



# Horizontal mergers with free entry<sup>☆</sup>

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## Abstract

We consider the impact of horizontal mergers in the presence of free entry and exit. In contrast to much of the previous literature on mergers, our model yields predictions that seem intuitively reasonable: with only moderate cost synergies mergers of a small number of industry participants are beneficial (even under quantity competition), there is no “free rider problem” in that insiders always benefit more than outsiders, and quantity-setting and price-setting games yield similar predictions about profitability. We also derive two welfare results that hold under quantity competition with homogeneous goods: If the initial, no-merger equilibrium is symmetric, then with free entry, (1) a horizontal merger has no impact on the equilibrium price and (2) all privately beneficial mergers are socially beneficial.

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## 1. Introduction

The literature on horizontal mergers has grown at a rapid pace over the last 20 years. Much of this work has been inspired by counter-intuitive results that arise in both quantity-setting and price-setting games. For example, in quantity-setting games without cost synergies, mergers that

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do not include a vast majority of industry participants are typically harmful for the insiders, whereas *all* mergers benefit the outsiders (Salant et al., 1983).<sup>1</sup> In contrast, mergers are always beneficial for the merging parties in price-setting games, although the outsiders gain more than the insiders (Deneckere and Davidson, 1985). This last result is especially troubling since it makes it difficult to explain how merger activity gets started — each firm would prefer to remain an outsider and free-ride off of the collusive behavior of the insiders.

Almost all of this literature assumes that the number of active firms is fixed exogenously.<sup>2</sup> While this may be appropriate for the short-run, it is clearly inappropriate for long-run industry analysis. In this paper, we analyze horizontal mergers in a simple model in which free entry and exit shapes the long-run industry structure and show that some of the troubling findings from earlier work disappear in this setting. Moreover, we find that when we allow for free entry, quantity-setting and price-setting games yield qualitatively similar predictions.

There is at least one other reason to analyze the impact of horizontal mergers in the presence of entry and exit. As emphasized by Werden and Froeb (1998), courts in the US have occasionally rejected merger challenges based on the logic that entry triggered by the merger would undo any anticompetitive effects that might arise.<sup>3</sup> Our model allows us to examine how the threat of entry affects the profitability of merger and its eventual impact on welfare. Our analysis suggests that mergers that are beneficial are likely to be welfare enhancing as well.

Our work is most closely related to Spector (2003) who uses a similar model to examine the price effects of horizontal mergers with free entry. Our papers differ, however, in some important ways. To begin with, Spector's main goal is fairly narrow in that he wants to show that without cost synergies, all *profitable* mergers in quantity-setting games must cause price to rise. In contrast, our focus is more general in that we want to show that Cournot and Bertrand models yield *intuitive and similar* predictions about horizontal mergers when free entry is allowed (which, as we noted above, is in stark contrast to previous work in this area). Second, Spector allows for increasing marginal costs and heterogeneity in costs across firms, whereas we assume that all firms initially face the same constant marginal cost. This difference in assumptions is important, since it implies that we are really examining very different industry structures. For example, we show that in our setting there are no

<sup>1</sup> There is a growing literature devoted to finding ways around this “merger paradox.” One typical approach is to assume that the merger changes the rules of the game (for example, Daughety (1990) analyzes a Stackelberg model with multiple followers and leaders in which a merger between two followers allows the new firm to become a leader). Another approach is to assume that a merger allows the new firm to use strategies that were not available before the merger (see, for example, Creane and Davidson, 2004 or Huck et al., *in press*). These papers assume that the number of industry participants is fixed.

<sup>2</sup> See Werden and Froeb (1998), Cabral (2003) and Spector (2003) for notable exceptions. The contributions of these papers are described below.

<sup>3</sup> The main goal of Werden and Froeb (1998) is to argue that mergers are not likely to induce new entry. To make this point, they use simulations of highly concentrated markets (no more than 8 incumbents) to calculate the price and profit effects of horizontal mergers in order to see if the effects will be sufficient to trigger entry by a new firm. They conclude that mergers in such markets are not likely to induce entry when the new firm must enter at the same scale as an incumbent. While these results are certainly intriguing, we view the issue of whether mergers trigger entry to be largely empirical. There are cases in which entry occurred shortly after a merger (e.g., Southwest's entry into the St. Louis market following the merger of TWA and Ozark) and other cases in which entry did not occur (there was no entry into the Minneapolis market after Northwest merged with Republic, apparently due to lack of gate space — see Kleit, 2001 for details). In addition, Cabral (2003) makes the valid point that new entry may not be necessary if a merger triggers expansion into new markets by existing firms (he points to the possibility that OfficeMax would have expanded had the Staples/Office Depot merger been approved). Unfortunately, we know of no empirical study that attempts to sort out the entry inducing effects of horizontal mergers.

beneficial mergers without cost synergies — it follows that the set of mergers that Spector analyzes is empty in our model. This is due to our assumption that all firms initially face the same marginal cost of production. In Spector's model, the fact that the firms differ in their initial cost structures allows for some beneficial mergers even in the absence of cost synergies. Finally, we note that Spector does not examine how the merger affects the profitability of the outsiders or social welfare, nor does he consider Bertrand competition.<sup>4</sup>

The paper divides into four additional sections. In Section 2, we introduce the basic model and derive two results that illustrate the dramatic way in which allowing free entry and exit can alter the model's predictions. In particular, we show that (a) if the firms compete in quantities and there are no sunk costs of entry, then *any* merger that results in *any* degree of cost synergy is beneficial; and, (b) with free entry there is never a free-rider problem. In the third section we turn to a more general analysis of mergers with free entry and examine conditions under which entry triggered by the merger may moderate its anticompetitive impact. One of the key results of Sections 2 and 3 is that, in stark contrast to the existing literature, *free entry* models with price and quantity competition yield *similar* predictions about the impact of mergers. In Section 4 we provide a more complete welfare analysis and derive two surprising results that hold when firms compete in quantities while producing homogeneous goods. To be precise, we show that if the initial, no-merger equilibrium is symmetric, then with free entry, (1) a horizontal merger has no impact on the equilibrium price and (2) all privately beneficial mergers are socially beneficial. We offer some concluding thoughts in Section 5.

