaining when $c > .1965$, consumers would oppose it – a conflict that does not arise when the firms differ only in labor productivity.

The reason for this is straightforward. As we have already noted, consumers only care about aggregate output. Since in section 3 there is no opportunity cost of labor, producer surplus is also determined solely by aggregate output – thus, consumer and producer surplus always move in the same direction. However, with differences in non-labor costs there is a cost from the inefficient firm employing additional labor, implying that producer surplus depends both on aggregate output and the distribution of labor across the firms. For a given output, if production is shifted to the inefficient firm, producer surplus decreases and this is precisely what sequential bargaining does: the inefficient firm pays a lower wage and so (all else equal) producers more. Hence, if a set of sequential wages generates the same aggregate output as a pattern wage, producer surplus (and total welfare) must be strictly lower in the former. Moreover, an arbitrarily small decrease in the inefficient firm’s wage could marginally increase output and still result in lower producer surplus and total welfare.
In the introduction we noted that the Howard Government in Australia recently adopted legislation aimed at outlawing pattern bargaining and that union leaders argued that the main goal of this legislation was to shift bargaining power towards firms. However, the fact that we find that sequential bargaining generates greater consumer welfare than pattern bargaining provides an intriguing alternative explanation for the Howard Government’s actions – the Workchoices legislation could have been an attempt to protect consumer’s interests. In addition, since consumers can be interpreted as downstream firms, our analysis also offers an explanation as to why some firms would voluntarily agree to pattern bargaining while downstream firms in other industries would lobby against it.

Turning next to total welfare, we see that for $c < .3202$ the most efficient mechanism is sequential bargaining with the inefficient firm as the target (see Appendix B). However, with sufficient firm-heterogeneity firms ($c \geq .3202$), pattern bargaining with the inefficient firm maximizes total welfare. It is worth noting that in the latter case both consumers and the union are against pattern bargaining. More generally, in contrast to the case in which the firms differ in labor productivity (Corollary 3.1), the mechanism that maximizes producer surplus can be at odds with the one that maximizes total welfare.

**Proposition 4.4:** When the firms differ in non-labor costs, consumer surplus is maximized by sequential bargaining with the inefficient firm as the target. This mechanism also maximizes total welfare when $c < .3202$. Otherwise, total welfare is maximized by pattern bargaining with the inefficient firm as the target.

**Corollary 4.1:** When the firms differ in non-labor costs, consumer surplus and total welfare are maximized by the same bargaining mechanism that maximizes producer surplus if $c < .1965$.

5. Conclusion

Pattern bargaining is a common negotiating strategy that is not well understood. The conventional wisdom is that this strategy is used by industry-wide unions to soften bargaining and secure higher wages. This intuition was confirmed in a recent paper by Marshall and Merlo (2004) in which
they show that unions prefer this bargaining mechanism to simultaneous and/or sequential bargaining. However, this does not explain why the firms are willing to accept such an arrangement. Building on the insights of Marshall and Merlo, we have presented a model in which the agents negotiate over wages and the bargaining mechanism. We have shown that when comparing sequential and pattern bargaining, there are competing forces at work for both the union and the firms. For the union, pattern bargaining is advantageous because it softens the negotiations with the firms by “taking labor out of competition,” resulting in higher industry wages. On the other hand, sequential bargaining allows the union to exploit difference across the firms by demanding different wages much in the same way that a monopolist would charge different prices to consumers with different levels of willingness to pay. When firms are similar, the benefits from wage discrimination are small and the union prefers pattern bargaining. The benefits tied to wage discrimination are much more important when the firms are sufficiently different, so that with sufficient firm-heterogeneity, the union prefers sequential bargaining.

In our model the relative competitive positions of the firms change over time due to random shocks to either labor productivity or non-labor costs. The firms may therefore prefer pattern bargaining because it allows the relatively more efficient firm to pay a lower wage than it would with sequential bargaining and this can lead to higher expected lifetime profits. On the other hand, as noted above, pattern bargaining softens negotiations and results in higher average wages. As a result, the firms may sometimes prefer sequential bargaining. In fact, we find that when firm-heterogeneity is weak (strong), the firms prefer sequential (pattern) bargaining. This is precisely the opposite of the way in which the union’s preferences are linked to the degree of firm-heterogeneity, implying that it may not be possible to find situations in which all agents prefer pattern bargaining to sequential.

