Rand Journal of Economics
Vol. 17, No. 3, Autumn 1986

Long-run competition in capacity, short-run competition in price, and the Cournot model

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In this article we investigate the nature of equilibrium in markets in which firms choose the scale of operation before they make pricing decisions. We analyze a duopoly model in which firms choose their capacities before engaging in Bertrand-like price competition. We demonstrate that the Cournot outcome is unlikely to emerge in such markets and that the equilibrium tends to be more competitive than the Cournot model would predict. In addition, our results indicate a tendency toward asymmetric firm sizes and price dispersion that results from the mixed strategies firms use in equilibrium.

1. Introduction

■ In a recent article in this Journal, Kreps and Scheinkman (1983) present and analyze a two-stage model of oligopoly in which firms choose their capacities before engaging in Bertrand-like price competition. Surprisingly, they are able to demonstrate that under certain assumptions the unique perfect equilibrium outcome of this game coincides with the Cournot outcome. This result, if robust, is important since it suggests that the Cournot equilibrium can be viewed as the result of price competition among firms, as long as they choose the scale of operation before they set prices. This provides a defense to the criticism that in Cournot's model prices are not set optimally.

The purpose of this article is to investigate in more detail the nature of equilibrium in markets in which firms make capacity decisions before they make pricing decisions. To do so we first demonstrate that the result of Kreps and Scheinkman (1983) depends critically on their assumption of how demand is rationed when the lower-priced firm cannot meet market demand. We accomplish this by suggesting an alternative rationing rule for which the Cournot outcome cannot emerge in equilibrium. In fact, we show that the same is true for virtually any other rationing rule. Moreover, we argue that when the rationing rule is allowed to be chosen endogenously, our preferred alternative is likely to be observed in equilibrium.

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The inspiration for this article arose in a discussion with Charles Wilson. Helpful suggestions and comments were also provided by William Brock, Michael Rothschild, Martin Osborne, José Scheinkman, two anonymous referees, and the Editorial Board. We are happy to have this opportunity to thank them.

The fact that the result of Kreps and Scheinkman (1983) is sensitive to the specification of residual demand does not imply, however, that their model is of no interest. In many markets firms must, for technological reasons, decide upon capacity long before they make their pricing decisions.[1] The nature of the equilibrium generated by models of this type is thus of great practical importance. Therefore, we derive the equilibrium in the two-stage game by using our alternative rationing rule and discuss its properties. One surprising result is that when capacity is relatively cheap, there exist two perfect equilibria in behavioral strategies. Despite the symmetric initial setup, these equilibria are asymmetric. Moreover, they involve firms' randomizing over prices, and occur at capacity levels exceeding the Cournot levels for both firms.

This striking difference from Kreps and Scheinkman (1983) occurs because in moving from their rationing rule to our alternative, the firms' ability to charge different prices increases as a more favorable contingent demand curve is left for the high-priced firm. For given values of capacities, then, firms earn larger profits. Moreover, under our preferred rationing rules the maximum of the first-stage profit function (as a function of the firm's capacity, and for a given capacity limit of its opponent) occurs at higher capacity levels. This lure of larger profits makes firms compete more intensely and results in higher equilibrium capacities.

Our results therefore tend to indicate that the Cournot model underestimates the degree of competitiveness in markets that are characterized by technological commitment. In addition, they point a natural tendency towards asymmetric firm sizes, even when no industry participant has any natural or technological advantage over his competitors. Finally, since firms use mixed strategies in equilibrium, our model predicts price dispersion. This price dispersion is persistent and could be interpreted as the result of periodic sales as in Varian (1980) or as the result of a game of incomplete information in which the firms' payoff functions are subject to very small privately observed random shocks as in Harsanyi (1973).

2. Rationing rules and contingent demand

■ Consider a market shielded from entry, in which two firms produce a homogeneous product at zero cost and face capacity constraints K_1 and K_2, respectively. It is well known that when the firms compete in prices, for a large portion of the parameter (i.e., capacity) space, equilibria will occur only in mixed strategies, with firms' almost certainly charging different prices (Shubik, 1959; Beckmann, 1965; Levitan and Shubik, 1972). When the lower-priced firm cannot meet its entire demand, the remaining firm's sales will depend upon the manner in which demand is rationed. Different rationing rules will clearly lead to different equilibria. The purpose of this section is to suggest alternative assumptions about rationing, the contingent demand curves they imply, and the instances in which these assumptions are appropriate. We can greatly facilitate this task by considering two possible interpretations of aggregate demand, which we then use to contrast the assumptions needed to generate various contingent demand curves.

Let us examine the rationing scheme advocated by Kreps and Scheinkman (1983). It has two possible interpretations. First, imagine that the market demand curve $D(p)$ is generated by a society of *identical* individual consumers, all of whom share the same downward-sloping demand curve. Consider the situation in which firm one charges a lower price than firm two ($p_1 < p_2$), but cannot meet all demand at that price (i.e., $D(p_1) > K_1$). Kreps and Scheinkman implicitly assume that firm one, realizing that there is excess demand for its product, places a limit on the number of units that each consumer may buy. In fact, they assume this limit to be K_1 (assuming the total measure of consumers to be one), so that

[1] In the automobile industry, for example, capacity decisions are made well over a year in advance of pricing decisions (Friedman, 1983).