## 2. The model

We assume that there is a large number of firms in the economy and that they each possess the same technology. Each firm faces two costs related to production: a constant marginal cost of  $c \geq 0$  and a fixed cost of  $F \geq 0$ . In addition, entry requires each firm to pay a sunk cost of  $S \geq 0$ . We assume that the firms produce homogeneous products and compete ala Cournot, although we also discuss the case of Bertrand competition (with the details provided in Appendix A). We also assume that there is free entry so that in equilibrium the profit of the marginal entrant (net of fixed and sunk costs) is equal to 0.

Denote the inverse market demand for the product by  $P(Q)$ , where  $Q$  is aggregate output, and assume that  $P'(Q) < 0$  and  $P''(Q) \leq 0$  for all  $Q$ .

We consider the following 4-stage game. In stage 1,  $M$  forward-looking firms enter the market if they expect it to be profitable to do so. We assume that they make this decision with the knowledge that they may merge in a later stage and that additional firms may enter at a later stage before production takes place. After these firms enter and the sunk costs have been paid, at stage 2, these  $M$  firms may (or may not) agree to merge. In stage 3, additional firms are free to enter with

<sup>4</sup> See also Cabral (2003) who investigates the impact of a bilateral merger in a 3-firm spatial model with a set-up quite different from ours. He assumes that in the initial equilibrium profits are driven to zero by free entry and that only two firms enter a particular market. He then assumes that these firms merge and asks what happens if the third firm (which is already in existence serving other markets) decides to expand by entering into the merged firms' market (additional entry by other firms is not allowed). His main goal is to show that the impact of the merger on price is fundamentally different when it induces an existing firm to enter the market.

<sup>5</sup> We thank the editor for suggesting this set-up in which only the potential insiders enter in stage 1. Of course, with complete information and no uncertainty, we could model this as a three stage game in which all entry takes place in stage 1 (provided that the firms correctly anticipate whether or not the merger will take place). However, our experience indicates that most readers are more comfortable with a set-up that allows for additional entry or exit after the merger has taken place.

$N$  denoting the total number of firms in the industry.<sup>5</sup> Since the merged firm will behave like a single firm, the active industry participants will be  $N - M + 1$ . To avoid strategic entry, which is not the focus of this paper, we assume that entry in stage 3 occurs sequentially.<sup>6</sup> Finally, in stage 4 the firms compete in output in standard Cournot fashion. In order to avoid integer problems, we follow the standard approach in the literature on oligopolistic interaction with free entry and treat the number of firms as a continuous variable (see, for example, Mankiw and Whinston, 1986 and Vives, 2002).<sup>7</sup>

If a merger occurs in stage 2, we assume that it has two effects. First, the insiders now choose their output to maximize joint profits. Second, the merger may result in cost synergies. This is modeled by assuming that if a merger occurs, the marginal cost of the insiders becomes  $\lambda c$  where  $\lambda \leq 1$  and that the insiders save by paying the fixed cost of production only once.

We are now in position to present two simple results that will illustrate just how much of a difference allowing for free entry can make for merger analysis. Our first result has to do with the case in which the sunk cost of entry is zero ( $S=0$ ). In this case, it is easy to show that if  $\lambda < 1$  then *any* merger must be beneficial for the merging parties. To see this, note that if no merger were to occur, every firm would earn zero profits (net of fixed costs). Now, suppose that a merger occurs so that a different number of firms enter the market. In fact, entry will occur until a typical outsider earns zero net profits. If we let  $q_O$  denote the output of a typical outsider, use  $q_I$  to represent the output of the merged firm (i.e., the insiders) and let  $n_O$  denote the number of outsiders in the free entry equilibrium, then this condition is given by

$$[P(n_O q_O + q_I) - c]q_O - F = 0 \quad (1)$$

Now, since the merged firm faces a lower marginal cost ( $\lambda c$ ) than the outsiders ( $c$ ) it follows that in the Cournot equilibrium  $q_I > q_O$ . Thus, we have

$$[P(n_O q_O + q_I) - \lambda c]q_I - F > [P(n_O q_O + q_I) - c]q_O - F = 0. \quad (2)$$

That is, the merger *must* be beneficial. This result is in stark contrast to the results derived by Salant, Switzer and Reynolds (1983) who analyze mergers in the Cournot model with a fixed number of firms. In that setting, *significant* cost synergies are required to make mergers of a small number of firms beneficial.

Our second result has to do with the free-rider problem. This problem arises in both quantity-setting and price-setting games when mergers do not lead to large cost savings. In those settings,

<sup>6</sup> If the firms make their entry decisions simultaneously, there may be multiple Nash equilibria for the entry game, creating a coordination problem with respect to the entry decision. As a result, firms enter the market with certain probabilities generated by the mixed strategy equilibrium. However, this coordination problem vanishes under sequential entry. Since strategic simultaneous entry would only complicate our analysis without adding anything to the main purpose of this paper, we consider only sequential entry. For more on the issues that arise with simultaneous entry see Cabral (2004) and the papers cited in the “grab-the-dollar” section of the paper.

<sup>7</sup> Since the stage 2 merger involves all the firms in the industry at that time, it may appear that we are only considering the impact of mergers to monopoly. However, since there is post-merger entry that takes place *before* production begins, the mergers are actually only partial in that outside firms are active in the market when production takes place. This point is made clearly in our earlier working paper in which we allowed for partial mergers in stage 2 followed by post-merger entry (or exit) in stage 3. That model yields qualitatively identical results. For details, see Davidson and Mukherjee (2004).

the outsiders *always* benefit more from a merger than do the insiders. We note that this problem disappears trivially when we allow for free entry. This is due to the fact that free entry drives the outsiders' net profits to zero both with and without the merger. Thus, the merger has no long run impact on the outsiders!

To summarize, we have

**Proposition 1.** If there are no sunk costs associated with entry, then *any* merger is beneficial for *any* degree of cost synergy.

**Proposition 2.** Mergers in the presence of free entry have no impact on the equilibrium profits earned by the outsiders.

We close this section by noting that these two propositions generalize easily to settings in which the firms compete in prices. To see this, assume that the firms produce symmetrically differentiated goods and that the fourth stage is characterized by Bertrand competition rather than Cournot (the model that we have in mind follows [Deneckere and Davidson, 1985](#) and is described in detail in Appendix A). Then, in the fourth stage of competition, the merged firm would always have the option of producing only one variety. If it did so, and if its costs did not fall as a result of the merger, then due to symmetry, it would earn the same profit as one of the outsiders, which must be zero due to free entry. Thus, if there are *any* cost synergies, the merged firm must be able to earn *positive* profits by producing only one variety. And, if the firm *chooses* to produce the other varieties, this action cannot lower profits so that the merger *must* be beneficial.<sup>8</sup> Proposition 2 generalizes trivially.

