Bubbles and Banks

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Abstract

We investigate the interaction between bank lending and the formation and bursting of asset price bubbles. In a model of sequential awareness and speculation à la Abreu and Brunnermeier (2003), we introduce a credit market where large creditors (i.e. banks) compete with small investors in financing households' asset purchases. Banks' ability to aggregate signals across borrowers allows them to acquire an informational advantage over their smaller counterparts. This advantage allows the bank to retrench credit on the eve of the crash, giving rise to a rich interaction between the bank's market share and the duration and magnitude of the bubble. When we allow banks to engage in loan securitization, we find that bubbles are prolonged and amplified by the banks' tendency to *evergreen* their loan portfolios.

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1 Introduction

The role of the banking system during boom-and-busts cycles is at the center of increasing attention among scholars and policy makers. Banks have often played a prominent role in recent booms and crises. In the U.S., Spanish, and Irish housing market experiences of the 2000s, and in the Japanese boom and crash of the late 1980s and 1990s, banks aggressively increased the amount of credit extended to households for house purchases, allegedly fuelling prolonged house price booms. In these episodes, the end of the boom and the collapse of house prices initiated a banking crisis and a long and painful recession.

The observed interaction between the dynamics of bank credit and asset price boom-and-busts elicits fundamental questions. Do banks play a special role in driving booms and busts in asset prices or does their behavior merely resemble that of dispersed lenders in the credit market, such as bond-holders and trade creditors? And, therefore, are financial systems characterized by a stronger relative importance of banks ("bank-based" financial systems) more vulnerable to the occurrence of asset price booms and busts than "market-based" financial systems, characterized by a stronger importance of non-intermediated credit by dispersed financiers? Finally, can financial regulation

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contribute to making the banking sector a stabilizer, rather than an amplifier, of boom-and-bust cycles?

In this paper, we address these questions by exploring the role of banks in the build-up and burst of asset price bubbles. Thus, in the paper we explicitly interpret asset price booms and busts as "bubbles". More precisely, we view the boom and bust through the lens of a model of bubble-riding as in Abreu and Brunnermeier (2003) and Doblas-Madrid (2012; DM henceforth), but we posit that investors buy the bubbly asset using funds borrowed in the credit market. As in DM, rational investors are asymmetrically informed about the time when a fundamental boom turns into a bubble. However, if the bubble grows fast enough, they find it optimal to continue bidding up assets they know to be overvalued. Investors thus ride bubbles hoping to make speculative profits and sell to a *greater fool* before the crash. Ex post, investors who observe signals early (i.e., before others) manage to realize profits, at the expense of investors who observe signals late, who get caught in the crash. Ex ante, investors are willing to attempt riding the bubble because they do not know the order in which they will observe signals.

We posit two types of lenders in the credit market, large and small. Large lenders, which we call "banks", can aggregate signals across their many credit market transactions. This conveys them an informational advantage over small (dispersed) lenders, who only serve one investor per period. Large lenders also resemble banks in a second dimension, besides the size of their portfolios: they are required—e.g., by regulatory constraints—to comply with a minimum capital requirement. We do not impose such a requirement on small lenders, whom we interpret as bondholders (or trade creditors) not subject to capital regulation. Our goal is to analyze the interaction between banks' relative importance in the credit market and the size and duration of asset price bubbles.

We construct an equilibrium in which small and large lenders follow the same strategies up to the period before the bubble peaks. In fact, both types of lenders fully serve investors' credit demand up to the limits imposed by capital regulation for banks and up to collateral constraints for small lenders. Crucially, however, large lenders are able to infer the period when the bubble peaks by aggregating information about borrowers' actions. Specifically, early-signal investors reduce their demand for credit right before the peak. Since large lenders observe the actions of a continuum of investors, they infer from this behavior that prices are about to peak and then collapse. Large lenders exploit this information to tailor their credit supply in such a way that they avoid credit losses when the bubble bursts. By contrast, dispersed lenders only serve one borrower per period and are then unable to infer the time of the crash.[1] Dispersed lenders will therefore fail to retrench credit and suffer credit losses. Importantly, given the different credit-supply choices of small and large lenders at the moment of the peak, the asset price at the peak is influenced by the relative importance of large lenders in the credit market. In turn, the peak asset price influences bubble duration through its impact on large lenders' returns during the bubbly period and, hence, through its impact on the dynamic evolution of the credit market share of large lenders. Bank capital is the key state variable driving this evolution. In the bubbly period preceding the peak, large lenders realize profits by charging an interest rate premium over the safe interest rate. This

[1] In the model, the only exception is represented by some lucky small lenders who happen to serve early-signal investors.

premium is charged by small lenders because in any period they fear that they might get caught in the crash. Although large lenders know that they will be able to pull out right before the crash, they can mimic the contract offered by small lenders and pocket the resulting profit without suffering losses at the crash stage. Because of the profits booked during the asset price boom, banks' capital will grow allowing them to take up a growing share of the credit market. Moreover, large lenders' profit margin during the boom is determined by the loss suffered by small lenders at the crash stage, which in turn depends on the length of the bubble. A rich interaction thus arises between large lenders' credit market share, and the duration and size of the bubble.

In an extension of the baseline model, we also consider an alternative version of the economy in which large lenders can securitize a portion of their loans, and sell them to small lenders. In this case, a bank can originate loans right before the peak with the only objective of selling them. In this extended setting, we assume that large lenders have a greater ability to repossess collateral from defaulted loans. This reduces, or even eliminates, their incentive to retrench credit and save their borrowers from the crash. This strategy allows large lenders to retain the size of their loan portfolios, instead of retrenching. This feature of the extended model is reminiscent of the *evergreening* problem emphasized by Caballero et al. (2008) in their analysis of the Japanese experience of the 1990s. A rich interaction arises between banks' incentives to engage in such an evergreening strategy and the size and duration of the bubble.

The remainder of the paper is organized as follows. In Section 2, we describe the model. In Section 3, we define equilibrium and conjecture an equilibrium strategy profile. In Section 4 we analyze the baseline case. In Section 5, we enrich the model to allow for loan securitization. Section 6 concludes. Technical proofs are in the Appendix.

2 The Model

2.1 Environment

Time is discrete and infinite with periods labeled $t = 0, 1, 2, ...$ Each period is subdivided into two subperiods: an asset market and a credit market. We consider a small open economy populated by a unit continuum of investors indexed by $i \in [0, 1]$ and a larger measure of lenders indexed by $j \in [0, J]$. All agents are risk neutral.

There is a final good (the numeraire) and two assets: a risky asset, which exists in unit supply and pays a single dividend of $d_{t_m} = \overline{F} R^{t_m}$ units of final good at the maturity date t_m; and a safe asset (a bond) which yields a gross return of R units of final good every period, with R exogenously determined in international markets. The timeline is split into phases, which refer to the value of the risky asset. There is a boom phase starting at $t = 0$ and ending at the (endogenous) crash time t_c, and a post-crash phase for all $t \geq t_c + 1$. The boom phase may in turn comprise a fundamental stage from $t = 0$ to $t = t_0$ and a bubble from $t_0 + 1$ to t_c.

As of time 0, agents do not precisely know the value of the discounted dividend $\overline{F}$ or the maturity date t_m of the risky asset. The value of $\overline{F}$ will be observed at t_m, but we will focus on situations where $\overline{F}$ becomes known before then. A crucial ingredient in our model is that, due to borrowing constraints, investors do not have enough funds to bid the asset price up to its expected level of $\overline{F}$ at $t = 0$. As credit constraints loosen, agents are able to gradually invest higher amounts in the risky asset, driving a boom in its price. Investors can predict the prices that will be observed as long as the boom lasts, but do not know the value of $\overline{F}$, and thus do not know how long the price boom will be justified by fundamentals. We denote by $t_0 \geq 0$ the number of periods it takes for the price to catch up with the present value of the dividend. Specifically, we define

$$t_0 \equiv \{t \leq t_c | p_t = \overline{F}R^t\}.$$

Following AB, we assume that nature draws t_0 from a geometric distribution with pdf

$$\psi(t_0) = (1-\lambda)\lambda^{t_0}, \quad \text{for } t_0 = 0, 1, ... \tag{1}$$

and $\lambda > 0$. If the price continues to boom past period t_0, a bubble will inflate.[2] Bubbly price gains are unjustified by fundamentals and bound to disappear when p_t crashes between periods t_c and $t_c + 1$.

2.2 Investors and information

Investor i begins period 0 endowed with a portfolio $(Rb_{i,0}, h_{i,0}, R_0^L l_{i,0})$, where $Rb_{i,0}$ denotes his initial holdings of the final good (holdings of the safe asset, $b_{i,0}$, plus interests), $h_{i,0}$ denotes his initial shares of the risky asset, and $R_0^L l_{i,0}$ the initial debt owed to a lender (loan, $l_{i,0}$, plus interests). We assume that all investors start with an endowment of $h_{i,0} = 1$ shares of the risky asset.

