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Incentives to form coalitions with Bertrand competition

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In this article we investigate the incentive to merge when firms that produce differentiated products engage in price competition. We demonstrate that mergers of any size are beneficial, and are so increasingly: large mergers yield higher profits than smaller ones. This is in contrast to the result that mergers tend to be disadvantageous in quantity-setting games. This qualitative difference follows from the fact that reaction functions are typically upward sloping in price games but downward sloping in quantity games. Thus, the reaction of outsiders reinforces the initial price increase that results from the merger.

1. Introduction

■ In the industrial organization literature, it is generally felt that even with constant returns to scale, mergers ought to be beneficial to the merging parties (Steiner, 1975, chaps. 2 and 3). Larger firms, it is argued, have more "market power," and can twist the terms of trade to their advantage, thereby securing for themselves a higher profit than if they acted independently.[1] This intuition is not very well captured in noncooperative oligopoly models. Salant, Switzer, and Reynolds (1983), Szidarowsky and Yakowitz (1982), and Davidson and Deneckere (1984) have all observed that mergers may reduce the joint profits of the participating firms. Following some earlier literature (Stigler, 1966), these articles identify a merger with a binding agreement between some firms to maximize profits jointly by correlating strategies and making side payments if necessary.[2] The merged entity is then treated as a multiplant Cournot player engaged in a noncooperative game against the remainder of the industry. Formally, a merger corresponds to a move from a Nash equilibrium in a given coalition structure to one in a coarser coalition structure.

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[1] We restrict our attention to horizontal mergers. We exclude returns to scale as a possible motive for merger, partly to concentrate on the "market power" incentives of merger and partly because there are some convincing arguments to the contrary (Williamson, 1967; Keren and Levhari, 1983).

[2] Alternatively, such an arrangement may represent a cartel.

At first glance, losses resulting from horizontal mergers seem surprising, since the coalition can always duplicate the actions chosen by its members before they joined. Such a choice would not be consistent with equilibrium, however, since the merged firm has an incentive to reduce output when other players maintain their premerger output levels. Because reaction functions are typically downward sloping in quantity-setting games, the reduction in output of the coalition is followed by an output expansion in the rest of the industry. In response, the coalition further reduces output, and so on. After all adjustments have taken place, each coalition member produces a smaller fraction of industry output. Even though total industry profits rise in the process, coalition members may end up losing because their share of the total pie decreased. For example, in the linear demand, constant marginal cost case studied by Salant *et al.* (1983), mergers are not beneficial unless they succeed in substantially monopolizing the industry by joining over 80% of all firms!

We argue that failure to explain the desirability of mergers results from an almost exclusive attention to quantity as the basic strategic variable under the firm's control.[3] We feel that price is a much more natural strategic variable than output, and that quantity-setting models are useful to the extent that they approach the insights gained from more complex price-setting models (Kreps and Scheinkman, 1983). Specifically, we consider a partial-equilibrium model of an industry consisting of symmetrically differentiated products, each one produced by a separate firm. Firms own the exclusive technology (patent) for the production of their particular brand, and operate at constant and identical average cost. When merging, firms are allowed to shut down the operation of some plants, but may not alter the characteristics of their products. We show that with price as a strategic variable, this model captures most traditional industrial organization insights. The reason our results differ from those obtained in the above-mentioned articles is that reaction functions are typically upward sloping in price-setting games while they are downward sloping in quantity-setting games.[4] This implies that the initial price increase of the coalition will be followed by a price increase on the part of its competitors. The coalition then further raises price, and so on. In equilibrium, after full adjustment has taken place, all prices in the industry will have risen, and all firms will be better off.

The remainder of the article is organized as follows. Section 2 analyzes the equilibrium distribution of prices and profits in an industry consisting of a single merged entity and a set of independent firms. With price as the strategic variable, mergers have very natural effects: mergers of any size are profitable, and are so increasingly, i.e., large mergers yield higher profits than smaller ones. In Section 3 we generalize these results to arbitrary coalition structures. Holding fixed a given coalition structure, we can rank the prices charged by different coalitions according to their sizes: larger coalitions charge higher prices but earn lower profits for their members. This is the expression of a free-rider problem: outsiders benefit more from increased concentration than do insiders. Mergers of coalitions, therefore, are always beneficial to the large party, but may be disadvantageous to the joining party, despite the fact that profits are superadditive.

2. Coalition structures with a single partnership

■ In a differentiated oligopoly the environment of each producer is described by his brand demand function and his cost function. We assume that each producer operates at a constant and identical marginal and average cost of c, and that the demand system is symmetric. Perhaps the simplest such specification of demand is Shubik's (1980):

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[3] Mergers in quantity-setting games can cause gains if the supply response of outsiders is limited by capacity constraints or marginal costs which rise sufficiently rapidly. For a model in this spirit, see Perry and Porter (1985).

[4] For other recent papers that emphasize this difference, see Fudenberg and Tirole (1984) and Eaton and Grossman (1984).

$$q_i(p_1, \ldots, p_N) = V - p_i - \gamma\left(p_i - \frac{1}{N}\sum_{j=1}^{N} p_j\right), \qquad i = 1, \ldots, N,$$

where p_i is the price charged and q_i is the quantity demanded of firm i's brand. The variable N refers to the number of brands (firms) in existence, and $\gamma \geqslant 0$ is a substitutability parameter. When γ approaches zero, goods become unrelated, and when γ approaches infinity, goods become perfect substitutes. Profits for the ith producer, π_i, can thus be expressed as:

$$\pi_i(p_1, \ldots, p_N) = (p_i - c)q_i(p_1, \ldots, p_N).$$

The proofs of the theorems in this section and in Section 2 exploit some of the particularities of the linear demand example. Our results are valid, however, under a fairly general set of assumptions (the details of which are relegated to the Appendix). At the end of each section, we discuss briefly which features of the linear example are essential for the analysis to generalize.