### 3. The entry and exit effects of mergers

In this section, we provide a more general analysis of the impact of mergers on industry structure. Since Proposition 1 takes care of the case in which  $S=0$ , we assume that  $S>0$  for the remainder of the paper. For illustrative purposes, we now assume that demand is linear. Accordingly, we assume that  $P(Q)=1-Q$ . It is straightforward to show that with this demand curve, profit and aggregate output in the no merger equilibrium are given by

$$\pi^* = \left[ \frac{1-c}{N+1} \right]^2 - F - S \quad (3)$$

$$Q^* = \frac{N(1-c)}{N+1} \quad (4)$$

In the absence of a merger, then in stage 3, firms would enter until  $\pi^*$  is driven to zero. Thus, without a merger the number of active firms in equilibrium would be given by

$$N^* = \frac{1-c}{\sqrt{F+S}} - 1 \quad (5)$$

<sup>8</sup> Note that this result depends upon our assumption that sunk costs are zero — if  $S>0$ , then the merged firm would have to cover higher sunk costs of entry than a typical outsider. We treat this case in Section 3.

Now, suppose instead that the  $M$  firms that enter in stage 1 merge in stage 2. Then, following entry in stage 3, the Cournot profits for the merged firm and a typical outsider are given by Eqs. (6) and (7), respectively<sup>9</sup>

$$\pi_1 = \left[ \frac{1 + (N-M)(1-\lambda)c - \lambda c}{N-M+2} \right]^2 - F - MS \tag{6}$$

$$\pi_0 = \left[ \frac{1 - (2-\lambda)c}{N-M+2} \right]^2 - F - S \tag{7}$$

The Cournot output for the merged firm and a typical outsider are given by Eqs. (8) and (9)

$$q_1 = \frac{1 + (N-M)c - (N-M+1)\lambda c}{N-M+2} \tag{8}$$

$$q_0 = \frac{1 - (2-\lambda)c}{N-M+2} \tag{9}$$

With free entry in stage 3, the final number of firms is determined by the zero profit for outsiders condition. Thus,  $N$  can be obtained by setting  $\pi_0$  (as defined in Eq. (7)) equal to zero and solving. This, however, is not the approach that we take.

Instead, we opt for an approach that we believe provides more insight. This approach also allows us to isolate the roles played by several of the key features of our model (e.g., cost synergies and fixed costs of production). We note that the analysis of a stage 2 merger can be simplified by first considering how the merger would have affected industry profits *if  $N$  had been set and held fixed at some initial, exogenously-determined value*. In that case, the merger is beneficial for the insiders if  $\pi_1 \geq M\pi^*$ ; it benefits the outsiders if  $\pi_0 \geq \pi^*$ ; and, the outsiders benefit more than the insiders if  $M\pi_0 \geq \pi_1$ . For a given value  $N$ , define  $\lambda_1(M|N)$  to be the value of  $\lambda$  such that  $\pi_1 = M\pi^*$ ; define  $\lambda_0(M|N)$  to be the value of  $\lambda$  such that  $\pi_0 = \pi^*$ ; and, define  $\hat{\lambda}(M|N)$  to be the value of  $\lambda$  such that  $M\pi_0 = \pi_1$ .

Consider first the case in which  $F=0$ . Then, from Eqs. (6) and (7) we have

$$\lambda_1(M|N) = \frac{(N+1)[1 + (N-M)c] - \sqrt{M}(N-M+2)(1-c)}{(N+1)(N-M+1)c} \tag{10}$$

$$\lambda_0(M|N) = \frac{(N+M)c + 1 - M}{(N+1)c} \tag{11}$$

$$\hat{\lambda}(M|N) = \frac{1 - \sqrt{M} + (N-M + 2\sqrt{M})c}{(N-M+1 + \sqrt{M})c} \tag{12}$$

<sup>9</sup> Note that from Eq. (9), for the outsiders to remain active after the merger we must have  $\lambda > 2 - (1/c)$ . With this restriction in place, Eq. (7) indicates that  $\pi_0$  is increasing in  $\lambda$ . We make use of this fact in the proof of Proposition 3.

It is straightforward to show that  $\lambda_I(M|N) > \hat{\lambda}(M|N) > \lambda_O(M|N)$  for all  $N$  and  $M$ . The reason for this ranking is easy to explain. When a horizontal merger occurs (with  $N$  held fixed), the insiders internalize the negative externalities that they impose on each other through quantity competition. If there are no cost synergies (i.e., if  $\lambda = 1$ ), then we know from Salant et al. (1983) that this internalization causes each insider to reduce production while each outsider responds by increasing output. As a result, the merger harms the insiders (unless  $M$  is sufficiently close to  $N$ ), and benefits the outsiders.<sup>10</sup> For  $\lambda < 1$ , the merger also generates cost synergies for the insiders. This reduction in their costs causes each insider to increase production, while the outsiders cut back their output levels. The impact on profits follows naturally — the insiders gain at the expense of the outsiders. When  $\lambda = \lambda_I(M|N)$ , the positive profit-effect generated by the cost savings just equals the negative profit-effect from the internalization of the externalities so that the insiders earn the same profits with or without the merger. This immediately implies that when  $\lambda = \lambda_I(M|N)$ , the insiders must be producing a lower total output than they would produce without the merger.<sup>11</sup> And, since the outsiders' best-reply mappings are downward sloping, they must produce more output than they would in the absence of the merger. Consequently, when  $\lambda = \lambda_I(M|N)$ , each outsider produces higher output and earns more profit than it would have had the merger not occurred. This implies that  $M\pi_O > M\pi^* = \pi_I$  at  $\lambda = \lambda_I(M|N)$  and hence,  $\hat{\lambda}(M|N)$  (which equates  $M\pi_O$  and  $\pi_I$ ), must be lower than  $\lambda_I(M|N)$ . Since  $\pi_I$  is negatively related to  $\lambda$  and  $\pi_O$  is positively related to  $\lambda$ , it is trivially true that  $M\pi_O$  must be greater than  $M\pi^*$  at  $\hat{\lambda}(M|N)$ . This immediately implies that  $\lambda_O(M|N)$ , which equates  $\pi_O$  and  $\pi^*$ , must be lower than  $\hat{\lambda}(M|N)$ .

With the following definition, we are now in position to analyze the impact of mergers with free entry. Note that this definition requires us to evaluate the critical values of  $\lambda$  with  $N = N^*$ , the number of firms that would enter in the absence of a merger.