As in AB, investors are asymmetrically informed: at time 0, different investors observe different signals about t_0. Specifically, the signal function $\nu : [0,1] \to \{t_0, ..., t_0+N-1\}$ divides investors into N types. The signal $\nu(i)$ reveals to investor i that the boom may only be justified by fundamentals up to period $\nu(i)$, but not longer. Since investors know that there are N types, investor i infers that t_0 cannot be below $\max\{0, \nu(i) - (N-1)\}$ or above $\nu(i)$.[3] The distribution of t_0 conditional on $\nu(i)$ is thus given by[4]

$$\psi(t_0|\nu(i)) = \begin{cases} \frac{\psi(t_0)}{\psi(\max\{0,\nu(i)-(N-1)\})+\cdots+\psi(\nu(i))} & \text{if } \max\{0, \nu(i) - (N-1)\} \leq t_0 \leq \nu(i) \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Private signals order investors along a line from earliest (i.e., most pessimistic about the asset value) to latest (i.e., most optimistic) signal. However, investors do not know their relative order in the line. Importantly, all investors—including those late in the line—assign positive probability to the event that they could be early.

[2] In our discrete-time model there has to be a "coincidence" for any given price to exactly equal $\overline{F}R^t$. A more general assumption would be to define t_0 as the first period with $p_t \geq \overline{F}R^t$. However, this complicates formulae substantially without yielding additional insight.

[3] The $\max\{0, \cdot\}$ operator captures the fact that, in the special case with $t_0 < N-1$, types with $\nu(i) < N-1$ know that they cannot be last in line, since $\nu(i) - (N-1) < 0$. For those types, the support of t_0 conditional on $\nu(i)$ is $\{0, ..., \nu(i)\}$ instead of $\{\nu(i) - (N-1), ..., \nu(i)\}$.

[4] Private signals about t_0 are equivalent to private signals about $\overline{F}$. In fact, signal $\nu(i)$ reveals that $\overline{F} \in \{p_{t_0}/R^{t_0} | \max\{0, \nu(i) - (N-1)\} \leq t_0 \leq \nu(i)\}$ with the probabilities of each possible value of $\overline{F}$ given by (2). This specification of signals is simply a discretized version of that in AB. Moreover, it resembles the timing devised by Moinas and Pouget (2012), who conduct experiments in which participants are uncertain about the order in which they move relative to each other.

2.3 Lenders

There is a large lender ($j = 0$) and a continuum of small lenders ($j \in (0, J]$). All lenders can borrow in the international market at the gross rate R. In turn, each small lender can finance one investor per period, while the large lender can finance a measure of investors. However, given that the large lender is a bank, its lending is subject to a regulatory capital requirement

$$D_t^0 \leq \psi L_t^0, \tag{3}$$

where L_t^0 and D_t^0 denote the lender 0's loans and liabilities, respectively. The parameter $\psi \in (0, 1)$ captures the regulatory risk weight assigned to loans.[5] Small lenders (e.g., proxying for bondholders) are not subject to capital requirements.

Each lender must satisfy the budget constraint

$$RD_t^j + L_{t+1}^j = D_{t+1}^j + \widetilde{\omega}_t^j R_t^L L_t^j. \tag{4}$$

which states that the repayment of liabilities (RD_t^j) and the extension of new loans (L_{t+1}^j) must be matched by the repayment of loans ($\widetilde{\omega}_t^j R_t^L L_t^j$) and by the new liabilities issued (D_{t+1}^j). In (4), $\widetilde{\omega}_t^j$ denotes the share of loans that are repaid.

2.4 Asset market

In period t, investors first trade in the asset market and then visit the credit market. Investor i enters the asset market with $Rb_{i,t} \geq 0$ units of final good, $h_{i,t} \geq 0$ units of the risky asset, and an outstanding loan of $R_t^L l_{i,t} \geq 0$ units owed to a lender, where R_t^L is the interest rate set when the loan was originated in the period $t-1$ credit market. We denote by $I_{i,t}$ the investor's information when she enters the asset market.

The asset market is modeled as a Shapley-Shubik trading post, where investors trade the risky asset for the final good. Investor i offers $s_{i,t} \in [0, h_{i,t}]$ risky shares for sale, and bids $m_{i,t} \in [0, Rb_{i,t}]$ units of the final good to buy risky shares.[6] We also allow for an exogenous net supply of shares of the risky asset $\theta(t) = \theta(1+\theta)^t$ coming from foreign investors in each period. We do not explicitly model this exogenous component of the supply: it only serves the technical purpose of guaranteeing a well defined price in the trading post even when all investors in our economy want to invest in the risk asset. The exogenous supply $\theta(t)$ could stem from portfolio rebalancing or liquidity needs of foreign investors.

In the trading post, agents enter orders to sell shares and submit bids. Thereafter, bids and offers are combined, and the price emerges as

$$p_t = \frac{M_t}{S_t + \theta(t)},$$

[5] Since bank capital (net worth) is about $K = L - D$, replacing into the constraint, we obtain

$$L_t (1 - \psi_l) \leq K_t.$$

Thus, ψ_l can be interpreted as the complement to 1 of the capital-to-loan regulatory ratio.

[6] Since $s_{i,t}$ cannot be negative, the risky asset cannot be shorted.

where M_t and S_t are the aggregates corresponding to $m_{i,t}$ and $s_{i,t}$, respectively. Trades are executed and agent i leaves the asset market with

$$\widetilde{h}_{i,t} = h_{i,t} + \frac{m_{i,t}}{p_t} - s_{i,t} \tag{5}$$

risky shares, that is, the initial holdings $h_{i,t}$ plus newly acquired shares $m_{i,t}/p_t$ minus sold shares $s_{i,t}$. After trading, final good (safe asset) holdings are given by

$$\widetilde{b}_{i,t} = Rb_{i,t} - m_{i,t} + p_t s_{i,t}, \tag{6}$$

where $Rb_{i,t}$ are the holdings entering the asset market, $m_{i,t}$ is the bid, and $p_t s_{i,t}$ is the revenue from selling $s_{i,t}$ shares.

2.5 Credit market

After trading in the asset market, investor i enters the credit market with a portfolio $(\widetilde{b}_{i,t}, \widetilde{h}_{i,t}, R_t^L l_{i,t})$. The investor repays or defaults on her outstanding loan from the previous period credit market $(R_t^L l_{i,t})$, takes a new loan $(l_{i,t+1})$, and consumes $(c_{i,t})$. As we detail below, the new loan $l_{i,t+1}$ is backed by the assets $h_{i,t+1}$ held by the investor at the end of the credit market.

Let $\omega_{i,t}$ denote the fraction of the outstanding loan $R_t^L l_{i,t}$ that the investor can repay without surrendering risky assets to the lender. If $\omega_{i,t} < 1$, the investor surrenders her risky assets; in turn, the lender can repossess the risky assets $(h_{i,t})$ backing the loan $R_t^L l_{i,t}$ up to the value of the unfulfilled repayment. We denote by $\widetilde{\omega}_{i,t}$ the fraction of the loan repaid after asset repossession. Investor i's ability to repay the loan without surrendering assets is governed by her budget constraint

$$c_{i,t} + b_{i,t+1} + \omega_{i,t} R_t^L l_{i,t} = \widetilde{b}_{i,t} + l_{i,t+1}. \tag{7}$$

Under this constraint, consumption $c_{i,t}$, next period's safe balances $b_{i,t+1} \geq 0$ and the repayment $\omega_{i,t} R_t^L l_{i,t}$ must be financed by safe balances carried over from the asset market $\widetilde{b}_{i,t}$, and by the new loan $l_{i,t+1}$.

Let us next consider the new loan $l_{i,t+1}$ that the investor can obtain.[7] As noted, due to the risk of voluntary default, this is limited. Specifically, an investor who can back the new loan with $h_{i,t+1}$ risky shares faces the following borrowing constraint

$$l_{i,t+1} \leq \min\{\overline{L}_t, \phi_t E[p_{t+1}|\widetilde{\mathcal{I}}_t^j] h_{i,t+1}\}, \tag{8}$$

where $\widetilde{\mathcal{I}}_t^j$ is the lender's information when the credit market opens. We treat $\overline{L}_t$ as an exogenous borrowing limit, capturing exogenous factors driving the investor's debt capacity (income limits, regulatory constraints, etc.) and let $\overline{L}_t$ grow at the rate $\gamma_l > 0$. We will treat $\overline{L}_t$ as the binding constraint during the boom, verifying that in equilibrium $\overline{L}_t < \phi_t E[p_{t+1}|\widetilde{\mathcal{I}}_t^j] h_{i,t+1}$ for all $t < t_c$. We will focus on the case in which, at the time of the crash, the drop in the asset price is large enough to reverse this inequality. Thus, from period t_c onward, $\phi_t E[p_{t+1}|\widetilde{\mathcal{I}}_t^j] h_{i,t+1} < \overline{L}_t$. This simplifies the interactions between borrowing constraints and asset prices.