Consider the situation in which a subset of the firms merges while the others continue to act independently. In particular, let $M < N$ denote the number of merging parties. Figure 1 illustrates why the merger destroys the original industry equilibrium in which every firm charges the same price p^o, given by:[5]

$$p^o = \frac{V}{2 + \gamma \dfrac{N-1}{N}}.$$

The curve dd' describes the market for product i, when all other prices are set at their premerger equilibrium value p^o. The DD' curve is Chamberlain's familiar "market demand curve," and corresponds to firm i's demand when all competitors charge the same price as it does. $d_M d'_M$, on the other hand, presents the intermediate situations in which only $(M - 1)$ firms (the coalition partners) follow i's price change; it lies between dd' and DD' and becomes steeper as M rises. It now becomes clear why the initial price-quantity pair (p^o, q^o) no longer represents a Nash equilibrium in the new coalition structure. When every outsider charges a price of p^o, the coalition has negative marginal revenue at q^o, and therefore has an incentive to raise price and reduce quantity to (p^1, q^1), where p^1, the best response of the merged entity to p^o charged by each outsider, is given by:

$$p^1 = \frac{V}{2} \frac{2 + \gamma \dfrac{2N - M - 1}{N}}{\left(2 + \gamma \dfrac{N-M}{N}\right)\left(1 + \gamma \dfrac{N-1}{N}\right)}.$$

No special assumptions are hidden in the geometry of Figure 1 to obtain this result: it is a simple consequence of gross substitutability. In the premerger situation, when a firm contemplated raising price, it did not care about the positive externality it would confer upon other industry members. In the coalition, this externality is internalized, and all coalition members set a higher price.

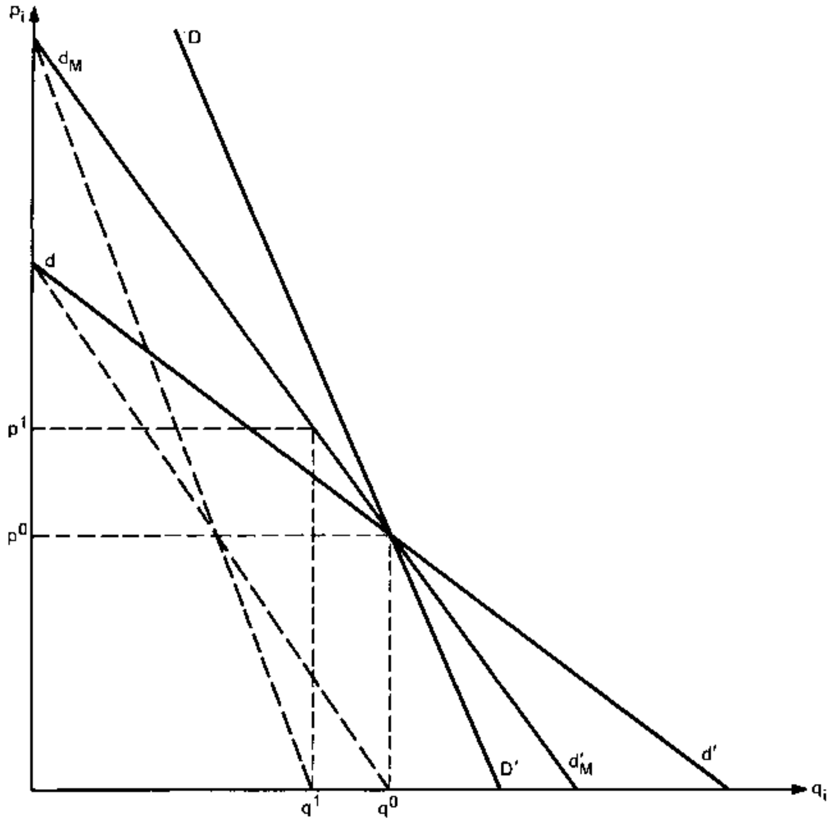
In fact, the same reasoning is valid for any price charged by outsiders. The reaction surface of the coalition thus lies strictly above the reaction surface that represents its members before they joined together. The reaction surfaces of the outsiders, on the other hand, remain unchanged. If we assume that coalition members charge identical prices $(p_1 = p_2 = \ldots p_M = p)$ and that outsiders charge identical prices $(p_{M+1} = \ldots p_N = r)$,[6] then

[5] In our computations and in Figure 1 we assumed that $c = 0$. This is without loss of generality, as we can always perform the transformation $\tilde{V} = V - c$, $\tilde{p} = p - c$, and $\tilde{r} = r - c$.

[6] Uniqueness of equilibrium implies that identical parties charge identical prices.

FIGURE 1

INITIAL PRICE INCREASE BY THE COALITION MEMBERS AS A CONSEQUENCE OF THE MERGER



the effects of the merger can be condensed into the simple two-dimensional diagram of Figure 2. The curve R_{N-M} shows the joint reaction of all outsiders to a price p charged by the coalition members. As emphasized in the Introduction, this reaction function is upward sloping:

$$R_{N-M}(p) = \frac{V + \gamma \dfrac{M}{N} p}{2 + \gamma \dfrac{N+M-1}{N}} .$$

Moreover, its slope is less than one. The curve $R_M(r)$ describes the premerger joint reaction of the first M firms to an outside price of r. It, too, is upward sloping:

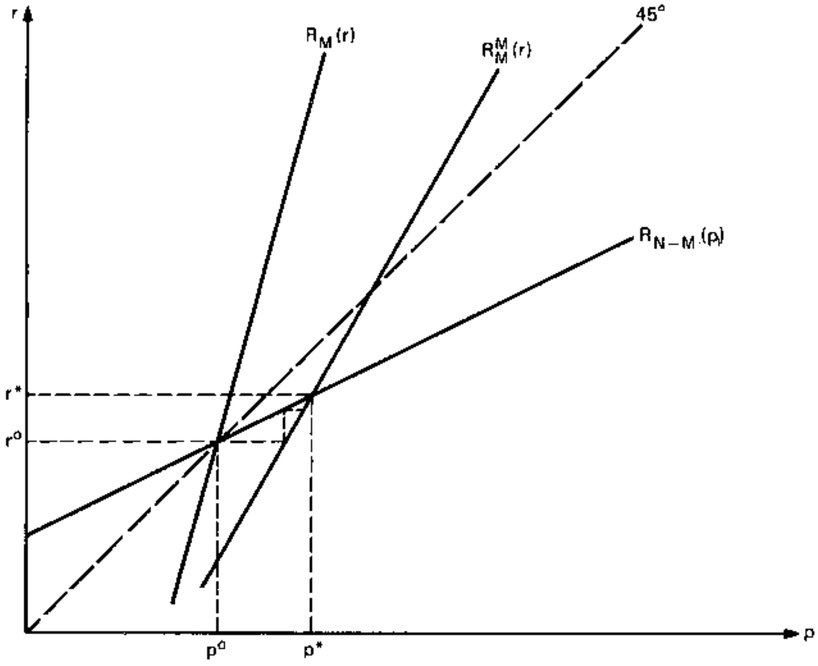
$$R_M(r) = \frac{V + \gamma \dfrac{N-M}{N} r}{2 + \gamma \dfrac{2N-M-1}{N}} .$$