**Definition.** The cost synergies associated with a merger of  $M$  firms are *modest* if  $\lambda \geq \hat{\lambda}(M|N^*)$ ; they are *moderate* if  $\lambda \in [\lambda_O(M|N^*), \hat{\lambda}(M|N^*)]$ ; and, they are *dramatic* if  $\lambda \leq \lambda_O(M|N^*)$ .

**Proposition 3.** With free entry and no fixed cost of production, a merger of  $M$  firms will not take place if cost synergies are modest and  $\lambda \geq \hat{\lambda} \equiv 1 - (\sqrt{S}/c)(\sqrt{M}-1)$ . If cost synergies are moderate or if they are modest with  $\lambda \leq \hat{\lambda}$ , the merger will occur and it will induce *more* firms (compared to no merger) to enter the market.<sup>12</sup> If the cost synergies are dramatic, the merger will occur and it will induce *fewer* firms (compared to no merger) to enter the market.

**Proof.** Suppose that the  $M$  firms merge in stage 2 and that in stage 3 firms enter until  $N = N^*$ . Then, if the cost synergies are modest with  $\lambda > \lambda_I(M|N^*)$ , it follows that  $\pi_O > 0 > \pi_I/M$ . This means that in the free entry equilibrium with the merger, more than  $N^*$  firms will have to enter the market

<sup>10</sup> If  $M$  is sufficiently close to  $N$ , the merger is beneficial for the insiders, implying that  $\lambda_I(M)$  is undefined.

<sup>11</sup> To see this, suppose that the insiders were to produce the same amount after the merger. This would imply that the outsiders would produce the same amount as well, resulting in the same price. However, since the insiders would now have lower costs, they would now be earning more profit than they would without the merger, violating the definition of  $\lambda_I(M|N)$ .

<sup>12</sup> We need to be precise here as to what we mean when we say that “more firms (compared to no merger) enter the market” since the merger effectively lowers the number of active firms. So, for example, suppose that  $M$  firms enter the market in stage 2, no merger occurs, and then  $N - M$  firms enter the market in stage 3. In this case, in the absence of a merger, there would be  $N$  active firms in equilibrium. Now, suppose that  $M$  firms enter in stage 2, these firms merge, and then entry occurs in stage 3. Our statement in Proposition 3 implies that more than  $N - M$  firms will enter in stage 3. However, due to the merger it would still be possible to have fewer than  $N$  firms active in the equilibrium since the merger reduces the number of firms that actually produce. In fact, we show in Proposition 4 below that this is always the case.

to drive  $\pi_O$  to zero. The entry of these extra outsiders lower  $\pi_I$  further, which implies that the merger harms the insiders.

Now, suppose that the cost synergies are modest with  $\lambda \leq \lambda_I(M|N^*)$ . Then it follows that if firms enter in stage 3 until  $N=N^*$ ,  $\pi_O > \pi_I/M > 0$ . This means that more than  $N^*$  firms will enter the market in the free entry equilibrium with the merger. The entry of these extra outsiders lowers both  $\pi_I/M$  and  $\pi_O$ , with  $\pi_O$  falling at a faster rate (from Eqs. (6) and (7)). Firms continue to enter until  $\pi_O=0$ , at which point  $N = \hat{N} = M-2 + [1-(2-\lambda)c]/\sqrt{S}$ . From Eq. (6),  $\pi_I(\hat{N}) \geq 0$  if and only if  $\lambda \leq \tilde{\lambda} = 1-(\sqrt{S}/c)(\sqrt{M}-1)$ . Furthermore, from Eqs. (10) and (12), we have

$$\lambda_I(M|N^*) = 1 - \frac{\sqrt{S}}{c} \left[ \frac{(\sqrt{M}-1)(1-c) - (M-1)\sqrt{MS}}{1-c-M\sqrt{S}} \right]$$

$$\lambda(M|N^*) = 1 - \frac{\sqrt{S}}{c} \left[ \frac{(\sqrt{M}-1)(1-c)}{1-c-\sqrt{M}(\sqrt{M}-1)} \right]$$

A straightforward comparison of  $\tilde{\lambda}$ ,  $\lambda_I(M|N^*)$  and  $\hat{\lambda}(M|N^*)$  reveals that it is always the case that  $\tilde{\lambda} \in [\hat{\lambda}(M|N^*), \lambda_I(M|N^*)]$ . It follows that in the free entry equilibrium with the merger and modest cost synergies,  $\pi_I(\hat{N})/M \geq 0 = \pi_O(\hat{N})$  if  $\lambda \leq \tilde{\lambda}$  and  $\pi_I(\hat{N})/M \leq 0 = \pi_O(\hat{N})$  if  $\lambda \geq \tilde{\lambda}$ . We conclude that when cost synergies are modest, the merger will benefit insiders if  $\lambda \leq \tilde{\lambda}$ , but it will harm insiders if  $\lambda \geq \tilde{\lambda}$ .<sup>13</sup>

Turn next to the case in which the cost synergies are moderate. Then, if firms enter in stage 3 until  $N=N^*$ , we will have  $\pi_I/M > \pi_O > 0$ . This means that more than  $N^*$  firms will enter the market in the free entry equilibrium with the merger. The entry of these extra outsiders lowers both  $\pi_I/M$  and  $\pi_O$ ; but, since  $\pi_O$  falls faster than  $\pi_I/M$  (from Eqs. (6) and (7)), in equilibrium, firms enter such that  $\pi_I/M > \pi_O = 0$ . Thus, the insiders benefit from the merger and the merger induces more firms to enter the market.

Finally, consider the case in which the cost synergies are dramatic. Then, if firms enter in stage 3 until  $N=N^*$ , we will have  $\pi_I/M > 0 > \pi_O$ . This means that with the merger stage 3 entry will stop with  $\hat{N} < N^*$  firms in the industry. And, since  $\pi_I/M$  is decreasing in  $N$ , we have  $\pi_I(\hat{N})/M > \pi_I(N^*)/M > 0$ . As a result, the insiders gain from the merger and the merger results in fewer firms entering the market.

Proposition 3 indicates that the Court’s presumption that a merger will trigger additional entry that will reverse its anticompetitive effects may be misguided. While it is true that mergers in the presence of moderate cost synergies trigger additional entry, when the cost synergies are dramatic, mergers cause fewer firms to enter. This implies that the merger’s anticompetitive effects will be magnified by the entry decision of the outsiders. Nevertheless, as we show in the next section, such mergers will always be welfare improving because of the large cost savings that they generate.