[7] We assume that voluntary default can happen immediately after a loan is taken, but not at the repayment stage. At the repayment stage, the case of $\omega_{i,t} < 1$ reflects unvoluntary default due to inability to repay.

If the investor defaults, the amount of risky shares repossessed by the lender satisfies

$$h_{i,t}^{Seized} = \max\left\{\frac{(1-\omega_{i,t})R_t^L l_{i,t}}{p_{t+1}}, h_{i,t}\right\}. \tag{9}$$

and, hence, the investor exits the credit market with an amount of risky shares given by

$$\begin{cases} h_{i,t+1} = \widetilde{h}_{i,t} & if\ \omega_{i,t} = 1 \text{ (no default)} \\ h_{i,t+1} = h_{i,t} - h_{i,t}^{Seized} & if\ \omega_{i,t} < 1 \text{ (default)} \end{cases} \tag{10}$$

We can then distinguish the following cases:

1. Solvency. The outstanding loan is fully repaid ($\widetilde{\omega}_{i,t} = 1$). This happens if the sum of the investor's holdings of the safe asset $\widetilde{b}_{i,t}$ and new borrowing $l_{i,t+1}$ suffice to fully repay the loan,

   $$R_t^L l_{i,t} \leq \widetilde{b}_{i,t} + \min\{\overline{L}_t, \phi_t E[p_{t+1}|\widetilde{\mathcal{I}}_t^j]h_{i,t+1}\}.$$

   It also happens if the sum of the safe asset holdings and new borrowing fall short of the outstanding loan, but the risky assets are enough to cover the shortfall, that is

   $$h_{i,t} \geq \frac{(1-\omega_{i,t})R_t^L l_{i,t}}{p_t}.$$

2. Insolvency. The lender seizes all the assets backing the loan, but this is not enough to cover the repayment due. Formally,

   $$h_{i,t} < \frac{(1-\omega_{i,t})R_t^L l_{i,t}}{p_t}.$$

   We can then solve for $\widetilde{\omega}_{i,t}$ as follows

   $$\widetilde{\omega}_{i,t} = \frac{\widetilde{b}_{i,t} + p_{t+1}h_{i,t}}{R_t^L l_{i,t}}. \tag{11}$$

2.6 Credit contracts

Lenders post a schedule $R_{t+1}^L(l_{i,t+1})$ associating an interest rate to each loan size. Because there are more small lenders than investors, the interest rate is competed down to the break-even point of small lenders. In turn, the bank has the incentive to charge the same interest rate as the small lenders. In fact, the size of its loan portfolio is independent of the loan rate. Lenders also choose the LTV ratio ϕ_t. We focus on the scenario in which during the boom lenders choose $\phi_t > \overline{L}_t/p_{t+1}$, so that the exogenous borrowing constraint binds, while at the time of the crash they set $\phi_t < \overline{L}_t/p_{t+1}$.

Consider a lender granting a loan $l_{i,t+1}$ to investor i at time $t < t_m - 1$. The lender will collect a repayment of $\widetilde{\omega}_{i,t+1}R_{t+1}^L l_{i,t+1}$ in the credit market at $t+1$. Given information $\widetilde{\mathcal{I}}_t^j$, the interest rate R_{t+1}^L is given by

$$R_{t+1}^L = \frac{R}{E[\widetilde{\omega}_{i,t+1}]} = \frac{R}{E\left[\min\left\{1, \frac{\widetilde{b}_{i,t+1}+p_{t+2}h_{i,t+1}}{R_{t+1}^L l_{i,t+1}}\right\}\right]} \tag{12}$$

Borrowers flock to the bank in a queue. The bank announces an interest rate schedule $R^L_{t+1}(l_{i,t+1})$ but it can update the schedule after observing loan applications. Effectively, similar to Chari and Jaganatthan (1988) for example, the bank can be thought of as a large window where borrowers can observe each others' behavior, whereas small lenders can be thought of as islands, as information is concerned.[8]

3 Model Solution

In this section, we study the solution of the model. We first illustrate the equilibrium concept and then we investigate the dynamics over boom and crash periods.

3.1 Equilibrium concept

The equilibrium concept is Perfect Bayesian Equilibrium (PBE), consisting of strategies $\{a_{i,t}, \widetilde{a}_{i,t}\}$, where $a_{i,t} = (m_{i,t}, s_{i,t})$ and $\widetilde{a}_{i,t} = (c_{i,t}, l_{i,t+1}, b_{i,t+1})$, and beliefs $\{\mu_{i,t}, \widetilde{\mu}_{i,t}\}$ for all investors, lender beliefs $\mathcal{I}^j_t$, for all lenders, and equilibrium prices R^L_t and p_t.

In the asset market, investor i chooses trades $a_{i,t}$ to maximize her expected utility, subject to $m_{i,t} \in [0, Rb_{i,t}]$, $s_{i,t} \in [0, h_{i,t}]$, (5), and (6). In the credit market, she chooses $\widetilde{a}_{i,t} = (c_{i,t}, l_{i,t+1}, b_{i,t+1})$, subject to (9), (7), (10), and (8). Beliefs $\{\mu_{i,t}, \widetilde{\mu}_{i,t}\}$ must be consistent with equilibrium strategies and observables.

Small lenders choose the interest rate schedule R^L_{t+1} in order to maximize the stream of expected profits

$$\sum_{s=t}^{\infty} \frac{1}{R^{s-t}} E[\left(\widetilde{\omega}_{i,s} R^L_s - 1\right) l_{i,s}] \tag{13}$$

given their information $\widetilde{\mathcal{I}}^j_t$ ($j \in (0, J]$). Similarly, the bank ($j = 0$) maximizes its expected profits

$$\sum_{s=t}^{\infty} \int_{i \in [0,\xi_s]} \frac{1}{R^{s-t}} E[\left(\widetilde{\omega}_{i,s} R^L_s - 1\right) l_{i,s}] di \tag{14}$$

given its information $\widetilde{\mathcal{I}}^0_t$. Asset and credit markets clear every period.

We use a guess-and-verify procedure to find equilibria. Our guess is that, given the speed at which the bubble grows and the probability of a crash, investors will ride and fuel bubbles as long as prospective speculative gains in the event of being an early-signal investor outweigh the risk of losses in the event of being a late-signal investor. More precisely, our conjectured strategies are as follows:

> Investors plan to ride the bubble for τ^* periods. That is, investor i plans to—unless the bubble bursts before then—borrow and invest into the risky asset as much as she can while $t < \nu(i) + \tau^*$. After the bubble has been pricked at time t_c, the fundamental value of the risky asset becomes known and investors set $m_{i,t}$ at whatever level necessary to equate p_t to fundamental value.

[8] In Chari and Jaganatthan (1988) and a broad strand of banking literature, bank clients can observe the length of queues and amounts of withdrawals.

If all agents follow the conjectured strategies, the price booms until period $t_c = t_0 + \tau^*$, at which point it crashes.

REMARK 4: DOESNT THE PRICE CRASH AT $t_c + 1$? AGAIN I AM A BIT CONFUSED BY CALLING CRASH TIME THE PEAK...

For $t < t_c$, all types of investors are fully invested in the bubble. The boom concludes at $t_c = t_0 + \tau^*$ when investors of type $\nu(i) = t_0$ sell, exiting the market. Their sales affect the asset price, revealing to others that $t = t_c$ and $t_0 = t_c - \tau^*$.[9] At this point, some investors default on their loans (while no investor defaults during the boom or in the post-crash phase). The uncertainty about t_0 and the dividend d_{t_m} is fully resolved when the price is observed at time t_c. Since the dividend is paid at date t_m when the asset market opens, from the crash until the maturity date, i.e., for $t \in \{t_c + 1, ..., t_m - 1\}$, the price of the risky asset is given by $p_t = d_{t_m}/R^{t_m - t}$, $l_{i,t+1} = p_{t+1} h_{i,t+1}$, the loan rate is R and all lenders earn zero profits.

REMARK 5: SHOULDNT WE WRITE ...for $t \in \{t_{c+1}, ..., t_m - 1\}$?

Afterwards, the price falls to zero.