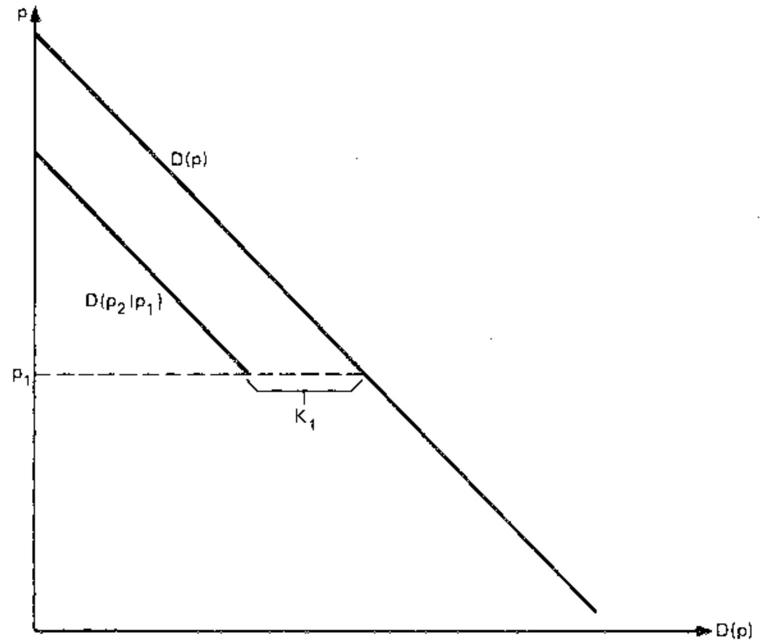
each customer can purchase the limit. Since the top portion of each consumer's demand curve is satisfied, sales for firm two will be given by $D(p_2) - K_1$. In other words, its contingent demand curve is obtained by shifting the market demand curve K_1 units to the left at each price above p_1 (see Figure 1).

In the second interpretation $D(p)$ is the summation of inelastic demands of *heterogeneous* consumers, all of whom wish to purchase one unit of the good, provided the price is below their reservation value. In this case, however, the lower-priced firm would have to serve the consumers with the highest reservation values first.[2]

The rationing rule chosen by Kreps and Scheinkman (1983) is an extreme one, in the sense that it leads to the worst contingent demand curve for firm two. With the first interpretation of aggregate demand this results because each consumer's willingness to pay is decreased equally. With the second interpretation this results because the lower-priced firm removes the most attractive group of customers from the market. We now wish to suggest an alternative rule that leads to the best contingent demand curve for firm two (under the first interpretation of aggregate demand).[3] When customers have identical downward-sloping demand curves, this rule rations output by allowing customers to make unlimited purchases as long as output is available. If output is sold on a first-come-first-served basis, those that arrive late are unable to purchase any output at all. The consumers left for the higher-priced firm would then be a random sample of the consumer population, and, thus, sales for firm two would amount to $D(p_2|p_1) = D(p_2)[1 - (K_1/D(p_1))]$ for all $p_2 > p_1$ (see Figure 2). In the heterogeneous consumer model the same residual demand curve will arise if we assume

FIGURE 1

THE KREPS–SCHEINKMAN CONTINGENT DEMAND CURVE

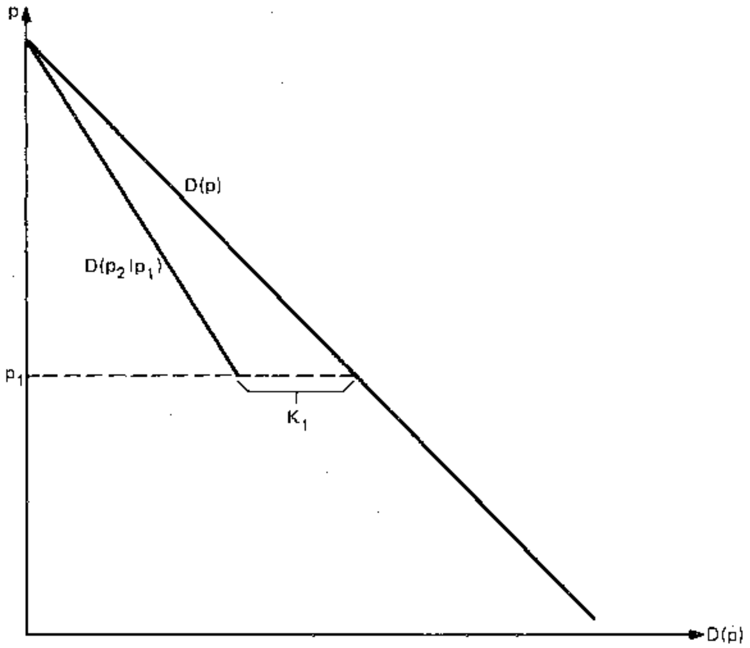


[2] Perry (1984) justifies his use of this contingent demand curve by assuming that resale is possible. We choose to rule out resale because in many markets it is infeasible (e.g., service industries) and in most others it occurs at only a trivial level.

[3] With the second interpretation of aggregate demand this is, strictly speaking, not true. In principle, it is possible that low reservation price customers arrive at the lower-period store first, so the high reservation price customers are left for the higher-priced firm.

FIGURE 2

THE BECKMANN CONTINGENT DEMAND CURVE



that customers arrive at random. Following a suggestion by Shubik (1959), Beckmann (1965) used this contingent demand curve to study equilibria in price-setting games[4] with symmetric capacity constraints. We shall, therefore, refer to this specification as the Beckmann contingent demand curve.