Both curves intersect at the premerger equilibrium price p^o. Since coalition members raise price when they merge, the R_M curve shifts to the right to R_M^M:

$$R_M^M(r) = \frac{V + \gamma \left[\dfrac{N-M}{N}\right] r}{2\left[1 + \gamma \dfrac{N-M}{N}\right]} .$$

FIGURE 2

DYNAMIC ADJUSTMENT PROCESS AND FINAL EQUILIBRIUM IN THE INDUSTRY WHEN A SUBSET OF M FIRMS MERGES



This leads to a new equilibrium price configuration of p^* for coalition members and r^* for outsiders:

$$p^* = \frac{2N + \gamma(2N-1)}{4N + 2\gamma(3N - M - 1) + \gamma^2\left(\frac{N-M}{N}\right)(2N + M - 2)} V$$

$$r^* = \frac{2N + \gamma(2N-M)}{4N + 2\gamma(3N - M - 1) + \gamma^2\left(\frac{N-M}{N}\right)(2N + M - 2)} V.$$

p^* exceeds r^* because $R_{N-M}(p)$ has a slope uniformly less than one. Two conclusions immediately follow from this analysis.

Theorem 1. A merger of any size $2 \leqslant M \leqslant N$ is profitable to each of the merging parties. Outsiders take a free ride and earn larger profits than do insiders.

Proof. Decompose the price change into two stages. First, all outsiders raise price from p^o to r^*. By gross substitutability, this raises demand for all coalition members, and hence benefits them. Facing r^*, the old price p^o is no longer profit-maximizing. Raising it to p^* must benefit the coalition. To see that an outsider takes a free ride, simply observe that he shares $(N - 2)$ competitors with the coalition member, and therefore faces the same prices in those markets. His remaining competitor, being a coalition member, charges the price p^*. The coalition member, on the other hand, is confronted with a remaining competitor (an outsider) charging the lower price r^*. Thus, the outsider's profit function strictly dominates that of a coalition member. Since coalition members do not set price at their individual profit-maximizing level while outsiders do, members must earn less. *Q.E.D.*

In fact, $\pi_i^C(M)$, the per member profit of a size M coalition can be calculated to be

$$\pi_i^C(M) = V^2\left[\frac{2N + \gamma(2N - 1)}{4N + 2\gamma(3N - M - 1) + \gamma^2\left(\frac{N - M}{N}\right)(2N + M - 2)}\right]^2\left[1 + \gamma\frac{N - M}{N}\right],$$

whereas each outsider earns profits equal to

$$\pi_i^o = V^2\left[\frac{2N + \gamma(2N - M)}{4N + 2\gamma(3N - M - 1) + \gamma^2\left(\frac{N - M}{N}\right)(2N + M - 2)}\right]^2\left[1 + \gamma\frac{N - 1}{N}\right].$$

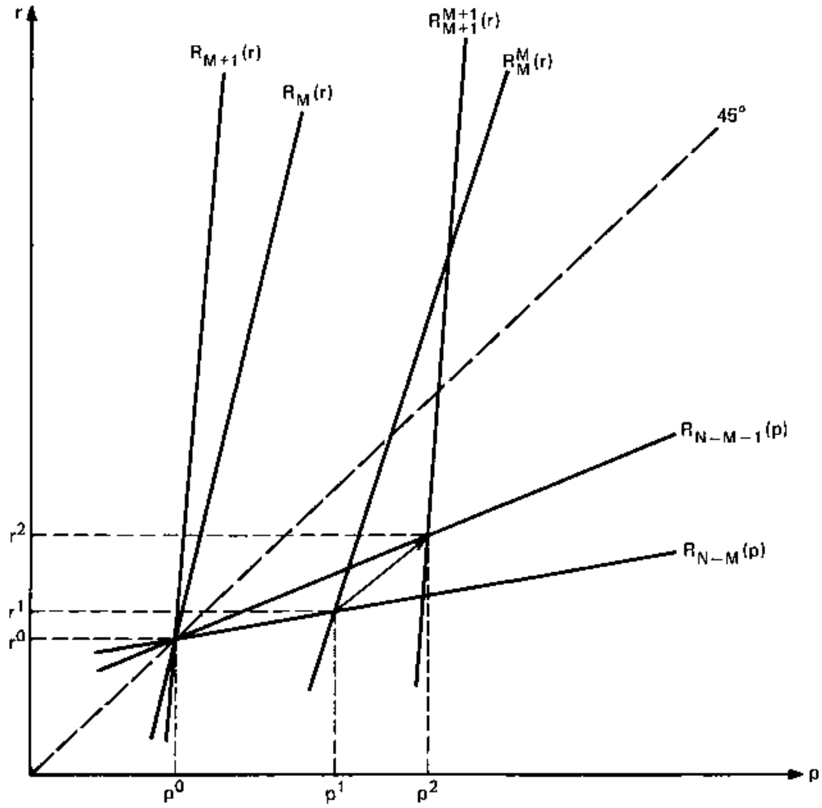
Actually, we can push our diagramatical analysis somewhat further.

Theorem 2. Mergers are increasingly profitable, i.e., $\pi_i^C(M + 1) > \pi_i^C(M)$.