3.2 Boom dynamics

During boom periods $t = 0, ..., t_c - 1$, all investors buy. Therefore, in period t the supply of the risky asset is $\theta(t)$. With investors at the maximum long position, $M_t = RB_t$ and $B_t = \overline{L}_0(1+\gamma_l)^t - R^L_{t-1}\overline{L}_0(1+\gamma_l)^{t-1}$. Therefore, the asset price is

$$p_t = \frac{M_t}{S_t + \theta(t)} = \frac{R\left[\overline{L}_0(1+\gamma_l)^t - R^L_{t-1}\overline{L}_0(1+\gamma_l)^{t-1}\right]}{\theta(1+\theta)^t}.$$

Dividing through by p_{t-1} yields

$$\frac{p_t}{p_{t-1}} = \frac{\left[R\left[\overline{L}_0(1+\gamma_l)^t - R^L_{t-1}\overline{L}_0(1+\gamma_l)^{t-1}\right]\right]}{(1+\theta)\left[R\left[\overline{L}_0(1+\gamma_l)^{t-1} - R^L_{t-2}\overline{L}_0(1+\gamma_l)^{t-2}\right]\right]}.$$

For any boom period $t \neq \tau^*$, $R^L_{t-1} = R^L_{t-2}$, and $p_t/p_{t-1} = (1+\gamma_l)/(1+\theta)$. At $t = \tau^*$, instead, p_t/p_{t-1} is a bit lower, since $R^L_{t-1} > R^L_{t-2}$.

To compute the interest rate, let us recall expression (12). Since $t_c = t_0 + \tau^*$, and $t_0 \geq 0$, it follows that, for $t < \tau^*$, the loan rate is R. Once $t \geq \tau^*$,

$$R^L_t = \frac{R - \pi \frac{p_{t_{c+1}} h_{i,t_c}}{l_{i,t_c}}}{1-\pi}, \tag{15}$$

where π is the probability that a small lender makes a loss, i.e., the probability that its borrower does not repay the outstanding loan in full. In turn, this probability is given by

$$\pi = \begin{cases} 0 \text{ if } t < \tau^* \\ \frac{N-1}{N}(1-\lambda) \text{ if } \tau^* \leq t < t_c. \end{cases} \tag{16}$$

[9] The trading protocol is such that agents submit orders first, and observe the price second. Thus, unlike in a Walrasian setting, buyers get stuck with the bubble at t_c because they cannot change their orders after observing the price.

Comparing (15) with (12), the reader can note that we have set $\widetilde{b}_{i,t_c}$ to zero. This is because in equilibrium, in the crash period all agents caught in the crash are fully invested in the risky asset, so their holdings $\widetilde{b}_{i,t_c}$ of the safe asset when exiting the asset market are equal to zero.

3.3 Bank behavior during the boom

When an investor demands less credit than she can obtain (a small loan just sufficient to repay the outstanding debt, $l_{i,t+1} = R_t^L \overline{L}_t$, instead of the maximum loan possible $\overline{L}_{t+1}$), this reveals to her lender that the next period will be the crash period, and that investor i is a low-signal agent who will sell all her shares $s_{i,t+1} = h_{i,t+1}$ at the peak price $p_{t+1} = p_{t_c}$. In this case, the lender's expected profit (per customer) is

$$E\left[\pi_{t+1} | \widetilde{\mathcal{I}}_t^j, l_{i,t+1} = R_t^L \overline{L}_t)\right] = \left[R_{t+1}^L (R_t^L \overline{L}_t) - R\right] l_{i,t+1} > 0. \tag{17}$$

After observing a mass of borrowers' loan applications, the bank knows with certainty whether next period will be the crash period or not. If some borrower demands to borrow less than $\overline{L}_{t+1}$, the bank updates the offered schedule, thus communicating to its borrowers that the next period is the crash. The bank's borrowers will then borrow the smallest loan necessary to repay their outstanding loans. In this case, the bank's expected profit is given by (17). Instead, when all borrowers demand the maximum possible loan, the bank knows that next period is not the crash and its expected profit per customer equals (17) substituting $R_{t+1}^L (R_t^L \overline{L}_t)$ with $R_{t+1}^L (\overline{L}_{t+1})$.

Notice that small lenders earn small positive profits every period from period τ^* until period $t_c - 1$ and large losses at time t_c. The large lender posts the same schedule as small lenders and (correctly) expects to extract a surplus as long as $t < t_c - 1$. At time $t_c - 1$, the large lender reduces the size of each loan in order to avoid credit losses, exploiting its informational advantage.

REMARK 5: SHOULDNT WE WRITE ...for $t \in \{t_{c+1}, ..., t_m - 1\}$?

3.4 Bank's market share

We now derive the law of motion of the market share of the bank. Recall that the capital constraint of the bank reads

$$D_t^0 \leq \psi L_t^0. \tag{18}$$

Moreover, the bank must satisfy the budget constraint

$$RD_t^0 + L_{t+1}^0 = D_{t+1}^0 + R_t^L L_t^0. \tag{19}$$

After substituting the former (taken with the equality sign) into the latter, we can solve for the growth rate of the bank loans as

$$\frac{L_{t+1}^0}{L_t^0} = \frac{R_t^L - R\psi}{1 - \psi}.$$

The market share of the bank ξ_t is given by the ratio between the bank's loans L_t^0 and the aggregate borrowing $\overline{L}_t$ (i.e., $\xi_t = L_t^0 / \overline{L}_t$). Thus, the gross growth rate of ξ_t is

$$\frac{\xi_t}{\xi_{t-1}} = \frac{R_t^L - R\psi}{(1 - \psi)(1 + \gamma_l)},$$

which implies that $\xi_t > \xi_{t-1}$ if

$$R_t^L > R\psi + (1-\psi)(1+\gamma_l).$$

Intuitively, the bank's market share grows if the bank's profits are large enough, i.e., if R_t^L is sufficiently larger than R, if the capital requirement is not large (i.e., $1-\psi$ is low) and if the price growth rate is not high. In turn, however, the interest rate charged by the bank depends on the magnitude of losses at the time of the crash, which is itself a function of the price growth rate γ_l. Thus, the price growth rate has two opposite effects on the bank's market share.[10] We will return to this point below.

3.5 Peak and crash

At time t_c, the mass of risky shares for sale increases from $\theta(t)$ to $\theta(t) + \xi_{t_c} + (1-\xi_{t_c})/N$ and the mass of buyers falls to $(1-1/N)(1-\xi_{t_c})$. In fact, all the borrowers of the bank as well as the borrowers of small lenders who anticipate the crash sell their shares of the risky asset. Therefore, instead of growing at the rate $1+\gamma_l$, at t_c the price of the risky asset grows at the rate

$$\begin{aligned} \frac{p_{t_c}}{p_{t_c-1}} &= \frac{\theta(1-1/N)(1-\xi_{t_c})}{\theta + \xi_{t_c} + (1-\xi_{t_c})/N} \frac{1+\gamma_l}{1+\theta} \\ &= K(\theta, N, \xi_{t_c})(1+\gamma_l), \end{aligned}$$

where the term $K(\theta, N, \xi_{t_c}) \equiv K < 1$ captures a kink in the price path at time t_c. The exponential growth from the boom slows down at time t_c due to the selling pressure of the investors who exit the market in (correct) anticipation of the crash. Since the bank rescues its clients by pulling out credit when it observes the retrenchment of some borrowers, the magnitude of the kink is positively related to the credit market share of the bank. As $1/N$ and ξ_{t_c} approach 0, $K(\theta, N, \xi_{t_c})$ approaches one. That is, when the market share of the bank is smaller, the kink is less pronounced and p_{t_c} is closer to $p_{t_c-1}(1+\gamma_l)$.

Bubble duration τ^* is pinned down by the willingness of investors to ride the bubble, which means buying risky assets using maximum leverage until they sell at time $\nu(i)+\tau^*$. From the point of view of investor i, the probability of a crash is zero while $t < \nu(i) - (N-1) + \tau^*$. As the investor's selling date nears, i.e. between $\nu(i) - (N-1) + \tau^*$ and $\nu(i) - 1 + \tau^*$, the investor assigns positive (and growing) probability to the event that the bubble is pricked in the asset market. The crash risk is highest at $t = \nu(i) - 1 + \tau^*$, since by this point all values of t_0 except for $\nu(i)-1$ and $\nu(i)$ have been ruled out. Given the prior distribution of t_0, the probabilities of these two values are, respectively, $1/(1+\lambda)$ and $\lambda/(1+\lambda)$.