Many other reasonable rationing rules could be considered. For example, in the homogeneous consumer case, one could consider the set of rules of the form: "each consumer is limited to $X(p_i)$ units with service on a first-come-first-served basis," where $X(p_i) \in [K_i, D(p_i)]$. The Kreps and Scheinkman (1983) and Beckmann (1965) rules form the bounds for this set with increases in $X(p_i)$ leading to more favorable residual demand curves for the higher-priced firm. Below we shall argue that when firms choose $X(p_i)$ in a profit-maximizing manner, Beckmann's (1965) contingent demand curve will arise in equilibrium. First, however, we wish to demonstrate that with any rule in this class, or, in fact, virtually any other one, the Cournot outcome cannot emerge as an equilibrium of the two-stage game.

Let $D(p_i|p_j)$ denote any downward-sloping contingent demand curve for firm i that is twice differentiable except at $p_i = p_j$. Then $D(p_i|p_j)$ satisfies the inequality $D(p_i) - K_j \leq D(p_i|p_j) \leq \min\,(D(p_j) - K_j,\, D(p_i))$ for $p_i > p_j$, and $D(p_i|p_j) = D(p_i)$ for $p_i < p_j$. For $p_i = p_j$ we assume that $D(p_i|p_j) = \min\,[K_i, \max\,(D(p_i)/2,\, D(p_i) - K_j)]$. Also, let $D_K(p_i|p_j)$ denote Kreps and Scheinkman's (1983) contingent demand curve, let $D_B(p_i|p_j)$ denote Beckmann's (1965) curve, let p^c denote the Cournot price for the demand curve $D(p)$ (assuming a cost of production equal to the cost of capacity), and let $K^c = q^c \equiv D(p^c)/2$ denote the Cournot capacity, output, respectively, for each duopolist. With the following definition, we can prove the desired result.

[4] His analysis contains some errors, however, as for certain values of capacity his proposed equilibrium distribution functions take on values exceeding one.

Definition. Let f, g: $R \to R$ be functions that are continuously differentiable, except at a finite number of common discontinuity points. Then f is locally distinct to the right from g at the point y if $\lim_{x \downarrow y} f'(x) \neq \lim_{x \downarrow y} g'(x)$.

Theorem 1. Suppose that $D(p_i|p_j)$ is locally distinct to the right from $D_K(p_i|p_j)$ at $p_i = p_j = p^c$ when $K_1 = K_2 = K^c$. Then the Cournot outcome cannot emerge in the equilibrium of the two-stage game with $D(p_i|p_j)$ as the contingent demand curve.

Proof. To prove this theorem it is sufficient to establish that $p_2 = p^c$ is not an optimal response to $p_1 = p^c$ when $K_1 = K_2 = K^c$. If each firm installs capacity K^c and charges the Cournot price, the market will clear with each firm's selling at capacity. Because of this, all contingent demand curves intersect at the point (K^c, p^c) and are right continuous at $p_2 = p^c$. Since $D(p_2|p^c)$ is locally distinct to the right from $D_K(p_2|p^c)$ and $D(p_2|p^c) \geqslant D_K(p_2|p^c)$, it follows that

$$\lim_{p_2 \downarrow p^c} D'(p_2|p^c) < \lim_{p_2 \downarrow p^c} D'_K(p_2|p^c). \tag{1}$$

Let us assume that production is costless (the general case is analogous). Then profits for firm two are given by $D(p_2|p^c)p_2$. From Kreps and Scheinkman (1983) we know that

$$\lim_{p_2 \downarrow p^c} D'_K(p_2|p^c)p_2 + D_K(p_2|p^c) = 0. \tag{2}$$

Differentiating firm two's profit function and using (1), (2), and the fact that $D_K(p^c|p^c) = D(p^c|p^c)$, we obtain:

$$\lim_{p_2 \downarrow p^c} D'(p_2|p^c)p_2 + D(p_2|p^c) > \lim_{p_2 \downarrow p^c} D'_K(p_2|p^c)p_2 + D_K(p_2|p^c) = 0.$$

Thus, firm two can increase its profits by raising p_2 above p^c. *Q.E.D.*

The intuition for this result is clear: since any contingent demand curve that is locally distinct to the right from the Kreps and Scheinkman curve at the Cournot point must be steeper at this point, firms have an incentive to raise price above the Cournot level.

Since the equilibrium is sensitive to the specification of contingent demand, it is important to know which assumption is most appropriate. If $D(p)$ represents the sum of the inelastic demands of heterogeneous consumers, then firms cannot easily influence the manner in which demand is rationed. The contingent demand curve then simply depends upon the arrival process of consumers at the lower-priced firm. This process is influenced by the location of consumers with respect to firms, the speed with which consumers gain price information, transportation costs and similar factors that are, for the most part, unaffected by firm behavior. In such a case the Beckmann contingent demand curve seems most appropriate, since it amounts to an assumption of symmetric treatment of consumers. The Kreps and Scheinkman curve, on the other hand, assumes that consumers with the highest reservation prices are always served first.

When consumers have identical downward-sloping demand curves, however, firms may affect the contingent demand curve by limiting the number of units each consumer may purchase. We close this section by offering an informal argument that implies that when firms choose the rationing rule in a profit-maximizing way, the Beckmann contingent demand curve will emerge in equilibrium. This result, coupled with the fact that firms cannot influence the rationing of output among customers with inelastic demand curves, suggests that as long as a positive fraction of all consumers has downward-sloping demand, firm behavior will lead to Beckmann rationing.