Using the Marshall-Merlo framework (linear demand and constant costs), we show that when the firms differ only in labor productivity, the trade-offs are such that without side-payments, pattern bargaining cannot arise in equilibrium (Proposition 3.3). Moreover, when side payments are possible, pattern bargaining can be an equilibrium outcome, but only when it is the firms’ most preferred mechanism (Proposition 3.1). In contrast, when the firms differ in non-labor costs, the trade-offs are
different and pattern bargaining can emerge in equilibrium even when side payments are ruled out (Proposition 4.3). We also showed that sequential bargaining is the mechanism that maximizes consumer surplus. Thus, there are cases in which the union and the firms both prefer pattern bargaining while consumers would oppose it (Proposition 4.4). This provides a new explanation for recent legislation in Australia aimed at eliminating pattern bargaining.

Appendices

A. Analytic Solutions

For concreteness, we provide analytic solutions to the general model in which the firms differ in both labor productivity and non-labor costs. Thus, following the notation in the text we have $x_a = \ell_a$ and $x_b = t\ell_b$, firm a’s non-labor costs are zero, and firm b incurs a non-labor cost of $c$ per unit of output. This implies that in the Cournot equilibrium output and employment are given by

(A.1) \[ x_a = \ell_a = (1/3t)[t(1 - 2w_a + c) + w_b] \]

(A.2) \[ x_b = t\ell_b = (1/3t)[t(1 + w_a - 2c) - 2w_b] \]

Profits and the union’s wage bill are still given by (3) and (4), respectively.

Case 1: Pattern Bargaining with Firm a as the Target

With a common wage, the wage bill becomes

(A.3) \[ \theta = (w/3t^2)[t^2(1 + c) + t(1 - 2c) - 2w(1 - t + t^2)] \]

When it bargains with firm a, it is straightforward to show that the union’s threat point is $(3/32)(1 - c)^2$. Thus, the wage maximizes

(A.4) \[ \left\{ \frac{w}{3t^2} \right\} \left[ t^2(1 + c) + t(1 - 2c) - 2w(1 - t + t^2) \right] - \frac{3(1 - c)^2}{32} \] \[ = \left\{ \frac{w(1 - 2t) + t(l + c)}{3t} \right\} \]

If we let $H = \left\{ \frac{w}{3t^2} \right\} \left[ t^2(1 + c) + t(1 - 2c) - 2w(1 - t + t^2) \right] - \frac{3(1 - c)^2}{32}$, the first-order-condition is
\[ \frac{H^{-1/2}}{3t^2} \left\{ \frac{w(1-2t)+t(1+c)}{3t} \right\} \left[ t^2(1+c) + t(1-2c) - 4w(1-t+t^2) \right] = H^{1/2} \frac{1-2t}{3t} ; \text{ which reduces to} \]

(A.5) \[ A_1 w^2 + B_1 w + C_1 = 0 \]

where \( A_1 = 4(2t-1)(1-t+t^2) ; \) \( B_1 = -2t(1+c)(t^2-t+1)-(3/2)[t^2(1+c)+t(1-2c)](2t-1) ; \) and

\[ C_1 = (t/2)(1+c)[t^2(1+c)+t(1-2c)] + (9/32)t^2(1-c)^2(2t-1) \]

We can use the quadratic formula to get the wage, which we denote by \( w^{p(a)} \).

**Case 2: Pattern Bargaining with Firm b as the Target**

The wage bill is still given by (A.3) and the union’s threat point is obtained by setting \( c = 0 \). So, in this case, the wage must maximize

(A.6) \[ \left\{ \frac{w}{3t^2} \left[ t^2(1+c) + t(1-2c) - 2w(1-t+t^2) \right] - \frac{3}{32} \right\}^{1/2} \left\{ \frac{w(t-2)+t(1-2c)}{3t} \right\} \]

The first-order condition is given by

\[ \frac{H^{1/2}}{6t^2} \left\{ \frac{w(t-2)+t(1-2c)}{3t} \right\} \left[ t(1+c) + t(1-2c) - 4w(1-t+t^2) \right] = H^{1/2} \frac{2-2t}{3t} . \]

After substituting for \( H \) we obtain

(A.7) \[ A_2 w^2 + B_2 w + C_2 = 0 \]

where \( A_2 = 4(2-t)(1-t+t^2) ; \) \( B_2 = -2t(1-2c)(t^2-t+1)-(3/2)(2-t)[t(1+c)+1-2c] ; \) and

\[ C_2 = (t^2/2)\{(1-2c)[t(1+c)+1-2c] + (9/16)(2-t) \} \]

We can use the quadratic formula to get the wage, which we denote by \( w^{p(b)} \).