Let us first consider the case in which investor i chooses to preemptively exit the market at time t. She will make this decision in the credit market in $t-1$, where she will borrow just enough to roll over the loan taken at $t-2$, i.e., $l_{i,t} = R^L l_{i,t-1}$. The previous loan $l_{i,t-1}$ was given by the

[10] The bank could conceivably influence bubble duration via its lending choice. If the bank lends below the amount allowed by the capital constraint, it gives up profitable loans. On the other hand, it could have some gain by inducing bigger expected losses, because its size will grow more slowly. We will henceforth assume (and later verify) that the latter effect is always dominated by the former.

maximum allowed by the borrowing constraint, i.e., $l_{i,t-1} = \overline{L}_{t-2}$ at the interest rate R^L. The loan $l_{i,t} = R^L \overline{L}_{t-2}$ taken at $t-1$ is smaller than the maximum allowed $\overline{L}_{t-1}$ **because** $R^L < 1+\gamma_l$. Consequently, the interest rate $\underline{R}^L$ for loan $l_{i,t}$ will also be lower than if the borrowing constraint at $t-1$ was binding, that is, $\underline{R}^L < R^L$. If investor i sells at time t, after rolling over her debt at $t-1$, her expected payoff will be as follows. If t_0 equals $\nu(i)-1$, there will be one type, as well as the bank customers, leaving the market at $t-1$ and the price will show a kink K. If, on the other hand, t_0 equals $\nu(i)$, agent i will be selling too early, and the price will exhibit no kink. Regardless of whether there is a kink or not, investor i will fully repay $\underline{R}^L R^L \overline{L}_{t-2}$.

Let us next consider the case in which investor i chooses instead to follow equilibrium strategies by continuing to lever up at $t-1$ in order to buy assets at t. She will then take the maximum loan at $t-1$, $l_{i,t} = \overline{L}_{t-1}$, at the interest rate R^L to buy assets in time t's asset market. Whether this results in profits or losses depends on t_0. If t_0 equals $\nu(i)-1$, there will be one type, as well as the bank-customers, leaving the market at t and the price will show a kink K. In this case, all uncertainty will be revealed, and all agents will know that the bubble has been pricked. Therefore, in the asset market, the collateral value of the risky shares will be given by the post-crash price, and those agents who are still in the market will default.In this case, the payoff of investor i will be given by $\max\left\{0, \left(\frac{1+\gamma_l}{R}\right)^{\nu(i)-1}\left[h_{i,t-1} + \frac{\overline{L}_{t-1} - R^L \overline{L}_{t-2}}{K(1+\gamma_l)^t}\right] - R^L \overline{L}_{t-1}\right\}$, where the max{.} operator captures the fact that some losses are shifted onto the lender. If t_0 is $\nu(i)$ the investor will benefit from one more period of appreciation and repay her debt in full.

The choice to sell preemptively or to continue to ride the bubble is made in the credit market at time $t-1 = \nu(i) - 2 + \tau^*$. The investor finds it optimal to buy if

$$\begin{aligned} & E[p_t]h_{i,t-1} - \underline{R}^L R^L \overline{L}_{t-2} \\ \leq \quad & \frac{1}{1+\lambda}\max\left\{0, F\left(\frac{1+\gamma_l}{R}\right)^{\nu(i)-1}\left[h_{i,t-1} + \frac{\overline{L}_{t-1} - R^L \overline{L}_{t-2}}{(1+\gamma_l)^t}\right] - R^L \overline{L}_{t-1}\right\} \\ & + \frac{\lambda}{1+\lambda}\left(\frac{[p_{t+1}|t_0 = \nu(i)]}{R}\left[h_{i,t-1} + \frac{\overline{L}_{t-1} - R^L \overline{L}_{t-2}}{(1+\gamma_l)^{\nu(i)+\tau^*}}\right] - \frac{\underline{R}^L}{R} R^L \overline{L}_{t-1}\right). \end{aligned} \tag{20}$$

In equilibrium, losing investors shift risk to lenders, and thus the max term equals zero. Recall also that $\overline{R}^L$ is in turn an increasing function of τ^*,

$$R^L = \frac{R - \pi \frac{p_{t_c+1} h_{i,t_c}}{l_{i,t_c}}}{1-\pi} = \frac{R - \pi\Lambda\left(\frac{1+\gamma_l}{R(1+\theta)}\right)^{-\tau^*}}{1-\pi},$$

where Λ is a constant. Henceforth, for expositional simplicity, we redefine $\Omega(.) = \Lambda\left(\frac{1+\gamma_l}{R(1+\theta)}\right)^{-\tau^*}$. Note that $\Omega(.)$ takes values in the $[0, R]$ interval, being monotonically increasing in $\left(\frac{1+\gamma_l}{R(1+\theta)}\right)^{-\tau^*}$ which reflects the drop in the risky asset's price in at the time of the crash.

4 Equilibrium and Experiments

In this section, we summarize the equilibrium system and then conduct comparative statics exercises. Our focus is on the impact that shocks to the size of the banking system can have on the duration and size of bubbles.

4.1 Equilibrium system

After some algebra (see the Supplement for details), the inequality in (20) for the bubble duration simplifies to

$$K\left[(1+\gamma_l)-R^L\right][\lambda(1+\gamma_l)-R] \geq \lambda R(1+\gamma_l)-R^L\left[\lambda R+\theta \underline{R}^L\left(\frac{1+\lambda}{1+\gamma_l}-\frac{\lambda}{R}\right)\right] \tag{21}$$

The system that characterizes the solution consists of (21) together with the price growth rate in the period of crash

$$\frac{p_{t_c+1}}{p_{t_c}}=\frac{1}{K}\left(\frac{1+\gamma_l}{R}\right)^{-\tau^*}, \tag{22}$$

the expression for the kink

$$K=\frac{\theta(1-1/N)(1-\xi_{t_c})}{\theta+\xi_{t_c}+(1-\xi_{t_c})/N}, \tag{23}$$

the growth rate of the bank's market share

$$\frac{\xi_t}{\xi_{t-1}}=\frac{R_t^L-R\psi}{(1-\psi)\,(1+\gamma_l)}, \tag{24}$$

the expression for the loan rate

$$R_t^L=\frac{R-\pi\Omega(.)}{1-\pi}, \tag{25}$$

and that for the probability of (partial) default on loans

$$\pi=\begin{cases} 0 \text{ if } t<\tau^* \\ (1-\lambda)\frac{N-1}{N} \text{ if } t \geq \tau^*. \end{cases} \tag{26}$$

The bank's market share at the moment of the crash plays a crucial role in our equilibrium system. The larger this share, the more pronounced the deceleration in the price growth rate (the kink) at $t_c$and, hence, the harder it is to satisfy the bubble duration condition in (21).

Substituting the loan rate into the growth rate of the bank's market share, we obtain

$$\frac{\xi_t}{\xi_{t-1}}=g_\xi=\begin{cases} \frac{R}{1+\gamma_l} \text{ if } t<\tau^*\,(1+\gamma_l) \\ \frac{\frac{R-\pi\Omega(.)}{1-\pi}-R\psi}{(1-\psi)(1+\gamma_l)} \text{ if } t \geq \tau^*. \end{cases} \tag{27}$$

From here, we can rewrite the crash-time (t_c) market share of the bank as

$$\xi_{t_c}=\xi_0\underbrace{\left(\frac{R}{1+\gamma_l}\right)^{\tau^*-1}}_{\text{Zero-crash-risk phase}}\underbrace{\left(\frac{R[1-(1-\pi)\psi]-\pi\Omega(.)}{(1-\pi)\,(1-\psi)\,(1+\gamma_l)}\right)^{t_0}}_{\text{Crash-risk phase}}. \tag{28}$$

There are τ^* periods—where τ^* is the duration of the bubble—with zero crash risk, during which the bank's market share shrinks because the bank's loan portfolio grows at the rate $R-1$ in a credit market growing at the rate γ_l (to fix ideas, we label this "extensive margin" effect). There are also t_0 periods during which small lenders fear a crash and the bank makes profits. The length of this phase, t_0, is independent of the length of the bubble. However, as (28) shows, a longer bubble duration τ^* increases the size of the loss expected by small lenders, allowing the bank to charge a higher loan rate during the crash-risk phase and, hence, realize larger profits (we label this "intensive margin" effect). Whether during the crash-risk phase the bank's market share expands or shrinks depends on whether the following condition holds,

$$R[1-(1-\pi)\psi]-\pi\Omega(.) > (1-\pi)(1-\psi)(1+\gamma_l).$$

If the condition holds the profits realized by the bank are large enough that the bank's market share expands. Otherwise, the bank's market share drops during the crash-risk phase, too. We call the equation in (28) that relates the market share ξ_{t_c} of the bank to the bubble duration τ^* the bank share, BS, locus. It is easy to see that in the (ξ_{t_c}, τ^*) space the BS locus has horizontal intercept given by $\xi_{t_c} = \xi_0 \frac{1+\gamma_l}{R} \left\{ \frac{R[1-(1-\pi)\psi]}{(1-\psi)(1-\pi)(1+\gamma_l)} \right\}^{t_0}$ and a vertical asymptote given by the vertical (τ^*) axis. The slope of the BS locus is governed by the extensive and intensive margin effects: the extensive margin tends to make the locus downward sloping, the intensive margin tends to make it upward sloping.