<sup>13</sup> Note that  $\tilde{\lambda}$  is decreasing in  $M$ , so that a merger of more firms requires *stronger* cost synergies for the merger to be profitable. This result is an artifact of our assumption that the degree of the cost synergy triggered by the merger is independent of  $M$ . To see this, note that a merger of two firms is sufficient to attain the cost synergies. Including more firms in the merger then generates the type of losses that were first discussed by Salant, Switzer and Reynolds (1983). Thus, for profitability, a merger of three firms will typically require greater cost synergies than a bilateral merger. Of course, one would expect that a merger of three firms would generate *greater* cost synergies than a bilateral merger. Thus, if we were interested in using our model to endogenize the merger process, we would first want to relax our assumption that  $\lambda$  is independent of  $M$ .

Before moving on, it is useful to point out that since  $\hat{\lambda}(M) < 1$  for all  $M$ , there is a simple corollary to Proposition 3 — mergers that purely increase market power without lowering costs are never beneficial.<sup>14</sup>

**Corollary.** If there are no cost synergies associated with the merger, then it cannot be beneficial for the insiders.<sup>15</sup>

At this point we can relax some of our assumptions to see if our qualitative results are affected. We begin by following Mankiw and Whinston (1986), who acknowledge the implications of the ‘integer constraint’ on their results. It should be clear that if cost synergies are dramatic, the integer constraint implies that there may not be *any* entry following the merger, since there may not be room for another firm in the market. This makes our result for dramatic cost synergies stronger. On the other hand, if the cost synergies are modest, the integer constraint may lead to a profitable merger for values of  $\lambda$  (slightly) above  $\hat{\lambda}$ . To see this, note that with the integer constraint in place, entry may not reduce the profit of the outsiders all the way to zero so that the equilibrium number of firms with the merger may be less than  $\tilde{N}$ . Moreover, since the profit of the merged firm increases with lower values of  $N$ , it is immediate that  $\pi_1 \geq 0$  at  $\hat{\lambda}$ , thus increasing the possibility of profitable merger. Hence, for Proposition 3, the new wording with an integer constraint would be: If cost synergies are moderate or not very modest, then the merger will occur and it will never induce *fewer* firms to enter the market. If the cost synergies are dramatic, then the merger will occur and it will never induce *more* firms to enter the market.

We turn next to our assumption that  $F=0$ . So, suppose instead that  $F>0$  so that the merger allows the insiders save on fixed costs. This makes it more likely that a merger with  $N$  held fixed will be beneficial. It also makes it more likely that when  $N$  is held fixed the insiders will benefit more from the merger than the outsiders. On the other hand, changes in  $F$  do not affect the profits earned by the outsiders. It follows that  $\lambda_I(M|N^*)$  and  $\hat{\lambda}(M|N^*)$  are increasing in  $F$  whereas  $\lambda_O(M|N^*)$  is independent of  $F$ . However, since the ordering of  $\lambda_I(M|N^*)$ ,  $\hat{\lambda}(M|N^*)$  and  $\lambda_O(M|N^*)$  does not change, the basic message of Proposition 3 is not altered.

Finally, suppose, as we did with Propositions 1 and 2, that instead of assuming that the firms compete in quantities, we had assumed that they produce differentiated goods and compete in prices (as in Deneckere and Davidson, 1985). Then, as we show in Appendix A, the only difference would be that with  $N$  fixed, all mergers would be beneficial to the insiders (i.e.,  $\lambda_I(M|N^*)$  would be greater than 1 for all  $M$ ). However, for fixed  $N$ , it would still be the case that when cost synergies are modest the outsiders would earn more than the insiders and when cost synergies rise above a certain level, the insiders’ profits would surpass the outsiders. Moreover, there would still be a level of cost synergies that would result in losses for the outsiders. Thus, there would be natural analogs to  $\hat{\lambda}(M|N^*)$  and  $\lambda_O(M|N^*)$  and their order would be preserved. It follows that the basic message of Proposition 3 is independent of the type of competition that the firms are engaged in: When cost synergies are quite weak, mergers do not benefit the insiders; with moderate cost synergies, insiders gain and the merger triggers entry; and, with dramatic cost synergies, the merger benefits insiders and results in fewer active firms.<sup>16</sup>

<sup>14</sup> As we noted in the introduction, Spector (2003) shows that without cost synergies, all beneficial mergers must cause price to rise. This corollary indicates that in our model, without cost synergies, there are no beneficial mergers. It follows that in our model the set of mergers that Spector examines is empty.

<sup>15</sup> Appendix B shows that the above corollary is a general result and does not depend on the linear demand function.

<sup>16</sup> An analysis of bilateral mergers in the Deneckere and Davidson (1985) model is presented in Appendix A to illustrate these points.

The fact that Propositions 1–3 hold in both quantity and price-setting games is surprising for at least two reasons. First, previous work has suggested that price-setting and quantity-setting models yield very different predictions about the impact of horizontal mergers. Our analysis indicates that this is not the case when we allow for free entry. Second, our results indicate that with price competition and free entry not all mergers are beneficial. If cost synergies are modest (i.e.,  $\lambda > \hat{\lambda}(M)$ ), then *some* mergers that would be beneficial with  $N$  fixed are no longer beneficial. This is due to the fact that such mergers would also benefit the outsiders and would therefore result in more firms entering in Stage 3. In fact, just as in the quantity setting game, firms would enter in Stage 3 until  $\pi_O = 0 > \pi_I/M$ ; implying that the merger would not occur (we make this explicit in Appendix A by providing a proof of the Corollary to Proposition 3 for the price-setting game). This result contrasts sharply with the predictions of [Deneckere and Davidson \(1985\)](#) who studied mergers in price setting games with a fixed number of firms.

In closing this section, we want to emphasize that we view this latter result as the main message of this section. After all, one of the main predictions of Proposition 3, that mergers are unprofitable when cost synergies are weak, simply reproduces the existing result from the original Salant–Switzer–Reynolds model. However, as we emphasize above, the standard result from price-setting games, that insiders always benefit from a merger, does not generalize to models with free entry. Thus, allowing for free entry greatly simplifies merger analysis — either price or quantity competition can be assumed without altering the predictions of the model. We know of no other class of models in which price and quantity competition yields similar prediction about horizontal mergers.