Consider, then, a three-stage game in which the first and third stages consist of capacity and price competition, respectively, as described above. In stage two, however, firms simultaneously and independently announce a rationing rule that will be used when there is excess demand for their product (which, in equilibrium, only occurs when they are charging the lower price). This rule is stated in the form of a function, $X_i(p_i)$ for firm i, which denotes the maximum number of units each consumer may purchase, as a function of its price. We wish to argue that in equilibrium both firms will choose $X_i(p_i)$ to be nonbinding. To see this suppose that K_1 and K_2 have already been chosen (in stage one) and that at least one of the firms has chosen a value for $X_i(p_i)$ that is less than $D(p_i)$ for some open set of p_i's. By altering $X_i(p_i)$ firm i can affect the contingent demand curve faced by firm j in the third stage of the game. Higher values of $X_i(p_i)$ lead to more favorable residual demand curves, and therefore induce firm j to charge higher prices in the third stage. These higher prices benefit firm i, since it will be lower-priced more often. In addition, the equilibrium price or price distribution for firm i associated with the original functions $X_i(p)$ and $X_j(p)$ is no longer optimal. Adjusting its own price distribution clearly benefits firm i. This firm can therefore increase its profits by increasing $X_i(p)$ and, in equilibrium, no limit will be placed on consumer purchases. As noted above, this amounts to the Beckmann contingent demand curve.

3. Equilibrium with Beckmann rationing

■ In this section we proceed with the formal analysis of the two-stage game with our alternative rationing rule. Let us remind the reader briefly about the basic set-up.[5] In the first stage firms independently and simultaneously choose capacity, which is available at a constant marginal cost of c_1. Firms then simultaneously and independently quote a price and supply the demand they face at a constant marginal cost of c_2, up to the capacities chosen in the first round. Without loss of generality we let $c_2 = 0$. According to the Beckmann contingent demand assumption, demand for firm i's product as a function of p_i, and for a given value of p_j, is:

$$D(p_i|p_j) = \begin{cases} D(p_i) & \text{if} \quad p_i < p_j; \\ \min\left[K_i, \max\left(\dfrac{D(p_i)}{2}, D(p_i) - K_j\right)\right] & \text{if} \quad p_i = p_j; \\ \max\left[0, D(p_i)\left(1 - \dfrac{K_j}{D(p_j)}\right)\right] & \text{if} \quad p_i > p_j. \end{cases} \tag{3}$$

We find the subgame-perfect equilibria in behavioral strategies of the two-stage game by backward induction. First, we compute the Nash equilibria in prices for given capacity combinations. This yields reaction functions in capacity space, from which we can deduce the solution to the two-stage game.

□ **The price subgame.** Let K_1 and K_2 denote the capacities chosen in the first stage, and assume without loss of generality that $0 < K_1 \leqslant K_2$. Before describing the equilibria in the subgames, we place some minor restrictions on demand.

Assumption 1. $D(p)$ is differentiable and strictly decreasing in p.

Assumption 2. There exists a $p_0 > 0$ such that $D(p) = 0$ if $p \geqslant p_0$ and $D(p) > 0$ if $p < p_0$, and $D(0) < \infty$.

Assumption 3. The revenue function, $pD(p)$, is single peaked and attains a unique maximum at p^m.

Assumption 4. $pD(p)$ is strictly concave for $p < p^m$.

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[5] To our knowledge, the two-stage game was first suggested, in its current form, by Sherman (1972, p. 65 ff). To simplify the complex analysis involved, however, he reduced its formulation to a simple matrix game.

Hence, we assume a finite choke price, bounded demand, and a revenue function that has a unique peak, and is strictly concave to the left of this peak. Letting $P(\cdot)$ be the inverse industry demand function so that $P(K_1 + K_2)$ indicates the price that clears the market when both firms produce at capacity,[6] we can now state the following theorem.

Theorem 2. For each pair (K_1, K_2) with $0 < K_1 \leqslant K_2$ there exists a unique Nash equilibrium in prices. If $K_1 \geqslant D(0)$, the equilibrium is in pure strategies with each firm's charging a price of zero and earning zero profits. If $P(K_1 + K_2) \geqslant p^m$, the equilibrium is in pure strategies with each firm's charging $P(K_1 + K_2)$ and earning profits of $K_i P(K_1 + K_2)$. Otherwise, the equilibrium is in nondegenerate mixed strategies. Moreover, equilibrium profits are continuous in K_1 and K_2.

Existence of equilibrium in this type of game is nontrivial since $D(p_i|p_j)$ is not continuous at $p_i = p_j$. Fortunately, the (mixed-strategy) Nash equilibrium existence theorem of Dasgupta and Maskin (1986) for games exhibiting certain kinds of discontinuities covers our subgames. Their proof, however, is nonconstructive. In the Appendix we show how to compute the equilibria. Here we only provide an intuitive (but informal) discussion of the properties of the equilibrium.

When $K_1 \geqslant D(0)$, each firm possesses enough capacity to serve the entire market, and hence the standard Bertrand result applies. When $K_1 < D(0)$, however, the large firm has an incentive to form a price umbrella under which the small firm can live. This is most easily seen when K_1 is very small: in that case the large firm may as well ignore the lower-priced competitor since he is of insufficient size. Of course, a pure-strategy equilibrium of this type will not exist. The mixed-strategy solution, on the other hand, does have this feature: the large firm has a masspoint at the upper end of the support (the monopoly price) that becomes larger as K_1 decreases. Finally, when both K_1 and K_2 are small (so that $P(K_1 + K_2) \geqslant p^m$), the monopoly (for a monopolist with capacity $K_1 + K_2$), duopoly, and competitive equilibria coincide. All involve auctioning off all the output that can be produced.