Proof. Figure 3 presents the graphical argument. Curves R_M, R_M^M, and R_{N-M} are as before. Curve R_{N-M-1} corresponds to the joint reaction of $(N - M - 1)$ outsiders to a price p, charged by the rest of the industry; by assumption it cuts the 45° line at (p^o, r^o). Furthermore, it is a relatively straightforward matter to establish that $R_{N-M-1}(p) \gtreqless R_{N-M}(p)$ as $p \gtreqless p^o$.[7]

FIGURE 3

COMPARISON OF THE EFFECTS OF A MERGER OF SIZE M WITH THOSE OF SIZE (M + 1)



[7] Since p^o is the unique premerger equilibrium, $p^o = R_{N-M}(p^o) = R_{N-M-1}(p^o)$. When $p > p^o$, the optimal joint reaction of all outsiders to a coalition of size M charging the price p, $R_{N-M}(p)$, is less than p (see footnote 11). Thus, if an extra outsider joins the coalition and raises price from $R_{N-M}(p)$ to p, the remaining $(N - M - 1)$ outsiders have an incentive to raise price above $R_{N-M}(p)$. The same reasoning establishes $R_{N-M-1}(p) < R_{N-M}(p)$ for $p < p^o$.

Observe also that $R^{M+1}_{M+1}(r) > R^M_M(r)$ for all r such that $R^{M+1}_{M+1}(r) > r$.[8] The intuition for this inequality is clear: if a coalition member were to leave a size $(M + 1)$ coalition, the remaining coalition partners would have a double incentive to reduce price. First of all, the new outsider lowers his price from $R^{M+1}_{M+1}(r)$ to r. Second, they no longer take into account the negative externality they are conferring upon him when they lower price. Thus, $R^M_M(r) < R^{M+1}_{M+1}(r)$. All of this leads to the following analysis: the point (p^1, r^1) lies at the intersection of R_{N-M} and R^M_M and gives the equilibrium price configuration in an industry consisting of a coalition of size M and $(N - M)$ independent outsiders. Similarly, R^{M+1}_{M+1} and R_{N-M-1} intersect at (p^2, r^2), the equilibrium prices when the coalition has size $M + 1$. Clearly, $p^2 > p^1$ and $r^2 > r^1$. Again, we may decompose the price change into two stages. First, one extra member joins the coalition and raises price from r^1 to p^1. Then all remaining outsiders raise price to p^2, and finally, the coalition adjusts p^1 to the profit-maximizing level p^2. Each of these steps is beneficial to existing coalition members. *Q.E.D.*

Table 1 summarizes the results obtained for the linear example: it reports the equilibrium profits for outsiders and for coalition members as a function of γ, the substitutability parameter, and M, the size of the coalition. Obviously, mergers are most beneficial when γ takes on intermediate values. When γ is close to zero, goods are basically unrelated, and mergers yield only marginal benefits. When γ is very large, all products are very close substitutes, and mergers do not significantly reduce the degree of competition in the market.

TABLE 1 **Profits per Firm for Insiders (π^c_i) and Outsiders (π^o_i) as a Function of the Substitutability Parameter γ and the Coalition Size M in a 10-Firm Industry***

$M \backslash \gamma$	.25	.5	1	3	5	7	10	50	100	1000
1	2474	2416	2259	1675	1302	1060	826	208	108	11
	(2474)	(2416)	(2259)	(1675)	(1302)	(1060)	(826)	(208)	(108)	(11)
2	2475	2417	2263	1684	1312	1069	835	211	109	11
	(2476)	(2419)	(2267)	(1692)	(1320)	(1077)	(842)	(213)	(110)	(12)
3	2476	2420	2271	1705	1336	1093	856	218	113	12
	(2478)	(2426)	(2285)	(1731)	(1361)	(1116)	(876)	(224)	(116)	(12)
4	2477	2425	2285	1741	1376	1132	892	231	120	12
	(2481)	(2437)	(2312)	(1794)	(1431)	(1183)	(937)	(245)	(127)	(13)
5	2480	2432	2304	1793	1437	1193	949	251	131	13
	(2486)	(2452)	(2351)	(1891)	(1541)	(1292)	(1036)	(280)	(146)	(15)
6	2482	2442	2330	1866	1525	1284	1034	285	149	16
	(2492)	(2471)	(2403)	(2035)	(1713)	(1467)	(1201)	(344)	(181)	(19)
7	2486	2453	2362	1964	1651	1418	1166	342	181	19
	(2500)	(2496)	(2471)	(2245)	(1984)	(1757)	(1487)	(469)	(251)	(27)
8	2490	2466	2400	2095	1832	1621	1378	452	245	26
	(2509)	(2525)	(2559)	(2562)	(2438)	(2277)	(2041)	(771)	(428)	(47)
9	2495	2482	2446	2269	2098	1945	1750	731	421	49
	(2519)	(2561)	(2670)	(3060)	(3272)	(3357)	(3353)	(1934)	(1185)	(146)

* Figures in parentheses refer to profits for outsiders. All entries in the tables were scaled up by a factor of 1000, and were calculated on the basis of a demand intercept $V = 1$ and marginal cost $c = 0$ for all players.

[8] Since $R_{N-M}(p) < p$ for all $p < p^o$ (see footnote 11), the search for equilibria may be confined to the region below the 45° line.

At this point, it is instructive to step back for a moment to investigate which features of the linear model are crucial in obtaining the results of Theorems 1 and 2. Our argument basically consists of four steps.

(1) $R_M^M(r) > R^M(r)$.
(2) Reaction functions (coalitional and individual) are upward sloping.
(3) For any coalition structure, the equilibrium is unique.
(4) The curve $R_{N-M}(p)$ has a slope uniformly less than one.