#### 4. Welfare

We now consider the impact of the merger on social welfare, which consists of consumer surplus and profits. Since welfare analysis in price games is complicated by consumers' preferences for variety, we focus on our quantity-setting game in which firms produce homogeneous goods.<sup>17</sup> We begin by noting that in the free entry equilibrium with mergers firms enter until  $\pi_O = 0$ . From Eq. (7), this implies that

$$\hat{N} = \frac{1 - (2 - \lambda)c}{\sqrt{F} + S} + M - 2. \quad (13)$$

Given our assumption of homogeneous products, after the merger the insiders combine and form just one active firm. Thus, since there are  $\hat{N} - M$  outsiders, the total number of active firms becomes  $\hat{N} - M + 1$ . If we use Eqs. (7) and (13) we then have (we show in Appendix C that this result does not depend on the linearity of demand):

**Proposition 4.** A merger of  $M$  firms always lowers the number of active firms.

Proposition 4 indicates that even when the merger triggers entry, the number of new outsiders that enter is always smaller than  $M - 1$ . As a result, the merger always reduces the total fixed costs of production incurred by the industry. Moreover, cost synergies allow the merged firm to also lower their marginal costs. This means that society can now produce a given level of output at a lower social cost, suggesting that the merger might be welfare enhancing.

<sup>17</sup> The impact of mergers on welfare in the price-setting game is discussed briefly at the end of Appendix A.

To see if this is the case, we now turn to consumer surplus, which, given our specification of demand, is given by  $.5Q^2$ . From Eqs. (8) and (9), aggregate output with the merger is

$$\hat{Q} = (N-M)q_0 + q_1 = \frac{N-M + 1-(N-M)c-\lambda c}{N-M+2} \quad (14)$$

Using Eq. (13) to substitute for  $N$  then yields

$$\hat{Q} = 1-c-\sqrt{F+S} = Q^* \quad (15)$$

where the last equality follows from substituting Eq. (5) into Eq. (4). Since the free-entry aggregate output is the same with and without the merger, price and consumer surplus are unaffected by the merger. Thus, we have<sup>18</sup>

**Proposition 5.** With free entry, all privately beneficial mergers are socially beneficial.

**Proof.** From Eq. (15), consumer surplus is the same with and without the merger. From Proposition 2, all outsiders earn the same with and without the merger (they earn zero). And, since the merger is beneficial, the insiders must be better off with the merger. Hence, the sum of consumer surplus and profits must be higher with the merger.

Propositions 4 and 5 are strong results. Together they imply that even though profitable mergers always reduce the number of active firms, such mergers will always raise social welfare because of the cost synergies the generate.

While the proof of Proposition 5 is based on the assumption of linear demand, it is not hard to show that the result can be generalized somewhat. To see this, note that since entry occurs until a typical outsider earns zero profits, the equilibrium price is determined by the point at which a typical outsider's residual demand curve is just tangent to its average cost curve. Now, suppose that demand for firm  $i$ 's product is such that any change in any rival's output leaves the slope of firm  $i$ 's residual demand curve unaffected (as is the case with linear demand). Then, since a merger does not affect the outsiders' cost curves, nor, in this case, the slope of the outsiders' residual demand curves, the merger has no impact on aggregate output or the equilibrium price (this result is listed as Proposition 6 below)!<sup>19</sup> Thus, if the merger is privately beneficial (which means that there must be cost savings for the insiders) it must be socially beneficial.<sup>20</sup>

**Proposition 6.** If the slope of a typical outsider's residual demand curve is independent of its rivals' outputs, the merger has no impact on the equilibrium price.

As with Proposition 3, it is useful to explore how the presence of an integer constraint would affect our welfare results. To do so, let us look specifically for a case in which our welfare results could be reversed. Thus, assume, for simplicity, that in the absence of a merger free entry drives

<sup>18</sup> The welfare effects of a merger in the price-setting game are more complex due to the assumption of product differentiation. For example, if cost synergies are dramatic, the merger results in fewer firms and less variety for consumers. The impact on welfare then depends on how much the consumers value variety (see the discussion at the end of Appendix A for details).

<sup>19</sup> It should be clear that even if the demand curve is non-linear such as  $P=A-f(Q)$ , the slope of the residual demand curve of the typical outsider, who stays in the market irrespective of merger, remains the same at a given price. For example, if  $B$  denotes the total output of the other firms, then the residual demand curve for the concerned firm is  $P=A-f(Q-B)$  or  $Q-B=f^{-1}(A-P)$ . Hence, slope of the residual demand, that is,  $\frac{\partial P}{\partial(Q-B)} = f'(f^{-1}(A-P))$ , remains the same for a given price.

<sup>20</sup> We are deeply grateful to Hiroshi Ohta for this insight. See [McGuire and Ohta \(2005\)](#) for a proof of this type applied in a different context.

the profit of each firm to zero (so that without the merger  $N=N^*$ ); whereas, due to the integer constraint, free entry stops short of driving the profit of the outsider to zero with a merger (so that with the merger,  $N$  is the largest integer below  $\hat{N}$ ). Then, since the effective number of firms under merger is lower than that without a merger and since aggregate output would be the same with and without the merger if there were no integer constraint (see Eq. (15)), in this situation, total output with a merger would be lower than the total output would be in the absence of a merger. This implies that a merger increases price and reduces consumer surplus. Thus, with free entry and a binding integer constraint, there *may be* a welfare trade-off in that a merger may reduce consumer surplus while increasing industry profit. If the loss in consumer surplus is large enough, welfare may fall as a result of a merger and our welfare result will be overturned. Of course, if the cost synergies associated with the merger are dramatic, then the merger is likely to increase welfare even with integer constraint. Moreover, for a given level of cost synergies, if the sunk cost of entry is sufficiently low (so that the number of firms is large enough to generate only a small merger-induced change in consumer surplus), then welfare is (once again) likely to be higher under merger.

## 5. Conclusion

In this paper we have analyzed a very simple model of horizontal mergers in the presence of free entry and exit. We choose to work with such a simple model in order to make the comparison of our results to others in the merger literature (in particular, [Salant et al., 1983](#) and [Deneckere and Davidson, 1985](#)) as straight forward as possible. And, although we relied on an assumption of linear demand to illustrate some of our basic points, we have provided general proofs of most of our results in Appendices B and C.

Our key result is that when we allow for free entry and exit, most of the counter-intuitive results that have plagued the literature on horizontal mergers disappear. Mergers in quantity-setting games may be beneficial, even in absence of dramatic cost synergies, the free-rider problem that makes it difficult to explain why mergers occur vanishes, and quantity and price-setting games yield similar predictions about the impact of merger activity on profitability.