□ **Equilibrium in the full two-stage game.** Subtracting the cost of capacity from the profit functions of the capacity-constrained subgame yields continuous payoff functions in the reduced game from which we can, in principle, calculate reaction functions. Unfortunately, for many capacity combinations the equilibria in the subgames occur in mixed strategies and, in such cases, no closed-form solutions for the profit functions are available.[7] To gain some insight into the nature of the equilibrium, we solve numerically for the reduced-game profit function for the special case where demand is linear, which after an appropriate choice of units we can write as $D(p) = 1 - p$.

We obtain the reaction function for firm i, $K_i^*(K_j)$, by selecting the value of K_i that maximizes firm i's payoff (net of capacity cost) on the assumption that firm j installs K_j units of capacity. Any equilibria in the two-stage game that involve pure strategies in capacity[8] correspond to a pair (K_1^*, K_2^*) such that $K_i^*(K_j^*) = K_i^*$ for $i = 1, 2$—in other words, an intersection point of the reaction curves (best-reply mappings).

Figure 3 illustrates the case where $c_1 = 0$. Discontinuities in the reaction curves, caused by secondary maxima in the profit function, preclude the existence of symmetric equilibria when capacity is relatively cheap. For $c_1 = 0$, there is a unique pair of asymmetric solutions in behavioral strategies. They involve firms' choosing capacity levels roughly equal to .43

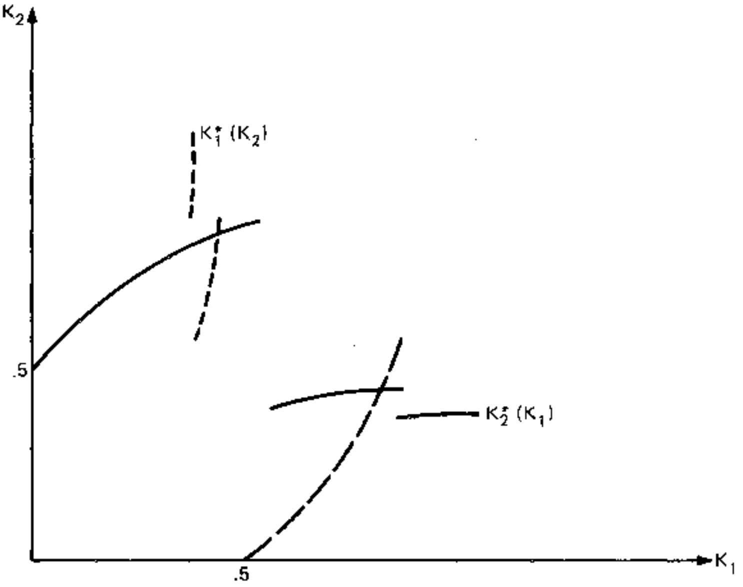
[6] When $K_1 + K_2 \geq D(0)$, we define $P(K_1 + K_2)$ to be equal to zero.

[7] If $\phi_i(p)$ denotes the equilibrium price distribution for firm i and $\underline{p}$ denotes the lowest price in the support of $\phi_i(p)$, then $\phi_1(\underline{p}) = 0$ defines $\underline{p}$ implicitly (see the Appendix for details). Profits are then given by $\pi_i = \underline{p} \min (D(\underline{p}), K_i)$. Unfortunately, it is the equation $\phi_1(\underline{p}) = 0$ that cannot be solved for analytically.

[8] It is hard to imagine that firms would resort to randomization for such long-term choices as the technology to be adopted in production.

FIGURE 3

REACTION FUNCTIONS IN THE BECKMANN GAME ($c_1 = 0$)



and .86, so that one firm is approximately twice the size of the other. Both capacities exceed the Cournot-Nash levels chosen in the Kreps and Scheinkman game (.33). Profit for the large firm is roughly the same as it would be in the Cournot equilibrium, but the small firm's profits are considerably smaller (.056 versus .111). Joint profits are, therefore, substantially lower than the Cournot model would predict. Finally, observe that the price-subgame corresponding to these capacities involves randomization over prices.

Increasing the cost of capacity makes the payoff functions concave, shifts the reaction curves down, and causes the discontinuities to occur at smaller capacity levels. If the cost of capacity is large enough, the secondary maxima disappear, and the reaction functions become continuous, downward-sloping curves. In this case the solution is symmetric. The capacity levels still exceed those obtained in the Kreps and Scheinkman game, but the difference diminishes as c_1 increases. For instance, if $c_1 = .1$, equilibrium capacities of .34 compare to the Cournot-Nash capacity of .30. Profits, however, are still considerably lower (.079 in the Beckmann game as compared with .09 in the Kreps and Scheinkman game), and firms continue to use mixed strategies in their pricing decisions. This remains true until the cost of capacity becomes so large that firms choose an aggregate capacity level below the level that a monopolist with zero marginal cost would choose.