After replacing for the loan rate R_t^L and the kink K into (21) taken with the equality sign, we also obtain a second locus that pins down the maximum size of the bank consistent with investors' incentives to ride bubbles (i.e., with the bubble riding condition). We call this second locus bubble duration, BD, locus. The bubble duration τ^* can affect the BD locus through its effect on the interest rate R^L: in particular, ex ante a higher interest rate R^L has an ambiguous effect on investors' incentive to ride bubbles.[11] Precisely, the BD locus is upward sloping if $\lambda/(1+\lambda) > (1+\gamma_l)/R$, **CHECK THIS** downward sloping otherwise. For high values of τ^*, since the effect of τ^* on the interest rate fades, the BD locus tends to becomes a vertical line in the (ξ_{t_c}, τ^*) space. Precisely, it has a vertical asymptote for a positive value of the bank's market share (see the Supplement).

Figures 2 and 3 plots the BD and the BS locus. Possible bubble durations and banks' market shares lie on the sections of the BS locus to the left of the BD locus. In what follows, we conduct experiments first studying scenarios in which the extensive margin effect is the only force at work. We then consider cases in which the intensive margin exerts an influence.

4.2 Extensive margin effects

We start with considering scenarios in which the extensive margin effect is the only force at work. Specifically, the BS locus crosses the BD locus only once, in its downward sloping part (see Figure 2 for an illustration). To understand such scenarios it is useful to work with an approximation of the equilibrium system. In particular, we consider the case in which the two loci cross for a bubble

[11] Ex ante, a higher interest rate R^L has an ambiguous effect on investors' incentive to ride bubbles. The interest rate is fully repaid only in case of preemptive exit, while if an investor is caught in a crash he does not repay his lender. Through this "risk-shifting" channel a higher interest rate encourages investors to ride bubbles. However, staying in the market implies getting bigger loans. Through this channel a higher interest rate discourages from riding bubbles.

duration long enough that the fraction of the price that vanishes in the crash is close to one. In this scenario, the term $\Omega(.)$ is close to zero so that the equilibrium system can be approximated by

$$K\left[(1+\gamma_l)-R^L\right]\left[\lambda(1+\gamma_l)-R\right]\geq\lambda R(1+\gamma_l)-R^L\left[\lambda R+\theta\underline{R}^L\left(\frac{1+\lambda}{1+\gamma_l}-\frac{\lambda}{R}\right)\right].$$

$$\xi_{t_c}=\xi_0\left(\frac{R}{1+\gamma_l}\right)^{\tau^*-1}\Psi^{t_0}, \tag{29}$$

where

$$R_t^L = \frac{R}{1-\pi}, \tag{30}$$

$$\Psi = \frac{R[1-(1-\pi)\psi]}{(1-\psi)(1-\pi)(1+\gamma_l)} \tag{31}$$

$$\pi = (1-\lambda)\frac{N-1}{N}, \tag{32}$$

with K satisfying (23). Let us start from studying the approximated BS locus

$$\xi_{t_c}=\xi_0\left(\frac{R}{1+\gamma_l}\right)^{\tau^*-1}\Psi^{t_0}. \tag{BD}$$

Taking logarithms

$$\frac{\ln\xi_{t_c}-\ln\Psi^{t_0}-\ln\xi_0}{\ln R-\ln(1+\gamma_l)}+1=\tau^*,$$

so the slope of the BS locus is $\frac{\ln\xi_{t_c}}{\ln R-\ln(1+\gamma_l)}$. Thus, the BS locus is downward sloping in the (ξ_{t_c},τ^*) space. This reflects the first of the two mechanisms described above, the extensive margin effect: a longer bubble duration implies a longer phase in which no crash is expected and, hence, the bank's market share drops.[12] By contrast, the intensive margin effect is muted in this scenario. Turning next to the BD locus, after substituting for R_t^L and K, one can notice that for the BD condition to hold the market share of the bank must not exceed an upper threshold. Thus, the BD locus is a vertical straight line in the (ξ_{t_c},τ^*) space, which has horizontal intercept that solves (21) with the equality sign. Overall, together the BS and the BD define a maximum market share of the banking sector consistent with the formation of bubbles and a minimum bubble duration, pinned down by the intersection of the two loci. If bubbles lasted less than this minimum duration, banks' market share would expand too much for bubbles to arise.

Next, we can perform comparative statics exercises to examine how shocks that alter the slope and position of the two loci modify the minimum duration of bubbles and the maximum size of the banking sector consistent with bubble formation. Our main interest is in the effects of shocks that influences the relative presence of banks in the economy, such as a shock to bank capital regulation. Let us consider the impact of a regulatory increase in ψ (i.e., a drop in the bank capital requirement). This shock will increase the Ψ term, shifting the BS locus upward. By contrast, it will not alter the position of the BD locus. The shock, therefore, will leave the maximum market

[12] We are instead missing the intensive margin component in the fear period (during which the gradient of growth of the bank's share positively depends on the bubble duration through the interest rate). Clearly, another way to silent the intensive effect is to assume that t_0 takes very low (in the limit 0) values.

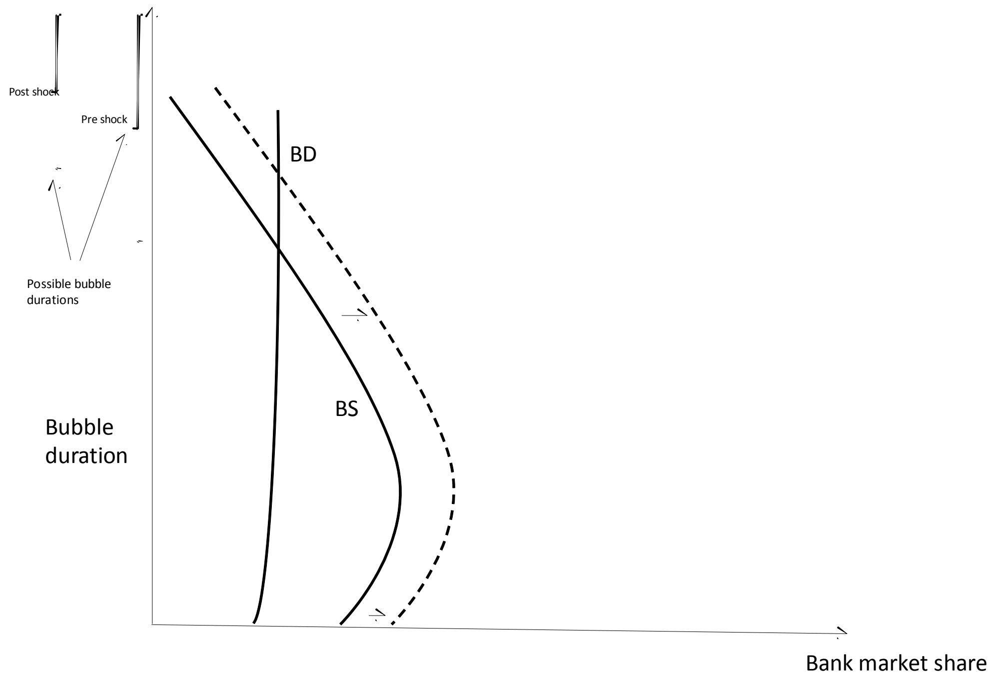

Figure 1: Relaxing capital regulation. Extensive margin

share of the bank unchanged, but it will produce an increase of the minimum length τ^* of the bubble. Intuitively, the increase in ψ will increase the market share of the bank for any given bubble duration. However, since the maximum market share of the bank consistent with bubble formation remains unchanged, bubbles must have longer duration (recall that longer bubbles imply a longer period over which the bank's market share shrinks). A change in N, which proxies for the degree of dispersion of information, produces the same effect as a change in ψ. In particular, such a shock only alters the position of the BS locus, while leaving the BD locus unchanged. We obtain that a drop in N shifts the BS locus upward and hence tends to make bubbles longer.[13] Figure 2 illustrates these comparative statics exercises.

The intuition for these results is immediate. A relaxation of capital regulation tends to expand the market share of the bank, for any given bubble duration. However, this share cannot grow too large: if the share becomes too large, the deceleration of price growth (kink) at the crash stage will become too pronounced and investors will have no incentive to ride bubbles. As a result, the bank's market share must grow less during the boom period. In this scenario this is possible only if bubbles become longer, because there will then be a longer risk-free period during which the bank's market share shrink. The insight that emerges is then that, although banks can use their information to rescue customers, an expansion of the banking sector will tend to make observed bubble durations and, hence, price market crashes, larger.