The first step ensures that if the initial industry equilibrium is at (p^o, r^o), and a merger occurs, the initial reaction of the new coalition is to raise price from $p^o = R^M(r^o)$ to $R_M^M(r^o)$. The second step guarantees that the adjustment $p_t = R(p_{t-1})$ that occurs as a consequence of the merger, starting at the initial equilibrium, is stable.[9] Indeed, according to this process, adjustments occur alternately: $p_o = (p^o, r^o)$, $p_1 = (R_M^M(r^o), r^o)$, $p_2 = (R_M^M(r^o), R_{N-M}(R_M^M(r^o)))$, etc. Since $R_M^M(r^o) > R_M(r^o) = p^o$, the sequence p_t is increasing. If the strategy spaces are compact, this sequence must converge. Continuity of the reaction functions then ensures that its accumulation point is an equilibrium, and step (3) guarantees that this is the equilibrium for the new coalition structure. Note that we do not require the postmerger adjustment process to be stable from any initial condition, but only from the premerger equilibrium. Nor do we require the premerger adjustment process to be stable. In fact, known sufficient conditions for the stability of either process (Friedman, 1977, p. 74) are overly restrictive, and not even satisfied for our simple example.[10]

The fourth and final step establishes that in equilibrium the coalition charges a higher price than outsiders do. This enables us to rank prices and profits of coalition members and outsiders. Theorem 3 of Section 3 (and Theorem A2 in the Appendix) show that prices can be ranked quite generally, and independently of any assumptions on the slope of R_{N-M}.[11] Hence, we need not make any special assumptions to ensure the implications of step (4). Furthermore, Theorem A1 in the Appendix proves, under fairly general conditions, that the reaction function of the coalition dominates that of its members before they merged. Thus, we need make no special assumptions to establish the first part of our argument either. Of the remaining two steps, the uniqueness requirement seems impossible to dispense with if we want the comparison between the pre- and the postmerger situation to be determinate. Hence, the crucial ingredient of our analysis is that reaction functions are upward sloping.[12] While this is the typical case in price-setting games (and holds for a rich class of functional forms, including most econometric demand systems that satisfy the assumptions of gross substitutability and symmetry), it may nevertheless be that reaction functions have a downward sloping portion over part of the parameter space. In such cases it may be that some mergers are not beneficial.[13]

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[9] We do not attach any descriptive significance to this adjustment process since our model is static. Rather, we use it as a technical device to analyze global changes in the equilibrium. Alternatively, one can regard it as a computational procedure for finding the postmerger equilibrium.

[10] Whether our assumptions imply global stability of either adjustment process remains an open question, except in the case of $N = 2$, where uniqueness and monotonicity suffice for global stability of the adjustment process. For the general case ($N > 2$), one can prove that stability obtains from the regions $\{p: R_i(p) \geqslant p_i, \forall\, i\}$ and $\{p: R_i(p) \leqslant p_i, \forall\, i\}$. The boundary conditions of Assumption 3 in the Appendix imply that these regions are nonempty.

[11] In fact, all that is needed is that $R_{N-M}(p) < p$ for $p > p^o$. This is actually implied by our assumption of a unique premerger equilibrium. If $R_{N-M}(p) > p$ for some $p > p^o$, R_{N-M} would have to cross the 45° line in the interval (p^o, p), which violates uniqueness.

[12] For the comparative statics to make sense, we also need to assume uniqueness of the equilibrium in any coalition structure. Note that in the linear case this is guaranteed by step (3).

[13] Conditions under which the reaction functions may have downward sloping portions are discussed in the Appendix. Conversely, if reaction functions in quantity-setting games have upward sloping portions, mergers may be beneficial.

3. General coalition structures

■ This section is devoted to the study of Nash equilibria in price games with general coalition structures. First, we characterize the distribution of prices and profits within a particular coalition structure. As in Section 2, larger coalitions charge higher prices, but earn lower profits per member than smaller ones. We then prove that if a merger occurs, all prices in the industry rise. Consequently, everybody gains from the merger, except possibly the smallest of the two joining parties. The smaller party may lose, despite the fact that profits are superadditive.

Theorem 3. Let $F = \{B_1, B_2, \ldots, B_k\}$ be a partition of the player set, with $n_1 \geqslant n_2 \ldots \geqslant n_k$, where n_i refers to the size of coalition B_i. Then $p_1 \geqslant p_2 \geqslant \ldots \geqslant p_k$ with $p_i > p_{i+1}$ if $n_i > n_{i+1}$.

Proof. We shall prove that $p_i \geqslant p_j$ when $i > j$. Suppose not. Then there exist $i < j$ such that $p_i < p_j$. The profit function for coalition B_i, $\pi(B_i)$, is given by:

$$\pi(B_i) = n_i\left\{(p_i - c)\left[V - p_i\left(1 + \gamma\left(1 - \frac{n_i}{N}\right)\right) + \frac{\gamma}{N}\sum_{j \notin B_i} p_j\right]\right\}.$$

Differentiating this expression with respect to p_i and setting the derivative equal to zero yield:

$$q_i(p_1, \ldots, p_n) = (p_i - c)\left[1 + \gamma\left(1 - \frac{n_i}{N}\right)\right].$$

Since $n_i \geqslant n_j$ and $p_i < p_j$, this implies $q_i < q_j$.

On the other hand, the definition of q_i,

$$q_i = V - p_i - \gamma\left(p_i - \frac{1}{N}\sum_{l=1}^{N} p_l\right),$$

yields

$$q_i - q_j = (p_j - p_i)(1 + \gamma),$$

which contradicts $q_i < q_j$. *Q.E.D.*

A simple corollary to Theorem 3 is the following.

Corollary. Let π_i denote the payoff to a member of B_i. Then $\pi_i \leqslant \pi_{i+1}$ (with equality if and only if $n_i = n_{i+1}$).

Proof. Members of B_m and B_{m+1} differ only in two prices in their profit functions. Suppress all other arguments except these two, and let the first argument in the profit function refer to the own price and the second argument to the opponent's price. Then

$$\pi_{i+1}(p_{i+1}, p_i) \geqslant \pi_{i+1}(p_i, p_i) = \pi_i(p_i, p_i) > \pi_i(p_i, p_{i+1}).$$

The first inequality obtains because $p_i \geqslant p_{i+1} > \underset{x}{\operatorname{argmax}}\, \pi_{i+1}(x, p_i)$[14] and because the individual payoff functions are single-peaked. This is illustrated in Figure 4. The second inequality follows from gross substitutability. *Q.E.D.*

Even though it is no longer possible to write analytical expressions for equilibrium payoffs in games with arbitrary coalition structures, Table 2 provides some numerical examples for the linear demand case.