**Case 3: Sequential Bargaining with Firm a Going First**

We start by looking at the bargaining between the union and firm b with \( w_a \) fixed. The wage bill is now given by

(A.8) \[ \theta(w_b \big| w_a) = (1/3t^2)\left[w_a t^2(1-2w_a + c) + w_b t(1-2c+2w_a) - 2w_b^2 \right] \]
If the union fails to reach an agreement with firm $b$, firm $a$ becomes a monopolist and the union’s wage bill is $(w_a / 2)(1 - w_a)$, so this is the union’s threat point. So, $w_b$ maximizes

$$\left\{ \frac{1}{3t^2} \left[ w_a t^2 (1 - 2w_a + c) + w_b t (1 + 2w_a - 2c) - 2w_b^2 \right] - \frac{w_a}{2} (1 - w_a) \right\}^{1/2} \left\{ \frac{t(1 + w_a - 2c) - 2w_b}{3t} \right\}.$$  

If we let $G = \left\{ \frac{1}{3t^2} \left[ w_a t^2 (1 - 2w_a + c) + w_b t (1 + 2w_a - 2c) - 2w_b^2 \right] - \frac{w_a}{2} (1 - w_a) \right\}$, then the first-order condition is given by $\frac{G^{-1/2}}{6t^2} \left\{ \frac{t(1 + w_a - 2c) - 2w_b}{3t} \right\} [t(1 + 2w_a - 2c) - 4w_b] = \frac{2G^{1/2}}{3t}$. Simplifying and substituting for $G$ yields $16w_a^2 - 2t[5 + 8w_1 - 10c]w_2 + t^2 [(1 - 2c)^2 + 5w_1 (1 - 2c) + 4w_1^2] = 0$. The quadratic can now be applied to obtain

(A.9) $w_b(w_a) = (t/8)(1 - 2c + 4w_a)$

We can now turn to the union’s negotiations with firm $a$. Plugging (A.9) back into (A.8) yields the wage bill as a function of $w_a$ -- we obtain

(A.10) $\theta(w_a) = (1/32)[(1 - 2c)^2 + 16w_a - 16w_a^2]$  

Using (A.9), firm $a$’s profits become

(A.11) $\pi_a(w_a) = \left[ \frac{3 + 2c - 4w_a}{8} \right]^2$  

If the union fails to reach an agreement with firm $a$, firm $b$ is a monopolist and so the union’s threat point is (as with pattern bargaining) $(3/32)(1 - c)^2$. This means that $w_a$ maximizes

$$\left\{ \frac{1}{32} [(1 - 2c)^2 + 16w_a - 16w_a^2] - \frac{3(1 - c)^2}{32} \right\}^{1/2} \left[ \frac{3 + 2c - 4w_a}{8} \right].$$

If we let $Z = \left\{ \frac{1}{32} [(1 - 2c)^2 + 16w_a (1 - w_a)] - \frac{3(1 - c)^2}{32} \right\}$, then the first-order-condition reduces to

$$[3 + 2c - 4w_a](1 - 2w_a) = 16Z;$$ which can be simplified to

(A.12) $A_3 w_a^2 + B_3 w_a + C_3 = 0$
Where \( A_3 \equiv 32 \); \( B_3 = -4(9 + 2c) \); and \( C_3 = 8 + 2c - c^2 \). We can use the quadratic formula to get the wage, which we denote by \( w_a^{(a)} \). Substituting this value into (A.9) yields \( w_b^{(a)} \).

**Case 4: Sequential Bargaining with Firm b Going First**

We start by looking at the bargaining between the union and firm \( a \) with \( w_b \) fixed. The wage bill is now given by

\[
\theta(w_a\mid w_b) = (1/3t^2) \{w_b[t(1 - 2c) - 2w_a] + w_a[t(2w_b + t(1 + c)) - 2t^2w_a^2]\}
\]

If the union fails to reach an agreement with firm \( a \), firm \( b \) becomes a monopolist and the union’s wage bill is \((w_b/2t^2)[t(1 - c) - w_b]\), so, this is the union’s threat point. Thus, \( w_a \) maximizes

\[
\left\{ \left[ \frac{1}{3t^2} \right] w_b[t(1 - 2c) - 2w_a] + w_a[t(2w_b + t(1 + c)) - 2t^2w_a^2] - \frac{w_b}{2t^2} [t(1 - c) - w_b]\right\}^{1/2} \left\{ \frac{t(1 - c - 2w_a) + w_b}{3t}\right\}
\]