□ **Interpretation and discussion.** At this point it is instructive to step back and consider what these results imply about the Kreps and Scheinkman result as well as what they mean on their own. First, we consider the Kreps and Scheinkman result. One would not expect a model in which the scale of operation is chosen before price competition takes place to duplicate the Cournot outcome exactly. From that perspective, the Kreps and Scheinkman result is merely an accident, but one might have hoped that equilibria in such games would be "sufficiently close" to the Cournot outcome to justify the widespread use of that model. In our opinion, however, when capacity is not overly expensive, equilibria may differ markedly from the Cournot outcome, both qualitatively and quantitatively. Plant sizes are much larger in the Beckmann game than in their Cournot counterpart, even if one takes into account a certain amount of "overcapacity" in the industry. The distribution of prices and output may also be rather different. For instance, when $c_1 = 0$, there is a 92% chance in the Beckmann game that output is higher and that price is lower than the corresponding Cournot-Nash levels. Profits are substantially lower as well. This suggests that the equilibrium is more competitive than the Cournot model would predict. Finally, welfare results derived from a model in which Beckmann rationing is assumed are likely to differ from those obtained by using the Kreps and Scheinkman rule, since these rationing rules reflect different degrees of price discrimination.

What then, can we say about the nature of equilibrium in markets in which capacity decisions are made before pricing decisions? First, the equilibrium is "more competitive" than the Cournot outcome. The Cournot equilibrium therefore places an upper bound on the degree of competitiveness in static models of such markets.[9] Second, when capacity is relatively cheap, firms choose different scales of operation despite the fact that the model is symmetric. This suggests that there might be a natural tendency toward asymmetries in such markets.

The third, and perhaps most interesting, result is that in equilibrium firms use mixed strategies when setting prices.[10] There are two interpretations of mixed strategies in the literature; the first is due to Varian (1980). While the competitive and Cournot models predict that firms will charge the same price, Varian notes that most markets are characterized by a large degree of price dispersion. He also points out that any model that yields equilibrium price dispersion in which firms use pure strategies seems to be a rather implausible explanation of the persistence of price dispersion, since one would expect consumers to learn about prices over time and stop frequenting high-priced stores. As an alternative, Varian presents a model in which firms use mixed strategies in equilibrium and interprets the price randomization as a strategy of having randomly chosen sales. If all firms use mixed strategies, then at any point in time the market would be characterized by price dispersion. This situation could easily persist, because owing to the *intentional* price fluctuations by each firm, consumers cannot learn anything about future prices from the observation of current prices. In Varian's model firms use mixed strategies (or sales) to price discriminate between informed and uninformed consumers. In our model the randomization allows firms to capture a greater share of consumer surplus.[11]

The second interpretation is due to Harsanyi (1973), who demonstrated that the mixed-strategy equilibria of a game may be viewed as the limit of the pure-strategy equilibria of a related "disturbed game" as the disturbances vanish. The disturbed game is equivalent to the initial game, except that in the disturbed game each player's payoff function is subject to a small random disturbance, the value of which is known only to him. Thus, each player has exact information about his own payoff function, but has only imprecise knowledge of his opponents' payoff functions. Harsanyi shows that the pure-strategy equilibria of the disturbed game converge to the mixed-strategy equilibria of the initial game as the random disturbances go to zero. The mixed strategies that are used in our model may therefore be viewed as pure strategies emanating from a game of incomplete information in which firms'

[9] If the game is repeated, tacit or explicit collusion could lead to less competitive equilibria (Davidson and Deneckere, 1984).

[10] Unless, of course, capacity is very expensive.

[11] An anonymous Associate Editor has raised the following objection to the mixed strategies employed in this article and in Kreps and Scheinkman (1983). In any pure-strategy equilibrium firms never "regret" the price they charge *ex post*. This is a result of the fact that firm i faces no *ex ante* uncertainty about its competitor's price. Thus, once that price is revealed, the firm learns nothing of value and has no regret about the price it charged. In mixed-strategy equilibria, however, we only require that the firms are *ex ante* indifferent about charging any price in their support. Once firm i finds out what price its competitor is charging, it may want to change its own price (under the assumption that its competitor's price will not change). It can be argued, therefore, that we are assuming that consumers can respond to market prices faster than firms.

profit functions are subject to small random shocks (due, perhaps, to random fluctuations in the cost of production).[12]

4. Conclusions

■ The Kreps and Scheinkman (1983) game has two alternative interpretations. In the first, c_1 is thought of as representing production costs (rather than capacity costs), and c_2 refers to distribution costs (rather than production costs). This produces a model very close in spirit to Cournot's original. Producers first make independent production decisions. After learning how much each produces, firms market their output by choosing price. In the Kreps and Scheinkman model firms find it optimal to compete the price down to the market-clearing level, and aggregate output coincides with its counterpart in a Cournot model with constant marginal cost $c_1 + c_2$. Our results indicate that this result is not, in general, true. Firms may find it profitable to produce more, and settle at an equilibrium in which some produced output is left unsold.

A second interpretation, apparently endorsed by Maskin and Tirole (1982), holds that quantity competition is merely a surrogate for long-run competition through the choice of technological scale. What we really mean when we say that a firm is choosing a production quantity is that it is choosing a long-run cost curve appropriate for that level. In the short run, competition occurs through price, but price policy and accompanying output decisions are not independent. They are jointly constrained by steeply rising costs beyond the normal scale of operation. While this interpretation "justifies" treating profit as a function of quantity, our results indicate that none of the specific predictions or welfare results of the Cournot model need hold.

Appendix

■ First, we prove existence and uniqueness of the pure-strategy equilibria (Lemma 1 below). For those capacity combinations (K_1, K_2) for which no pure-strategy equilibria exist, we show how to compute equilibrium distributions. We then establish that the resulting distributions indeed form an equilibrium pair. The argument leading to uniqueness is rather involved; the interested reader is referred to Deneckere (1983) or Allen and Hellwig (1984).