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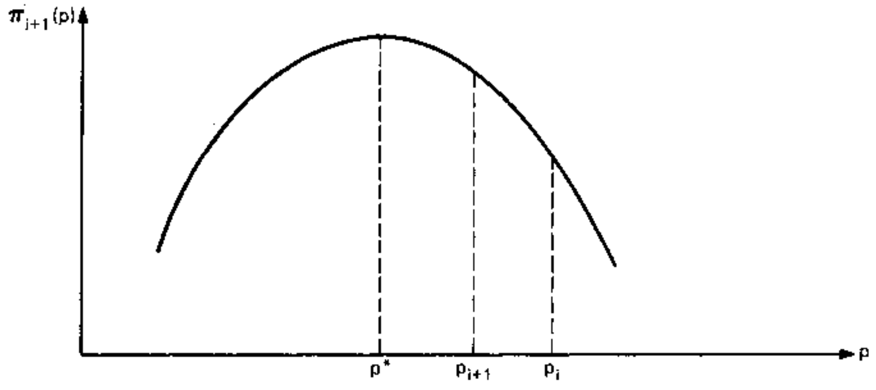
[14] Price p_{i+1} exceeds the price a member of coalition B_{i+1} would set if he ignored the negative externality he imposes on fellow coalition members (this price is referred to as p^* in Figure 4, which illustrates the payoff of a member of coalition B_{i+1}).

FIGURE 4

PAYOFF OF A MEMBER OF COALITION B_{i+1} AS A FUNCTION OF ITS PRICE, WHEN ALL OTHER INDUSTRY PARTICIPANTS ARE CHARGING EQUILIBRIUM PRICES



Theorem 3 and its corollary provide us with a ranking of profits and prices for an industry with any given coalition structure. It does not, however, allow us to compare prices and profits across different coalition structures. In particular, we would like to evaluate the effect of a merger, i.e., a move from a given coalition structure to a coarser one. Suppose, for instance, that in the coalition structure $F = \{B_1, \ldots, B_k\}$, B_i and B_j join together. As in Section 2, we can easily establish that the initial reaction of B_i and B_j to the absorption of the negative externality they imposed upon each other is to increase price above the maximum of p_i and p_j (see Theorem A1 in the Appendix). Since all coalitional reaction functions are upward sloping, stability of the alternate adjustment process then implies that all prices in the industry rise. We are now ready to establish the main proposition of this section.

Theorem 4. If $n_i > n_j$, then a merger of B_i and B_j is beneficial to everybody in the industry, except possibly B_j.

Proof. Let $k \neq i, j$. Since all prices in the industry rise, B_k gains. A further source of benefit to B_k is that p_k is no longer profit-maximizing, given the new prices of other industry members. For a B_i member, decompose the adjustment in three steps: (i) B_j members raise price from p_j to p_i; (ii) all other industry members raise their price to the new equilibrium level; and (iii) p_i adjusts to its new equilibrium level. Each of these steps benefits B_i members. The first step, however, hurts B_j members. *Q.E.D.*

Theorem 4 only suggests the *possibility* that the smallest of the two joining parties may lose as a consequence of the merger. Table 2, on the other hand, provides a numerical example of this phenomenon. Consider the industry structure consisting of the coalitions $\{1, 2, 3\}$ and $\{4\}$. For low values of the substitutability parameter ($\gamma = .5$ and $\gamma = 1$), player 4 earns more by going it alone than by joining the three other players and receiving his share of monopoly profits. This should not come entirely as a surprise, as the small coalition is taking a free ride on the high prices charged by the large coalition. When goods are close substitutes, however, (e.g., $\gamma = 5$), the presence of even two competing coalitions leads to excessive competition, and player 4 would be better off joining the large coalition. Observe also that the table illustrates a principle which can be derived from the Corollary to Theorem 3: whenever the industry contains two coalitions of the same size, these coalitions would find it in their interest to join together. Thus, for example, coalitions $\{1, 2\}$ and $\{3, 4\}$ always want to merge, independently of the substitutability parameter γ.

While coalition B_i may lose as a consequence of a merger with B_j if $n_i > n_j$, we may

TABLE 2 **Distribution of Profits in a 4-Firm Industry as a Function of the Substitutability Parameter γ and the Coalition Structure***

γ \ Player	1	2	3	4
.5	2438	2438	2438	2438
	[2445	2445]	2461	2461
	[2469	2469]	[2469	2469]
	[2466	2466	2466]	2513
	[2500	2500	2500	2500]
1	2419	2419	2419	2419
	[2337	2337]	2375	2375
	[2400	2400]	[2400	2400]
	[2398	2398	2398]	2521
	[2500	2500	2500	2500]
5	1437	1437	1437	1437
	[1525	1525]	1616	1616
	[1728	1728]	[1728	1728]
	[1795	1795	1795]	2232
	[2500	2500	2500	2500]

* Coalitions are bracketed. All entries in the tables were scaled up by a factor of 1000, and were calculated on the basis of a demand intercept $V = 1$ and marginal cost $c = 0$ for all players.

nevertheless prove that the sum of the profits of members of coalition B_i and B_j increases owing to the merger.

Corollary. Let $\pi(B_i \cup B_j)$ be the profits of $B_i \cup B_j$ after a merger of B_i and B_j occurs, and let $\pi(B_i)$ and $\pi(B_j)$ be the premerger profits of the participating coalitions. Then $\pi(B_i \cup B_j) > \pi(B_i) + \pi(B_j)$.

Proof. We know that in equilibrium coalitions other than B_i and B_j raise price. This benefits both B_i and B_j members. Adjusting the premerger profits to their postmerger values for $B_i \cup B_j$ is by definition profitable. *Q.E.D.*

Since industry profits are maximal when all industry members join together, one might conjecture that firms would merge all the way to monopoly if they were allowed to combine freely. Due to the free-rider problem mentioned in the Corollary of Theorem 3, this is not entirely obvious: every firm would like the other ones to join first so it could make higher profits during the transition. Moreover, despite the fact that profits are superadditive, a large coalition may not be able to bribe a smaller one to join it. Since joining partners are *ex post* indistinguishable from existing partners, they cannot credibly be offered more than their proportional share of the profits of the resulting coalition. Small coalitions thus may not be able to earn their marginal contribution. It is conceivable, therefore, that some industry members will refuse to cooperate and remain "small." Considerations such as these require explicit modelling of the coalition formation process. In Deneckere and Davidson (1983) we model this process as a two-stage noncooperative game. In a first stage firms choose a particular coalition to which they want to belong. Given each player's preferences, the rules of the game then determine which coalitions actually form. This coalition structure becomes binding in the second stage, in which firms compete in prices. We show that depending upon the coalition formation rules, the subgame perfect equilibria of this game may involve either the monopoly equilibrium or some less concentrated industry structure in which some small players refuse to join the grand coalition. Nevertheless, only very concentrated industry structures obtain, and when brands become good substitutes, monopoly is the only equilibrium.