[13] We can also consider shocks to parameters that affect not only the BS locus but also the BD locus, such as γ_l and R. An increase in γ_l will....

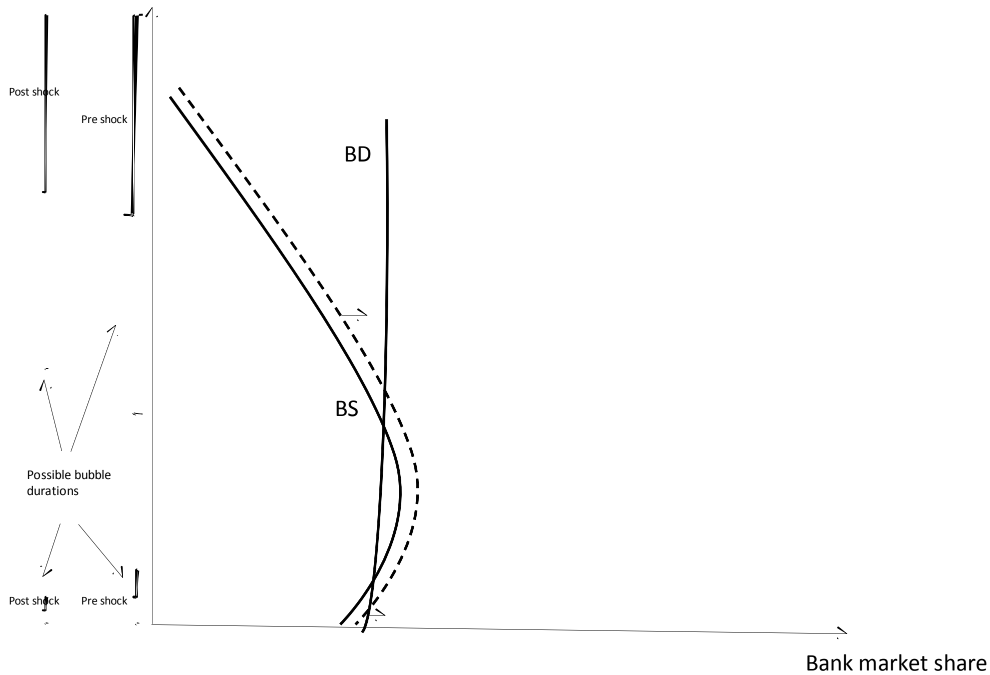

Figure 2: Relaxing capital regulation. Extensive and intensive margin

4.3 Intensive margin effects

We now consider scenarios in which not only the extensive margin channel but also the intensive margin channel plays a role. Precisely, the BS locus is upward sloping for low values of bubble duration τ^* and it crosses the BD locus twice, in its upward part and in its downward part (see Figure 3 for an illustration). Recall that the intuition for the upward sloping part of the BS locus is that, for low enough τ^*, the intensive margin effect of a longer bubble duration dominates: a longer bubble implies larger losses for small lenders and, hence, allows the bank to realize larger profits and grow more during the crash-risk phase.

We can then repeat the same comparative statics exercises performed in the last section. For example, consider a shock to bank capital regulation, in the form of a regulatory increase in ψ (i.e., a drop in the bank capital requirement). This shock tends to shift the BS locus to the right. In this scenario, the shock makes more likely the occurrence of longer bubbles or shorter bubbles, or to put it differently, increases the dispersion in possible bubble durations. A similar effect obtains following a change in N, our proxy for for the degree of dispersion of information.

The intuition for these results can be grasped by recalling our discussion in the previous section. When capital regulation loosens, and banks tend to grow larger, the bank's market share must grow less during the boom period. This is key for preserving investors' incentive to ride bubbles (otherwise the deceleration of price growth at the peak stage would become too pronounced and investors would have no incentive to ride bubbles). What differs from the previous section is how this more contained growth of banks' market share can be achieved. In the previous section, this could occur only with a lengthening of bubble durations, which implied longer (risk-free) periods of shrinkage of the bank's market share. In this case where intensive margin effects matter too, this can also occur with shorter bubbles. In fact, shorter bubbles will imply lower losses for small lenders. Hence, the bank will charge a lower interest rate during the crash-risk phase and hence will grow less during that phase (recall that in this scenario, for short bubbles the intensive margin effect dominates over the extensive margin effect).

5 Extensions

We explore two extensions of the baseline environment. In the first, we allow the bank to engage in a (primitive form of) securitization. In the second extension, we allow the bank to monitor assets. At the end of this section, we discuss further possible variations of the baseline structure.

5.1 Securitization

In this extension, we allow the bank to sell (securitize) the loans that it originates. We posit that when the bank originates loans and securitizes them, it retains an advantage in repossessing and liquidating the risky assets backing the loans. In the event of loan default, the bank can then extract a surplus by "selling" its superior liquidation skills to small lenders (similar to Diamond and Rajan, 2002). We are going to see that the amount of loans that the bank has the incentive to securitize depends on the surplus it is able to extract, which in turn depends on (and influences) the evolution of the asset price.

Formally, we now assume that if a small lender repossesses risky shares backing a loan it purchased from the bank, a portion ω of their value is lost in transaction costs. If instead the small lender resorts to the bank to repossess the assets, the bank can repossess the assets on behalf of the small lender without facing transaction costs. Further, the bank extracts the entire surplus associated with its superior liquidation skills (i.e., the bank has full bargaining power). The price of a unit loan granted to an investor is then given by

$$p_L = 1 - \pi(1-\omega^L)p_{t_c+1}h_{i,t+1}. \tag{33}$$

Notice that small lenders account for the fact that the bank will extract surplus from its liquidation skills and factor this in the price they are willing to pay for the loan. However, crucially the small lenders do have less precise information on whether a crash will occur and, hence, assets will be liquidated. The bank disburses one unit to grant the loan and at the moment of the crash know that with certainty a crash will occur, so its expected return is

$$p_L - 1 + (1-\omega^L)p_{t_c+1}h_{i,t+1} = (1-\pi)(1-\omega^L)p_{t_c+1}h_{i,t+1}$$

The total expected net profit for a bank that securitizes S units of loans is then

$$S_{t_c}(1-\pi)(1-\omega^L)p_{t_c+1}h_{i,t+1} - \left[\frac{aS_{t_c}^2}{2} - \underline{a}S_{t_c}\right], \tag{34}$$

which pins down the optimal crash-period amount of securitization

$$S_{t_c} = \frac{(1-\pi)(1-\omega^L)p_{t_c+1}h_{i,t+1}}{a} + \frac{\underline{a}}{a}. \tag{35}$$

Before the crash, the bank knows that there will be no repossessions, and thus issues the amount of securities that maximizes

$$\underline{a}S_t - \frac{aS_t^2}{2}, \tag{36}$$

i.e., $\underline{a}/a$. From the above, one can see that securitization grows on the eve of the crash. [This is

correct as long as the size of the expected loss of little lenders is ε, independent of the crash.]

The kink is modified in the presence of securitization, since the bank need not rescue its customers. In particular, the fraction of bank customers who sell the risky asset in the period before the crash is now given by $\widehat{\xi}_{t_c} = \max\{0, \xi_{t_c} - \frac{S_{t_c}}{L}\}$. As a result, the kink becomes

$$\widehat{K} = \frac{(1-1/N)(1-\widehat{\xi}_{t_c})}{\theta(t) + \widehat{\xi}_{t_c} + (1-\widehat{\xi}_{t_c})/N}, \tag{37}$$

5.2 Monitoring

In the baseline model, we assumed that lenders did not need to invest in monitoring when granting loans. Inspired by Diamond and Rajan (2001), we now posit that lenders can invest in monitoring collateral in order to acquire better ability to repossess and liquidate it in the market. Since banks are able to aggregate signals and infer when the bubble will burst, they are able to choose the monitoring effort more efficiently. This conveys them an advantage over small lenders. Formally, we assume that a fraction of the repossession value of risky assets is lost in the form of transaction costs. Lenders can lower the value of ω by exerting monitoring effort. Precisely, the monitoring cost reads

$$C(\omega) = c\frac{(1-\omega)^2}{2}. \tag{38}$$

In addition, the cost of monitoring a unit of collateral involves a component independent of monitoring but increasing in the size of the loan portfolio, which captures limited ability to monitor large portfolios. This cost arises only if $\omega < 1$.