If we let \( M = \left\{ \left[ \frac{1}{3t^2} \right] w_b[t(1 - 2c) - 2w_a] + w_a[t(2w_b + t(1 + c)) - 2t^2w_a^2] - \frac{w_b}{2t^2} [t(1 - c) - w_b]\right\} \), then the first-order condition can be written as \([t(1 - 2w_a + c) + w_a] [2w_a + t(1 + c) - 4tw_b] = 12t^2M\). If we now substitute for \( M \) and simplifying we obtain

\[
16t^2w_a^2 - 2t[5t(1 + c) + 8w_b]w_a + t^2(1 + c)^2 + 5t(1 + c)w_b + 4w_b^2 = 0
\]

Applying the quadratic formula yields

\[
w_a(w_b) = (1/8t)[t(1 + c) + 4w_b]
\]

We are now ready to look at the bargaining between firm \( b \) and the union. Using (A.14) the union’s wage bill as a function of \( w_b \) is given by

\[
\theta(w_b) = \frac{1}{32t^2} \left\{ t^2(1 + c)^2 + 16w_b[t(1 - c) - 16w_b^2] \right\}
\]

and firm \( b \)’s profits are

\[
\pi(w_b) = \left( \frac{t(3 - 5c) - 4w_b}{8t} \right)^2
\]
So, from (A.15) and (A.16), \( w_b \) maximizes

\[
\left\{ \frac{1}{32t^2} \left[ t^2 (1 + c)^2 + 16w_b[t(1-c) - w_b] \right] - \frac{3}{32} \right\}^{1/2} \left\{ \frac{t(3-5c) - 4w_b}{8t} \right\}
\]

If we let \( K = \left\{ \frac{1}{32t^2} \left[ t^2 (1 + c)^2 + 16w_b[t(1-c) - w_b] \right] - \frac{3}{32} \right\} \), then the first order condition reduces to

\[
[t(1-c) - 2w_b][t(3 - 5c) - 4w_b] = 16t^2 K.
\]

After substituting for \( K \) and simplifying we get

(A.17) \( A_4w_b^2 + B_4w_b + C_4 = 0 \)

where \( A_4 = 32 \); \( B_4 = -4t(9 - 11c) \); and \( C_4 = t^2 \{ 8 - 18c + 9c^2 \} \). We can use the quadratic formula to get the wage, which we denote by \( w_b^{(b)} \). Substitution of this value into (A.14) then yields \( w_a^{(b)} \).

With the wages determined, (3), (4), (A.1) and (A.2) can be used to determine profits and the wage bill under each of the bargaining mechanisms.

B. Preference Orderings when the Firms Differ in Non-Labor Costs

In all cases, we must have \( c \leq .342 \). With this in mind, for the union we have

\[
\begin{align*}
\theta^{p(a)} & \geq \theta^{p(b)} \geq \theta^{s(a)} \geq \theta^{s(b)} \quad \text{if } c \leq .2702 \\
\theta^{p(a)} & \geq \theta^{s(a)} \geq \theta^{p(b)} \geq \theta^{s(b)} \quad \text{if } .2702 \leq c \leq .3127 \\
\theta^{p(a)} & \geq \theta^{s(a)} \geq \theta^{s(b)} \geq \theta^{p(b)} \quad \text{if } .3127 \leq c
\end{align*}
\]

Turn next to the firms. We obtain

\[
\begin{align*}
\Pi^{s(b)} & \geq \Pi^{s(a)} \geq \Pi^{p(b)} \geq \Pi^{p(a)} \quad \text{if } c \leq .2052 \\
\Pi^{s(b)} & \geq \Pi^{p(b)} \geq \Pi^{s(a)} \geq \Pi^{p(a)} \quad \text{if } .2052 \leq c \leq .2630 \\
\Pi^{p(b)} & \geq \Pi^{s(b)} \geq \Pi^{s(a)} \geq \Pi^{p(a)} \quad \text{if } .2630 \leq c \leq .2646 \\
\Pi^{p(b)} & \geq \Pi^{s(b)} \geq \Pi^{p(a)} \geq \Pi^{s(a)} \quad \text{if } .2646 \leq c
\end{align*}
\]

For Producer Surplus (PS) we have

\[
PS^{s(b)} \geq PS^{s(a)} \geq PS^{p(b)} \geq PS^{p(a)} \quad \text{if } c \leq .1197
\]
\[ PS^{s(b)} \geq PS^{p(b)} \geq PS^{s(a)} \geq PS^{p(a)} \quad \text{if } .1197 \leq c \leq .1523 \]
\[ PS^{s(b)} \geq PS^{p(b)} \geq PS^{s(a)} \geq PS^{p(a)} \quad \text{if } .1523 \leq c \leq .1965 \]
\[ PS^{p(b)} \geq PS^{s(b)} \geq PS^{p(a)} \geq PS^{s(a)} \quad \text{if } .1965 \leq c \leq .2458 \]
\[ PS^{p(b)} \geq PS^{p(a)} \geq PS^{s(b)} \geq PS^{s(a)} \quad \text{if } .2458 \leq c \]