Lemma 1. In any pure-strategy equilibrium $p_1 = p_2 = P(K_1 + K_2)$. Moreover, pure-strategy equilibria exist if and only if $K_1, K_2 \geqslant D(0)$ or $P(K_1 + K_2) \geqslant p^m$.

Proof. Assume first that $p_i < p_j$. Then p_i must be less than $P(K_i)$; if not, $D(p_j|p_i) = 0$. This yields an immediate contradiction if $p_i > 0$, for then firm j could undercut i and make positive profits. If $p_i = 0$, then $p_j > 0$ by assumption, and hence i could make positive profits by raising price. Thus, $p_i < P(K_i)$ and firm i earns p_iK_i. Since the latter expression is increasing in p_i, we have a contradiction. Thus, $p_i = p_j = p$ in any pure-strategy equilibrium.

In order that neither firm have an incentive to lower price below p, it must be that both sell at capacity, or $p = P(K_1 + K_2)$ (if $p < P(K_1 + K_2)$, firms could increase profits by raising price). If firm j raises its price p_j above p, it earns

$$p_jD(p_j)\max\left(0, 1 - \frac{K_i}{D(p)}\right).$$

Neither firm will have an incentive to raise price only if $D(p) \leqslant K_i \; \forall i$, in which case $p = 0$ and $K_i \geqslant D(0) \; \forall i$, or if $P(K_1 + K_2) \geqslant p^m$. *Q.E.D.*

If (K_1, K_2) does not belong to the pure-strategy region indicated in Lemma 1, an equilibrium must be found in mixed strategies. Lemma 2 tells us how to calculate the equilibrium in this region.

Lemma 2. Assume that $K_1 < D(0)$ and that (K_1, K_2) satisfy $P(K_1 + K_2) < p^m$. Then there exists a unique pair of solutions (ϕ_1, ϕ_2) to the following differential equations:

$$-\phi_j'(p)Z_3(p) + (1 - \phi_j(p))Z_i(p) = -\pi_i Z_4(p) \qquad \text{for} \qquad i, j = 1, 2 \tag{A1}$$

with boundary conditions $\phi_1(p^m) = 1$, $\phi_2(\underline{p}) = 0$ that satisfy $\phi_1(\underline{p}) = 0$, and

[12] This interpretation suffers from the same "regret" properties discussed in footnote 11.

$$Z_i(p) = \frac{d}{dp}\min\,[K_i/D(p), 1] \qquad i = 1, 2$$

$$Z_3(p) = \min\left[1, \frac{K_i}{D(p)}, \frac{K_j}{D(p)}, \frac{K_i + K_j - D(p)}{D(p)}\right]$$

$$Z_4(p) = \frac{D(p) + pD'(p)}{[pD(p)]^2}$$

$$\pi_i = \underline{p}\min\,(D(\underline{p}), K_i).$$

Moreover, the pair (ϕ_1, ϕ_2) is a solution to the subgame starting at (K_1, K_2).

Proof. First, let us show that ϕ_1 is a proper distribution function for any positive choice of π_2. Observe that $Z_i(p) \geqslant 0$ for $i = 1, 2$ and that Z_4 and Z_3 are positive on $(P(K_1 + K_2), p^m)$. Inspection of (A1) then reveals that $\phi_1'(p)$ is strictly positive on this interval (except at $P(K_2)$, where it is not defined), and that $\phi_1'(p) \to \infty$ as $p \to P(K_1 + K_2)$. Thus, $\underline{p} > P(K_1 + K_2)$ is well defined and satisfies all the requirements of a distribution function, whatever the initial choice of π_2. Let us denote the dependence of $\underline{p}$ on π_2 as $\underline{p} = g(\pi_2)$. One readily checks that $P(K_1 + K_2) < g(\cdot) < p^m$ and that g is continuous (in fact, it is monotone). Hence, the function $\zeta: p \to g(p\min\,(D(p), K_2))$ is continuous and maps $[P(K_1 + K_2), p^m]$ into itself. By the Brouwer fixed-point theorem, ζ has a fixed point. In other words, there exists at least one $\underline{p}$ such that $\underline{p} = g(\pi_2)$ and $\pi_2 = \underline{p}\min\,(D(\underline{p}), K_2)$ hold simultaneously.

Next, we establish that there exists at most one such $\underline{p}$, so that the conditions of Lemma 2 indeed determine a unique pair (ϕ_1, ϕ_2). Let $f_1(p)$ be the solution to

$$f_1'(p)Z_3(p) + f_1(p)Z_2(p) = -Z_4(p)$$

with boundary condition $f_1(p^m) = 0$. $\underline{p}$ will be unique if there exists a unique solution to the equation

$$p\min\,(D(p), K_2)f_1(p) = 1$$

that satisfies $p < p^m$. Some elementary algebra reveals that the derivative of the left-hand side of this equation is negative for all p such that $p\min\,(D(p), K_2)f_1(p) \leqslant 1$, which implies the desired property.