One last remark: while Theorem 3 exploits the special structure of the linear demand system, it is valid in much greater generality, as proved in Theorem A2 of the Appendix. Since the remaining results of this section did not specifically invoke the linearity assumption, and required only those assumptions made explicit in the Appendix (mainly compactness of strategy spaces, uniqueness of equilibrium, and upward sloping coalitional reaction functions), they are valid at the same level of generality.

4. Conclusion

■ The incentive to merge in noncooperative oligopoly models depends on the interaction of two basic forces. First, a merger allows coalition partners to absorb a negative externality. This beneficial effect is present both in price-setting and quantity-setting games. Second, the merger elicits a spiral of responses from rival firms. In price-setting games, this response tends to be beneficial to coalition partners because reaction functions are upward sloping. In such an environment, then, firms are likely to have an incentive to merge. In quantity-setting games, on the other hand, the response of other industry members tends to hurt coalition partners because in these games reaction functions are typically downward sloping. Which force dominates depends on the degree of product substitutability, and on the steepness of the marginal cost curves. When opponents produce goods which are very close substitutes to the coalition's brands, and when they have a fairly flat marginal cost curve,[15] they react vigorously to any output reduction. Mergers in quantity-setting games are thus likely to be disadvantageous when products are not differentiated much, and when marginal costs do not rise very rapidly, as observed by Salant *et al.* (1983), and subsequently by Szidarowsky and Yakowitz (1982) and Davidson and Deneckere (1984). Moreover, this counterintuitive phenomenon does not disappear unless goods are close to being unrelated or firms face serious capacity bottlenecks.

Price-setting games, on the other hand, seem to capture traditional industrial organization insights rather well. Under certain plausible conditions on the demand system, mergers are always beneficial to existing members and become more profitable as the size of the merger increases. The resulting industrial concentration confers large positive externalities on other industry members, so that coalitions producing a small number of varieties earn more than larger ones. Not surprisingly, short of antitrust policy, the industry would concentrate almost completely towards monopoly.

Appendix

■ In this Appendix we make some assumptions on the brand demand system, which ensure that the results obtained in Sections 2 and 3 generalize.

Assumption A1. Let $A = \{p \geqslant 0: q_i(p) > 0, \forall\, i = 1, \ldots, N\}$, where $q_i(p) = q_i(p_1, \ldots, p_N)$ is the ith firm's demand function. Then $q_i(p)$ is twice continuously differentiable on A, with $\partial q_i/\partial p_j > 0$ and $\partial q_i/\partial p_i < 0$ on A. Moreover, A is convex and bounded.

Assumption A2. $q_i(p_i, p_{-i}) = q_j(p_j, p_{-j})$ if $p_i = p_j$ and $p_{-i} = p_{-j}$, where $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N)$.

Assumptions A1 and A2 yield an environment which is similar to that of the models of monopolistic competition of Chamberlain (1956), Spence (1979), Dixit and Stiglitz (1977), and Hart (1983): we have a symmetrically differentiated industry, in which all goods are gross substitutes, and in which each firm's demand is downward sloping.

[15] For an elaboration on this point, see Perry and Porter (1985). They introduce rising marginal cost curves in a homogeneous product model, and find that mergers are beneficial if costs rise sharply enough. This result should not be very surprising: in the limit, as the marginal cost curve becomes very steep, capacity limitations will prevent outsiders from expanding, and mergers will thus be advantageous to the joining parties.

It is basically the framework adopted by Friedman (1977). Assumption A3 below guarantees the existence of a pure-strategy Nash equilibrium in prices.

Assumption A3. The profit functions $\pi_i(p_i, p_{-i}) = (p_i - c)q_i(p)$ are single peaked[16] in p_i for all p_{-i} such that $(p_i, p_{-i}) \in A \cap \{p | p_i \geqslant c, \forall\, i\}$, where c is the constant marginal production cost common to all producers. Moreover, if we define $p_{\bar{i}}$ as $\inf\{p_i\colon q_i(p_i, ci_{N-1}) > 0\}$, where i_{N-1} is an $(N-1)$ vector with all components equal to one, then $p_{\bar{i}} > c$ for all i.

The last requirement of Assumption A3 ensures that firm i's demand remains positive when all its opponents are charging a price equal to marginal cost, and hence that all equilibria are interior.

Let $\{B_1, B_2, \ldots, B_m\}$ be a coalition structure, i.e., a partition of the player set, and let the cardinality of B_i be indicated by n_i. We may write the payoff to coalition B_i as

$$\pi_{B_i}(p) = \sum_{k \in B_i} (p_k - c)q_k(p_{B_i}, p_{-B_i}),$$

where p_{B_i} is a vector consisting of the components $\{p_j, j \in B_i\}$, and p_{-B_i} is a vector containing the components $\{p_j, j \notin B_i\}$. For arbitrary coalition structures, the assumption parallel to Assumption A3 becomes:

Assumption A3a. $\pi_{B_i}(p)$ is single peaked in p_{B_i} for any p_{-B_i} such that $(p_{B_i}, p_{-B_i}) \in A \cap \{p\colon p_i \geqslant c\ \forall\ i\}$. Moreover, $p_{\bar{i}} > c$ for all i.

Finally, we require that coalitional reaction functions[17] be upward sloping. For coalition B_j:

Assumption A4. $\delta q_i/\delta p_s(p) + (p_k - c) \sum_{k \in B_i} \delta^2 q_k/\delta p_i \delta p_s(p) > 0$, $\forall\ p = (R_{B_i}(p_{-B_i}), p_{-B_i})$, where $i \in B_j$, $s \notin B_j$, and $R_{B_i}(p_{-B_i})$ is the price vector charged by coalition B_i in response to an outside price vector p_{-B_i}.