For the bank, the optimal choice of monitoring collateral is given by the solution to

$$\max_{\omega} \phi\,(1-\omega)\,p_{t_l} - c\frac{(1-\omega)^2}{2} - \frac{aS^2}{2} \tag{39}$$

where p_{t_l} the liquidation price. The solution is given by

$$1-\omega^B = \begin{cases} 0 & \text{if } t+1 \neq t_c \\ \frac{\phi p_{t_c+1}}{c} & \text{if } t+1 = t_c. \end{cases}.$$

A small lender can also monitor but it must invest in monitoring without knowing whether repossession will occur. With probability π there will be repossession and the investment in monitoring will pay off in terms of a fuller recovery of the collateral. Otherwise, there will be no repossession and the monitoring effort will be wasted. The maximization problem of the small lender $j > 0$ is thus given by

$$\max_{\omega} (1-\omega)\,\pi\,[\phi p_{t_c+1}|t+1=t_c] - c\frac{(1-\omega)^2}{2}, \tag{40}$$

and is thus solved when

$$1-\omega^L = \pi\frac{[\phi p_{t_c+1}|t+1=t_c]}{c}.$$

The loan R_t^L will then be obtained from the small lenders' zero-profit condition

$$R_t^L = \frac{R}{1-\pi+\pi\frac{(1-\omega^L)p_{t_c+1}\widetilde{h}_{i,t_c}}{R_{t_c}^L l_{i,t_c}}} + C(\omega^L). \tag{41}$$

5.3 Further extensions

The model can be extended in further directions. For example, in the baseline model the bank does not hold and trade risky assets. We could however posit that the bank has an initial stock (H_0) of the risky asset. The bank's budget and capital constraints would then read

$$D_t^0 \leq \psi_L L_t^0 + \psi_H p_t H_t, \tag{42}$$

$$RD_t^j + L_{t+1}^j + p_t(H_{t+1}-H_t) = D_{t+1}^j + R_t^L(1-\widetilde{\omega}_t^j)L_t^j. \tag{43}$$

where H_t denotes the bank's holding of the risky asset at time t and the parameters ψ_L and ψ_H capture the regulatory risk weights assigned to loans and asset holdings.[14] The presence of bank's asset holdings gives an extra—and exponentially growing—boost to the growth rate of the bank's market share. Unlike in the baseline case, for some parameter constellations the bank's market share could rise during the zero-crash-risk phase.

6 Conclusion

We investigate the role of bank lending in fueling asset price booms that ultimately overshoot past fundamental value, inflate bubbles and lead to crises. Our model of bubbles is based on the speculative framework first proposed by Abreu and Brunnermeier (2003) and further developed by Doblas-Madrid (2012), and Doblas-Madrid and Lansing (2014). In our model, bubbles are fuelled from credit issued by banks and by small, dispersed, investors in financing households' asset purchases. Due to their size, banks are exposed to a large number of borrowers, which gives them an informational advantage over small lenders. This advantage allows them to retrench credit right before the crash, and to invest in monitoring only on the eve of the crash. Because banks profit from their ability to better time the market, they have an incentive to inflate bubbles. Moreover, this effect is exacerbated when banks engage in loan securitization, as banks tend to *evergreen* their loan portfolios, extending the boom in the short run at the expense of a bigger crash.

[14] Since bank capital (net worth) is about $K = L - D$, replacing into the constraint, we obtain

$$L_t\,(1-\psi_l) \leq K_t.$$

Thus, ψ_l can be interpreted as the complement to 1 of the capital-to-loan regulatory ratio.

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7 Supplement

7.1 Derivation of the interest rate

For any boom period $t \neq \tau^*$, $R^L_{t-1} = R^L_{t-2}$, $p_t/p_{t-1} = 1 + \gamma_l$. At $t = \tau^*$, instead, p_t/p_{t-1} is a bit lower, since $R^L_{t-1} > R^L_{t-2}$. To compute the interest rate, let us recall expression (12). Since $t_c = t_0 + \tau^*$, and $t_0 \geq 0$, it follows that, for $t < \tau^*$, the lending rate is R. Once $t \geq \tau^*$. Notice that we can rewrite the formula in (16), as follows

$$R^L_t = \frac{R - \pi \frac{p_{t_{c+1}} \widetilde{h}_{i,t_c}}{l_{i,t_c}}}{1 - \pi} \tag{44}$$

and, after substituting for $p_{t_{c+1}}$, $\widetilde{h}_{i,t_c}$, and l_{i,t_c},

$$R^L_t = \frac{R - \pi \Lambda \left(\frac{1+\gamma_l}{R(1+\theta)} \right)^{-\tau^*}}{1 - \pi}, \tag{45}$$

where Λ is a constant.

7.2 Derivation of the BD condition

The starting point in the derivation is the following condition

$$\begin{aligned} & E[p_t] h_{i,t-1} - \underline{R}^L R^L \overline{L}_{t-2} \\ \leq \; & \frac{1}{1+\lambda} \max \left\{ 0, F \left(\frac{1+\gamma_l}{R} \right)^{\nu(i)-1} \left[h_{i,t-1} + \frac{\overline{L}_{t-1} - R^L \overline{L}_{t-2}}{(1+\gamma_l)^t} \right] - R^L \overline{L}_{t-1} \right\} \\ & + \frac{\lambda}{1+\lambda} \left(\frac{[p_{t+1} | t_0 = \nu(i)]}{R} \left[h_{i,t-1} + \frac{\overline{L}_{t-1} - R^L \overline{L}_{t-2}}{(1+\gamma_l)^{\nu(i)+\tau^*}} \right] - \frac{\underline{R}^L}{R} R^L \overline{L}_{t-1} \right). \end{aligned} \tag{46}$$

In equilibrium, losing investors shift risk to lenders, and thus the max{} term equals zero, which allows us to rewrite the above as

$$E[p_t] h_{i,t-1} - \underline{R}^L R^L \overline{L}_{t-2} \leq \frac{\lambda}{1+\lambda} \left(\frac{[p_{t+1} | t_0 = \nu(i)]}{R} h_{i,t} - \frac{\underline{R}^L}{R} R^L \overline{L}_{t-1} \right). \tag{47}$$

We next substitute $h_{i,t}, \overline{L}_t$ for $(1+\theta)^t$ and $\overline{L}_0 (1+\gamma_l)^t$ to obtain

$$E[p_t](1+\theta)^{t-1} - \underline{R}^L R^L \overline{L}_0 (1+\gamma_l)^{t-2} \leq \frac{\lambda}{1+\lambda} \left(\frac{[p_{t+1} | t_0 = \nu(i)]}{R} (1+\theta)^t - \frac{\underline{R}^L}{R} R^L \overline{L}_0 (1+\gamma_l)^{t-1} \right). \tag{48}$$

Next, we substitute the expected price $E[p_t]$ and $[p_{t+1} | t_0 = \nu(i)]$ for their values

$$\begin{aligned} & \left[\frac{1}{1+\lambda} \frac{(1-1/N)(1-\xi_{t_c})}{\theta + \xi_{t_c} + (1-\xi_{t_c})/N} + \frac{\lambda}{1+\lambda} \frac{1}{\theta} \right] R \left[\overline{L}_0 (1+\gamma_l)^t - R^L \overline{L}_0 (1+\gamma_l)^{t-1} \right] - \underline{R}^L R^L \overline{L}_0 (1+\gamma_l)^{t-2} \\ \leq \; & \frac{\lambda}{1+\lambda} \left(\frac{(1-1/N)(1-\xi_{t_c})}{\theta + \xi_{t_c} + (1-\xi_{t_c})/N} \left[\overline{L}_0 (1+\gamma_l)^{t+1} - R^L \overline{L}_0 (1+\gamma_l)^t \right] - \frac{\underline{R}^L}{R} R^L \overline{L}_0 (1+\gamma_l)^{t-1} \right). \end{aligned} \tag{49}$$

Dividing through by $\overline{L}_0(1+\gamma_l)^{t-1}/(1+\lambda)$ yields

$$\begin{aligned} & \left[\frac{(1-1/N)(1-\xi_{t_c})}{\theta+\xi_{t_c}+(1-\xi_{t_c})/N}+\frac{\lambda}{\theta}\right] R\left[(1+\gamma_l)-R^L\right]-(1+\lambda)\frac{\underline{R}^L R^L}{1+\gamma_l} \\ \leq\quad & \lambda\left(\frac{(1-1/N)(1-\xi_{t_c})}{\theta+\xi_{t_c}+(1-\xi_{t_c})/N}(1+\gamma_l)\left[(1+\gamma_l)-R^L\right]-\frac{\underline{R}^L R^L}{R}\right), \end{aligned} \tag{50}$$

and substibuting the kink K for its value we obtain

$$\begin{aligned} & (K+\lambda)\, R\left[(1+\gamma_l)-R^L\right]-(1+\lambda)\theta\frac{\underline{R}^L R^L}{1+\gamma_l} \\ \leq\quad & \lambda\left(K(1+\gamma_l)\left[(1+\gamma_l)-R^L\right]-\theta\frac{\underline{R}^L R^L}{R}\right). \end{aligned} \tag{51}$$