To complete the proof of the first part of the lemma, we still need to establish that ϕ_2 is a proper distribution function. Since $\phi_2' > 0$ for all p in $[\underline{p}, p^m]$ for which $\phi_2(p) < 1$, we only need to show that $\lim_{p\uparrow p^m}\phi_2(p) \leqslant 1$. Assume to the contrary that $\phi_2(\hat{p}) = 1$ for some $\hat{p} \leqslant p^m$. First, we establish that if there exists a $\hat{p}$, then $\underline{p} < P(K_2)$. The integrated form of (A1) is

$$\pi_i = pD(p)\int_{\underline{p}}^{p}\max\left\{0, 1 - \frac{K_j}{D(Z)}\right\}d\phi_j(Z) + p\min\,\{D(p), K_i\}(1 - \phi_j(p)). \tag{A2}$$

If $\underline{p} \geqslant P(K_2)$, then (A2) becomes, for $i = 1$,

$$\pi_1 = p\min\,\{D(p), K_i\}\,(1 - \phi_2(p)).$$

Evaluating this expression at $\underline{p}$ and $\hat{p}$, we obtain different values—a contradiction.

Next, we show that $\phi_2'(p) < \phi_1'(p)$ for all $p \in [\underline{p}, P(K_2)]$. From (A1) we have

$$-\phi_2'(p)Z_3(p) + (1 - \phi_2(p))Z_1(p) = \frac{\pi_1}{\pi_2}[-\phi_1'(p)Z_3(p) + (1 - \phi_1(p))Z_2(p)].$$

Letting $p \downarrow \underline{p}$, we obtain

$$\phi_2'(\underline{p}) = \frac{\pi_1}{\pi_2}\phi_1'(\underline{p}) + \left[Z_1(\underline{p}) - \frac{\pi_1}{\pi_2}Z_2(\underline{p})\right]\frac{1}{Z_3(\underline{p})}.$$

Since $\underline{p} < P(K_2) < P(K_1)$, $\pi_1/\pi_2 = K_1/K_2$. Moreover, $Z_i(p) = d(K_i/D(p))/dp$, which implies that $Z_1(\underline{p}) - (K_1/K_2)Z_2(\underline{p}) = 0$. Thus, $\phi_2(p) < \phi_2'(p)$ and $\phi_2 < \phi_1$ for p close to $\underline{p}$. Repeating the same argument as above, we can show that $\phi_2'(p) < \phi_1'(p)$ if $\phi_2(p) \leqslant \phi_1(p)$ and $p < P(K_2)$. Thus, ϕ_1 and ϕ_2 cannot intersect for $p \leqslant P(K_2)$, and so $\phi_2 < \phi_1$ on $[\underline{p}, P(K_2)]$. Since $\phi_2(\hat{p}) = 1$, there exists a $p > P(K_2)$ such that $\phi_1(p) = \phi_2(p)$. Let $\tilde{p}$ be any such intersection point. We shall now show that necessarily $\tilde{p} \leqslant P(K_2)$. Thus, the assumption $\phi_2(\hat{p}) = 1$ for some $\hat{p} \leqslant p^m$ will lead to a contradiction.

Define $m_i(p) = 1/\pi_i\int_{\underline{p}}^{p}\max\,\{0, 1 - (K_j/D(Z))\}d\phi_j(Z)$. If $\tilde{p} \in [P(K_2), P(K_1) \wedge p^m)$, we have

$$\min\,(K_1, D(\tilde{p})) = K_1 \qquad \text{and} \qquad \min\,(D(\tilde{p}), K_2) = D(\tilde{p}) < K_2.$$

From (A2) we have

$$1 = pD(p)m_1(P(K_2)) + \frac{pK_1}{\pi_1}(1 - \phi_2(p)) = pD(p)m_2(p) + \frac{pD(p)}{\pi_2}(1 - \phi_1(p)),$$

which implies that

$$m_1(P(K_2)) - m_2(\tilde{p}) = [1 - \phi_1(\tilde{p})]\left[\frac{1}{\pi_2} - \frac{K_1}{\pi_1 D(\tilde{p})}\right] < 0.$$

If $\tilde{p} \in [P(K_1) \wedge p^m, p^m)$,

$$1 = pD(p)m_1(P(K_2)) + \frac{pK_1}{\pi_1}(1 - \phi_2(p)) = pD(p)m_2(p) + \frac{pK_2}{\pi_2}(1 - \phi_1(p)),$$

which implies that

$$m_1(P(K_2)) - m_2(\tilde{p}) = [1 - \phi_1(\tilde{p})]\left[\frac{1}{\pi_2} - \frac{1}{\pi_1}\right] < 0.$$

Furthermore, we have

$$0 > m_1(P(K_2)) - m_2(\tilde{p}) \geqslant m_1(P(K_2)) - m_2(\hat{p}) = \frac{1}{\pi_2}(1 - \phi_1(\hat{p})) > 0.$$

This contradiction establishes that $\phi_2(p^m) < 1$ if $K_2 > K_1$. This implies that the large firm always has a masspoint at the upper boundary of the support.

To complete the proof of Lemma 2 we still need to show that profits are constant almost everywhere on the support, and lower everywhere else. We can easily check that since $\underline{p} > P(K_1 + K_2)$, profits on the support are given in (A2), and hence are constant almost everywhere on $[\underline{p}, p^m)$. Profits for firm i, when $p > p^m$, are

$$pD(p)\int_{\underline{p}}^{p^m} \max\left(0, 1, -\frac{K_j}{D(Z)}\right) d\phi_j(Z),$$

which is decreasing in p since $p > p^m$, and when $p \leqslant \underline{p}$ profits are

$$p \min(D(p), K_i),$$

which is increasing in p. *Q.E.D.*

Finally, let us remark that the differential equations in (A1) are linear. A closed-form solution for the distribution functions is therefore available (parametrically in $\underline{p}$). But the equation $\phi_i(\underline{p}) = 0$ is highly nonlinear, and we cannot obtain a closed-form solution for $\underline{p}$.

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