Assumption A4 is crucial to our results, but seems quite reasonable. Gross substitutability implies that $\delta q_i/\delta p_s > 0$ for all p in A. While it is reasonable to expect $\delta^2 q_i/\delta p_m \delta p_k$ to be positive for $m \neq k \neq i$, we may expect the same term to be negative for $k \neq m \neq i$.[18] All we require is that the latter term not exceed the sum of the other terms in Assumption A4 (and this only along reaction surfaces).

Below, we shall also assume that equilibrium is unique for all possible coalition structures. This assumption facilitates the comparison of equilibria in different coalition structures.

Assumption A5. For any coalition structure, the best reply map $R\colon A \to A$ has a unique fixed point.

Theorem A1 shows that the initial reaction of coalition partners to a merger is to raise price. (We prove the theorem here only for the case of a single coalition, with the rest of the industry unorganized. The general case is analogous.)

Theorem A1. $R_M^M(r) > R_M(r)$.

Proof. The premerger first-order condition for a coalition member when outsiders charge a price r is given by

$$q_i(\cdot) + \frac{\delta q_i}{\delta p_i}(\cdot)(R_M(ri_{N-M}) - c) = 0, \tag{A1}$$

while the postmerger conditions are

$$q_i(\cdot) + \sum_{j \in M} \frac{\delta q_j}{\delta p_i}(\cdot)(R_M^M(ri_{N-M}) - c) = 0. \tag{A2}$$

All arguments are evaluated at $(R_M(ri_{N-M}), ri_{N-M})$ in (A1) and $(R_M^M(ri_{N-M}), ri_{N-M})$ in (A2). If R_M^M were equal to R_M, the second expression would be positive when the first was equal to zero. Single peakedness of π_M then implies that $R_M^M(ri_{N-M}) > R_M(ri_{N-M})$. *Q.E.D.*

Theorem A2 below generalizes Theorem 3 of Section 3.

Theorem A2. Let $F = \{B_1, B_2, \ldots, B_k\}$ be a partition of the player set, with $n_1 \geqslant n_2 \geqslant \ldots \geqslant n_k$. Then $p_1 \geqslant p_2 \geqslant \ldots \geqslant p_k$ with $p_i > p_{i+1}$ if $n_i > n_{i+1}$.

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[16] A function $f\colon B \subset \mathbb{R}^n \to \mathbb{R}$ is single peaked if and only if there exists a unique x^* in B such that for all y such that $x^* + y \in B$ such that it is true that the gradient of $f(x^* + y)y < 0$. This property is more general than quasi concavity, since it does not require level curves to be convex.

[17] The coalitional reaction function $R_{B_i}\colon R^{N-n_i} \to A$ for coalition B_i is defined as

$$R_{B_i}(p_{-B_i}) = \arg\max_x \pi_{B_i}(x, p_{-B_i}).$$

For a coalition structure $\{B_1, \ldots, B_m\}$ the best reply map $R\colon A \to A$ is then defined as $R = (R_{B_1}, \ldots, R_{B_m})$.

[18] A unit price increase should reduce a firm's demand more as an opponent's price increases, because it then has a larger customer base. Similarly, a firm should obtain more of an opponent's customers if the former increases its price when the rest of the industry charges higher prices.

Proof. Without loss of generality, we prove $p_1 \geqslant p_2$. Fix all other prices at their equilibrium values, and from here on suppress them as an argument. Let

$$q_i^*(x, y) = q_i(\underbrace{x, x, \dots, x}_{n_1}, \underbrace{y, y, \dots, y}_{n_2}) \text{ and similarly for } \left(\frac{\delta q_i}{\delta p_j}\right)^*,$$

where x refers to the price charged by B_1, and y to the price charged by B_2 members. Define

$$h_1^*(x, y) = q_1^*(x, y) + \left(\frac{\delta q_i}{\delta p_j}\right)^*(x, y)(x - c) + (n_1 - 1)(x - c)\left(\frac{\delta q_i}{\delta p_j}\right)^*(x, y)$$

$$h_2^*(x, y) = q_2^*(x, y) + \left(\frac{\delta q_i}{\delta p_j}\right)^*(x, y)(y - c) + (n_2 - 1)(y - c)\left(\frac{\delta q_i}{\delta p_j}\right)^*(x, y)$$

$$f(x, y) = h_1^*(x, y) - h_2^*(x, y),$$

where for each h_k $(k = 1, 2)$: $i, j \in B_k$ $(i \neq j)$.

Let us assume, to the contrary, that $p_1 < p_2$. Then $f(p_1, p_2) = 0$ and $f(p_1, p_1) > 0$, since

$$f(p, p) = (p - c)(n_1 - n_2)(\delta q_i/\delta p_j)^*(p, p) > 0 \text{ if } n_1 > n_2.$$

Moreover, there exists $\epsilon > 0$ such that $f(p_1, p_2 - \epsilon) < 0$. This follows from the fact that the reaction function of B_1 is upward sloping and that the profit function of B_2 is single peaked in p_2. Since A is convex, by the mean value theorem there exists a $\tilde{p}_2 < p_2$ such that $f(p_1, \tilde{p}_2) = 0$. Using the single peakedness of B_2's profit function once more, we obtain $h_2^*(p_1, \tilde{p}_2) > 0$. Finally, we also claim that $h_1^*(p_1, \tilde{p}_2) < 0$, which contradicts the fact that $f(p_1, \tilde{p}_2) = 0$. This establishes the proposition that $p_1 \geqslant p_2$. To prove our claim, we note that $\tilde{p}_1$, the optimal reaction of B_1 against $\tilde{p}_2$ (which exists by Assumption A3), is less than p_1, because reaction functions are upward sloping (Assumption A4). Single peakedness of B_1's payoff function then establishes that $h_1^*(p_1, \tilde{p}_2) < h_1^*(\tilde{p}_1, \tilde{p}_2) = 0$. Finally, we note that $p_1 = p_2$ is ruled out if $n_1 > n_2$, since then $f(p, p) > 0$. *Q.E.D.*

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