For Consumer Surplus (CS) we have
\[ CS^{s(b)} \geq CS^{s(a)} \geq CS^{p(b)} \geq CS^{p(a)} \quad \text{for all } c \]

Finally, for Total Welfare (W)
\[ W^{s(b)} \geq W^{s(a)} \geq W^{p(b)} \geq W^{p(a)} \quad \text{if } c \leq .2655 \]
\[ W^{s(b)} \geq W^{p(b)} \geq W^{s(a)} \geq W^{p(a)} \quad \text{if } .2655 \leq c \leq .3202 \]
\[ W^{p(b)} \geq W^{s(b)} \geq W^{s(a)} \geq W^{p(a)} \quad \text{if } .3202 \leq c \]

C. Negotiating over mechanism and timing jointly

In this part of the Appendix we consider a richer negotiating environment in which the agents can bargain over the mechanism and the target jointly. We formalize this notion by allowing for an initial stage in which the agents can propose both a bargaining mechanism and a target – for example, the union could propose that pattern bargaining with the efficient firm as the target be adopted. If all agents agree, the proposal is adopted; otherwise, the firms revert to the same three stage game analyzed in Sections 3 and 4. For concreteness, we focus on the case in which side payments cannot be used and the firms differ in non-labor costs.\(^{29}\) Thus, the equilibrium described in Proposition 4.3 serves as the default if the agents cannot reach agreement in the initial stage.

To pin down the possible outcomes, it is sufficient to focus on the cases in which one side has complete control over the target. We begin with the case in which the union selects the target \((q = 1)\).

From Proposition 4.3, this implies that the equilibrium in the sub-game is \(S(E)\). Now note that for \(c \in\)

\(^{29}\) It is easy to show that generalizing the model in this manner has no effect on the outcome when the firms differ only in labor productivity. In that case, sequential bargaining is the equilibrium outcome in the sub-game.
S(E) dominates S(I) for the union, so we can eliminate S(I) from consideration. In addition, for the firms S(E) dominates P(E), so we can eliminate P(E) from consideration. That leaves P(I) which dominates S(E) for both the union and the firms, and so this will be the equilibrium outcome.\textsuperscript{30}

Turn next to the case in which the union selects the target firm and \( c \geq .2646 \). From Proposition 4.3 equilibrium in the sub-game is P(E). Since this is the best outcome for the union, the union will reject any other combination. Finally, consider the case in which the firms are relatively homogenous (\( c < .2052 \)). From Proposition 4.3, the sub-game equilibrium is S(E). From Appendix B, for these values of \( c \), the only better outcome for the firms is S(I). However, since S(I) is dominated by the equilibrium in the sub-game for the union, the joint negotiations would not alter the outcome. Hence, for \( c > .2052 \) and \( q = 1 \), pattern bargaining arises with the joint negotiations over mechanism and target.

Completing the analysis, consider what occurs when the firm chooses the target (\( q = 0 \)). Since the equilibrium in the sub-game, S(I), is the best possible outcome for the firms for \( c < .2630 \), allowing for an new initial stage will not alter the final equilibrium. For \( c \in [.2630, .3127] \), equilibrium in the sub-game is P(I). Since this is also the best outcome for the firms in this range, bargaining over both the mechanism and the target has no impact. Finally, for \( c > .3127 \), equilibrium in the sub-game is S(I). For \( c \in [.3127, .3380] \), the firms can only do better with P(I), but since this would always make the union worse off, it cannot arise. For \( c > .3380 \), the firms would also be better off with P(E), which is the union’s most preferred structure. Hence, we equilibrium would be characterized by P(E). To summarize

**Proposition C.1:** If the firms differ in non-labor costs and bargain over the mechanism and target jointly, then for all \( c \geq .2052 \), there exist values of \( q \) that yield pattern bargaining as the equilibrium outcome.

\textsuperscript{30} With productivity differences, such a voting mechanism would have no affect on the outcome because \{sequential, \( \bullet \}\), the sub-game outcome, is dominant for the firms if \( t > .7267 \), while if \( t < .5785 \) \{sequential, \( \bullet \}\) is dominant for the union; for \( t \in (.5785,.7267) \) \{sequential, \( \bullet \}\) is only dominated by \{pattern, efficient\} for the union, but \{pattern, efficient\} is dominated by \{sequential, \( \bullet \}\) for the firms, so the firms would never agree to \{pattern, efficient\}. 

40
References