The other main result of our paper is that the possibility of entry greatly simplifies the welfare analysis of horizontal mergers. In our quantity-setting model with homogeneous goods, *all* privately beneficial mergers are also socially beneficial *regardless* of the degree of cost synergies. The fact that we are able to obtain such a strong result may be due to our assumption that all firms are initially symmetric; however, it clearly indicates that additional research on the welfare effects of horizontal mergers in the presence of free entry is warranted.

Finally, although we have shown that free entry can significantly affect many of the previous results on the impact of horizontal mergers, it is worth acknowledging that our analysis has abstracted from several issues that need to be dealt with in the future. For example, we have examined the impact of a single merger between an exogenously given number of firms, and have not allowed the outsiders to react by merging as well. Such counter-mergers will clearly affect the profitability of the initial merger and alter the equilibrium number of firms in the market. They may have important implications for welfare as well. So, in the future it would be useful to allow for multiple mergers and use such a model to determine the number of merged firms endogenously. Hence, one possible extension of this paper is to investigate industry dynamics with multiple mergers and free entry. Such an analysis, which would make industry structure endogenous, could be viewed as an extension of the related literature on endogenous coalition structure (see, for example, [Ray and Vohra, 1997, 1999](#)).

## Appendix A. An analysis of mergers with Bertrand competition

Our goal is to show that the basic results of Proposition 3 extend to price-setting games (note that since we are concerned with Proposition 3, we set  $F=0$ ). We follow Deneckere and Davidson by assuming that the demand for good  $i$  is given by  $q_i=1-p_i+\gamma(\bar{p}-p_i)$ , where  $\bar{p}$  denotes the average price charged in the industry and  $\gamma \geq 0$  is a parameter that measures product differentiation ( $\gamma=\infty$  indicates that the goods are perfect substitutes and  $\gamma=0$  indicates that the goods are unrelated). In what follows, we compare the outcome of the 4-stage game in which no merger occurs with the outcome that occurs when firms 1 and 2 merge in Stage 2. The general case in which  $M$  firms merge in Stage 2 is qualitatively similar but results in more complex expressions.

For notational convenience, we begin by defining  $\bar{c} \equiv \frac{1}{N} \sum_j c_j$ ,  $z \equiv \frac{\gamma}{N}$  and  $\sigma \equiv 1 + \gamma - z$ . Then, when no merger occurs, it is straightforward to show that the equilibrium price for each firm is given by  $p_i = \frac{1+\sigma c}{1+\sigma}$  and that profit per firm is

$$\pi^* = \sigma \left[ \frac{1-c}{1+\sigma} \right]^2 - S. \quad (\text{A.1})$$

Now, consider the case in which firms 1 and 2 merge in Stage 2. Then the first order conditions for the two insiders are given by

$$1 + \lambda c(1 + \gamma) + \gamma \bar{p} - 2p_1(1 + \gamma) + z(p_1 - \lambda c) + z(p_2 - \lambda c) = 0 \quad (\text{A.2})$$

$$1 + \lambda c(1 + \gamma) + \gamma \bar{p} - 2p_2(1 + \gamma) + z(p_1 - \lambda c) + z(p_2 - \lambda c) = 0, \quad (\text{A.3})$$

whereas the first order condition for each outsider is

$$1 + c(1 + \gamma) + \gamma \bar{p} - 2p_j(1 + \gamma) + z(p_j - c) = 0 \quad (\text{A.4})$$

Summing over all firms and then dividing by  $N$  yields:

$$1 + \sigma \bar{c} - (1 + \sigma) \bar{p} + \frac{z}{N} (p_1 + p_2 - 2\lambda c) = 0 \quad (\text{A.5})$$

Summing Eqs. (A.2) and (A.3) yields

$$1 + \lambda c(\sigma - z) + \gamma \bar{p} - \sigma(p_1 + p_2) = 0 \quad (\text{A.6})$$

We can now solve Eqs. (A.5) and (A.6) for  $\bar{p}$  and  $(p_1 + p_2)$ . We obtain

$$\bar{p} = \frac{(\sigma + \frac{z}{N}) + \sigma^2 \bar{c} - \lambda c \frac{z}{N} (1 + \gamma)}{\sigma(1 + \sigma) - z^2} \quad (\text{A.7})$$

and

$$p_1 + p_2 = \frac{(1 + \gamma + \sigma) + \gamma \sigma \bar{c} + \lambda c [(1 + \sigma)(\sigma - z) - 2z^2]}{\sigma(1 + \sigma) - z^2}. \quad (\text{A.8})$$

We can now use Eq. (A.7) to substitute for  $\bar{p}$  in Eq. (A.4) and solve for a typical outsider's price:

$$p_j = \frac{\sigma(1 + \gamma + \sigma) + \gamma\sigma^2\bar{c} + \sigma[\sigma(1 + \sigma) - z^2]c - \lambda c(1 + \gamma)z^2}{(1 + \gamma + \sigma)[\sigma(1 + \sigma) - z^2]} \quad (\text{A.9})$$

Turn next to profits and start with the insiders. It is straightforward to show that the first order conditions for firms 1 and 2 can be written as:

$$q_1 = \sigma(p_1 - c_1) - z(p_2 - c_2)$$

$$q_2 = \sigma(p_2 - c_2) - z(p_1 - c_1)$$

It follows that the merged firm's profits can be written as

$$\pi_1 + \pi_2 = \sigma[(p_1 - \lambda c)^2 + (p_2 - \lambda c)^2] - 2z(p_1 - \lambda c)(p_2 - \lambda c) - 2S$$

With symmetry, we then have

$$\pi_1/2 = .5(\pi_1 + \pi_2) = (\sigma - z)(p_1 - \lambda c)^2 - S$$

Substitution from Eq. (A.8) then yields

$$\pi_1/2 = (\sigma - z) \left[ \frac{1 + \gamma + \sigma + \gamma\sigma\bar{c} - \lambda c(1 + \sigma)(\sigma + z)}{2[\sigma(1 + \sigma) - z^2]} \right]^2 - S \quad (\text{A.10})$$

For the outsiders, their first order condition is equivalent to  $q_i = \sigma(p_i - c)$ . So that we may write profits as

$$\pi_O = \sigma(p_i - c)^2 - S$$

Substituting from Eq. (A.8) then yields

$$\pi_O = \sigma \left[ \frac{\sigma(1 + \gamma + \sigma) + \gamma\sigma^2\bar{c} - (1 + \gamma)[\sigma(1 + \sigma) - z^2]c - \lambda c(1 + \gamma)z^2}{(1 + \gamma + \sigma)[\sigma(1 + \sigma) - z^2]} \right]^2 - S \quad (\text{A.11})$$

In what follows, we make use of the fact that with the merger

$$\bar{c} = \frac{(N-2)c + 2\lambda c}{N} = \frac{1}{\gamma} [(\gamma - 2z)c + 2z\lambda c] \quad (\text{A.12})$$

Now, we know from [Deneckere and Davidson \(1985\)](#) that with  $N$  fixed, all mergers are beneficial. Thus,  $\lambda_1 > 1$ . Furthermore, using Eqs. (A.10) (A.11) (A.12) we have:

$$\lambda_O = \max \left\{ \frac{c\sigma[2\sigma(1 + \sigma) - z\gamma] - z(1 + \gamma + \sigma)}{c(1 + \sigma)[2\sigma^2 - (1 + \gamma)z]}, 0 \right\}; \quad (\text{A.13})$$

and from Eq. (A.1) and Eqs. (A.11)–(A.12) we have

$$\hat{\lambda} = \max \left\{ \frac{\eta_1 - c\eta_2}{c(\eta_3 + \eta_4)}, 0 \right\} \quad (\text{A.14})$$

where

$$\psi \equiv \sqrt{\frac{\sigma - z}{4\sigma}}; \eta_1 = (1 + \gamma + \sigma)[\sigma(1 - \psi) - \psi(1 + \gamma)];$$

$$\eta_2 = (1 + \gamma)[\sigma(1 + \sigma) - z^2] + (\gamma - 2z)[\psi\sigma(1 + \gamma) - \sigma^2(1 - \psi)];$$

$$\eta_3 = z[(1 + \gamma)z - 2\sigma^2]; \text{ and, } \eta_4 = \psi(1 + \gamma + \sigma)[2\sigma z - (1 + \sigma)(\sigma + z)].$$

We know that  $\lambda_1 > 1$ . Further, at  $\lambda_0$ , we have  $\pi^* = \pi_0$  and  $\pi^* < \frac{\pi_1}{2}$ , since  $\lambda_0 < 1$ . Therefore,  $\pi_0 < \frac{\pi_1}{2}$ . Since,  $\pi_1$  is negatively related to  $\lambda$  and  $\pi_0$  is positively related to  $\lambda$ , it implies that  $\hat{\lambda}$  (where  $\pi_0 = \frac{\pi_1}{2}$ ) is greater than  $\lambda_0$ . It can also be checked that  $\hat{\lambda} < 1$ .

Finally, to show that the Corollary to Proposition 3 generalizes to price setting games, assume that the merger does not generate cost synergies and set  $c=0$  for simplicity. In this case, the merger-induced profit functions simplify to

$$\pi_0(N) = \left[ 1 + \gamma \frac{N-1}{N} \right] \left\{ \frac{N(1 + \gamma) - \gamma}{2N + 3\gamma(N-1) + \gamma^2(N-2)} \right\}^2 - S \quad (\text{A.15})$$

$$\pi_1(N)/2 = \left[ 1 + \gamma \frac{N-2}{N} \right] \left\{ \frac{2N(1 + \gamma) - \gamma}{2[2N + 3\gamma(N-1) + \gamma^2(N-2)]} \right\}^2 - S \quad (\text{A.16})$$

With free entry, firms enter until Eq. (A.15) is driven to zero; and, as in the text, we use  $\hat{N}$  to denote this number of firms. Then, a straightforward comparison of Eqs. (A.15) and (A.16) reveals that  $\pi_0(N) > \pi_1(N)/2$  for all  $N$  such that  $N > \gamma/(1 + \gamma)$ ; and, since  $\gamma/(1 + \gamma) \in [0, 1]$ , it must be the case that  $\pi_0(\hat{N}) > \pi_1(\hat{N})/2$ . Thus, without cost synergies, a bilateral merger in a price setting game with free entry will not be profitable.

The above analysis shows that the effects of free entry on profits under Bertrand competition are similar to those generated by Cournot competition. Like Cournot competition, a merger creates a trade-off between the cost savings generated by cost synergies and the effect of increased market concentration. However, with Bertrand competition and differentiated goods a new force arises — a merger is likely to alter the amount of variety offered in the market. With product differentiation and Bertrand competition, though the merged firms choose prices to maximize their joint profits, *all* the varieties of the merged firms will be produced in equilibrium. So, if the total number of firms is increased due to merger, then the merger increases the number of varieties produced in the market. In this situation, even though the merger may increase prices due to market concentration, it provides benefits by lowering costs (via cost synergy) and increasing the amount of variety available to consumers. But, if merger reduces the number of firms in the economy,<sup>21</sup> it generates negative effects

<sup>21</sup> Note that we refer to the total number of firms (and not the active number of firms), which considers each constituent of the merged firms separately. Hence, if the total number of firms (which also equals the total number of varieties) falls, this results in a reduction in the number of varieties.

by increasing market concentration and reducing variety and these must be weighed against the positive benefits triggered by the reduction in costs. Therefore, whether merger is welfare improving under Bertrand competition depends on the relative strengths of the three factors such as market concentration, the degree of cost synergy and the number of product varieties.

### **Appendix B. A generalization of the corollary to proposition 3**

If there is no synergy under merger, the merged firm and the outside firms produce at the marginal cost  $c$ . So, the operating profit (i.e., revenue minus total variable cost of production) of the merged firm and an outsider firm is the same. Since the net profit of an outsider is zero in equilibrium and the amount of sunk cost plus the fixed cost incurred by the merged firm is greater compared to a typical outsider firm, the net profit of the merged firm is negative. Therefore, merger is never profitable without any synergy.

### **Appendix C. A generalization of proposition 4**

In case of free entry, the net profit of a typical firm under non-cooperation is zero and the zero profit condition determines  $N^*$  as the equilibrium number of firms.

If there is a merger of  $M$  firms, the net profit of the outsiders becomes zero in the free entry equilibrium. However, under merger, the operating profit of an outsider is lower than that of under non-cooperation whenever there is synergy under merger. Hence, the zero profit condition of the free-entry equilibrium is more binding under merger than non-cooperation, which implies that the total number of active firms is lower under merger than non-cooperation.

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